Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/43-1.2.2.6-P-x-d-x-^m-a+b-x^2+c-x^4-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [145]. This is test number [43].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (145)	0.00 (0)
Mathematica	100.00 (145)	0.00 (0)
Maple	98.62 (143)	1.38 (2)
Mupad	98.62 (143)	1.38 (2)
Giac	98.62 (143)	1.38 (2)
Fricas	86.21 (125)	13.79 (20)
Sympy	55.17 (80)	44.83 (65)
Maxima	50.34 (73)	49.66 (72)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

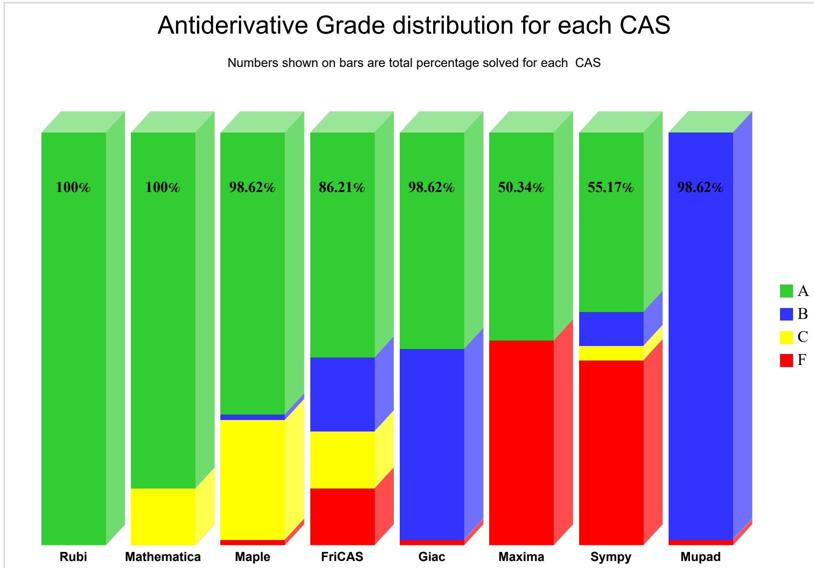
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

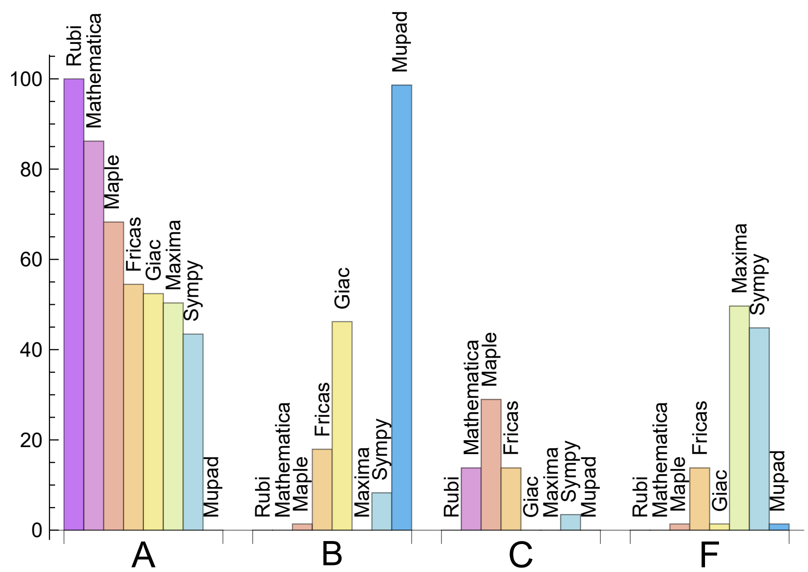
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	86.207	0.000	13.793	0.000
Maple	68.276	1.379	28.966	1.379
Fricas	54.483	17.931	13.793	13.793
Giac	52.414	46.207	0.000	1.379
Maxima	50.345	0.000	0.000	49.655
Sympy	43.448	8.276	3.448	44.828
Mupad	0.000	98.621	0.000	1.379

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Mupad	2	0.00	100.00	0.00
Giac	2	100.00	0.00	0.00
Fricas	20	10.00	90.00	0.00
Sympy	65	1.54	98.46	0.00
Maxima	72	77.78	0.00	22.22

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.15
Maxima	0.24
Mathematica	0.31
Rubi	0.57
Giac	0.74
Sympy	3.73
Mupad	6.56
Fricas	10.67

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	105.48	0.95	65.00	0.89
Mathematica	202.84	0.97	139.00	1.00
Maple	202.96	0.94	101.00	0.83
Rubi	207.48	1.03	179.00	1.00
Sympy	1048.58	3.99	71.00	0.98
Giac	1770.90	5.37	228.00	1.20
Mupad	4593.26	13.26	176.00	0.96
Fricas	31130.94	122.35	143.00	1.27

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

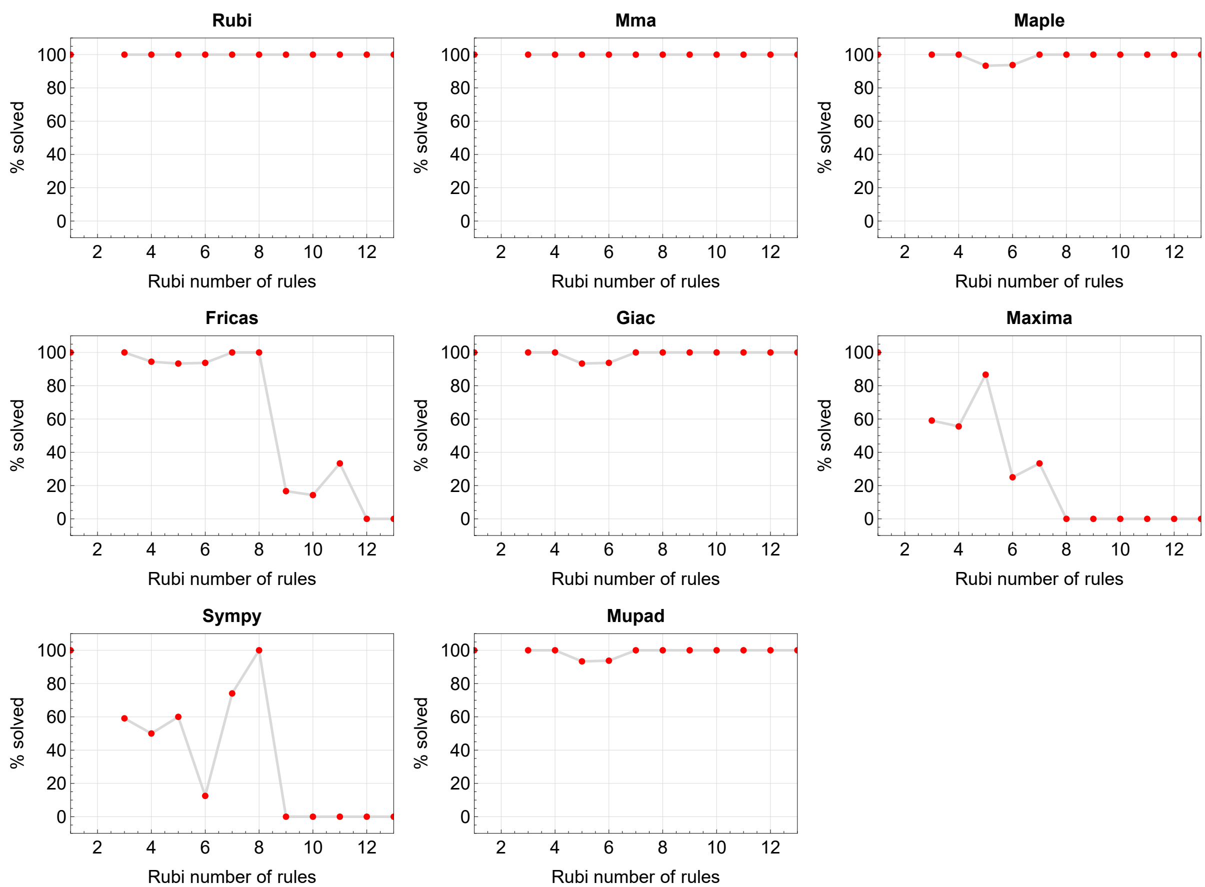


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

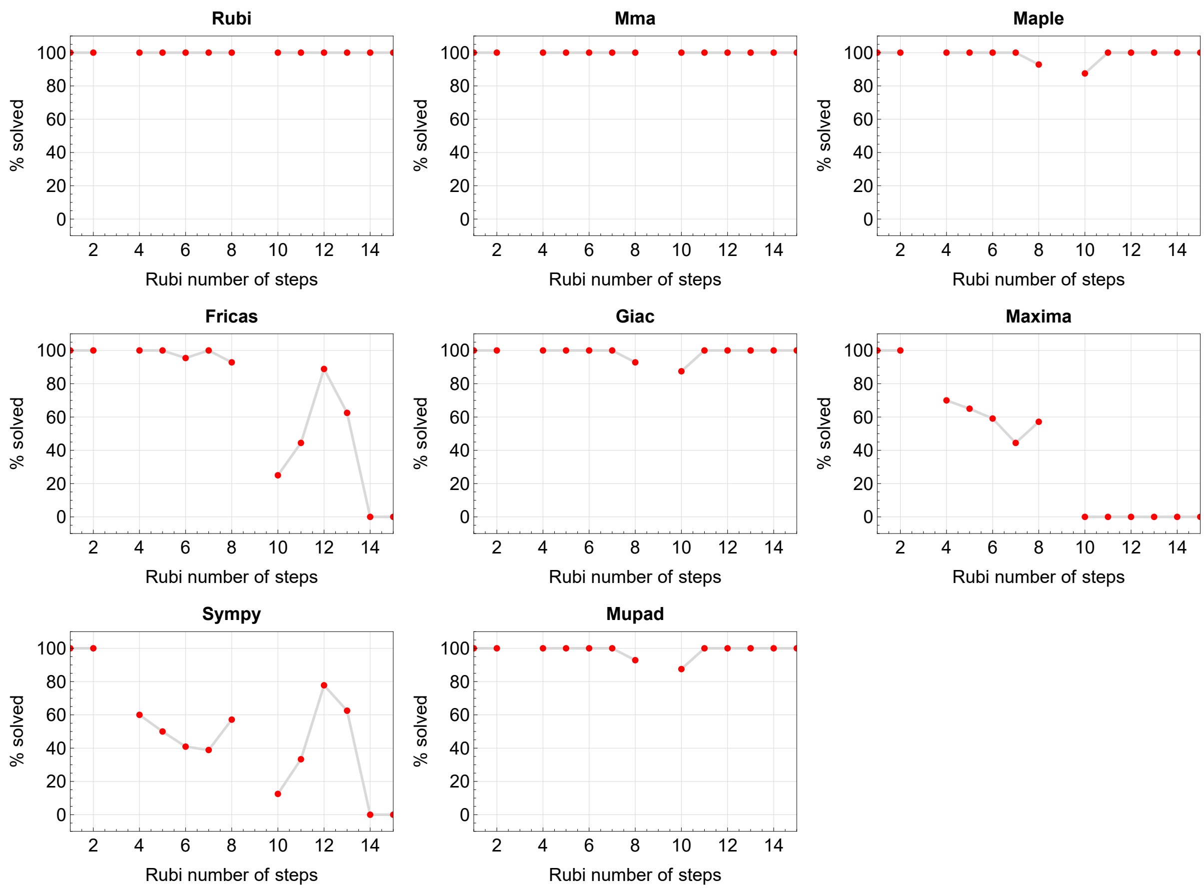


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

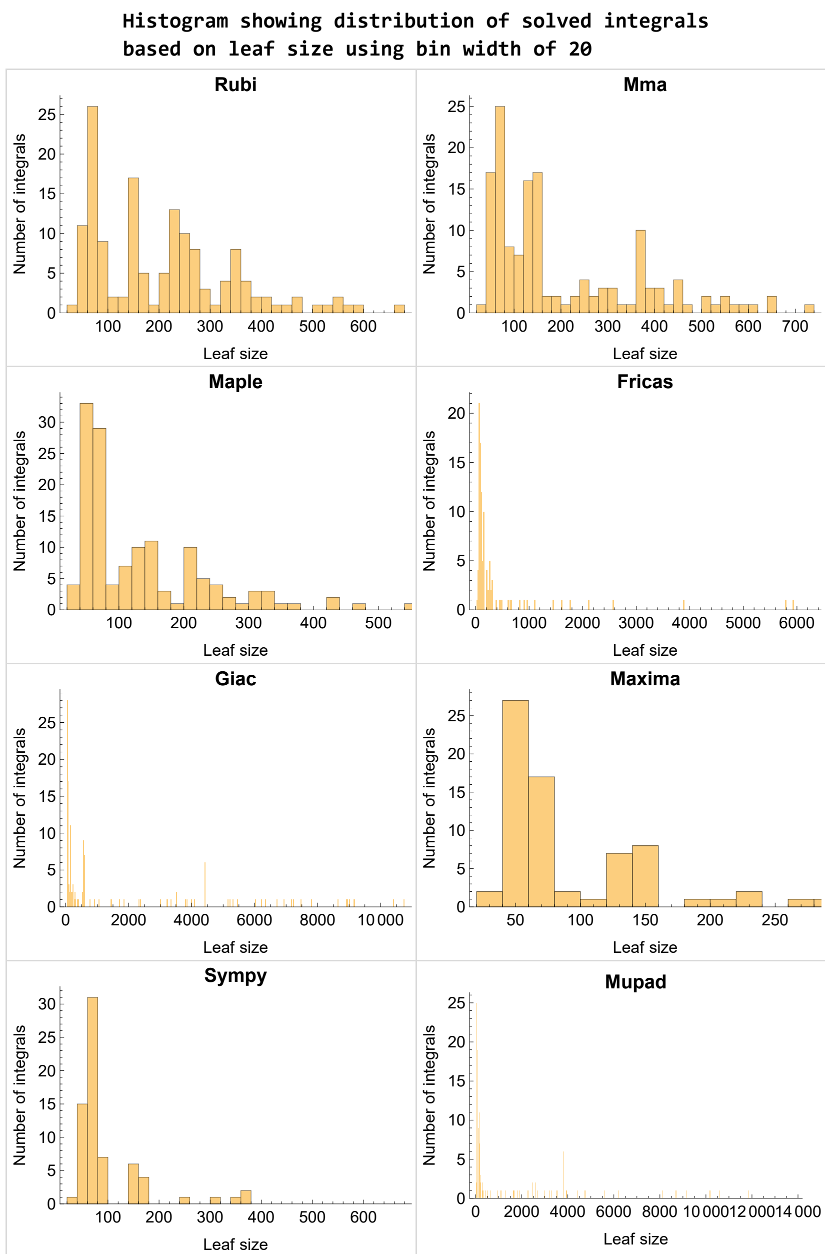


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

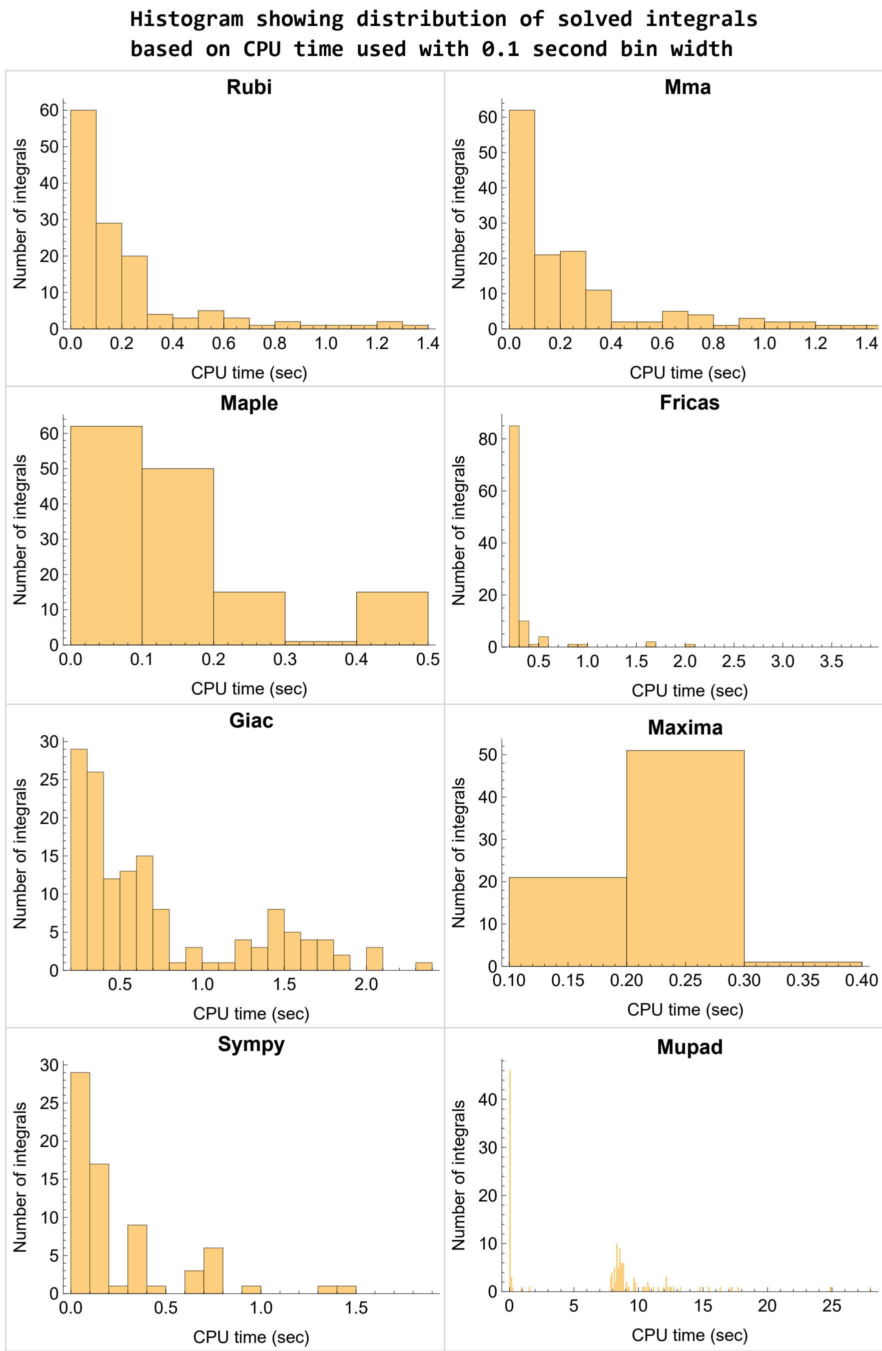


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

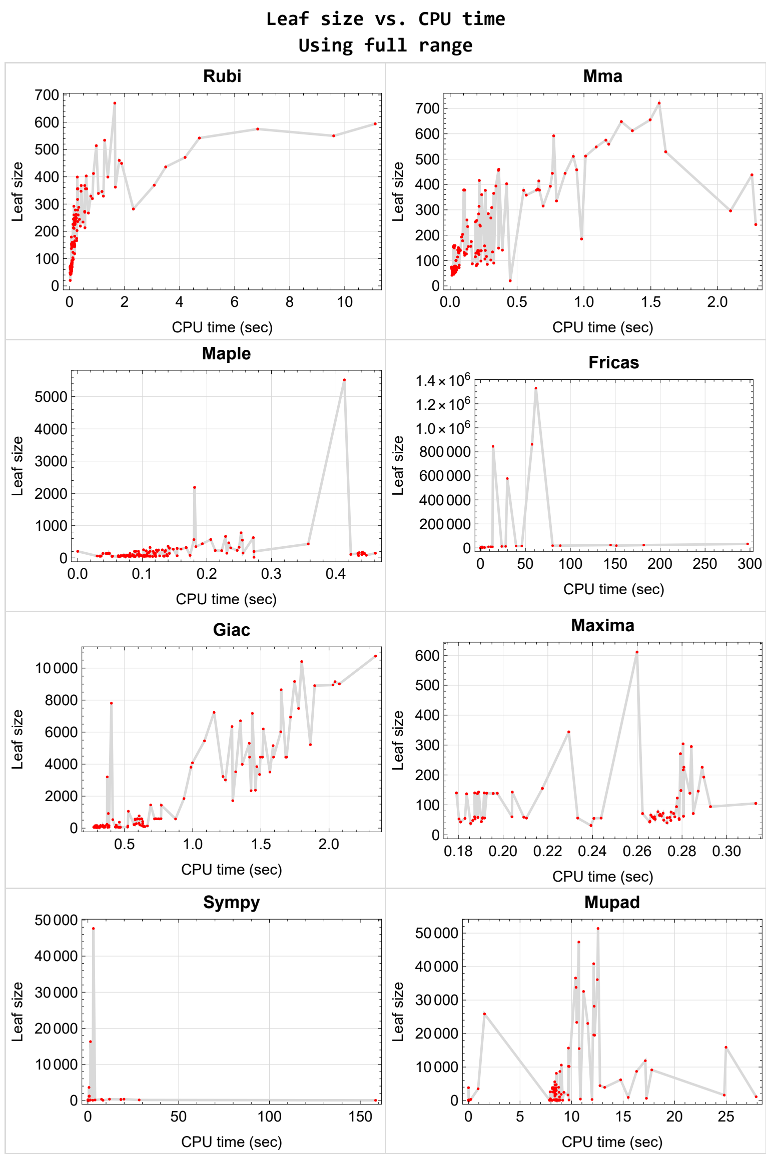


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {40}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

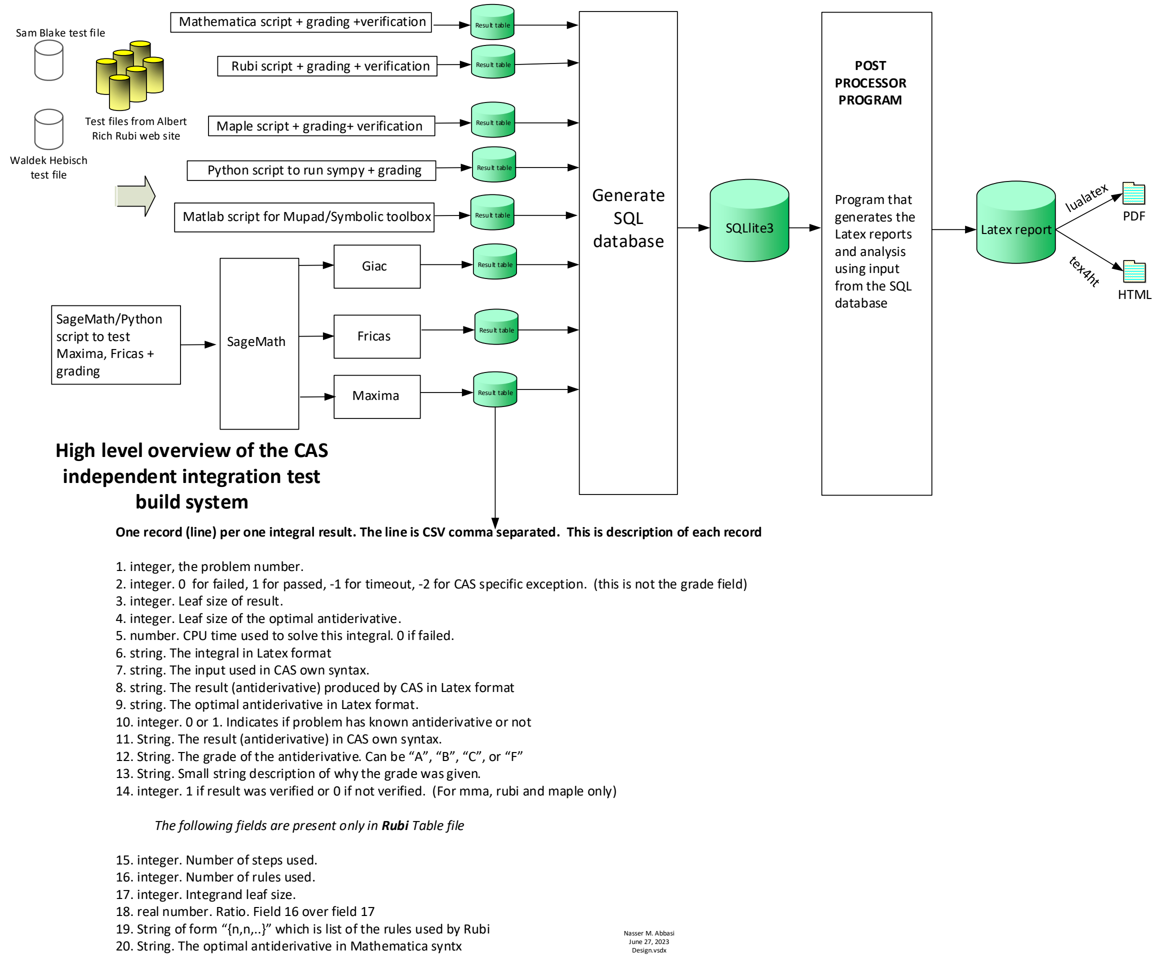
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { }

C grade { 40, 41, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 26, 27, 28, 34, 35, 36, 39, 47, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { 37, 38 }

C grade { 21, 22, 23, 24, 25, 29, 30, 31, 32, 33, 42, 43, 44, 45, 46, 55, 56, 57, 68, 69, 70, 71, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 135 }

F normal fail { 40, 41 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { 37, 38, 39, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 127, 128, 129, 130 }

C grade { 22, 23, 24, 25, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

F normal fail { 40, 41 }

F(-1) timedout fail { 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 42, 43, 44, 45, 46, 126 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 37, 38, 39, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade { }

C grade { }

F normal fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130 }

F(-1) timedout fail { }

F(-2) exception fail { 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 125 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 132, 133, 134, 139, 140 }

B grade { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 141, 142, 143, 144, 145 }

C grade { }

F normal fail { 40, 41 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

C grade { }

F normal fail { }

F(-1) timedout fail { 40, 41 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 115, 118, 119, 123, 124 }

B grade { 37, 38, 39, 110, 113, 114, 116, 117, 120, 121, 122, 131 }

C grade { 132, 133, 134, 135, 142 }

F normal fail { 40 }

F(-1) timedout fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 125, 126, 127, 128, 129, 130, 136, 137, 138, 139, 140, 141, 143, 144, 145 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	60	68	64	62
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.92	0.86	0.84
time (sec)	N/A	0.055	0.013	0.112	0.187	0.241	0.021	0.292	0.020

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	60	68	64	62
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.92	0.86	0.84
time (sec)	N/A	0.039	0.010	0.104	0.204	0.265	0.020	0.288	0.016

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	61	59
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.88	0.86
time (sec)	N/A	0.023	0.012	0.064	0.191	0.322	0.026	0.300	0.016

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	55	55	63	60	57
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.97	0.92	0.88
time (sec)	N/A	0.026	0.014	0.030	0.183	0.248	0.070	0.292	0.020

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	55	62	58	57	56
N.S.	1	1.00	1.00	0.90	0.87	0.98	0.92	0.90	0.89
time (sec)	N/A	0.033	0.018	0.034	0.191	0.243	0.078	0.315	0.021

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	55	62	61	58	56
N.S.	1	1.00	0.92	0.92	0.87	0.98	0.97	0.92	0.89
time (sec)	N/A	0.033	0.029	0.036	0.241	0.265	0.154	0.299	0.019

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	55	56	62	63	56	55
N.S.	1	1.00	0.95	0.87	0.89	0.98	1.00	0.89	0.87
time (sec)	N/A	0.035	0.032	0.035	0.244	0.247	0.303	0.355	0.018

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	56	56	62	63	57	56
N.S.	1	1.00	0.98	0.89	0.89	0.98	1.00	0.90	0.89
time (sec)	N/A	0.034	0.021	0.036	0.233	0.270	0.962	0.304	0.025

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	56	62	66	57	56
N.S.	1	1.00	1.00	0.89	0.89	0.98	1.05	0.90	0.89
time (sec)	N/A	0.033	0.040	0.034	0.210	0.254	2.701	0.305	7.909

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	59	59	62	70	60	60
N.S.	1	1.00	1.00	0.87	0.87	0.91	1.03	0.88	0.88
time (sec)	N/A	0.031	0.036	0.036	0.209	0.247	8.313	0.294	7.867

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	143	168	154	141
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.06	0.97	0.89
time (sec)	N/A	0.153	0.036	0.118	0.204	0.273	0.030	0.386	7.975

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	143	163	154	141
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.03	0.97	0.89
time (sec)	N/A	0.097	0.028	0.120	0.189	0.254	0.029	0.289	0.056

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	140	140	165	151	138
N.S.	1	1.00	1.00	0.90	0.91	0.91	1.07	0.98	0.90
time (sec)	N/A	0.071	0.023	0.142	0.187	0.252	0.032	0.291	0.055

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	150	138	138	138	156	149	135
N.S.	1	1.00	1.00	0.92	0.92	0.92	1.04	0.99	0.90
time (sec)	N/A	0.074	0.030	0.039	0.196	0.252	0.135	0.297	7.863

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	145	142	137	145	156	147	135
N.S.	1	1.00	1.00	0.98	0.94	1.00	1.08	1.01	0.93
time (sec)	N/A	0.082	0.066	0.049	0.184	0.256	0.139	0.300	0.059

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	139	140	139	145	153	148	135
N.S.	1	1.00	0.93	0.94	0.93	0.97	1.03	0.99	0.91
time (sec)	N/A	0.083	0.076	0.048	0.197	0.263	0.232	0.358	0.053

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	151	141	140	145	160	146	137
N.S.	1	1.00	1.01	0.95	0.94	0.97	1.07	0.98	0.92
time (sec)	N/A	0.093	0.061	0.046	0.179	0.257	0.402	0.282	0.034

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	130	139	139	145	153	142	134
N.S.	1	1.00	0.88	0.94	0.94	0.98	1.03	0.96	0.91
time (sec)	N/A	0.089	0.065	0.046	0.193	0.243	1.325	0.300	0.033

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	142	135	138	145	155	140	136
N.S.	1	1.00	0.99	0.94	0.97	1.01	1.08	0.98	0.95
time (sec)	N/A	0.099	0.057	0.049	0.189	0.254	3.945	0.301	0.031

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	144	136	140	145	158	141	136
N.S.	1	1.00	0.97	0.91	0.94	0.97	1.06	0.95	0.91
time (sec)	N/A	0.094	0.066	0.044	0.192	0.278	28.254	0.293	7.869

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	460	118	0	0	0	5304	2588
N.S.	1	1.00	1.36	0.35	0.00	0.00	0.00	15.65	7.63
time (sec)	N/A	1.048	0.363	0.120	0.000	0.000	0.000	1.415	7.917

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	377	86	0	1329593	0	3519	2696
N.S.	1	1.00	1.36	0.31	0.00	4782.71	0.00	12.66	9.70
time (sec)	N/A	0.294	0.263	0.103	0.000	61.661	0.000	1.315	8.366

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	360	71	0	861800	0	3843	1890
N.S.	1	1.00	1.33	0.26	0.00	3191.85	0.00	14.23	7.00
time (sec)	N/A	0.553	0.235	0.099	0.000	57.378	0.000	1.470	8.511

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	240	52	0	845032	0	2368	5594
N.S.	1	1.00	1.08	0.23	0.00	3789.38	0.00	10.62	25.09
time (sec)	N/A	0.151	0.222	0.073	0.000	13.964	0.000	1.460	8.370

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	234	48	0	578003	0	1714	3942
N.S.	1	1.00	1.11	0.23	0.00	2739.35	0.00	8.12	18.68
time (sec)	N/A	0.161	0.131	0.053	0.000	29.860	0.000	1.294	8.749

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	285	251	0	0	0	2339	2258
N.S.	1	1.00	1.24	1.10	0.00	0.00	0.00	10.21	9.86
time (sec)	N/A	0.172	0.285	0.102	0.000	0.000	0.000	1.428	8.368

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	315	325	0	0	0	3505	2588
N.S.	1	1.00	1.21	1.25	0.00	0.00	0.00	13.48	9.95
time (sec)	N/A	0.310	0.695	0.112	0.000	0.000	0.000	1.566	8.160

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	377	300	0	0	0	3353	3563
N.S.	1	1.00	1.31	1.04	0.00	0.00	0.00	11.64	12.37
time (sec)	N/A	0.334	0.550	0.142	0.000	0.000	0.000	1.490	8.141

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	444	249	0	0	0	5217	4754
N.S.	1	1.00	1.08	0.60	0.00	0.00	0.00	12.66	11.54
time (sec)	N/A	0.868	0.859	0.131	0.000	0.000	0.000	1.863	8.513

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	358	213	0	0	0	3227	3278
N.S.	1	1.00	1.03	0.61	0.00	0.00	0.00	9.30	9.45
time (sec)	N/A	0.416	0.569	0.106	0.000	0.000	0.000	1.222	8.495

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	204	0	0	0	4438	3835
N.S.	1	1.00	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.578	0.646	0.112	0.000	0.000	0.000	1.593	8.491

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	335	198	0	0	0	3014	3198
N.S.	1	1.00	1.06	0.62	0.00	0.00	0.00	9.51	10.09
time (sec)	N/A	0.290	0.795	0.273	0.000	0.000	0.000	1.240	8.311

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	393	232	0	0	0	5156	4707
N.S.	1	1.00	1.07	0.63	0.00	0.00	0.00	14.01	12.79
time (sec)	N/A	0.551	0.749	0.247	0.000	0.000	0.000	1.590	8.399

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	458	566	0	0	0	6021	8129
N.S.	1	1.00	1.14	1.40	0.00	0.00	0.00	14.94	20.17
time (sec)	N/A	0.606	0.946	0.180	0.000	0.000	0.000	1.646	8.529

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	514	559	667	0	0	0	9013	8684
N.S.	1	1.00	1.09	1.30	0.00	0.00	0.00	17.54	16.89
time (sec)	N/A	0.972	1.185	0.229	0.000	0.000	0.000	2.076	8.872

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	655	778	0	0	0	6939	10595
N.S.	1	1.00	1.23	1.46	0.00	0.00	0.00	12.99	19.84
time (sec)	N/A	1.274	1.495	0.253	0.000	0.000	0.000	1.718	9.009

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	296	5520	611	3898	47658	7808	2443
N.S.	1	1.00	0.74	13.83	1.53	9.77	119.44	19.57	6.12
time (sec)	N/A	0.281	2.096	0.413	0.260	0.379	3.032	0.404	9.254

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	185	2187	344	1603	16323	3203	1314
N.S.	1	1.00	0.71	8.41	1.32	6.17	62.78	12.32	5.05
time (sec)	N/A	0.158	0.982	0.181	0.229	0.329	1.424	0.373	8.386

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	90	136	155	444	3628	914	527
N.S.	1	1.00	0.66	0.99	1.13	3.24	26.48	6.67	3.85
time (sec)	N/A	0.065	0.326	0.081	0.218	0.300	0.626	0.381	7.950

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	368	368	438	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	2.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	685	670	242	0	0	0	0	0	0
N.S.	1	0.98	0.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.644	2.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	204	0	0	0	4438	3835
N.S.	1	1.00	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.622	0.667	0.000	0.000	0.000	0.000	1.683	0.002

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	204	0	0	0	4438	3835
N.S.	1	1.00	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.286	0.109	0.094	0.000	0.000	0.000	1.689	8.213

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	204	0	0	0	4438	3835
N.S.	1	1.00	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.286	0.103	0.089	0.000	0.000	0.000	1.512	8.127

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	204	0	0	0	4438	3835
N.S.	1	1.00	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.298	0.102	0.090	0.000	0.000	0.000	1.417	8.194

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	204	0	0	0	4438	3835
N.S.	1	1.00	1.06	0.57	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.297	0.103	0.088	0.000	0.000	0.000	1.497	8.304

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	260	330	0	900	0	297	2972
N.S.	1	1.00	0.95	1.21	0.00	3.30	0.00	1.09	10.89
time (sec)	N/A	0.549	0.126	0.250	0.000	0.582	0.000	0.633	8.519

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	193	224	0	677	0	208	2295
N.S.	1	1.00	0.95	1.10	0.00	3.33	0.00	1.02	11.31
time (sec)	N/A	0.270	0.085	0.223	0.000	0.500	0.000	0.612	8.368

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	136	146	0	473	0	137	1689
N.S.	1	1.00	0.94	1.01	0.00	3.28	0.00	0.95	11.73
time (sec)	N/A	0.184	0.061	0.231	0.000	0.368	0.000	0.668	8.383

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	100	101	0	318	0	97	1081
N.S.	1	1.00	0.97	0.98	0.00	3.09	0.00	0.94	10.50
time (sec)	N/A	0.118	0.043	0.138	0.000	0.343	0.000	0.641	8.831

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	178	99	0	309	0	96	3927
N.S.	1	1.00	1.84	1.02	0.00	3.19	0.00	0.99	40.48
time (sec)	N/A	0.136	0.093	0.085	0.000	0.490	0.000	0.649	13.214

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	203	132	0	399	0	131	4437
N.S.	1	1.00	1.72	1.12	0.00	3.38	0.00	1.11	37.60
time (sec)	N/A	0.184	0.093	0.095	0.000	0.546	0.000	0.631	12.759

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	314	203	0	609	0	206	6187
N.S.	1	1.00	1.80	1.17	0.00	3.50	0.00	1.18	35.56
time (sec)	N/A	0.257	0.216	0.123	0.000	0.815	0.000	0.573	14.765

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	416	294	0	834	0	303	9141
N.S.	1	1.00	1.70	1.20	0.00	3.42	0.00	1.24	37.46
time (sec)	N/A	0.359	0.218	0.154	0.000	1.674	0.000	0.606	17.782

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	456	164	0	15467	0	7235	23332
N.S.	1	1.00	1.24	0.44	0.00	41.92	0.00	19.61	63.23
time (sec)	N/A	3.077	0.361	0.110	0.000	39.652	0.000	1.157	10.502

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	365	100	0	9364	0	5454	15674
N.S.	1	1.00	1.29	0.35	0.00	33.21	0.00	19.34	55.58
time (sec)	N/A	2.320	0.326	0.092	0.000	8.913	0.000	1.086	9.699

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	258	68	0	5788	0	4082	10209
N.S.	1	1.00	1.18	0.31	0.00	26.43	0.00	18.64	46.62
time (sec)	N/A	0.392	0.198	0.077	0.000	4.341	0.000	0.998	9.697

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	253	220	0	5930	0	3984	10170
N.S.	1	1.00	1.19	1.03	0.00	27.84	0.00	18.70	47.75
time (sec)	N/A	0.559	0.189	0.106	0.000	1.663	0.000	1.364	9.773

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	284	244	0	9850	0	3804	15505
N.S.	1	1.00	1.06	0.91	0.00	36.89	0.00	14.25	58.07
time (sec)	N/A	0.684	0.213	0.122	0.000	11.664	0.000	0.987	10.739

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	394	360	0	15830	0	6710	23019
N.S.	1	1.00	1.20	1.09	0.00	48.12	0.00	20.40	69.97
time (sec)	N/A	1.229	0.342	0.140	0.000	45.944	0.000	1.350	11.573

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	309	432	0	2111	0	415	3499
N.S.	1	1.00	0.97	1.35	0.00	6.60	0.00	1.30	10.93
time (sec)	N/A	0.829	0.311	0.357	0.000	0.526	0.000	0.633	0.939

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	236	309	0	1455	0	271	2450
N.S.	1	1.00	1.00	1.31	0.00	6.17	0.00	1.15	10.38
time (sec)	N/A	0.296	0.225	0.237	0.000	0.353	0.000	0.584	8.790

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	175	228	0	970	0	191	1651
N.S.	1	1.00	1.06	1.38	0.00	5.88	0.00	1.16	10.01
time (sec)	N/A	0.190	0.158	0.213	0.000	0.325	0.000	0.615	9.658

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	130	139	0	650	0	137	342
N.S.	1	1.00	1.06	1.13	0.00	5.28	0.00	1.11	2.78
time (sec)	N/A	0.122	0.065	0.108	0.000	0.288	0.000	0.524	0.236

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	268	228	0	1103	0	224	8706
N.S.	1	1.00	1.61	1.37	0.00	6.64	0.00	1.35	52.45
time (sec)	N/A	0.258	0.304	0.136	0.000	0.968	0.000	0.569	16.310

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	403	316	0	1764	0	280	11879
N.S.	1	1.00	1.72	1.35	0.00	7.54	0.00	1.20	50.76
time (sec)	N/A	0.487	0.422	0.168	0.000	2.081	0.000	0.587	17.156

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	592	466	0	2567	0	521	15905
N.S.	1	1.00	1.80	1.42	0.00	7.80	0.00	1.58	48.34
time (sec)	N/A	0.771	0.774	0.234	0.000	4.471	0.000	0.579	24.981

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	648	323	0	18909	0	8946	33799
N.S.	1	1.00	1.18	0.59	0.00	34.38	0.00	16.27	61.45
time (sec)	N/A	9.599	1.280	0.168	0.000	80.289	0.000	2.029	10.441

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	511	242	0	12597	0	7479	25862
N.S.	1	1.00	1.17	0.56	0.00	28.89	0.00	17.15	59.32
time (sec)	N/A	3.493	0.921	0.125	0.000	23.673	0.000	1.777	1.559

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	414	200	0	8951	0	6200	19494
N.S.	1	1.00	1.14	0.55	0.00	24.73	0.00	17.13	53.85
time (sec)	N/A	1.664	0.663	0.135	0.000	13.281	0.000	1.517	12.240

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	382	200	0	8991	0	6348	19589
N.S.	1	1.00	1.10	0.58	0.00	25.99	0.00	18.35	56.62
time (sec)	N/A	1.177	0.655	0.117	0.000	11.610	0.000	1.288	12.155

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	444	438	0	13111	0	7173	28164
N.S.	1	1.00	1.11	1.10	0.00	32.86	0.00	17.98	70.59
time (sec)	N/A	1.391	0.764	0.193	0.000	27.996	0.000	1.438	12.193

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	575	575	548	570	0	19333	0	8649	36097
N.S.	1	1.00	0.95	0.99	0.00	33.62	0.00	15.04	62.78
time (sec)	N/A	6.838	1.092	0.206	0.000	88.785	0.000	1.649	12.499

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	58	82	61	63	57
N.S.	1	1.00	0.91	0.82	0.85	1.21	0.90	0.93	0.84
time (sec)	N/A	0.089	0.022	0.086	0.189	0.241	0.080	0.274	0.036

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	51	53	77	56	58	53
N.S.	1	1.00	1.00	0.84	0.87	1.26	0.92	0.95	0.87
time (sec)	N/A	0.073	0.022	0.079	0.180	0.256	0.067	0.279	0.024

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	46	48	72	48	53	47
N.S.	1	1.00	1.00	0.85	0.89	1.33	0.89	0.98	0.87
time (sec)	N/A	0.071	0.017	0.088	0.186	0.258	0.067	0.376	0.022

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	41	43	67	44	45	43
N.S.	1	1.00	1.00	0.84	0.88	1.37	0.90	0.92	0.88
time (sec)	N/A	0.056	0.018	0.063	0.181	0.253	0.069	0.293	8.487

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	38	57	36	40	37
N.S.	1	1.00	1.00	0.86	0.90	1.36	0.86	0.95	0.88
time (sec)	N/A	0.035	0.014	0.065	0.185	0.252	0.065	0.304	0.029

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	44	71	41	47	40
N.S.	1	1.00	1.00	0.86	1.00	1.61	0.93	1.07	0.91
time (sec)	N/A	0.053	0.015	0.074	0.190	0.251	0.072	0.291	0.025

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	45	53	92	51	53	50
N.S.	1	1.00	0.91	0.82	0.96	1.67	0.93	0.96	0.91
time (sec)	N/A	0.070	0.018	0.081	0.187	0.255	0.086	0.300	8.523

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	50	56	97	56	66	55
N.S.	1	1.00	0.88	0.78	0.88	1.52	0.88	1.03	0.86
time (sec)	N/A	0.075	0.020	0.082	0.192	0.255	0.087	0.296	0.026

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	56	58	79	68	58	58
N.S.	1	1.00	1.01	0.80	0.83	1.13	0.97	0.83	0.83
time (sec)	N/A	0.061	0.034	0.110	0.274	0.257	0.092	0.311	0.035

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	58	49	51	74	54	51	50
N.S.	1	1.00	1.02	0.86	0.89	1.30	0.95	0.89	0.88
time (sec)	N/A	0.049	0.031	0.112	0.279	0.254	0.100	0.315	0.032

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	46	48	69	54	48	48
N.S.	1	1.00	1.02	0.82	0.86	1.23	0.96	0.86	0.86
time (sec)	N/A	0.048	0.029	0.104	0.275	0.249	0.091	0.317	0.029

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	50	41	43	64	48	43	42
N.S.	1	1.00	1.02	0.84	0.88	1.31	0.98	0.88	0.86
time (sec)	N/A	0.044	0.027	0.089	0.266	0.259	0.089	0.317	0.039

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	46	38	40	59	46	40	40
N.S.	1	1.00	0.96	0.79	0.83	1.23	0.96	0.83	0.83
time (sec)	N/A	0.018	0.028	0.088	0.273	0.246	0.087	0.294	0.045

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	43	45	68	49	45	45
N.S.	1	1.00	0.96	0.81	0.85	1.28	0.92	0.85	0.85
time (sec)	N/A	0.049	0.034	0.116	0.266	0.268	0.096	0.292	0.045

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	48	52	79	56	52	51
N.S.	1	1.00	0.90	0.77	0.84	1.27	0.90	0.84	0.82
time (sec)	N/A	0.054	0.038	0.121	0.268	0.255	0.102	0.300	8.507

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	61	53	57	84	61	57	57
N.S.	1	1.00	0.88	0.77	0.83	1.22	0.88	0.83	0.83
time (sec)	N/A	0.060	0.041	0.134	0.273	0.254	0.109	0.286	0.047

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	58	62	89	66	62	61
N.S.	1	1.00	1.01	0.76	0.82	1.17	0.87	0.82	0.80
time (sec)	N/A	0.062	0.039	0.139	0.281	0.253	0.116	0.386	8.742

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	71	61	71	114	75	61	70
N.S.	1	1.00	0.88	0.75	0.88	1.41	0.93	0.75	0.86
time (sec)	N/A	0.072	0.042	0.127	0.262	0.244	0.110	0.288	9.143

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	66	58	68	109	76	58	68
N.S.	1	1.00	0.82	0.72	0.85	1.36	0.95	0.72	0.85
time (sec)	N/A	0.064	0.040	0.120	0.270	0.261	0.107	0.283	0.032

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	53	63	104	70	53	63
N.S.	1	1.00	0.80	0.71	0.84	1.39	0.93	0.71	0.84
time (sec)	N/A	0.060	0.040	0.107	0.271	0.255	0.109	0.308	0.040

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	50	60	99	65	50	59
N.S.	1	1.00	0.76	0.69	0.83	1.38	0.90	0.69	0.82
time (sec)	N/A	0.052	0.042	0.101	0.267	0.260	0.111	0.277	8.562

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	50	60	99	66	50	60
N.S.	1	1.00	0.78	0.69	0.83	1.38	0.92	0.69	0.83
time (sec)	N/A	0.047	0.042	0.095	0.277	0.246	0.109	0.292	0.045

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	50	60	99	65	50	59
N.S.	1	1.00	0.78	0.69	0.83	1.38	0.90	0.69	0.82
time (sec)	N/A	0.024	0.040	0.101	0.268	0.251	0.106	0.289	0.046

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	63	56	65	108	71	55	65
N.S.	1	1.00	0.80	0.71	0.82	1.37	0.90	0.70	0.82
time (sec)	N/A	0.067	0.050	0.133	0.270	0.255	0.120	0.298	8.839

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	61	72	119	76	62	71
N.S.	1	1.00	0.91	0.71	0.84	1.38	0.88	0.72	0.83
time (sec)	N/A	0.080	0.040	0.134	0.272	0.258	0.124	0.286	0.048

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	73	66	77	124	82	67	77
N.S.	1	1.00	0.78	0.71	0.83	1.33	0.88	0.72	0.83
time (sec)	N/A	0.088	0.053	0.154	0.269	0.256	0.130	0.296	0.054

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	71	71	95	87	76	75
N.S.	1	1.00	0.91	0.83	0.83	1.10	1.01	0.88	0.87
time (sec)	N/A	0.091	0.032	0.067	0.276	0.246	0.079	0.439	0.038

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	66	66	90	80	71	69
N.S.	1	1.00	0.90	0.81	0.81	1.11	0.99	0.88	0.85
time (sec)	N/A	0.087	0.022	0.063	0.266	0.244	0.078	0.433	0.033

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	66	62	59	85	73	66	65
N.S.	1	1.00	0.89	0.84	0.80	1.15	0.99	0.89	0.88
time (sec)	N/A	0.081	0.021	0.079	0.267	0.251	0.076	0.523	0.027

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	56	54	80	68	54	60
N.S.	1	1.00	0.94	0.86	0.83	1.23	1.05	0.83	0.92
time (sec)	N/A	0.069	0.020	0.071	0.279	0.251	0.078	0.470	8.608

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	51	49	70	60	49	69
N.S.	1	1.00	1.00	0.88	0.84	1.21	1.03	0.84	1.19
time (sec)	N/A	0.043	0.016	0.054	0.272	0.249	0.076	0.460	0.030

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	72	58	55	84	65	62	59
N.S.	1	1.00	1.09	0.88	0.83	1.27	0.98	0.94	0.89
time (sec)	N/A	0.069	0.067	0.068	0.267	0.255	0.083	0.451	8.656

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	101	63	66	105	76	66	68
N.S.	1	1.00	1.42	0.89	0.93	1.48	1.07	0.93	0.96
time (sec)	N/A	0.087	0.045	0.073	0.270	0.246	0.097	0.438	0.037

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	68	71	110	80	79	72
N.S.	1	1.00	1.02	0.85	0.89	1.38	1.00	0.99	0.90
time (sec)	N/A	0.088	0.068	0.072	0.285	0.252	0.095	0.438	0.038

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	114	73	76	115	85	84	78
N.S.	1	1.00	1.31	0.84	0.87	1.32	0.98	0.97	0.90
time (sec)	N/A	0.097	0.055	0.072	0.275	0.241	0.112	0.441	8.641

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	145	79	0	215	71	585	171
N.S.	1	1.00	0.58	0.32	0.00	0.87	0.29	2.36	0.69
time (sec)	N/A	0.230	0.126	0.174	0.000	0.249	0.344	0.594	0.075

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	132	72	0	210	1205	576	164
N.S.	1	1.00	0.56	0.30	0.00	0.89	5.08	2.43	0.69
time (sec)	N/A	0.196	0.105	0.087	0.000	0.247	0.739	0.628	8.618

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	129	69	0	205	60	573	162
N.S.	1	1.00	0.56	0.30	0.00	0.88	0.26	2.47	0.70
time (sec)	N/A	0.183	0.105	0.081	0.000	0.251	0.329	0.593	0.072

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	121	64	0	222	51	566	156
N.S.	1	1.00	0.54	0.28	0.00	0.99	0.23	2.52	0.69
time (sec)	N/A	0.196	0.109	0.076	0.000	0.270	0.330	0.611	0.089

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	115	61	0	217	1185	565	153
N.S.	1	1.00	0.51	0.27	0.00	0.97	5.29	2.52	0.68
time (sec)	N/A	0.157	0.189	0.075	0.000	0.256	0.694	0.582	8.652

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	126	63	0	230	1192	572	159
N.S.	1	1.00	0.55	0.28	0.00	1.00	5.21	2.50	0.69
time (sec)	N/A	0.200	0.121	0.099	0.000	0.253	0.765	0.577	0.101

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	131	68	0	245	60	579	165
N.S.	1	1.00	0.55	0.29	0.00	1.03	0.25	2.43	0.69
time (sec)	N/A	0.184	0.210	0.099	0.000	0.246	0.353	0.595	0.099

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	140	73	0	252	1202	584	171
N.S.	1	1.00	0.57	0.30	0.00	1.03	4.91	2.38	0.70
time (sec)	N/A	0.206	0.207	0.106	0.000	0.257	0.772	0.625	8.765

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	156	82	0	282	1204	588	184
N.S.	1	1.00	0.64	0.34	0.00	1.16	4.95	2.42	0.76
time (sec)	N/A	0.212	0.154	0.095	0.000	0.257	0.754	0.729	0.083

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	155	79	0	279	82	585	182
N.S.	1	1.00	0.64	0.33	0.00	1.15	0.34	2.42	0.75
time (sec)	N/A	0.206	0.136	0.076	0.000	0.252	0.369	0.751	8.896

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	138	74	0	272	71	580	176
N.S.	1	1.00	0.59	0.31	0.00	1.16	0.30	2.47	0.75
time (sec)	N/A	0.203	0.222	0.077	0.000	0.248	0.344	0.722	8.853

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	129	71	0	267	1198	577	173
N.S.	1	1.00	0.54	0.30	0.00	1.12	5.03	2.42	0.73
time (sec)	N/A	0.185	0.204	0.075	0.000	0.256	0.697	0.874	0.127

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	133	71	0	277	1200	577	174
N.S.	1	1.00	0.54	0.29	0.00	1.13	4.88	2.35	0.71
time (sec)	N/A	0.196	0.201	0.079	0.000	0.250	0.721	0.754	0.111

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	129	71	0	277	1195	577	173
N.S.	1	1.00	0.52	0.29	0.00	1.12	4.82	2.33	0.70
time (sec)	N/A	0.172	0.192	0.070	0.000	0.251	0.701	0.720	8.700

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	140	73	0	290	75	582	179
N.S.	1	1.00	0.55	0.29	0.00	1.15	0.30	2.30	0.71
time (sec)	N/A	0.223	0.263	0.096	0.000	0.270	0.364	0.738	8.872

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	139	78	0	305	80	589	185
N.S.	1	1.00	0.53	0.30	0.00	1.16	0.31	2.25	0.71
time (sec)	N/A	0.238	0.233	0.098	0.000	0.261	0.367	0.768	8.752

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	142	151	0	486	0	144	1834
N.S.	1	1.00	0.95	1.01	0.00	3.26	0.00	0.97	12.31
time (sec)	N/A	0.187	0.075	0.256	0.000	0.335	0.000	0.638	9.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	721	348	0	0	0	10752	47339
N.S.	1	1.00	1.21	0.59	0.00	0.00	0.00	18.10	79.70
time (sec)	N/A	11.107	1.562	0.183	0.000	0.000	0.000	2.341	10.708

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	575	274	0	23774	0	9152	36589
N.S.	1	1.00	1.22	0.58	0.00	50.48	0.00	19.43	77.68
time (sec)	N/A	4.197	1.164	0.160	0.000	181.702	0.000	2.044	10.390

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	512	269	0	19375	0	8905	32587
N.S.	1	1.00	1.14	0.60	0.00	43.15	0.00	19.83	72.58
time (sec)	N/A	1.891	1.012	0.148	0.000	151.256	0.000	1.895	11.165

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	529	550	0	23991	0	9167	40860
N.S.	1	1.00	1.15	1.20	0.00	52.15	0.00	19.93	88.83
time (sec)	N/A	1.805	1.611	0.255	0.000	144.800	0.000	1.747	12.149

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	612	629	0	33432	0	10411	51386
N.S.	1	1.00	1.13	1.16	0.00	61.68	0.00	19.21	94.81
time (sec)	N/A	4.717	1.362	0.272	0.000	297.365	0.000	1.799	12.571

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	31	31	54	58	31
N.S.	1	1.00	1.00	1.05	1.55	1.55	2.70	2.90	1.55
time (sec)	N/A	0.023	0.449	0.273	0.239	0.248	158.450	0.346	8.107

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	278	149	145	295	138	367	228	287
N.S.	1	1.32	0.71	0.69	1.40	0.66	1.75	1.09	1.37
time (sec)	N/A	0.211	0.362	0.461	0.284	0.263	19.879	0.372	8.419

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	213	116	109	217	104	367	164	215
N.S.	1	1.34	0.73	0.69	1.36	0.65	2.31	1.03	1.35
time (sec)	N/A	0.138	0.271	0.423	0.281	0.261	11.903	0.324	8.289

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	149	80	73	139	71	350	102	143
N.S.	1	1.37	0.73	0.67	1.28	0.65	3.21	0.94	1.31
time (sec)	N/A	0.086	0.196	0.435	0.284	0.274	7.520	0.337	8.359

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	151	106	143	105	80	304	189	161
N.S.	1	1.62	1.14	1.54	1.13	0.86	3.27	2.03	1.73
time (sec)	N/A	0.109	0.260	0.442	0.313	0.268	18.143	0.440	9.754

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	155	85	137	123	98	0	374	422
N.S.	1	1.57	0.86	1.38	1.24	0.99	0.00	3.78	4.26
time (sec)	N/A	0.177	0.278	0.434	0.278	0.275	0.000	0.462	10.856

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	182	102	143	193	102	0	767	932
N.S.	1	1.44	0.81	1.13	1.53	0.81	0.00	6.09	7.40
time (sec)	N/A	0.185	0.304	0.444	0.290	0.268	0.000	0.607	15.498

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	248	141	179	271	137	0	1434	1621
N.S.	1	1.17	0.67	0.84	1.28	0.65	0.00	6.76	7.65
time (sec)	N/A	0.249	0.390	0.441	0.279	0.273	0.000	0.770	24.813

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	245	134	161	226	134	0	177	1132
N.S.	1	1.13	0.62	0.75	1.05	0.62	0.00	0.82	5.24
time (sec)	N/A	0.141	0.317	0.436	0.281	0.284	0.000	0.340	27.906

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	179	98	128	146	100	0	115	651
N.S.	1	1.40	0.77	1.00	1.14	0.78	0.00	0.90	5.09
time (sec)	N/A	0.065	0.230	0.444	0.287	0.266	0.000	0.334	17.263

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	155	86	116	94	90	0	239	306
N.S.	1	1.52	0.84	1.14	0.92	0.88	0.00	2.34	3.00
time (sec)	N/A	0.082	0.203	0.441	0.293	0.271	0.000	0.372	11.990

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	81	107	94	90	257	530	138
N.S.	1	1.00	0.52	0.68	0.60	0.57	1.64	3.38	0.88
time (sec)	N/A	0.092	0.196	0.440	0.278	0.282	18.200	0.414	8.747

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	87	82	148	76	0	1055	146
N.S.	1	1.00	0.54	0.51	0.92	0.48	0.00	6.59	0.91
time (sec)	N/A	0.104	0.166	0.447	0.279	0.269	0.000	0.528	8.491

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	124	118	226	110	0	1451	218
N.S.	1	1.00	0.55	0.52	1.00	0.49	0.00	6.42	0.96
time (sec)	N/A	0.123	0.216	0.441	0.289	0.317	0.000	0.692	8.531

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	158	154	304	144	0	1847	290
N.S.	1	1.00	0.54	0.53	1.04	0.49	0.00	6.33	0.99
time (sec)	N/A	0.156	0.257	0.441	0.280	0.352	0.000	0.936	8.564

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [35] had the largest ratio of [.4642999999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	26	0.038
2	A	2	1	1.00	24	0.042
3	A	2	1	1.00	23	0.043
4	A	2	1	1.00	26	0.038
5	A	2	1	1.00	26	0.038
6	A	2	1	1.00	26	0.038
7	A	2	1	1.00	26	0.038
8	A	2	1	1.00	26	0.038
9	A	2	1	1.00	26	0.038
10	A	2	1	1.00	26	0.038
11	A	2	1	1.00	28	0.036
12	A	2	1	1.00	26	0.038
13	A	2	1	1.00	25	0.040
14	A	2	1	1.00	28	0.036
15	A	2	1	1.00	28	0.036
16	A	2	1	1.00	28	0.036
17	A	2	1	1.00	28	0.036
18	A	2	1	1.00	28	0.036
19	A	2	1	1.00	28	0.036
20	A	2	1	1.00	28	0.036
21	A	13	11	1.00	28	0.393
22	A	12	11	1.00	28	0.393
23	A	11	10	1.00	28	0.357
24	A	10	9	1.00	26	0.346

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	8	7	1.00	25	0.280
26	A	12	10	1.00	28	0.357
27	A	13	12	1.00	28	0.429
28	A	13	11	1.00	28	0.393
29	A	11	10	1.00	28	0.357
30	A	10	9	1.00	28	0.321
31	A	10	9	1.00	28	0.321
32	A	10	9	1.00	26	0.346
33	A	10	9	1.00	25	0.360
34	A	14	12	1.00	28	0.429
35	A	15	13	1.00	28	0.464
36	A	15	13	1.00	28	0.464
37	A	2	1	1.00	30	0.033
38	A	2	1	1.00	30	0.033
39	A	2	1	1.00	28	0.036
40	A	8	5	1.00	30	0.167
41	A	10	6	0.98	30	0.200
42	A	10	9	1.00	28	0.321
43	A	11	10	1.00	30	0.333
44	A	11	10	1.00	31	0.323
45	A	11	10	1.00	34	0.294
46	A	11	10	1.00	34	0.294
47	A	7	6	1.00	30	0.200
48	A	7	6	1.00	30	0.200
49	A	7	6	1.00	30	0.200
50	A	7	6	1.00	28	0.214
51	A	7	6	1.00	30	0.200
52	A	7	6	1.00	30	0.200
53	A	7	6	1.00	30	0.200
54	A	7	6	1.00	30	0.200
55	A	5	3	1.00	30	0.100
56	A	5	3	1.00	30	0.100
57	A	5	3	1.00	27	0.111
58	A	5	3	1.00	30	0.100
59	A	5	3	1.00	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	5	3	1.00	30	0.100
61	A	8	7	1.00	30	0.233
62	A	7	7	1.00	30	0.233
63	A	6	6	1.00	30	0.200
64	A	5	5	1.00	28	0.179
65	A	8	7	1.00	30	0.233
66	A	8	7	1.00	30	0.233
67	A	8	7	1.00	30	0.233
68	A	6	4	1.00	30	0.133
69	A	6	4	1.00	30	0.133
70	A	4	3	1.00	30	0.100
71	A	4	3	1.00	27	0.111
72	A	6	4	1.00	30	0.133
73	A	6	4	1.00	30	0.133
74	A	7	5	1.00	31	0.161
75	A	7	5	1.00	31	0.161
76	A	7	5	1.00	31	0.161
77	A	7	5	1.00	31	0.161
78	A	5	4	1.00	29	0.138
79	A	4	3	1.00	31	0.097
80	A	4	3	1.00	31	0.097
81	A	4	3	1.00	31	0.097
82	A	6	4	1.00	31	0.129
83	A	6	4	1.00	31	0.129
84	A	6	4	1.00	31	0.129
85	A	6	4	1.00	31	0.129
86	A	4	3	1.00	28	0.107
87	A	5	3	1.00	31	0.097
88	A	5	3	1.00	31	0.097
89	A	5	3	1.00	31	0.097
90	A	5	3	1.00	31	0.097
91	A	7	5	1.00	31	0.161
92	A	7	5	1.00	31	0.161
93	A	7	5	1.00	31	0.161
94	A	5	4	1.00	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	5	4	1.00	31	0.129
96	A	5	4	1.00	28	0.143
97	A	6	3	1.00	31	0.097
98	A	6	3	1.00	31	0.097
99	A	6	3	1.00	31	0.097
100	A	8	7	1.00	31	0.226
101	A	8	7	1.00	31	0.226
102	A	8	7	1.00	31	0.226
103	A	8	7	1.00	31	0.226
104	A	6	6	1.00	29	0.207
105	A	8	7	1.00	31	0.226
106	A	8	7	1.00	31	0.226
107	A	8	7	1.00	31	0.226
108	A	8	7	1.00	31	0.226
109	A	12	7	1.00	31	0.226
110	A	12	7	1.00	31	0.226
111	A	12	7	1.00	31	0.226
112	A	12	7	1.00	31	0.226
113	A	10	6	1.00	28	0.214
114	A	12	7	1.00	31	0.226
115	A	12	7	1.00	31	0.226
116	A	12	7	1.00	31	0.226
117	A	13	8	1.00	31	0.258
118	A	13	8	1.00	31	0.258
119	A	13	8	1.00	31	0.258
120	A	11	7	1.00	31	0.226
121	A	11	7	1.00	31	0.226
122	A	11	7	1.00	28	0.250
123	A	13	7	1.00	31	0.226
124	A	13	7	1.00	31	0.226
125	A	7	6	1.00	33	0.182
126	A	6	4	1.00	35	0.114
127	A	6	4	1.00	35	0.114
128	A	4	3	1.00	32	0.094
129	A	6	4	1.00	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	6	4	1.00	35	0.114
131	A	1	1	1.00	42	0.024
132	A	5	4	1.32	35	0.114
133	A	4	3	1.34	35	0.086
134	A	4	3	1.37	33	0.091
135	A	6	5	1.62	35	0.143
136	A	6	6	1.57	35	0.171
137	A	6	6	1.44	35	0.171
138	A	7	7	1.17	35	0.200
139	A	6	6	1.13	35	0.171
140	A	5	5	1.40	32	0.156
141	A	5	5	1.52	35	0.143
142	A	5	5	1.00	35	0.143
143	A	4	4	1.00	35	0.114
144	A	5	5	1.00	35	0.143
145	A	6	5	1.00	35	0.143

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 74

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/8
*B*c*x^8+1/9*c*C*x^9
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

```
[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(aAx^2 + aBx^3 + (Ab + aC)x^4 + bBx^5 + (Ac + bC)x^6 + Bcx^7 + cCx^8\right) dx$$
$$= \frac{1}{3}aAx^3 + \frac{1}{4}aBx^4 + \frac{1}{5}(Ab + aC)x^5 + \frac{1}{6}bBx^6 + \frac{1}{7}(Ac + bC)x^7 + \frac{1}{8}Bcx^8 + \frac{1}{9}cCx^9$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^2 (A + Bx + Cx^2) (a + bx^2 + cx^4) \, dx = \frac{1}{3}aAx^3 + \frac{1}{4}aBx^4 + \frac{1}{5}(Ab + aC)x^5 + \frac{1}{6}bBx^6 + \frac{1}{7}(Ac + bC)x^7 + \frac{1}{8}Bcx^8 + \frac{1}{9}cCx^9$$

```
[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{aA\,x^3}{3} + \frac{aB\,x^4}{4} + \frac{(Ab+Ca)x^5}{5} + \frac{bB\,x^6}{6} + \frac{(Ac+Cb)x^7}{7} + \frac{Bc\,x^8}{8} + \frac{cC\,x^9}{9}$	61
norman	$\frac{cC\,x^9}{9} + \frac{Bc\,x^8}{8} + \left(\frac{Ac}{7} + \frac{Cb}{7}\right)x^7 + \frac{bB\,x^6}{6} + \left(\frac{Ab}{5} + \frac{Ca}{5}\right)x^5 + \frac{aB\,x^4}{4} + \frac{aA\,x^3}{3}$	63
gosper	$\frac{1}{9}cC\,x^9 + \frac{1}{8}Bc\,x^8 + \frac{1}{7}x^7Ac + \frac{1}{7}x^7Cb + \frac{1}{6}bB\,x^6 + \frac{1}{5}x^5Ab + \frac{1}{5}x^5Ca + \frac{1}{4}aB\,x^4 + \frac{1}{3}aA\,x^3$	65
risch	$\frac{1}{9}cC\,x^9 + \frac{1}{8}Bc\,x^8 + \frac{1}{7}x^7Ac + \frac{1}{7}x^7Cb + \frac{1}{6}bB\,x^6 + \frac{1}{5}x^5Ab + \frac{1}{5}x^5Ca + \frac{1}{4}aB\,x^4 + \frac{1}{3}aA\,x^3$	65
parallelrisch	$\frac{1}{9}cC\,x^9 + \frac{1}{8}Bc\,x^8 + \frac{1}{7}x^7Ac + \frac{1}{7}x^7Cb + \frac{1}{6}bB\,x^6 + \frac{1}{5}x^5Ab + \frac{1}{5}x^5Ca + \frac{1}{4}aB\,x^4 + \frac{1}{3}aA\,x^3$	65

```
[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/8
*B*c*x^8+1/9*c*C*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx = \frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{7}\,(Cb+Ac)x^7+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^4 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx = \frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Bbx^6}{6}+\frac{Bcx^8}{8}+\frac{Ccx^9}{9}+x^7\left(\frac{Ac}{7}+\frac{Cb}{7}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)

[Out] A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 + x**7*(A*c/7 + C*b/7) + x**5*(A*b/5 + C*a/5)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx = \frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{7}\,(Cb+Ac)x^7+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^4 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx = \frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{7}\,Cbx^7+\frac{1}{7}\,Acx^7+\frac{1}{6}\,Bbx^6+\frac{1}{5}\,Cax^5+\frac{1}{5}\,Abx^5+\frac{1}{4}\,Bax^4+\frac{1}{3}\,Aax^3$$

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/7*C*b*x^7 + 1/7*A*c*x^7 + 1/6*B*b*x^6 + 1/5*C
*a*x^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx = \frac{C\,c\,x^9}{9}+\frac{B\,c\,x^8}{8}+\left(\frac{A\,c}{7}+\frac{C\,b}{7}\right)\,x^7+\frac{B\,b\,x^6}{6}+\left(\frac{A\,b}{5}+\frac{C\,a}{5}\right)\,x^5+\frac{B\,a\,x^4}{4}+\frac{A\,a\,x^3}{3}$$

```
[In] int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((A*b)/5 + (C*a)/5) + x^7*((A*c)/7 + (C*b)/7) + (A*a*x^3)/3 + (B*a*x^4)
/4 + (B*b*x^6)/6 + (B*c*x^8)/8 + (C*c*x^9)/9
```

3.2 $\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/7
*B*c*x^7+1/8*c*C*x^8
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1642}

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

```
[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(aAx + aBx^2 + (Ab + aC)x^3 + bBx^4 + (Ac + bC)x^5 + Bcx^6 + cCx^7\right) dx \\
&= \frac{1}{2}aAx^2 + \frac{1}{3}aBx^3 + \frac{1}{4}(Ab + aC)x^4 + \frac{1}{5}bBx^5 + \frac{1}{6}(Ac + bC)x^6 + \frac{1}{7}Bcx^7 + \frac{1}{8}cCx^8
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$
\int x\left(A + Bx + Cx^2\right)\left(a + bx^2 + cx^4\right) dx = \frac{1}{2}aAx^2 + \frac{1}{3}aBx^3 + \frac{1}{4}(Ab + aC)x^4 + \frac{1}{5}bBx^5 + \frac{1}{6}(Ac + bC)x^6 + \frac{1}{7}Bcx^7 + \frac{1}{8}cCx^8
$$

```
[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{aA\,x^2}{2} + \frac{aB\,x^3}{3} + \frac{(Ab+Ca)x^4}{4} + \frac{bB\,x^5}{5} + \frac{(Ac+Cb)x^6}{6} + \frac{Bc\,x^7}{7} + \frac{cC\,x^8}{8}$	61
norman	$\frac{cC\,x^8}{8} + \frac{Bc\,x^7}{7} + \left(\frac{Ac}{6} + \frac{Cb}{6}\right)x^6 + \frac{bB\,x^5}{5} + \left(\frac{Ab}{4} + \frac{Ca}{4}\right)x^4 + \frac{aB\,x^3}{3} + \frac{aA\,x^2}{2}$	63
gosper	$\frac{1}{8}cC\,x^8 + \frac{1}{7}Bc\,x^7 + \frac{1}{6}x^6Ac + \frac{1}{6}x^6Cb + \frac{1}{5}bB\,x^5 + \frac{1}{4}x^4Ab + \frac{1}{4}x^4Ca + \frac{1}{3}aB\,x^3 + \frac{1}{2}aA\,x^2$	65
risch	$\frac{1}{8}cC\,x^8 + \frac{1}{7}Bc\,x^7 + \frac{1}{6}x^6Ac + \frac{1}{6}x^6Cb + \frac{1}{5}bB\,x^5 + \frac{1}{4}x^4Ab + \frac{1}{4}x^4Ca + \frac{1}{3}aB\,x^3 + \frac{1}{2}aA\,x^2$	65
parallelrisch	$\frac{1}{8}cC\,x^8 + \frac{1}{7}Bc\,x^7 + \frac{1}{6}x^6Ac + \frac{1}{6}x^6Cb + \frac{1}{5}bB\,x^5 + \frac{1}{4}x^4Ab + \frac{1}{4}x^4Ca + \frac{1}{3}aB\,x^3 + \frac{1}{2}aA\,x^2$	65

```
[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/7
*B*c*x^7+1/8*c*C*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{8}\,Ccx^8+\frac{1}{7}\,Bcx^7+\frac{1}{5}\,Bbx^5+\frac{1}{6}\,(Cb+Ac)x^6+\frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^3
 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{Aax^2}{2}+\frac{Bax^3}{3}+\frac{Bbx^5}{5}+\frac{Bcx^7}{7}+\frac{Ccx^8}{8}+x^6\left(\frac{Ac}{6}+\frac{Cb}{6}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

```
[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + B*b*x**5/5 + B*c*x**7/7 + C*c*x**8/8 + x**6*(A*c/
6 + C*b/6) + x**4*(A*b/4 + C*a/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{8}\,Ccx^8+\frac{1}{7}\,Bcx^7+\frac{1}{5}\,Bbx^5+\frac{1}{6}\,(Cb+Ac)x^6+\frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^3
 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{8}\,Ccx^8+\frac{1}{7}\,Bcx^7+\frac{1}{6}\,Cbx^6+\frac{1}{6}\,Acx^6+\frac{1}{5}\,Bbx^5+\frac{1}{4}\,Cax^4+\frac{1}{4}\,Abx^4+\frac{1}{3}\,Bax^3+\frac{1}{2}\,Aax^2$$

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/6*C*b*x^6 + 1/6*A*c*x^6 + 1/5*B*b*x^5 + 1/4*C
*a*x^4 + 1/4*A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{C\,c\,x^8}{8}+\frac{B\,c\,x^7}{7}+\left(\frac{A\,c}{6}+\frac{C\,b}{6}\right)\,x^6+\frac{B\,b\,x^5}{5}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^4+\frac{B\,a\,x^3}{3}+\frac{A\,a\,x^2}{2}$$

```
[In] int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^4*((A*b)/4 + (C*a)/4) + x^6*((A*c)/6 + (C*b)/6) + (A*a*x^2)/2 + (B*a*x^3)
/3 + (B*b*x^5)/5 + (B*c*x^7)/7 + (C*c*x^8)/8
```

3.3 $\int (A+Bx+Cx^2)(a+bx^2+cx^4)\, dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right) dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4 +\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*x
^6+1/7*c*C*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1671}

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right) dx = \frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2 +\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

```
[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(aA + aBx + (Ab + aC)x^2 + bBx^3 + (Ac + bC)x^4 + Bcx^5 + cCx^6\right) dx \\ &= aAx + \frac{1}{2}aBx^2 + \frac{1}{3}(Ab + aC)x^3 + \frac{1}{4}bBx^4 + \frac{1}{5}(Ac + bC)x^5 + \frac{1}{6}Bcx^6 + \frac{1}{7}cCx^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \left(A + Bx + Cx^2\right)\left(a + bx^2 + cx^4\right) dx = aAx + \frac{1}{2}aBx^2 + \frac{1}{3}(Ab + aC)x^3 + \frac{1}{4}bBx^4 + \frac{1}{5}(Ac + bC)x^5 + \frac{1}{6}Bcx^6 + \frac{1}{7}cCx^7$$

```
[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$aAx + \frac{Ba\,x^2}{2} + \frac{(Ab+Ca)x^3}{3} + \frac{bB\,x^4}{4} + \frac{(Ac+Cb)x^5}{5} + \frac{Bc\,x^6}{6} + \frac{cC\,x^7}{7}$	58
norman	$\frac{cC\,x^7}{7} + \frac{Bc\,x^6}{6} + \left(\frac{Ac}{5} + \frac{Cb}{5}\right)x^5 + \frac{bB\,x^4}{4} + \left(\frac{Ab}{3} + \frac{Ca}{3}\right)x^3 + \frac{Ba\,x^2}{2} + aAx$	60
gosper	$\frac{1}{7}cC\,x^7 + \frac{1}{6}Bc\,x^6 + \frac{1}{5}x^5Ac + \frac{1}{5}x^5Cb + \frac{1}{4}bB\,x^4 + \frac{1}{3}x^3Ab + \frac{1}{3}x^3Ca + \frac{1}{2}Ba\,x^2 + aAx$	62
risch	$\frac{1}{7}cC\,x^7 + \frac{1}{6}Bc\,x^6 + \frac{1}{5}x^5Ac + \frac{1}{5}x^5Cb + \frac{1}{4}bB\,x^4 + \frac{1}{3}x^3Ab + \frac{1}{3}x^3Ca + \frac{1}{2}Ba\,x^2 + aAx$	62
parallelrisch	$\frac{1}{7}cC\,x^7 + \frac{1}{6}Bc\,x^6 + \frac{1}{5}x^5Ac + \frac{1}{5}x^5Cb + \frac{1}{4}bB\,x^4 + \frac{1}{3}x^3Ab + \frac{1}{3}x^3Ca + \frac{1}{2}Ba\,x^2 + aAx$	62

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*x
^6+1/7*c*C*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{4}Bbx^4+\frac{1}{5}(Cb+Ac)x^5+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ab)x^3+Aax$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^2
 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=Aax+\frac{Bax^2}{2}+\frac{Bbx^4}{4}+\frac{Bcx^6}{6}+\frac{Ccx^7}{7}+x^5\left(\frac{Ac}{5}+\frac{Cb}{5}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + x**5*(A*c/5 + C
*b/5) + x**3*(A*b/3 + C*a/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{4}Bbx^4+\frac{1}{5}(Cb+Ac)x^5+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ab)x^3+Aax$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^2
 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{5}Cbx^5+\frac{1}{5}Acx^5+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/5*C*b*x^5 + 1/5*A*c*x^5 + 1/4*B*b*x^4 + 1/3*C
*a*x^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{C\,c\,x^7}{7}+\frac{B\,c\,x^6}{6}+\left(\frac{A\,c}{5}+\frac{C\,b}{5}\right)x^5+\frac{B\,b\,x^4}{4}+\left(\frac{A\,b}{3}+\frac{C\,a}{3}\right)x^3+\frac{B\,a\,x^2}{2}+A\,a\,x$$

```
[In] int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((A*b)/3 + (C*a)/3) + x^5*((A*c)/5 + (C*b)/5) + A*a*x + (B*a*x^2)/2 + (
B*b*x^4)/4 + (B*c*x^6)/6 + (C*c*x^7)/7
```

3.4 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx$

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx = aBx + \frac{1}{2}(Ab+aC)x^2 + \frac{1}{3}bBx^3 + \frac{1}{4}(Ac+bC)x^4 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6 + aA\log(x)$$

```
[Out] a*B*x+1/2*(A*b+C*a)*x^2+1/3*b*B*x^3+1/4*(A*c+C*b)*x^4+1/5*B*c*x^5+1/6*c*C*x
^6+a*A*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx = \frac{1}{2}x^2(aC+Ab) + aA\log(x) + aBx + \frac{1}{4}x^4(Ac+bC) + \frac{1}{3}bBx^3 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(aB + \frac{aA}{x} + (Ab + aC)x + bBx^2 + (Ac + bC)x^3 + Bcx^4 + cCx^5\right) dx \\
&= aBx + \frac{1}{2}(Ab + aC)x^2 + \frac{1}{3}bBx^3 + \frac{1}{4}(Ac + bC)x^4 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6 + aA\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x} \, dx = aBx + \frac{1}{2}(Ab + aC)x^2 + \frac{1}{3}bBx^3 \\
+ \frac{1}{4}(Ac + bC)x^4 + \frac{1}{5}Bcx^5 + \frac{1}{6}cCx^6 + aA\log(x)
\end{aligned}
$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result	size
norman	$\left(\frac{Ab}{2} + \frac{Ca}{2}\right) x^2 + \left(\frac{Ac}{4} + \frac{Cb}{4}\right) x^4 + Bax + \frac{Bb\,x^3}{3} + \frac{Bc\,x^5}{5} + \frac{cC\,x^6}{6} + aA\ln(x)$	58
default	$\frac{cC\,x^6}{6} + \frac{Bc\,x^5}{5} + \frac{Ac\,x^4}{4} + \frac{Cb\,x^4}{4} + \frac{Bb\,x^3}{3} + \frac{Ab\,x^2}{2} + \frac{Ca\,x^2}{2} + Bax + aA\ln(x)$	60
risch	$\frac{cC\,x^6}{6} + \frac{Bc\,x^5}{5} + \frac{Ac\,x^4}{4} + \frac{Cb\,x^4}{4} + \frac{Bb\,x^3}{3} + \frac{Ab\,x^2}{2} + \frac{Ca\,x^2}{2} + Bax + aA\ln(x)$	60
parallelrisch	$\frac{cC\,x^6}{6} + \frac{Bc\,x^5}{5} + \frac{Ac\,x^4}{4} + \frac{Cb\,x^4}{4} + \frac{Bb\,x^3}{3} + \frac{Ab\,x^2}{2} + \frac{Ca\,x^2}{2} + Bax + aA\ln(x)$	60

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*A*b+1/2*C*a)*x^2+(1/4*A*c+1/4*C*b)*x^4+B*a*x+1/3*B*b*x^3+1/5*B*c*x^5+1
/6*c*C*x^6+a*A*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = \frac{1}{6}Ccx^6+\frac{1}{5}Bcx^5+\frac{1}{3}Bbx^3+\frac{1}{4}(Cb+Ac)x^4 + Bax+\frac{1}{2}(Ca+Ab)x^2+Aa\log(x)$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2
*(C*a + A*b)*x^2 + A*a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = Aa\log(x)+Bax+\frac{Bbx^3}{3}+\frac{Bcx^5}{5}+\frac{Ccx^6}{6} + x^4\left(\frac{Ac}{4}+\frac{Cb}{4}\right)+x^2\left(\frac{Ab}{2}+\frac{Ca}{2}\right)$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + x**4*(A*c/4 + C
*b/4) + x**2*(A*b/2 + C*a/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = \frac{1}{6}Ccx^6+\frac{1}{5}Bcx^5+\frac{1}{3}Bbx^3+\frac{1}{4}(Cb+Ac)x^4 + Bax+\frac{1}{2}(Ca+Ab)x^2+Aa\log(x)$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2
*(C*a + A*b)*x^2 + A*a*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = \frac{1}{6}Ccx^6+\frac{1}{5}Bcx^5+\frac{1}{4}Cbx^4+\frac{1}{4}Acx^4+\frac{1}{3}Bbx^3+\frac{1}{2}Cax^2+\frac{1}{2}Abx^2+Bax+Aa\log(|x|)$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/4*C*b*x^4 + 1/4*A*c*x^4 + 1/3*B*b*x^3 + 1/2*C
*a*x^2 + 1/2*A*b*x^2 + B*a*x + A*a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = x^2\left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)+x^4\left(\frac{A\,c}{4}+\frac{C\,b}{4}\right)+B\,a\,x+\frac{B\,b\,x^3}{3}+\frac{B\,c\,x^5}{5}+\frac{C\,c\,x^6}{6}+A\,a\,\ln(x)$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x)
```

```
[Out] x^2*((A*b)/2 + (C*a)/2) + x^4*((A*c)/4 + (C*b)/4) + B*a*x + (B*b*x^3)/3 + (
B*c*x^5)/5 + (C*c*x^6)/6 + A*a*log(x)
```

3.5 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

```
[Out] -a*A/x+(A*b+C*a)*x+1/2*b*B*x^2+1/3*(A*c+C*b)*x^3+1/4*B*c*x^4+1/5*c*C*x^5+a*
B*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(Ab\left(1+\frac{aC}{Ab}\right)+\frac{aA}{x^2}+\frac{aB}{x}+bBx+(Ac+bC)x^2+Bcx^3+cCx^4\right)dx$$
$$= -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3 + \frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{cC\,x^5}{5}+\frac{Bc\,x^4}{4}+\frac{Ac\,x^3}{3}+\frac{Cb\,x^3}{3}+\frac{bB\,x^2}{2}+Abx+Cax+aB\ln(x)-\frac{aA}{x}$	57
risch	$\frac{cC\,x^5}{5}+\frac{Bc\,x^4}{4}+\frac{Ac\,x^3}{3}+\frac{Cb\,x^3}{3}+\frac{bB\,x^2}{2}+Abx+Cax+aB\ln(x)-\frac{aA}{x}$	57
norman	$\frac{\left(\frac{Ac}{3}+\frac{Cb}{3}\right)x^4+(Ab+Ca)x^2-Aa+\frac{Bb\,x^3}{2}+\frac{Bc\,x^5}{4}+\frac{cC\,x^6}{5}}{x}+aB\ln(x)$	61
parallelrisch	$\frac{12cC\,x^6+15Bc\,x^5+20Ac\,x^4+20Cb\,x^4+30Bb\,x^3+60Ab\,x^2+60Ba\ln(x)x+60Ca\,x^2-60Aa}{60x}$	67

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*c*C*x^5+1/4*B*c*x^4+1/3*A*c*x^3+1/3*C*b*x^3+1/2*b*B*x^2+A*b*x+C*a*x+a*B
*ln(x)-a*A/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = \frac{12\,Ccx^6+15\,Bcx^5+30\,Bbx^3+20\,(Cb+Ac)x^4+60\,Bax\log(x)+60\,(Ca+Ab)x^2-60\,Aa}{60\,x}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c*x^6 + 15*B*c*x^5 + 30*B*b*x^3 + 20*(C*b + A*c)*x^4 + 60*B*a*x*
log(x) + 60*(C*a + A*b)*x^2 - 60*A*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = -\frac{Aa}{x}+Ba\log(x)+\frac{Bbx^2}{2}+\frac{Bcx^4}{4}+\frac{Ccx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Cb}{3}\right)+x(Ab+Ca)$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 +
C*b/3) + x*(A*b + C*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Ccx^5+\frac{1}{4}\,Bcx^4+\frac{1}{2}\,Bbx^2+\frac{1}{3}\,(Cb+Ac)x^3+Ba\log(x)+(Ca+Ab)x-\frac{Aa}{x}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/3*(C*b + A*c)*x^3 + B*a*log(x)
+ (C*a + A*b)*x - A*a/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Ccx^5+\frac{1}{4}\,Bcx^4+\frac{1}{3}\,Cbx^3+\frac{1}{3}\,Acx^3 + \frac{1}{2}\,Bbx^2+Cax+Abx+Ba\log\left(|x|\right)-\frac{Aa}{x}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*b*x^3 + 1/3*A*c*x^3 + 1/2*B*b*x^2 + C*a*x
 + A*b*x + B*a*log(abs(x)) - A*a/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = x\,(A\,b+C\,a)+x^3\left(\frac{A\,c}{3}+\frac{C\,b}{3}\right)-\frac{A\,a}{x} + \frac{B\,b\,x^2}{2}+\frac{B\,c\,x^4}{4}+\frac{C\,c\,x^5}{5}+B\,a\,\ln\left(x\right)$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x*(A*b + C*a) + x^3*((A*c)/3 + (C*b)/3) - (A*a)/x + (B*b*x^2)/2 + (B*c*x^4)
/4 + (C*c*x^5)/5 + B*a*log(x)
```

3.6 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3} \, dx = -\frac{aA}{2x^2} - \frac{aB}{x} + bBx + \frac{1}{2}(Ac+bC)x^2 + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4 + (Ab+aC)\log(x)$$

```
[Out] -1/2*a*A/x^2-a*B/x+b*B*x+1/2*(A*c+C*b)*x^2+1/3*B*c*x^3+1/4*c*C*x^4+(A*b+C*a
)*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3} \, dx = \log(x)(aC+Ab) - \frac{aA}{2x^2} - \frac{aB}{x} + \frac{1}{2}x^2(Ac+bC) + bBx + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3)/3 + (c*C
*x^4)/4 + (A*b + a*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(bB + \frac{aA}{x^3} + \frac{aB}{x^2} + \frac{Ab + aC}{x} + (Ac + bC)x + Bcx^2 + cCx^3\right) dx \\ &= -\frac{aA}{2x^2} - \frac{aB}{x} + bBx + \frac{1}{2}(Ac + bC)x^2 + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4 + (Ab + aC)\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^3} dx = -\frac{a(A + 2Bx)}{2x^2} + \frac{1}{12}x\left(6b(2B + Cx) + cx\left(6A + 4Bx + 3Cx^2\right)\right) + (Ab + aC)\log(x)$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -1/2*(a*(A + 2*B*x))/x^2 + (x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C*x^2
)))/12 + (A*b + a*C)*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{cC\,x^4}{4} + \frac{Bc\,x^3}{3} + \frac{Ac\,x^2}{2} + \frac{Cb\,x^2}{2} + Bbx + (Ab + Ca)\ln(x) - \frac{aA}{2x^2} - \frac{aB}{x}$	58
risch	$\frac{cC\,x^4}{4} + \frac{Bc\,x^3}{3} + \frac{Ac\,x^2}{2} + \frac{Cb\,x^2}{2} + Bbx + \frac{-Bax - \frac{1}{2}Aa}{x^2} + A\ln(x)\,b + C\ln(x)\,a$	58
norman	$\frac{\left(\frac{Ac}{2} + \frac{Cb}{2}\right)x^4 + Bb\,x^3 - \frac{Aa}{2} - Bax + \frac{Bc\,x^5}{3} + \frac{cC\,x^6}{4}}{x^2} + (Ab + Ca)\ln(x)$	59
parallelrisch	$\frac{3cC\,x^6 + 4Bc\,x^5 + 6Ac\,x^4 + 6Cb\,x^4 + 12A\ln(x)x^2b + 12Bb\,x^3 + 12C\ln(x)x^2a - 12Bax - 6Aa}{12x^2}$	69

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*C*x^4+1/3*B*c*x^3+1/2*A*c*x^2+1/2*C*b*x^2+B*b*x+(A*b+C*a)*ln(x)-1/2*a
*A/x^2-a*B/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = \frac{3\,Ccx^6+4\,Bcx^5+12\,Bbx^3+6\,(Cb+Ac)x^4+12\,(Ca+Ab)x^2\log(x)-12\,Bax-6\,Aa}{12\,x^2}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*x^6 + 4*B*c*x^5 + 12*B*b*x^3 + 6*(C*b + A*c)*x^4 + 12*(C*a + A*
b)*x^2*log(x) - 12*B*a*x - 6*A*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = Bbx+\frac{Bcx^3}{3}+\frac{Ccx^4}{4}+x^2\left(\frac{Ac}{2}+\frac{Cb}{2}\right)+(Ab+Ca)\log(x)+\frac{-Aa-2Bax}{2x^2}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] B*b*x + B*c*x**3/3 + C*c*x**4/4 + x**2*(A*c/2 + C*b/2) + (A*b + C*a)*log(x)
 + (-A*a - 2*B*a*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}\,Ccx^4+\frac{1}{3}\,Bcx^3+Bbx+\frac{1}{2}\,(Cb+Ac)x^2+(Ca+Ab)\log(x)-\frac{2\,Bax+Aa}{2\,x^2}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + B*b*x + 1/2*(C*b + A*c)*x^2 + (C*a + A*b)*log(x
) - 1/2*(2*B*a*x + A*a)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}\,Ccx^4 + \frac{1}{3}\,Bcx^3 + \frac{1}{2}\,Cbx^2 + \frac{1}{2}\,Acx^2 + Bbx + (Ca+Ab)\log(|x|) - \frac{2\,Bax+Aa}{2\,x^2}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + 1/2*C*b*x^2 + 1/2*A*c*x^2 + B*b*x + (C*a + A*b)
*log(abs(x)) - 1/2*(2*B*a*x + A*a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = x^2\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right) - \frac{\frac{A\,a}{2}+B\,a\,x}{x^2} + \ln(x)\,\left(A\,b+C\,a\right) + B\,b\,x + \frac{B\,c\,x^3}{3} + \frac{C\,c\,x^4}{4}$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x)
```

```
[Out] x^2*((A*c)/2 + (C*b)/2) - ((A*a)/2 + B*a*x)/x^2 + log(x)*(A*b + C*a) + B*b*
x + (B*c*x^3)/3 + (C*c*x^4)/4
```

3.7 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx = -\frac{aA}{3x^3} - \frac{aB}{2x^2} - \frac{Ab+aC}{x} + (Ac+bC)x + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3 + bB\log(x)$$

```
[Out] -1/3*a*A/x^3-1/2*a*B/x^2+(-A*b-C*a)/x+(A*c+C*b)*x+1/2*B*c*x^2+1/3*c*C*x^3+b
*B*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx = -\frac{aC+Ab}{x} - \frac{aA}{3x^3} - \frac{aB}{2x^2} + x(Ac+bC) + bB\log(x) + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x + (B*c*x^2)/
2 + (c*C*x^3)/3 + b*B*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(Ac\left(1+\frac{bC}{Ac}\right)+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB}{x}+Bcx+cCx^2\right)dx\\ &= -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+(Ac+bC)x+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3+bB\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = -\frac{Ab}{x}+Acx+bCx+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3 - \frac{a(2A+3x(B+2Cx))}{6x^3}+bB\log(x)$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + A*c*x + b*C*x + (B*c*x^2)/2 + (c*C*x^3)/3 - (a*(2*A + 3*x*(B +
 2*C*x)))/(6*x^3) + b*B*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{cC\,x^3}{3}+\frac{B\,x^2c}{2}+Acx+Cbx+bB\ln\left(x\right)-\frac{aB}{2x^2}-\frac{Ab+Ca}{x}-\frac{aA}{3x^3}$	55
risch	$\frac{cC\,x^3}{3}+\frac{B\,x^2c}{2}+Acx+Cbx+\frac{(-Ab-Ca)x^2-\frac{Bax}{2}-\frac{Aa}{3}}{x^3}+bB\ln\left(x\right)$	56
norman	$\frac{(-Ab-Ca)x^2+(Ac+Cb)x^4-\frac{Aa}{3}-\frac{Bax}{2}+\frac{Bc\,x^5}{2}+\frac{cC\,x^6}{3}}{x^3}+bB\ln\left(x\right)$	59
parallelrisch	$\frac{2cC\,x^6+3Bc\,x^5+6Ac\,x^4+6Bb\ln(x)x^3+6Cb\,x^4-6Ab\,x^2-6Ca\,x^2-3Bax-2Aa}{6x^3}$	67

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c*C*x^3+1/2*B*x^2*c+A*c*x+C*b*x+b*B*ln(x)-1/2*a*B/x^2-(A*b+C*a)/x-1/3*a
*A/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{2\,Ccx^6+3\,Bcx^5+6\,Bbx^3\log(x)+6\,(Cb+Ac)x^4-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*x^6 + 3*B*c*x^5 + 6*B*b*x^3*log(x) + 6*(C*b + A*c)*x^4 - 3*B*a*x
 - 6*(C*a + A*b)*x^2 - 2*A*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = Bb\log(x)+\frac{Bcx^2}{2}+\frac{Ccx^3}{3}+x(Ac+Cb)+\frac{-2Aa-3Bax+x^2(-6Ab-6Ca)}{6x^3}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] B*b*log(x) + B*c*x**2/2 + C*c*x**3/3 + x*(A*c + C*b) + (-2*A*a - 3*B*a*x +
x**2*(-6*A*b - 6*C*a))/(6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}\,Ccx^3+\frac{1}{2}\,Bcx^2+Bb\log(x)+(Cb+Ac)x-\frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + B*b*log(x) + (C*b + A*c)*x - 1/6*(3*B*a*x + 6*(
C*a + A*b)*x^2 + 2*A*a)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}Ccx^3 + \frac{1}{2}Bcx^2 + Cbx + Acx + Bb\log(|x|) - \frac{3\,Bax + 6\,(Ca+Ab)x^2 + 2\,Aa}{6\,x^3}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + C*b*x + A*c*x + B*b*log(abs(x)) - 1/6*(3*B*a*x
+ 6*(C*a + A*b)*x^2 + 2*A*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = x\,(A\,c+C\,b) - \frac{(A\,b+C\,a)\,x^2 + \frac{B\,a\,x}{2} + \frac{A\,a}{3}}{x^3} + \frac{B\,c\,x^2}{2} + \frac{C\,c\,x^3}{3} + B\,b\,\ln(x)$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x)
```

```
[Out] x*(A*c + C*b) - ((A*a)/3 + x^2*(A*b + C*a) + (B*a*x)/2)/x^3 + (B*c*x^2)/2 +
 (C*c*x^3)/3 + B*b*log(x)
```

3.8 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5} dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5} dx = -\frac{aA}{4x^4} - \frac{aB}{3x^3} - \frac{Ab+aC}{2x^2} - \frac{bB}{x} + Bcx + \frac{1}{2}cCx^2 + (Ac+bC)\log(x)$$

```
[Out] -1/4*a*A/x^4-1/3*a*B/x^3+1/2*(-A*b-C*a)/x^2-b*B/x+B*c*x+1/2*c*C*x^2+(A*c+C*
b)*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5} dx = -\frac{aC+Ab}{2x^2} - \frac{aA}{4x^4} - \frac{aB}{3x^3} + \log(x)(Ac+bC) - \frac{bB}{x} + Bcx + \frac{1}{2}cCx^2$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x + B*c*x + (c
*C*x^2)/2 + (A*c + b*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(Bc + \frac{aA}{x^5} + \frac{aB}{x^4} + \frac{Ab + aC}{x^3} + \frac{bB}{x^2} + \frac{Ac + bC}{x} + cCx\right) dx \\ &= -\frac{aA}{4x^4} - \frac{aB}{3x^3} - \frac{Ab + aC}{2x^2} - \frac{bB}{x} + Bcx + \frac{1}{2}cCx^2 + (Ac + bC)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^5} dx = -\frac{a(3A + 4Bx + 6Cx^2)}{12x^4} + \frac{-Ab - 2bBx + cx^3(2B + Cx)}{2x^2} + (Ac + bC)\log(x)$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -1/12*(a*(3*A + 4*B*x + 6*C*x^2))/x^4 + (-(A*b) - 2*b*B*x + c*x^3*(2*B + C*
x))/(2*x^2) + (A*c + b*C)*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{cC\,x^2}{2} + Bcx + (Ac + Cb)\ln(x) - \frac{Ab+Ca}{2x^2} - \frac{bB}{x} - \frac{aA}{4x^4} - \frac{aB}{3x^3}$	56
risch	$\frac{cC\,x^2}{2} + Bcx + \frac{-Bb\,x^3 + \left(-\frac{Ab}{2} - \frac{Ca}{2}\right)x^2 - \frac{Bax}{3} - \frac{Aa}{4}}{x^4} + A\ln(x)\,c + C\ln(x)\,b$	57
norman	$\frac{\left(-\frac{Ab}{2} - \frac{Ca}{2}\right)x^2 + Bc\,x^5 - \frac{Aa}{4} - \frac{Bax}{3} - Bb\,x^3 + \frac{cC\,x^6}{2}}{x^4} + (Ac + Cb)\ln(x)$	59
parallelrisch	$\frac{6cC\,x^6 + 12A\ln(x)x^4c + 12Bc\,x^5 + 12C\ln(x)x^4b - 12Bb\,x^3 - 6Ab\,x^2 - 6Ca\,x^2 - 4Bax - 3Aa}{12x^4}$	69

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*C*x^2+B*c*x+(A*c+C*b)*ln(x)-1/2*(A*b+C*a)/x^2-b*B/x-1/4*a*A/x^4-1/3*a
*B/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \frac{6\,Ccx^6+12\,Bcx^5+12\,(Cb+Ac)x^4\log(x)-12\,Bbx^3-4\,Bax-6\,(Ca+Ab)x^2-3\,Aa}{12\,x^4}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(6*C*c*x^6 + 12*B*c*x^5 + 12*(C*b + A*c)*x^4*log(x) - 12*B*b*x^3 - 4*B
*a*x - 6*(C*a + A*b)*x^2 - 3*A*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = Bcx+\frac{Ccx^2}{2}+(Ac+Cb)\log(x) + \frac{-3Aa-4Bax-12Bbx^3+x^2(-6Ab-6Ca)}{12x^4}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] B*c*x + C*c*x**2/2 + (A*c + C*b)*log(x) + (-3*A*a - 4*B*a*x - 12*B*b*x**3 +
 x**2*(-6*A*b - 6*C*a))/(12*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}\,Ccx^2+Bcx+(Cb+Ac)\log(x) - \frac{12\,Bbx^3+4\,Bax+6\,(Ca+Ab)x^2+3\,Aa}{12\,x^4}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(x) - 1/12*(12*B*b*x^3 + 4*B*a*x + 6*(
C*a + A*b)*x^2 + 3*A*a)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}\,Ccx^2 + Bcx + (Cb+Ac)\log(|x|) - \frac{12\,Bbx^3 + 4\,Bax + 6\,(Ca+Ab)x^2 + 3\,Aa}{12\,x^4}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(abs(x)) - 1/12*(12*B*b*x^3 + 4*B*a*x
+ 6*(C*a + A*b)*x^2 + 3*A*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \ln(x)\ (A\,c+C\,b) - \frac{B\,b\,x^3 + \left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)\,x^2 + \frac{B\,a\,x}{3} + \frac{A\,a}{4}}{x^4} + B\,c\,x + \frac{C\,c\,x^2}{2}$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x)
```

```
[Out] log(x)*(A*c + C*b) - ((A*a)/4 + x^2*((A*b)/2 + (C*a)/2) + (B*a*x)/3 + B*b*x
^3)/x^4 + B*c*x + (C*c*x^2)/2
```

3.9 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx = -\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ab+aC}{3x^3}-\frac{bB}{2x^2}-\frac{Ac+bC}{x}+cCx+Bc\log(x)$$

```
[Out] -1/5*a*A/x^5-1/4*a*B/x^4+1/3*(-A*b-C*a)/x^3-1/2*b*B/x^2+(-A*c-C*b)/x+c*C*x+
B*c*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx = -\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -1/5*(a*A)/x^5 - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x^2) - (A*c
 + b*C)/x + c*C*x + B*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(cC + \frac{aA}{x^6} + \frac{aB}{x^5} + \frac{Ab + aC}{x^4} + \frac{bB}{x^3} + \frac{Ac + bC}{x^2} + \frac{Bc}{x}\right) dx \\ &= -\frac{aA}{5x^5} - \frac{aB}{4x^4} - \frac{Ab + aC}{3x^3} - \frac{bB}{2x^2} - \frac{Ac + bC}{x} + cCx + Bc\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^6} dx$$
$$= -\frac{12aA - 60cCx^6 + 30bx^3(B + 2Cx) + 5ax(3B + 4Cx) + 20Ax^2(b + 3cx^2)}{60x^5} + Bc\log(x)$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
[Out] -1/60*(12*a*A - 60*c*C*x^6 + 30*b*x^3*(B + 2*C*x) + 5*a*x*(3*B + 4*C*x) + 2
0*A*x^2*(b + 3*c*x^2))/x^5 + B*c*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$cCx + Bc\ln(x) - \frac{bB}{2x^2} - \frac{aA}{5x^5} - \frac{Ac+Cb}{x} - \frac{aB}{4x^4} - \frac{Ab+Ca}{3x^3}$	56
risch	$cCx + \frac{(-Ac-Cb)x^4 - \frac{Bb\,x^3}{2} + \left(-\frac{Ab}{3} - \frac{Ca}{3}\right)x^2 - \frac{Bax}{4} - \frac{Aa}{5}}{x^5} + Bc\ln(x)$	58
norman	$\frac{\left(-\frac{Ab}{3} - \frac{Ca}{3}\right)x^2 + (-Ac-Cb)x^4 + cC\,x^6 - \frac{Aa}{5} - \frac{Bax}{4} - \frac{Bb\,x^3}{2}}{x^5} + Bc\ln(x)$	60
parallelrisch	$-\frac{-60Bc\ln(x)x^5 - 60cC\,x^6 + 60Ac\,x^4 + 60Cb\,x^4 + 30Bb\,x^3 + 20Ab\,x^2 + 20Ca\,x^2 + 15Bax + 12Aa}{60x^5}$	67

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
[Out] c*C*x+B*c*ln(x)-1/2*b*B/x^2-1/5*a*A/x^5-(A*c+C*b)/x-1/4*a*B/x^4-1/3*(A*b+C*
a)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^6}\,dx = \frac{60\,Ccx^6+60\,Bcx^5\log(x)-30\,Bbx^3-60\,(Cb+Ac)x^4-15\,Bax-20\,(Ca+Ab)x^2-12\,Aa}{60\,x^5}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6 + 60*B*c*x^5*log(x) - 30*B*b*x^3 - 60*(C*b + A*c)*x^4 - 15
*B*a*x - 20*(C*a + A*b)*x^2 - 12*A*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^6}\,dx = Bc\log(x)+Ccx + \frac{-12Aa-15Bax-30Bbx^3+x^4(-60Ac-60Cb)+x^2(-20Ab-20Ca)}{60x^5}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] B*c*log(x) + C*c*x + (-12*A*a - 15*B*a*x - 30*B*b*x**3 + x**4*(-60*A*c - 60
*C*b) + x**2*(-20*A*b - 20*C*a))/(60*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^6}\,dx = Ccx+Bc\log(x)-\frac{30\,Bbx^3+60\,(Cb+Ac)x^4+15\,Bax+20\,(Ca+Ab)x^2+12\,Aa}{60\,x^5}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] C*c*x + B*c*log(x) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x + 20*
(C*a + A*b)*x^2 + 12*A*a)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx$$
$$= Ccx + Bc\log(|x|) - \frac{30\,Bbx^3 + 60\,(Cb+Ac)x^4 + 15\,Bax + 20\,(Ca+Ab)x^2 + 12\,Aa}{60\,x^5}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

```
[Out] C*c*x + B*c*log(abs(x)) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x
+ 20*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx$$
$$= C\,c\,x - \frac{(A\,c+C\,b)\,x^4 + \frac{B\,b\,x^3}{2} + \left(\frac{A\,b}{3}+\frac{C\,a}{3}\right)\,x^2 + \frac{B\,a\,x}{4} + \frac{A\,a}{5}}{x^5} + B\,c\,\ln(x)$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x)
```

```
[Out] C*c*x - ((A*a)/5 + x^2*((A*b)/3 + (C*a)/3) + x^4*(A*c + C*b) + (B*a*x)/4 +
(B*b*x^3)/2)/x^5 + B*c*log(x)
```

3.10 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7} dx$

Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7} dx = -\frac{aA}{6x^6} - \frac{aB}{5x^5} - \frac{Ab+aC}{4x^4} - \frac{bB}{3x^3} - \frac{Ac+bC}{2x^2} - \frac{Bc}{x} + cC\log(x)$$

```
[Out] -1/6*a*A/x^6-1/5*a*B/x^5+1/4*(-A*b-C*a)/x^4-1/3*b*B/x^3+1/2*(-A*c-C*b)/x^2-
B*c/x+c*C*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7} dx = -\frac{aC+Ab}{4x^4} - \frac{aA}{6x^6} - \frac{aB}{5x^5} - \frac{Ac+bC}{2x^2} - \frac{bB}{3x^3} - \frac{Bc}{x} + cC\log(x)$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -1/6*(a*A)/x^6 - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x^3) - (A*c
 + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{aA}{x^7}+\frac{aB}{x^6}+\frac{Ab+aC}{x^5}+\frac{bB}{x^4}+\frac{Ac+bC}{x^3}+\frac{Bc}{x^2}+\frac{cC}{x}\right)dx\\ &= -\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{bB}{3x^3}-\frac{Ac+bC}{2x^2}-\frac{Bc}{x}+cC\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$$
$$= -\frac{a(10A+3x(4B+5Cx))+5x^2(3A(b+2cx^2)+2x(2bB+3bCx+6Bcx^2))}{60x^6} + cC\log(x)$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -1/60*(a*(10*A + 3*x*(4*B + 5*C*x)) + 5*x^2*(3*A*(b + 2*c*x^2) + 2*x*(2*b*B
 + 3*b*C*x + 6*B*c*x^2))))/x^6 + c*C*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

method	result	size
default	$cC\ln(x)-\frac{Ac+Cb}{2x^2}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Bc}{x}-\frac{Ab+Ca}{4x^4}-\frac{bB}{3x^3}$	59
norman	$\frac{\left(-\frac{Ab}{4}-\frac{Ca}{4}\right)x^2+\left(-\frac{Ac}{2}-\frac{Cb}{2}\right)x^4-\frac{Aa}{6}-\frac{Bax}{5}-\frac{Bbx^3}{3}-Bcx^5}{x^6}+cC\ln(x)$	61
risch	$\frac{\left(-\frac{Ab}{4}-\frac{Ca}{4}\right)x^2+\left(-\frac{Ac}{2}-\frac{Cb}{2}\right)x^4-\frac{Aa}{6}-\frac{Bax}{5}-\frac{Bbx^3}{3}-Bcx^5}{x^6}+cC\ln(x)$	61
parallelrisch	$-\frac{-60Cc\ln(x)x^6+60Bcx^5+30Acx^4+30Cbx^4+20Bbx^3+15Abx^2+15Cax^2+12Bax+10Aa}{60x^6}$	67

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c*C*ln(x)-1/2*(A*c+C*b)/x^2-1/6*a*A/x^6-1/5*a*B/x^5-B*c/x-1/4*(A*b+C*a)/x^4
-1/3*b*B/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = \frac{60\,Ccx^6\log(x) - 60\,Bcx^5 - 20\,Bbx^3 - 30\,(Cb+Ac)x^4 - 12\,Bax - 15\,(Ca+Ab)x^2 - 10\,Aa}{60\,x^6}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6*log(x) - 60*B*c*x^5 - 20*B*b*x^3 - 30*(C*b + A*c)*x^4 - 12
*B*a*x - 15*(C*a + A*b)*x^2 - 10*A*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = Cc\log(x) + \frac{-10Aa - 12Bax - 20Bbx^3 - 60Bcx^5 + x^4(-30Ac - 30Cb) + x^2(-15Ab - 15Ca)}{60x^6}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] C*c*log(x) + (-10*A*a - 12*B*a*x - 20*B*b*x**3 - 60*B*c*x**5 + x**4*(-30*A*
c - 30*C*b) + x**2*(-15*A*b - 15*C*a))/(60*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = Cc\log(x) - \frac{60\,Bcx^5 + 20\,Bbx^3 + 30\,(Cb+Ac)x^4 + 12\,Bax + 15\,(Ca+Ab)x^2 + 10\,Aa}{60\,x^6}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] C*c*log(x) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B*a*x
+ 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$$
$$= Cc\log(|x|) - \frac{60\,Bcx^5+20\,Bbx^3+30\,(Cb+Ac)x^4+12\,Bax+15\,(Ca+Ab)x^2+10\,Aa}{60\,x^6}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] C*c*log(abs(x)) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B
*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$$
$$= C\,c\,\ln(x) - \frac{B\,c\,x^5+\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right)\,x^4+\frac{B\,b\,x^3}{3}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^2+\frac{B\,a\,x}{5}+\frac{A\,a}{6}}{x^6}$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x)
```

```
[Out] C*c*log(x) - ((A*a)/6 + x^2*((A*b)/4 + (C*a)/4) + x^4*((A*c)/2 + (C*b)/2) +
 (B*a*x)/5 + (B*b*x^3)/3 + B*c*x^5)/x^6
```

3.11 $\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 159

$$\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(A\left(b^2+2ac\right)+2abC\right)x^7+\frac{1}{8}B\left(b^2+2ac\right)x^8+\frac{1}{9}\left(2Abc+\left(b^2+2ac\right)C\right)x^9+\frac{1}{5}bBcx^{10}+\frac{1}{11}c(Ac+2bC)x^{11}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

```
[Out] 1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/3*a*b*B*x^6+1/7*(A*(2*a
*c+b^2)+2*a*b*C)*x^7+1/8*B*(2*a*c+b^2)*x^8+1/9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+
1/5*b*B*c*x^10+1/11*c*(A*c+2*C*b)*x^11+1/12*B*c^2*x^12+1/13*c^2*C*x^13
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1642}

$$\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}abBx^6+\frac{1}{11}cx^{11}(Ac+2bC)+\frac{1}{5}bBcx^{10}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

```
[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
(b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2Ax^2 + a^2Bx^3 + a(2Ab+aC)x^4 + 2abBx^5 + \left(A\left(b^2+2ac\right)+2abC\right)x^6 \right. \\
&\quad + B\left(b^2+2ac\right)x^7 + \left(2Abc+\left(b^2+2ac\right)C\right)x^8 + 2bBcx^9 + c(Ac+2bC)x^{10} \\
&\quad \left. + Bc^2x^{11} + c^2Cx^{12}\right) dx \\
&= \frac{1}{3}a^2Ax^3 + \frac{1}{4}a^2Bx^4 + \frac{1}{5}a(2Ab+aC)x^5 + \frac{1}{3}abBx^6 + \frac{1}{7}\left(A\left(b^2+2ac\right)+2abC\right)x^7 \\
&\quad + \frac{1}{8}B\left(b^2+2ac\right)x^8 + \frac{1}{9}\left(2Abc+\left(b^2+2ac\right)C\right)x^9 \\
&\quad + \frac{1}{5}bBcx^{10} + \frac{1}{11}c(Ac+2bC)x^{11} + \frac{1}{12}Bc^2x^{12} + \frac{1}{13}c^2Cx^{13}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2 dx &= \frac{1}{3}a^2Ax^3 + \frac{1}{4}a^2Bx^4 + \frac{1}{5}a(2Ab+aC)x^5 + \frac{1}{3}abBx^6 \\
&\quad + \frac{1}{7}\left(Ab^2+2aAc+2abC\right)x^7 + \frac{1}{8}B\left(b^2+2ac\right)x^8 \\
&\quad + \frac{1}{9}\left(2Abc+b^2C+2acC\right)x^9 + \frac{1}{5}bBcx^{10} \\
&\quad + \frac{1}{11}c(Ac+2bC)x^{11} + \frac{1}{12}Bc^2x^{12} + \frac{1}{13}c^2Cx^{13}
\end{aligned}$$

```
[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*b^2 + 2*a*A*c + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
b^2*C + 2*a*c*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
default	$\frac{c^2 C x^{13}}{13}+\frac{B c^2 x^{12}}{12}+\frac{(A c^2+2Cbc)x^{11}}{11}+\frac{bBc x^{10}}{5}+\frac{(2Abc+(2ac+b^2)C)x^9}{9}+\frac{B(2ac+b^2)x^8}{8}+\frac{(A(2ac+b^2)+2abC)x^7}{7}$
norman	$\frac{c^2 C x^{13}}{13}+\frac{B c^2 x^{12}}{12}+\left(\frac{1}{11}A c^2+\frac{2}{11}Cbc\right)x^{11}+\frac{bBc x^{10}}{5}+\left(\frac{2}{9}Abc+\frac{2}{9}acC+\frac{1}{9}b^2C\right)x^9+\left(\frac{1}{4}Bac+\frac{1}{8}B\right.$
gosper	$\frac{1}{13}c^2C x^{13}+\frac{1}{12}B c^2x^{12}+\frac{1}{11}x^{11}A c^2+\frac{2}{11}x^{11}Cbc+\frac{1}{5}bBc x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9acC+\frac{1}{9}x^9b^2C+$
risch	$\frac{1}{13}c^2C x^{13}+\frac{1}{12}B c^2x^{12}+\frac{1}{11}x^{11}A c^2+\frac{2}{11}x^{11}Cbc+\frac{1}{5}bBc x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9acC+\frac{1}{9}x^9b^2C+$
parallelrisch	$\frac{1}{13}c^2C x^{13}+\frac{1}{12}B c^2x^{12}+\frac{1}{11}x^{11}A c^2+\frac{2}{11}x^{11}Cbc+\frac{1}{5}bBc x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9acC+\frac{1}{9}x^9b^2C+$

```
[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c^2*C*x^13+1/12*B*c^2*x^12+1/11*(A*c^2+2*C*b*c)*x^11+1/5*b*B*c*x^10+1/
9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+1/8*B*(2*a*c+b^2)*x^8+1/7*(A*(2*a*c+b^2)+2*a*
b*C)*x^7+1/3*a*b*B*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\begin{aligned}\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx &= \frac{1}{13}Cc^2x^{13}+\frac{1}{12}Bc^2x^{12}\\ &+\frac{1}{5}Bbcx^{10}+\frac{1}{11}\left(2Cbc+Ac^2\right)x^{11}\\ &+\frac{1}{9}\left(Cb^2+2(Ca+Ab)c\right)x^9\\ &+\frac{1}{3}Babx^6+\frac{1}{8}\left(Bb^2+2Bac\right)x^8\\ &+\frac{1}{7}\left(2Cab+Ab^2+2Aac\right)x^7+\frac{1}{4}Ba^2x^4\\ &+\frac{1}{3}Aa^2x^3+\frac{1}{5}\left(Ca^2+2Aab\right)x^5\end{aligned}$$

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2)
*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 + 2*
B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A*a^
2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx=\frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Babx^6}{3}+\frac{Bbcx^{10}}{5}+\frac{Bc^2x^{12}}{12}+\frac{Cc^2x^{13}}{13}+x^{11}\left(\frac{Ac^2}{11}+\frac{2Cbc}{11}\right)+x^9\cdot\left(\frac{2Abc}{9}+\frac{2Cac}{9}+\frac{Cb^2}{9}\right)+x^8\left(\frac{Bac}{4}+\frac{Bb^2}{8}\right)+x^7\cdot\left(\frac{2Aac}{7}+\frac{Ab^2}{7}+\frac{2Cab}{7}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ca^2}{5}\right)$$

```
[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*c**2*x**12
/12 + C*c**2*x**13/13 + x**11*(A*c**2/11 + 2*C*b*c/11) + x**9*(2*A*b*c/9 +
2*C*a*c/9 + C*b**2/9) + x**8*(B*a*c/4 + B*b**2/8) + x**7*(2*A*a*c/7 + A*b**
2/7 + 2*C*a*b/7) + x**5*(2*A*a*b/5 + C*a**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{13}Cc^2x^{13}+\frac{1}{12}Bc^2x^{12}+\frac{1}{5}Bbcx^{10}+\frac{1}{11}\left(2\,Cbc+Ac^2\right)x^{11}+\frac{1}{9}\left(Cb^2+2\,(Ca+Ab)c\right)x^9+\frac{1}{3}Babx^6+\frac{1}{8}\left(Bb^2+2\,Bac\right)x^8+\frac{1}{7}\left(2\,Cab+Ab^2+2\,Aac\right)x^7+\frac{1}{4}Ba^2x^4+\frac{1}{3}Aa^2x^3+\frac{1}{5}\left(Ca^2+2\,Aab\right)x^5$$

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2)
*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 + 2*
B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A*a^
2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int x^2 (A + Bx + Cx^2) \left(a + bx^2 + cx^4\right)^2 dx = \frac{1}{13} Cc^2x^{13} + \frac{1}{12} Bc^2x^{12} + \frac{2}{11} Cbcx^{11} + \frac{1}{11} Ac^2x^{11} + \frac{1}{5} Bbcx^{10} + \frac{1}{9} Cb^2x^9 + \frac{2}{9} Cacx^9 + \frac{2}{9} Abcx^9 + \frac{1}{8} Bb^2x^8 + \frac{1}{4} Bacx^8 + \frac{2}{7} Cabx^7 + \frac{1}{7} Ab^2x^7 + \frac{2}{7} Aacx^7 + \frac{1}{3} Babx^6 + \frac{1}{5} Ca^2x^5 + \frac{2}{5} Aabx^5 + \frac{1}{4} Ba^2x^4 + \frac{1}{3} Aa^2x^3$$

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 2/11*C*b*c*x^11 + 1/11*A*c^2*x^11 + 1/5
*B*b*c*x^10 + 1/9*C*b^2*x^9 + 2/9*C*a*c*x^9 + 2/9*A*b*c*x^9 + 1/8*B*b^2*x^8
 + 1/4*B*a*c*x^8 + 2/7*C*a*b*x^7 + 1/7*A*b^2*x^7 + 2/7*A*a*c*x^7 + 1/3*B*a*
b*x^6 + 1/5*C*a^2*x^5 + 2/5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

$$\int x^2 (A + Bx + Cx^2) \left(a + bx^2 + cx^4\right)^2 dx = x^5 \left(\frac{C\,a^2}{5} + \frac{2\,A\,b\,a}{5}\right) + x^{11} \left(\frac{A\,c^2}{11} + \frac{2\,C\,b\,c}{11}\right) + x^7 \left(\frac{A\,b^2}{7} + \frac{2\,C\,a\,b}{7} + \frac{2\,A\,a\,c}{7}\right) + x^9 \left(\frac{C\,b^2}{9} + \frac{2\,A\,c\,b}{9} + \frac{2\,C\,a\,c}{9}\right) + \frac{A\,a^2\,x^3}{3} + \frac{B\,a^2\,x^4}{4} + \frac{B\,c^2\,x^{12}}{12} + \frac{C\,c^2\,x^{13}}{13} + \frac{B\,x^8\,(b^2 + 2\,a\,c)}{8} + \frac{B\,a\,b\,x^6}{3} + \frac{B\,b\,c\,x^{10}}{5}$$

```
[In] int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((C*a^2)/5 + (2*A*a*b)/5) + x^11*((A*c^2)/11 + (2*C*b*c)/11) + x^7*((A*
b^2)/7 + (2*A*a*c)/7 + (2*C*a*b)/7) + x^9*((C*b^2)/9 + (2*A*b*c)/9 + (2*C*a
*c)/9) + (A*a^2*x^3)/3 + (B*a^2*x^4)/4 + (B*c^2*x^12)/12 + (C*c^2*x^13)/13
+ (B*x^8*(2*a*c + b^2))/8 + (B*a*b*x^6)/3 + (B*b*c*x^10)/5
```

3.12 $\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 159

$$\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(A\left(b^2+2ac\right)+2abC\right)x^6+\frac{1}{7}B\left(b^2+2ac\right)x^7+\frac{1}{8}\left(2Abc+\left(b^2+2ac\right)C\right)x^8+\frac{2}{9}bBcx^9+\frac{1}{10}c(Ac+2bC)x^{10}+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

```
[Out] 1/2*a^2*A*x^2+1/3*a^2*B*x^3+1/4*a*(2*A*b+C*a)*x^4+2/5*a*b*B*x^5+1/6*(A*(2*a
*c+b^2)+2*a*b*C)*x^6+1/7*B*(2*a*c+b^2)*x^7+1/8*(2*A*b*c+(2*a*c+b^2)*C)*x^8+
2/9*b*B*c*x^9+1/10*c*(A*c+2*C*b)*x^10+1/11*B*c^2*x^11+1/12*c^2*C*x^12
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}abBx^5+\frac{1}{10}cx^{10}(Ac+2bC)+\frac{2}{9}bBcx^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

```
[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2Ax + a^2Bx^2 + a(2Ab + aC)x^3 + 2abBx^4 + \left(A\left(b^2 + 2ac\right) + 2abC\right)x^5 \right.\\ &\quad + B\left(b^2 + 2ac\right)x^6 + \left(2Abc + \left(b^2 + 2ac\right)C\right)x^7 + 2bBcx^8 + c(Ac + 2bC)x^9 \\ &\quad \left. + Bc^2x^{10} + c^2Cx^{11}\right) dx \\ &= \frac{1}{2}a^2Ax^2 + \frac{1}{3}a^2Bx^3 + \frac{1}{4}a(2Ab + aC)x^4 + \frac{2}{5}abBx^5 + \frac{1}{6}\left(A\left(b^2 + 2ac\right) + 2abC\right)x^6 \\ &\quad + \frac{1}{7}B\left(b^2 + 2ac\right)x^7 + \frac{1}{8}\left(2Abc + \left(b^2 + 2ac\right)C\right)x^8 \\ &\quad + \frac{2}{9}bBcx^9 + \frac{1}{10}c(Ac + 2bC)x^{10} + \frac{1}{11}Bc^2x^{11} + \frac{1}{12}c^2Cx^{12}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x\left(A + Bx + Cx^2\right)\left(a + bx^2 + cx^4\right)^2 dx &= \frac{1}{2}a^2Ax^2 + \frac{1}{3}a^2Bx^3 + \frac{1}{4}a(2Ab + aC)x^4 + \frac{2}{5}abBx^5 \\ &\quad + \frac{1}{6}\left(Ab^2 + 2aAc + 2abC\right)x^6 + \frac{1}{7}B\left(b^2 + 2ac\right)x^7 \\ &\quad + \frac{1}{8}\left(2Abc + b^2C + 2acC\right)x^8 + \frac{2}{9}bBcx^9 \\ &\quad + \frac{1}{10}c(Ac + 2bC)x^{10} + \frac{1}{11}Bc^2x^{11} + \frac{1}{12}c^2Cx^{12}\end{aligned}$$

```
[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ b^2*C + 2*a*c*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
default	$\frac{c^2 C x^{12}}{12}+\frac{B c^2 x^{11}}{11}+\frac{(A c^2+2Cbc) x^{10}}{10}+\frac{2bBc x^9}{9}+\frac{(2Abc+(2ac+b^2)C) x^8}{8}+\frac{B(2ac+b^2) x^7}{7}+\frac{(A(2ac+b^2)+2abC)}{6}$
norman	$\frac{c^2 C x^{12}}{12}+\frac{B c^2 x^{11}}{11}+\left(\frac{1}{10}A c^2+\frac{1}{5}Cbc\right) x^{10}+\frac{2bBc x^9}{9}+\left(\frac{1}{4}Abc+\frac{1}{4}acC+\frac{1}{8}b^2C\right) x^8+\left(\frac{2}{7}Bac+\frac{1}{7}\right.$
gosper	$\frac{1}{12}c^2C x^{12}+\frac{1}{11}B c^2x^{11}+\frac{1}{10}x^{10}A c^2+\frac{1}{5}x^{10}Cbc+\frac{2}{9}bBc x^9+\frac{1}{4}x^8Abc+\frac{1}{4}x^8acC+\frac{1}{8}x^8b^2C+$
risch	$\frac{1}{12}c^2C x^{12}+\frac{1}{11}B c^2x^{11}+\frac{1}{10}x^{10}A c^2+\frac{1}{5}x^{10}Cbc+\frac{2}{9}bBc x^9+\frac{1}{4}x^8Abc+\frac{1}{4}x^8acC+\frac{1}{8}x^8b^2C+$
parallelrisch	$\frac{1}{12}c^2C x^{12}+\frac{1}{11}B c^2x^{11}+\frac{1}{10}x^{10}A c^2+\frac{1}{5}x^{10}Cbc+\frac{2}{9}bBc x^9+\frac{1}{4}x^8Abc+\frac{1}{4}x^8acC+\frac{1}{8}x^8b^2C+$

```
[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c^2*C*x^12+1/11*B*c^2*x^11+1/10*(A*c^2+2*C*b*c)*x^10+2/9*b*B*c*x^9+1/8
*(2*A*b*c+(2*a*c+b^2)*C)*x^8+1/7*B*(2*a*c+b^2)*x^7+1/6*(A*(2*a*c+b^2)+2*a*b
*C)*x^6+2/5*a*b*B*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{12}\,Cc^2x^{12}+\frac{1}{11}\,Bc^2x^{11}+\frac{2}{9}\,Bbcx^9+\frac{1}{10}\,\left(2\,Cbc+Ac^2\right)x^{10}+\frac{1}{8}\,\left(Cb^2+2\,(Ca+Ab)c\right)x^8+\frac{2}{5}\,Babx^5+\frac{1}{7}\,\left(Bb^2+2\,Bac\right)x^7+\frac{1}{6}\,\left(2\,Cab+Ab^2+2\,Aac\right)x^6+\frac{1}{3}\,Ba^2x^3+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,\left(Ca^2+2\,Aab\right)x^4$$

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)*
x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2*B
*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*a^2
*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{2Babx^5}{5}+\frac{2Bbcx^9}{9}+\frac{Bc^2x^{11}}{11}+\frac{Cc^2x^{12}}{12}+x^{10}\left(\frac{Ac^2}{10}+\frac{Cbc}{5}\right)+x^8\left(\frac{Abc}{4}+\frac{Cac}{4}+\frac{Cb^2}{8}\right)+x^7\cdot\left(\frac{2Bac}{7}+\frac{Bb^2}{7}\right)+x^6\left(\frac{Aac}{3}+\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

```
[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*x**9/9 + B*c**2*x**11/11 + C*c**2*x**12/12 + x**10*(A*c**2/10 + C*b*c/5) + x**8*(A*b*c/4 + C*a*c/4 + C*b**2/8) + x**7*(2*B*a*c/7 + B*b**2/7) + x**6*(A*a*c/3 + A*b**2/6 + C*a*b/3) + x**4*(A*a*b/2 + C*a**2/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{12}Cc^2x^{12}+\frac{1}{11}Bc^2x^{11}+\frac{2}{9}Bbcx^9+\frac{1}{10}\left(2\,Cbc+Ac^2\right)x^{10}+\frac{1}{8}\left(Cb^2+2\,(Ca+Ab)c\right)x^8+\frac{2}{5}Babx^5+\frac{1}{7}\left(Bb^2+2\,Bac\right)x^7+\frac{1}{6}\left(2\,Cab+Ab^2+2\,Aac\right)x^6+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2+\frac{1}{4}\left(Ca^2+2\,Aab\right)x^4$$

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)*x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2*B*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{12}Cc^2x^{12}+\frac{1}{11}Bc^2x^{11}+\frac{1}{5}Cbcx^{10}+\frac{1}{10}Ac^2x^{10}+\frac{2}{9}Bbcx^9+\frac{1}{8}Cb^2x^8+\frac{1}{4}Cacx^8+\frac{1}{4}Abcx^8+\frac{1}{7}Bb^2x^7+\frac{2}{7}Bacx^7+\frac{1}{3}Cabx^6+\frac{1}{6}Ab^2x^6+\frac{1}{3}Aacx^6+\frac{2}{5}Babx^5+\frac{1}{4}Ca^2x^4+\frac{1}{2}Aabx^4+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2$$

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 1/5*C*b*c*x^10 + 1/10*A*c^2*x^10 + 2/9*
B*b*c*x^9 + 1/8*C*b^2*x^8 + 1/4*C*a*c*x^8 + 1/4*A*b*c*x^8 + 1/7*B*b^2*x^7 +
 2/7*B*a*c*x^7 + 1/3*C*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*A*a*c*x^6 + 2/5*B*a*b*
x^5 + 1/4*C*a^2*x^4 + 1/2*A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

$$\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=x^4\left(\frac{C\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^{10}\left(\frac{A\,c^2}{10}+\frac{C\,b\,c}{5}\right)+x^6\left(\frac{A\,b^2}{6}+\frac{C\,a\,b}{3}+\frac{A\,a\,c}{3}\right)+x^8\left(\frac{C\,b^2}{8}+\frac{A\,c\,b}{4}+\frac{C\,a\,c}{4}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,a^2\,x^3}{3}+\frac{B\,c^2\,x^{11}}{11}+\frac{C\,c^2\,x^{12}}{12}+\frac{B\,x^7\,\left(b^2+2\,a\,c\right)}{7}+\frac{2\,B\,a\,b\,x^5}{5}+\frac{2\,B\,b\,c\,x^9}{9}$$

```
[In] int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4*((C*a^2)/4 + (A*a*b)/2) + x^10*((A*c^2)/10 + (C*b*c)/5) + x^6*((A*b^2)/
6 + (A*a*c)/3 + (C*a*b)/3) + x^8*((C*b^2)/8 + (A*b*c)/4 + (C*a*c)/4) + (A*a
^2*x^2)/2 + (B*a^2*x^3)/3 + (B*c^2*x^11)/11 + (C*c^2*x^12)/12 + (B*x^7*(2*a
*c + b^2))/7 + (2*B*a*b*x^5)/5 + (2*B*b*c*x^9)/9
```

3.13 $\int (A+Bx+Cx^2)(a+bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4 +\frac{1}{5}\left(A\left(b^2+2ac\right)+2abC\right)x^5+\frac{1}{6}B\left(b^2+2ac\right)x^6 +\frac{1}{7}\left(2Abc+\left(b^2+2ac\right)C\right)x^7+\frac{1}{4}bBcx^8 +\frac{1}{9}c(Ac+2bC)x^9+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

```
[Out] a^2*A*x+1/2*a^2*B*x^2+1/3*a*(2*A*b+C*a)*x^3+1/2*a*b*B*x^4+1/5*(A*(2*a*c+b^2
)+2*a*b*C)*x^5+1/6*B*(2*a*c+b^2)*x^6+1/7*(2*A*b*c+(2*a*c+b^2)*C)*x^7+1/4*b*
B*c*x^8+1/9*c*(A*c+2*C*b)*x^9+1/10*B*c^2*x^10+1/11*c^2*C*x^11
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1671}

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right) +\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab) +\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}abBx^4+\frac{1}{9}cx^9(Ac+2bC) +\frac{1}{4}bBcx^8+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

```
[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b^
2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + (b^2 +
 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(a^2A + a^2Bx + a(2Ab + aC)x^2 + 2abBx^3 + \left(A\left(b^2 + 2ac\right) + 2abC\right)x^4 + B\left(b^2 + 2ac\right)x^5 + \left(2Abc + \left(b^2 + 2ac\right)C\right)x^6 + 2bBcx^7 + c(Ac + 2bC)x^8 + Bc^2x^9 + c^2Cx^{10}\right)dx$$

$$= a^2Ax + \frac{1}{2}a^2Bx^2 + \frac{1}{3}a(2Ab + aC)x^3 + \frac{1}{2}abBx^4 + \frac{1}{5}\left(A\left(b^2 + 2ac\right) + 2abC\right)x^5 + \frac{1}{6}B\left(b^2 + 2ac\right)x^6 + \frac{1}{7}\left(2Abc + \left(b^2 + 2ac\right)C\right)x^7 + \frac{1}{4}bBcx^8 + \frac{1}{9}c(Ac + 2bC)x^9 + \frac{1}{10}Bc^2x^{10} + \frac{1}{11}c^2Cx^{11}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int \left(A + Bx + Cx^2\right)\left(a + bx^2 + cx^4\right)^2 dx = a^2Ax + \frac{1}{2}a^2Bx^2 + \frac{1}{3}a(2Ab + aC)x^3 + \frac{1}{2}abBx^4 + \frac{1}{5}\left(Ab^2 + 2aAc + 2abC\right)x^5 + \frac{1}{6}B\left(b^2 + 2ac\right)x^6 + \frac{1}{7}\left(2Abc + b^2C + 2acC\right)x^7 + \frac{1}{4}bBcx^8 + \frac{1}{9}c(Ac + 2bC)x^9 + \frac{1}{10}Bc^2x^{10} + \frac{1}{11}c^2Cx^{11}$$

```
[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*b^2
 + 2*a*A*c + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + b^2*C
+ 2*a*c*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^2 C x^{11}}{11}+\frac{B c^2 x^{10}}{10}+\frac{(A c^2+2Cbc)x^9}{9}+\frac{bBc x^8}{4}+\frac{(2Abc+(2ac+b^2)C)x^7}{7}+\frac{B(2ac+b^2)x^6}{6}+\frac{(A(2ac+b^2)+2abC)x^5}{5}$
norman	$\frac{c^2 C x^{11}}{11}+\frac{B c^2 x^{10}}{10}+\left(\frac{1}{9}A c^2+\frac{2}{9}Cbc\right)x^9+\frac{bBc x^8}{4}+\left(\frac{2}{7}Abc+\frac{2}{7}acC+\frac{1}{7}b^2C\right)x^7+\left(\frac{1}{3}Bac+\frac{1}{6}B b^2\right.$
gosper	$\frac{1}{11}c^2C x^{11}+\frac{1}{10}B c^2x^{10}+\frac{1}{9}x^9A c^2+\frac{2}{9}x^9Cbc+\frac{1}{4}bBc x^8+\frac{2}{7}x^7Abc+\frac{2}{7}x^7acC+\frac{1}{7}x^7b^2C+\frac{1}{3}x^6$
risch	$\frac{1}{11}c^2C x^{11}+\frac{1}{10}B c^2x^{10}+\frac{1}{9}x^9A c^2+\frac{2}{9}x^9Cbc+\frac{1}{4}bBc x^8+\frac{2}{7}x^7Abc+\frac{2}{7}x^7acC+\frac{1}{7}x^7b^2C+\frac{1}{3}x^6$
parallelrisch	$\frac{1}{11}c^2C x^{11}+\frac{1}{10}B c^2x^{10}+\frac{1}{9}x^9A c^2+\frac{2}{9}x^9Cbc+\frac{1}{4}bBc x^8+\frac{2}{7}x^7Abc+\frac{2}{7}x^7acC+\frac{1}{7}x^7b^2C+\frac{1}{3}x^6$

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^2*C*x^11+1/10*B*c^2*x^10+1/9*(A*c^2+2*C*b*c)*x^9+1/4*b*B*c*x^8+1/7*(
2*A*b*c+(2*a*c+b^2)*C)*x^7+1/6*B*(2*a*c+b^2)*x^6+1/5*(A*(2*a*c+b^2)+2*a*b*C
)*x^5+1/2*B*a*b*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}Cc^2x^{11}+\frac{1}{10}Bc^2x^{10}+\frac{1}{4}Bbcx^8+\frac{1}{9}\left(2Cbc+Ac^2\right)x^9+\frac{1}{7}\left(Cb^2+2\left(Ca+Ab\right)c\right)x^7+\frac{1}{2}Babx^4+\frac{1}{6}\left(Bb^2+2Bac\right)x^6+\frac{1}{5}\left(2Cab+Ab^2+2Aac\right)x^5+\frac{1}{2}Ba^2x^2+Aa^2x+\frac{1}{3}\left(Ca^2+2Aab\right)x^3$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*x
^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B*a
*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x + 1
/3*(C*a^2 + 2*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2 dx = Aa^2x+\frac{Ba^2x^2}{2}+\frac{Babx^4}{2}+\frac{Bbcx^8}{4}+\frac{Bc^2x^{10}}{10}+\frac{Cc^2x^{11}}{11}+x^9\left(\frac{Ac^2}{9}+\frac{2Cbc}{9}\right)+x^7\cdot\left(\frac{2Abc}{7}+\frac{2Cac}{7}+\frac{Cb^2}{7}\right)+x^6\left(\frac{Bac}{3}+\frac{Bb^2}{6}\right)+x^5\cdot\left(\frac{2Aac}{5}+\frac{Ab^2}{5}+\frac{2Cab}{5}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x**10/10 +
C*c**2*x**11/11 + x**9*(A*c**2/9 + 2*C*b*c/9) + x**7*(2*A*b*c/7 + 2*C*a*c/7
 + C*b**2/7) + x**6*(B*a*c/3 + B*b**2/6) + x**5*(2*A*a*c/5 + A*b**2/5 + 2*C
*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2 dx = \frac{1}{11}Cc^2x^{11}+\frac{1}{10}Bc^2x^{10}+\frac{1}{4}Bbcx^8+\frac{1}{9}\left(2Cbc+Ac^2\right)x^9+\frac{1}{7}\left(Cb^2+2(Ca+Ab)c\right)x^7+\frac{1}{2}Babx^4+\frac{1}{6}\left(Bb^2+2Bac\right)x^6+\frac{1}{5}\left(2Cab+Ab^2+2Aac\right)x^5+\frac{1}{2}Ba^2x^2+Aa^2x+\frac{1}{3}\left(Ca^2+2Aab\right)x^3$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*x
^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B*a
*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x + 1
/3*(C*a^2 + 2*A*a*b)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,Cc^2x^{11}+\frac{1}{10}\,Bc^2x^{10}+\frac{2}{9}\,Cbcx^9+\frac{1}{9}\,Ac^2x^9+\frac{1}{4}\,Bbcx^8+\frac{1}{7}\,Cb^2x^7+\frac{2}{7}\,Cacx^7+\frac{2}{7}\,Abcx^7+\frac{1}{6}\,Bb^2x^6+\frac{1}{3}\,Bacx^6+\frac{2}{5}\,Cabx^5+\frac{1}{5}\,Ab^2x^5+\frac{2}{5}\,Aacx^5+\frac{1}{2}\,Babx^4+\frac{1}{3}\,Ca^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Ba^2x^2+Aa^2x$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 2/9*C*b*c*x^9 + 1/9*A*c^2*x^9 + 1/4*B*b
*c*x^8 + 1/7*C*b^2*x^7 + 2/7*C*a*c*x^7 + 2/7*A*b*c*x^7 + 1/6*B*b^2*x^6 + 1/
3*B*a*c*x^6 + 2/5*C*a*b*x^5 + 1/5*A*b^2*x^5 + 2/5*A*a*c*x^5 + 1/2*B*a*b*x^4
 + 1/3*C*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^9\left(\frac{A\,c^2}{9}+\frac{2\,C\,b\,c}{9}\right)+x^5\left(\frac{A\,b^2}{5}+\frac{2\,C\,a\,b}{5}+\frac{2\,A\,a\,c}{5}\right)+x^7\left(\frac{C\,b^2}{7}+\frac{2\,A\,c\,b}{7}+\frac{2\,C\,a\,c}{7}\right)+\frac{B\,a^2\,x^2}{2}+\frac{B\,c^2\,x^{10}}{10}+\frac{C\,c^2\,x^{11}}{11}+\frac{B\,x^6\,\left(b^2+2\,a\,c\right)}{6}+A\,a^2\,x+\frac{B\,a\,b\,x^4}{2}+\frac{B\,b\,c\,x^8}{4}$$

```
[In] int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3*((C*a^2)/3 + (2*A*a*b)/3) + x^9*((A*c^2)/9 + (2*C*b*c)/9) + x^5*((A*b^2
)/5 + (2*A*a*c)/5 + (2*C*a*b)/5) + x^7*((C*b^2)/7 + (2*A*b*c)/7 + (2*C*a*c)
/7) + (B*a^2*x^2)/2 + (B*c^2*x^10)/10 + (C*c^2*x^11)/11 + (B*x^6*(2*a*c + b
^2))/6 + A*a^2*x + (B*a*b*x^4)/2 + (B*b*c*x^8)/4
```

3.14 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} dx = {} & a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3 \\ & +\frac{1}{4}\left(A\left(b^2+2ac\right)+2abC\right)x^4+\frac{1}{5}B\left(b^2+2ac\right)x^5 \\ & +\frac{1}{6}\left(2Abc+\left(b^2+2ac\right)C\right)x^6 \\ & +\frac{2}{7}bBcx^7+\frac{1}{8}c(Ac+2bC)x^8 \\ & +\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}+a^2A\log(x)\end{aligned}$$

```
[Out] a^2*B*x+1/2*a*(2*A*b+C*a)*x^2+2/3*a*b*B*x^3+1/4*(A*(2*a*c+b^2)+2*a*b*C)*x^4
+1/5*B*(2*a*c+b^2)*x^5+1/6*(2*A*b*c+(2*a*c+b^2)*C)*x^6+2/7*b*B*c*x^7+1/8*c*
(A*c+2*C*b)*x^8+1/9*B*c^2*x^9+1/10*c^2*C*x^10+a^2*A*ln(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1642}

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} dx = {} & a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right) \\ & +\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab) \\ & +\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}abBx^3+\frac{1}{8}cx^8(Ac+2bC) \\ & +\frac{2}{7}bBcx^7+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}\end{aligned}$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 + 2*a*c) + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
10)/10 + a^2*A*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2B+\frac{a^2A}{x}+a(2Ab+aC)x+2abBx^2+\left(A\left(b^2+2ac\right)+2abC\right)x^3+B\left(b^2+2ac\right)x^4\right.\\&\quad\left.+\left(2Abc+\left(b^2+2ac\right)C\right)x^5+2bBcx^6+c(Ac+2bC)x^7+Bc^2x^8+c^2Cx^9\right)dx\\&= a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3+\frac{1}{4}\left(A\left(b^2+2ac\right)+2abC\right)x^4+\frac{1}{5}B\left(b^2+2ac\right)x^5\\&\quad+\frac{1}{6}\left(2Abc+\left(b^2+2ac\right)C\right)x^6+\frac{2}{7}bBcx^7+\frac{1}{8}c(Ac+2bC)x^8+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}\\&\quad+a^2A\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x}\,dx &= a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3\\&\quad+\frac{1}{4}\left(Ab^2+2aAc+2abC\right)x^4\\&\quad+\frac{1}{5}B\left(b^2+2ac\right)x^5+\frac{1}{6}\left(2Abc+b^2C+2acC\right)x^6\\&\quad+\frac{2}{7}bBcx^7+\frac{1}{8}c(Ac+2bC)x^8\\&\quad+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}+a^2A\log(x)\end{aligned}$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*b^2 + 2*a*A*c + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
10)/10 + a^2*A*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

method	result
norman	$\left(\frac{1}{8}A\,c^{2}+\frac{1}{4}Cbc\right)x^{8}+\left(Aab+\frac{1}{2}C\,a^{2}\right)x^{2}+\left(\frac{2}{5}Bac+\frac{1}{5}B\,b^{2}\right)x^{5}+\left(\frac{1}{2}Aac+\frac{1}{4}A\,b^{2}+\frac{1}{2}abC\right)x^{4}$
default	$\frac{c^{2}C\,x^{10}}{10}+\frac{B\,c^{2}x^{9}}{9}+\frac{A\,c^{2}x^{8}}{8}+\frac{Cbc\,x^{8}}{4}+\frac{2bBc\,x^{7}}{7}+\frac{Abc\,x^{6}}{3}+\frac{Cac\,x^{6}}{3}+\frac{C\,b^{2}x^{6}}{6}+\frac{2Bac\,x^{5}}{5}+\frac{B\,b^{2}x^{5}}{5}+\frac{Aac\,x^{4}}{2}$
risch	$\frac{c^{2}C\,x^{10}}{10}+\frac{B\,c^{2}x^{9}}{9}+\frac{A\,c^{2}x^{8}}{8}+\frac{Cbc\,x^{8}}{4}+\frac{2bBc\,x^{7}}{7}+\frac{Abc\,x^{6}}{3}+\frac{Cac\,x^{6}}{3}+\frac{C\,b^{2}x^{6}}{6}+\frac{2Bac\,x^{5}}{5}+\frac{B\,b^{2}x^{5}}{5}+\frac{Aac\,x^{4}}{2}$
parallelrisch	$\frac{c^{2}C\,x^{10}}{10}+\frac{B\,c^{2}x^{9}}{9}+\frac{A\,c^{2}x^{8}}{8}+\frac{Cbc\,x^{8}}{4}+\frac{2bBc\,x^{7}}{7}+\frac{Abc\,x^{6}}{3}+\frac{Cac\,x^{6}}{3}+\frac{C\,b^{2}x^{6}}{6}+\frac{2Bac\,x^{5}}{5}+\frac{B\,b^{2}x^{5}}{5}+\frac{Aac\,x^{4}}{2}$

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*A*c^2+1/4*C*b*c)*x^8+(A*a*b+1/2*C*a^2)*x^2+(2/5*B*a*c+1/5*B*b^2)*x^5+(
1/2*A*a*c+1/4*A*b^2+1/2*a*b*C)*x^4+(1/3*A*b*c+1/3*a*c*C+1/6*b^2*C)*x^6+B*a^
2*x+1/9*B*c^2*x^9+1/10*c^2*C*x^10+2/3*B*a*b*x^3+2/7*b*B*c*x^7+a^2*A*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x}\,dx &= \frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7\\&+\frac{1}{8}\,(2\,Cbc+Ac^2)x^8+\frac{1}{6}\,(Cb^2+2\,(Ca+Ab)c)x^6\\&+\frac{2}{3}\,Babx^3+\frac{1}{5}\,(Bb^2+2\,Bac)x^5\\&+\frac{1}{4}\,(2\,Cab+Ab^2+2\,Aac)x^4+Ba^2x\\&+Aa^2\log(x)+\frac{1}{2}\,(Ca^2+2\,Aab)x^2\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8
 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c
)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*
(C*a^2 + 2*A*a*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = Aa^2\log(x)+Ba^2x+\frac{2Babx^3}{3}+\frac{2Bbcx^7}{7}+\frac{Bc^2x^9}{9}+\frac{Cc^2x^{10}}{10}+x^8\left(\frac{Ac^2}{8}+\frac{Cbc}{4}\right)+x^6\left(\frac{Abc}{3}+\frac{Cac}{3}+\frac{Cb^2}{6}\right)+x^5\cdot\left(\frac{2Bac}{5}+\frac{Bb^2}{5}\right)+x^4\left(\frac{Aac}{2}+\frac{Ab^2}{4}+\frac{Cab}{2}\right)+x^2\left(Aab+\frac{Ca^2}{2}\right)$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c**2*x**9/9
+ C*c**2*x**10/10 + x**8*(A*c**2/8 + C*b*c/4) + x**6*(A*b*c/3 + C*a*c/3 + C
*b**2/6) + x**5*(2*B*a*c/5 + B*b**2/5) + x**4*(A*a*c/2 + A*b**2/4 + C*a*b/2
) + x**2*(A*a*b + C*a**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7+\frac{1}{8}\,(2\,Cbc+Ac^2)x^8+\frac{1}{6}\,(Cb^2+2\,(Ca+Ab)c)x^6+\frac{2}{3}\,Babx^3+\frac{1}{5}\,(Bb^2+2\,Bac)x^5+\frac{1}{4}\,(2\,Cab+Ab^2+2\,Aac)x^4+Ba^2x+Aa^2\log(x)+\frac{1}{2}\,(Ca^2+2\,Aab)x^2$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8
 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c
)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*
(C*a^2 + 2*A*a*b)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}Cc^2x^{10}+\frac{1}{9}Bc^2x^9+\frac{1}{4}Cbcx^8+\frac{1}{8}Ac^2x^8+\frac{2}{7}Bbcx^7+\frac{1}{6}Cb^2x^6+\frac{1}{3}Cacx^6+\frac{1}{3}Abcx^6+\frac{1}{5}Bb^2x^5+\frac{2}{5}Bacx^5+\frac{1}{2}Cabx^4+\frac{1}{4}Ab^2x^4+\frac{1}{2}Aacx^4+\frac{2}{3}Babx^3+\frac{1}{2}Ca^2x^2+Aabx^2+Ba^2x+Aa^2\log(|x|)$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 1/4*C*b*c*x^8 + 1/8*A*c^2*x^8 + 2/7*B*b*c
*x^7 + 1/6*C*b^2*x^6 + 1/3*C*a*c*x^6 + 1/3*A*b*c*x^6 + 1/5*B*b^2*x^5 + 2/5*
B*a*c*x^5 + 1/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*A*a*c*x^4 + 2/3*B*a*b*x^3 +
 1/2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*x + A*a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = x^2\left(\frac{C\,a^2}{2}+A\,b\,a\right)+x^8\left(\frac{A\,c^2}{8}+\frac{C\,b\,c}{4}\right)+x^4\left(\frac{A\,b^2}{4}+\frac{C\,a\,b}{2}+\frac{A\,a\,c}{2}\right)+x^6\left(\frac{C\,b^2}{6}+\frac{A\,c\,b}{3}+\frac{C\,a\,c}{3}\right)+\frac{B\,c^2\,x^9}{9}+\frac{C\,c^2\,x^{10}}{10}+A\,a^2\,\ln(x)+\frac{B\,x^5\,(b^2+2\,a\,c)}{5}+B\,a^2\,x+\frac{2\,B\,a\,b\,x^3}{3}+\frac{2\,B\,b\,c\,x^7}{7}$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x)
```

```
[Out] x^2*((C*a^2)/2 + A*a*b) + x^8*((A*c^2)/8 + (C*b*c)/4) + x^4*((A*b^2)/4 + (A
*a*c)/2 + (C*a*b)/2) + x^6*((C*b^2)/6 + (A*b*c)/3 + (C*a*c)/3) + (B*c^2*x^9
)/9 + (C*c^2*x^10)/10 + A*a^2*log(x) + (B*x^5*(2*a*c + b^2))/5 + B*a^2*x +
(2*B*a*b*x^3)/3 + (2*B*b*c*x^7)/7
```

3.15 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2} \, dx = -\frac{a^2A}{x} + a(2Ab+aC)x + abBx^2 + \frac{1}{3}\left(A\left(b^2+2ac\right)+2abC\right)x^3 + \frac{1}{4}B\left(b^2+2ac\right)x^4 + \frac{1}{5}\left(2Abc+\left(b^2+2ac\right)C\right)x^5 + \frac{1}{3}bBcx^6 + \frac{1}{7}c(Ac+2bC)x^7 + \frac{1}{8}Bc^2x^8 + \frac{1}{9}c^2Cx^9 + a^2B\log(x)$$

```
[Out] -a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*(A*(2*a*c+b^2)+2*a*b*C)*x^3+1/4*B*(2
*a*c+b^2)*x^4+1/5*(2*A*b*c+(2*a*c+b^2)*C)*x^5+1/3*b*B*c*x^6+1/7*c*(A*c+2*C*
b)*x^7+1/8*B*c^2*x^8+1/9*c^2*C*x^9+a^2*B*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2} \, dx = -\frac{a^2A}{x} + a^2B\log(x) + \frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right) + \frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right) + ax(aC+2Ab) + \frac{1}{4}Bx^4\left(2ac+b^2\right) + abBx^2 + \frac{1}{7}cx^7(Ac+2bC) + \frac{1}{3}bBcx^6 + \frac{1}{8}Bc^2x^8 + \frac{1}{9}c^2Cx^9$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c) + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^5)/5 + (
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(a(2Ab+aC)+\frac{a^2A}{x^2}+\frac{a^2B}{x}+2abBx+\left(A\left(b^2+2ac\right)+2abC\right)x^2+B\left(b^2+2ac\right)x^3 + \left(2Abc+\left(b^2+2ac\right)C\right)x^4+2bBcx^5+c(Ac+2bC)x^6+Bc^2x^7+c^2Cx^8\right)dx$$

$$= -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+2abC\right)x^3+\frac{1}{4}B\left(b^2+2ac\right)x^4+\frac{1}{5}\left(2Abc + \left(b^2+2ac\right)C\right)x^5+\frac{1}{3}bBcx^6+\frac{1}{7}c(Ac+2bC)x^7+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9+a^2B\log(x)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2 + \frac{1}{3}\left(Ab^2+2aAc+2abC\right)x^3+\frac{1}{4}B\left(b^2+2ac\right)x^4 + \frac{1}{5}\left(2Abc+b^2C+2acC\right)x^5 + \frac{1}{3}bBcx^6+\frac{1}{7}c(Ac+2bC)x^7 + \frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9+a^2B\log(x)$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*b^2 + 2*a*A*c + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^5)/5 + (
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\left(\frac{1}{7}A\,c^2+\frac{2}{7}Cbc\right)x^8+\left(\frac{1}{2}Bac+\frac{1}{4}B\,b^2\right)x^5+\left(\frac{2}{3}Aac+\frac{1}{3}A\,b^2+\frac{2}{3}abC\right)x^4+\left(\frac{2}{5}Abc+\frac{2}{5}acC+\frac{1}{5}b^2C\right)x^6+\left(2Aab+C\,a^2\right)x^2+Bab\,x^3-A\,a^2+}{x}$
default	$\frac{c^2C\,x^9}{9}+\frac{B\,c^2x^8}{8}+\frac{A\,c^2x^7}{7}+\frac{2Cbc\,x^7}{7}+\frac{bBc\,x^6}{3}+\frac{2Abc\,x^5}{5}+\frac{2Cac\,x^5}{5}+\frac{C\,b^2x^5}{5}+\frac{Bac\,x^4}{2}+\frac{B\,b^2x^4}{4}+\frac{2Aac\,x^3}{3}$
risch	$\frac{c^2C\,x^9}{9}+\frac{B\,c^2x^8}{8}+\frac{A\,c^2x^7}{7}+\frac{2Cbc\,x^7}{7}+\frac{bBc\,x^6}{3}+\frac{2Abc\,x^5}{5}+\frac{2Cac\,x^5}{5}+\frac{C\,b^2x^5}{5}+\frac{Bac\,x^4}{2}+\frac{B\,b^2x^4}{4}+\frac{2Aac\,x^3}{3}$
parallelrisch	$\frac{280c^2C\,x^{10}+315B\,c^2x^9+360A\,c^2x^8+720Cbc\,x^8+840bBc\,x^7+1008Abc\,x^6+1008Cac\,x^6+504C\,b^2x^6+1260Bac\,x^5+630B\,b^2x^5+1}{2520x}$

[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)

[Out] ((1/7*A*c^2+2/7*C*b*c)*x^8+(1/2*B*a*c+1/4*B*b^2)*x^5+(2/3*A*a*c+1/3*A*b^2+2/3*a*b*C)*x^4+(2/5*A*b*c+2/5*a*c*C+1/5*b^2*C)*x^6+(2*A*a*b+C*a^2)*x^2+B*a*b*x^3-A*a^2+1/8*B*c^2*x^9+1/9*c^2*C*x^10+1/3*b*B*c*x^7)/x+a^2*B*ln(x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{280\,Cc^2x^{10}+315\,Bc^2x^9+840\,Bbcx^7+360\,(2\,Cbc+Ac^2)x^8+504\,(Cb^2+2\,(Ca+Ab)c)x^6+2520\,Babx^3}{2}$$

[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")

[Out] 1/2520*(280*C*c^2*x^10 + 315*B*c^2*x^9 + 840*B*b*c*x^7 + 360*(2*C*b*c + A*c^2)*x^8 + 504*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2520*B*a*b*x^3 + 630*(B*b^2 + 2*B*a*c)*x^5 + 840*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 2520*B*a^2*x*log(x) - 2520*A*a^2 + 2520*(C*a^2 + 2*A*a*b)*x^2)/x

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = -\frac{Aa^2}{x} + Ba^2\log(x) + Babx^2 + \frac{Bbcx^6}{3} + \frac{Bc^2x^8}{8} + \frac{Cc^2x^9}{9} + x^7\left(\frac{Ac^2}{7} + \frac{2Cbc}{7}\right) + x^5 \cdot \left(\frac{2Abc}{5} + \frac{2Cac}{5} + \frac{Cb^2}{5}\right) + x^4\left(\frac{Bac}{2} + \frac{Bb^2}{4}\right) + x^3 \cdot \left(\frac{2Aac}{3} + \frac{Ab^2}{3} + \frac{2Cab}{3}\right) + x(2Aab + Ca^2)$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + B*a*b*x**2 + B*b*c*x**6/3 + B*c**2*x**8/8 + C*c
**2*x**9/9 + x**7*(A*c**2/7 + 2*C*b*c/7) + x**5*(2*A*b*c/5 + 2*C*a*c/5 + C*
b**2/5) + x**4*(B*a*c/2 + B*b**2/4) + x**3*(2*A*a*c/3 + A*b**2/3 + 2*C*a*b/
3) + x*(2*A*a*b + C*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}Cc^2x^9 + \frac{1}{8}Bc^2x^8 + \frac{1}{3}Bbcx^6 + \frac{1}{7}(2\,Cbc + Ac^2)x^7 + \frac{1}{5}(Cb^2 + 2\,(Ca + Ab)c)x^5 + Babx^2 + \frac{1}{4}(Bb^2 + 2\,Bac)x^4 + \frac{1}{3}(2\,Cab + Ab^2 + 2\,Aac)x^3 + Ba^2\log(x) - \frac{Aa^2}{x} + (Ca^2 + 2\,Aab)x$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/7*(2*C*b*c + A*c^2)*x^7 +
 1/5*(C*b^2 + 2*(C*a + A*b)*c)*x^5 + B*a*b*x^2 + 1/4*(B*b^2 + 2*B*a*c)*x^4
+ 1/3*(2*C*a*b + A*b^2 + 2*A*a*c)*x^3 + B*a^2*log(x) - A*a^2/x + (C*a^2 + 2
*A*a*b)*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^2}\,dx = \frac{1}{9}\,Cc^2x^9+\frac{1}{8}\,Bc^2x^8+\frac{2}{7}\,Cbcx^7+\frac{1}{7}\,Ac^2x^7+\frac{1}{3}\,Bbcx^6+\frac{1}{5}\,Cb^2x^5+\frac{2}{5}\,Cacx^5+\frac{2}{5}\,Abcx^5+\frac{1}{4}\,Bb^2x^4+\frac{1}{2}\,Bacx^4+\frac{2}{3}\,Cabx^3+\frac{1}{3}\,Ab^2x^3+\frac{2}{3}\,Aacx^3+Babx^2+Ca^2x+2\,Aabx+Ba^2\log\left(|x|\right)-\frac{Aa^2}{x}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 2/7*C*b*c*x^7 + 1/7*A*c^2*x^7 + 1/3*B*b*c*x
^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 2/5*A*b*c*x^5 + 1/4*B*b^2*x^4 + 1/2*B*
a*c*x^4 + 2/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3*A*a*c*x^3 + B*a*b*x^2 + C*a^2
*x + 2*A*a*b*x + B*a^2*log(abs(x)) - A*a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^2}\,dx = x^7\left(\frac{A\,c^2}{7}+\frac{2\,C\,b\,c}{7}\right)+x^3\left(\frac{A\,b^2}{3}+\frac{2\,C\,a\,b}{3}+\frac{2\,A\,a\,c}{3}\right)+x^5\left(\frac{C\,b^2}{5}+\frac{2\,A\,c\,b}{5}+\frac{2\,C\,a\,c}{5}\right)+x\left(C\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{x}+\frac{B\,c^2\,x^8}{8}+\frac{C\,c^2\,x^9}{9}+B\,a^2\,\ln\left(x\right)+\frac{B\,x^4\,\left(b^2+2\,a\,c\right)}{4}+B\,a\,b\,x^2+\frac{B\,b\,c\,x^6}{3}$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x)
```

```
[Out] x^7*((A*c^2)/7 + (2*C*b*c)/7) + x^3*((A*b^2)/3 + (2*A*a*c)/3 + (2*C*a*b)/3)
 + x^5*((C*b^2)/5 + (2*A*b*c)/5 + (2*C*a*c)/5) + x*(C*a^2 + 2*A*a*b) - (A*a
^2)/x + (B*c^2*x^8)/8 + (C*c^2*x^9)/9 + B*a^2*log(x) + (B*x^4*(2*a*c + b^2)
)/4 + B*a*b*x^2 + (B*b*c*x^6)/3
```

3.16 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3} dx = -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+2abBx+\frac{1}{2}\left(A\left(b^2+2ac\right)+2abC\right)x^2 +\frac{1}{3}B\left(b^2+2ac\right)x^3+\frac{1}{4}\left(2Abc+\left(b^2+2ac\right)C\right)x^4 +\frac{2}{5}bBcx^5+\frac{1}{6}c(Ac+2bC)x^6+\frac{1}{7}Bc^2x^7 +\frac{1}{8}c^2Cx^8+a(2Ab+aC)\log(x)$$

```
[Out] -1/2*a^2*A/x^2-a^2*B/x+2*a*b*B*x+1/2*(A*(2*a*c+b^2)+2*a*b*C)*x^2+1/3*B*(2*a
*c+b^2)*x^3+1/4*(2*A*b*c+(2*a*c+b^2)*C)*x^4+2/5*b*B*c*x^5+1/6*c*(A*c+2*C*b)
*x^6+1/7*B*c^2*x^7+1/8*c^2*C*x^8+a*(2*A*b+C*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3} dx = -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right) +\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab) +\frac{1}{3}Bx^3\left(2ac+b^2\right)+2abBx+\frac{1}{6}cx^6(Ac+2bC) +\frac{2}{5}bBcx^5+\frac{1}{7}Bc^2x^7+\frac{1}{8}c^2Cx^8$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^2
)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^4)/4 + (2*b*
B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2*x^7)/7 + (c^2*C*x^8)/8 + a*(2
*A*b + a*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(2abB + \frac{a^2A}{x^3} + \frac{a^2B}{x^2} + \frac{a(2Ab+aC)}{x} + \left(A\left(b^2+2ac\right)+2abC\right)x + B\left(b^2+2ac\right)x^2 \right. \\ &\quad \left. + \left(2Abc + \left(b^2+2ac\right)C\right)x^3 + 2bBcx^4 + c(Ac+2bC)x^5 + Bc^2x^6 + c^2Cx^7\right) dx \\ &= -\frac{a^2A}{2x^2} - \frac{a^2B}{x} + 2abBx + \frac{1}{2}\left(A\left(b^2+2ac\right)+2abC\right)x^2 \\ &\quad + \frac{1}{3}B\left(b^2+2ac\right)x^3 + \frac{1}{4}\left(2Abc+\left(b^2+2ac\right)C\right)x^4 + \frac{2}{5}bBcx^5 \\ &\quad + \frac{1}{6}c(Ac+2bC)x^6 + \frac{1}{7}Bc^2x^7 + \frac{1}{8}c^2Cx^8 + a(2Ab+aC)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx &= -\frac{a^2(A+2Bx)}{2x^2} \\ &\quad + \frac{1}{6}ax\left(6b(2B+Cx)+cx\left(6A+4Bx+3Cx^2\right)\right) \\ &\quad + \frac{1}{840}x^2\left(70b^2x(4B+3Cx)+56bcx^3(6B+5Cx)\right. \\ &\quad \left. +15c^2x^5(8B+7Cx)+140A\left(3b^2+3bcx^2+c^2x^4\right)\right) \\ &\quad + a(2Ab+aC)\log(x)\end{aligned}$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*(A + 2*B*x))/x^2 + (a*x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C
*x^2)))/6 + (x^2*(70*b^2*x*(4*B + 3*C*x) + 56*b*c*x^3*(6*B + 5*C*x) + 15*c^
2*x^5*(8*B + 7*C*x) + 140*A*(3*b^2 + 3*b*c*x^2 + c^2*x^4)))/840 + a*(2*A*b
+ a*C)*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

method	result
norman	$\frac{\left(\frac{1}{6}A\,c^{2}+\frac{1}{3}Cbc\right)x^{8}+\left(\frac{2}{3}Bac+\frac{1}{3}B\,b^{2}\right)x^{5}+\left(Aac+\frac{1}{2}A\,b^{2}+abC\right)x^{4}+\left(\frac{1}{2}Abc+\frac{1}{2}acC+\frac{1}{4}b^{2}C\right)x^{6}-\frac{A\,a^{2}}{2}-B\,a^{2}x+\frac{B\,c^{2}x^{9}}{7}+\frac{c^{2}C\,x^{10}}{8}+2}{x^{2}}$
default	$\frac{c^{2}C\,x^{8}}{8}+\frac{B\,c^{2}x^{7}}{7}+\frac{A\,c^{2}x^{6}}{6}+\frac{Cbc\,x^{6}}{3}+\frac{2bBc\,x^{5}}{5}+\frac{Abc\,x^{4}}{2}+\frac{Cac\,x^{4}}{2}+\frac{C\,b^{2}x^{4}}{4}+\frac{2Bac\,x^{3}}{3}+\frac{B\,b^{2}x^{3}}{3}+Aac\,x$
risch	$\frac{c^{2}C\,x^{8}}{8}+\frac{B\,c^{2}x^{7}}{7}+\frac{A\,c^{2}x^{6}}{6}+\frac{Cbc\,x^{6}}{3}+\frac{2bBc\,x^{5}}{5}+\frac{Abc\,x^{4}}{2}+\frac{Cac\,x^{4}}{2}+\frac{C\,b^{2}x^{4}}{4}+\frac{2Bac\,x^{3}}{3}+\frac{B\,b^{2}x^{3}}{3}+Aac\,x$
parallelrisch	$\frac{105c^{2}C\,x^{10}+120B\,c^{2}x^{9}+140A\,c^{2}x^{8}+280Cbc\,x^{8}+336bBc\,x^{7}+420Abc\,x^{6}+420Cac\,x^{6}+210C\,b^{2}x^{6}+560Bac\,x^{5}+280B\,b^{2}x^{5}+84}{840x^{2}}$

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((1/6*A*c^2+1/3*C*b*c)*x^8+(2/3*B*a*c+1/3*B*b^2)*x^5+(A*a*c+1/2*A*b^2+a*b*C
)*x^4+(1/2*A*b*c+1/2*a*c*C+1/4*b^2*C)*x^6-1/2*A*a^2-B*a^2*x+1/7*B*c^2*x^9+1
/8*c^2*C*x^10+2*B*a*b*x^3+2/5*b*B*c*x^7)/x^2+(2*A*a*b+C*a^2)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^3}\,dx$$
$$=\frac{105\,Cc^2x^{10}+120\,Bc^2x^9+336\,Bbcx^7+140\,(2\,Cbc+Ac^2)x^8+210\,(Cb^2+2\,(Ca+Ab)c)x^6+1680\,Babx^3}{8}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^2*x^10 + 120*B*c^2*x^9 + 336*B*b*c*x^7 + 140*(2*C*b*c + A*c^
2)*x^8 + 210*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 1680*B*a*b*x^3 + 280*(B*b^2 +
2*B*a*c)*x^5 + 420*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 840*B*a^2*x + 840*(C*a
^2 + 2*A*a*b)*x^2*log(x) - 420*A*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^3}\,dx = 2Babx+\frac{2Bbcx^5}{5}+\frac{Bc^2x^7}{7}+\frac{Cc^2x^8}{8} + a(2Ab+Ca)\log(x)+x^6\left(\frac{Ac^2}{6}+\frac{Cbc}{3}\right) + x^4\left(\frac{Abc}{2}+\frac{Cac}{2}+\frac{Cb^2}{4}\right)+x^3\cdot\left(\frac{2Bac}{3}+\frac{Bb^2}{3}\right) + x^2\left(Aac+\frac{Ab^2}{2}+Cab\right)+\frac{-Aa^2-2Ba^2x}{2x^2}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] 2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2*x**7/7 + C*c**2*x**8/8 + a*(2*A*b + C*a
)*log(x) + x**6*(A*c**2/6 + C*b*c/3) + x**4*(A*b*c/2 + C*a*c/2 + C*b**2/4)
+ x**3*(2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c + A*b**2/2 + C*a*b) + (-A*a**2
- 2*B*a**2*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{8}\,Cc^2x^8+\frac{1}{7}\,Bc^2x^7+\frac{2}{5}\,Bbcx^5 + \frac{1}{6}\,(2\,Cbc+Ac^2)x^6+\frac{1}{4}\,(Cb^2+2\,(Ca+Ab)c)x^4 + 2\,Babx+\frac{1}{3}\,(Bb^2+2\,Bac)x^3 + \frac{1}{2}\,(2\,Cab+Ab^2+2\,Aac)x^2 + (Ca^2+2\,Aab)\log(x)-\frac{2\,Ba^2x+Aa^2}{2\,x^2}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/6*(2*C*b*c + A*c^2)*x^6 +
 1/4*(C*b^2 + 2*(C*a + A*b)*c)*x^4 + 2*B*a*b*x + 1/3*(B*b^2 + 2*B*a*c)*x^3
+ 1/2*(2*C*a*b + A*b^2 + 2*A*a*c)*x^2 + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B
*a^2*x + A*a^2)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{8}\,Cc^2x^8 + \frac{1}{7}\,Bc^2x^7 + \frac{1}{3}\,Cbcx^6 + \frac{1}{6}\,Ac^2x^6 + \frac{2}{5}\,Bbcx^5 + \frac{1}{4}\,Cb^2x^4 + \frac{1}{2}\,Cacx^4 + \frac{1}{2}\,Abcx^4 + \frac{1}{3}\,Bb^2x^3 + \frac{2}{3}\,Bacx^3 + Cabx^2 + \frac{1}{2}\,Ab^2x^2 + Aacx^2 + 2\,Babx + \left(Ca^2 + 2\,Aab\right)\log\left(|x|\right) - \frac{2\,Ba^2x + Aa^2}{2\,x^2}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 1/3*C*b*c*x^6 + 1/6*A*c^2*x^6 + 2/5*B*b*c*x
^5 + 1/4*C*b^2*x^4 + 1/2*C*a*c*x^4 + 1/2*A*b*c*x^4 + 1/3*B*b^2*x^3 + 2/3*B*
a*c*x^3 + C*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x^2 + 2*B*a*b*x + (C*a^2 + 2*A*
a*b)*log(abs(x)) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^3}\,dx = x^6\left(\frac{A\,c^2}{6} + \frac{C\,b\,c}{3}\right) + \ln\left(x\right)\,\left(C\,a^2 + 2\,A\,b\,a\right) + x^2\left(\frac{A\,b^2}{2} + C\,a\,b + A\,a\,c\right) + x^4\left(\frac{C\,b^2}{4} + \frac{A\,c\,b}{2} + \frac{C\,a\,c}{2}\right) - \frac{\frac{A\,a^2}{2} + B\,a^2\,x}{x^2} + \frac{B\,c^2\,x^7}{7} + \frac{C\,c^2\,x^8}{8} + \frac{B\,x^3\,(b^2 + 2\,a\,c)}{3} + \frac{2\,B\,b\,c\,x^5}{5} + 2\,B\,a\,b\,x$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x)
```

```
[Out] x^6*((A*c^2)/6 + (C*b*c)/3) + log(x)*(C*a^2 + 2*A*a*b) + x^2*((A*b^2)/2 + A
*a*c + C*a*b) + x^4*((C*b^2)/4 + (A*b*c)/2 + (C*a*c)/2) - ((A*a^2)/2 + B*a^
2*x)/x^2 + (B*c^2*x^7)/7 + (C*c^2*x^8)/8 + (B*x^3*(2*a*c + b^2))/3 + (2*B*b
*c*x^5)/5 + 2*B*a*b*x
```

3.17 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4} \, dx = -\frac{a^2A}{3x^3} - \frac{a^2B}{2x^2} - \frac{a(2Ab+aC)}{x} + \left(A(b^2+2ac)+2abC\right)x + \frac{1}{2}B(b^2+2ac)x^2 + \frac{1}{3}\left(2Abc+(b^2+2ac)C\right)x^3 + \frac{1}{2}bBcx^4 + \frac{1}{5}c(Ac+2bC)x^5 + \frac{1}{6}Bc^2x^6 + \frac{1}{7}c^2Cx^7 + 2abB\log(x)$$

```
[Out] -1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+(A*(2*a*c+b^2)+2*a*b*C)*x+1/2*
B*(2*a*c+b^2)*x^2+1/3*(2*A*b*c+(2*a*c+b^2)*C)*x^3+1/2*b*B*c*x^4+1/5*c*(A*c+
2*C*b)*x^5+1/6*B*c^2*x^6+1/7*c^2*C*x^7+2*a*b*B*ln(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4} \, dx = -\frac{a^2A}{3x^3} - \frac{a^2B}{2x^2} + \frac{1}{3}x^3\left(C(2ac+b^2)+2Abc\right) + x\left(A(2ac+b^2)+2abC\right) - \frac{a(aC+2Ab)}{x} + \frac{1}{2}Bx^2(2ac+b^2) + 2abB\log(x) + \frac{1}{5}cx^5(Ac+2bC) + \frac{1}{2}bBcx^4 + \frac{1}{6}Bc^2x^6 + \frac{1}{7}c^2Cx^7$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^2 + 2*a*c)
 + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^3)
/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)/
7 + 2*a*b*B*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(Ab^2\left(1+\frac{2a(Ac+bC)}{Ab^2}\right)+\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(2Ab+aC)}{x^2}+\frac{2abB}{x}+B\left(b^2+2ac\right)x \right.\\ &\qquad \left. +\left(2Abc+\left(b^2+2ac\right)C\right)x^2+2bBcx^3+c(Ac+2bC)x^4+Bc^2x^5+c^2Cx^6\right)dx\\ &= -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+\left(A\left(b^2+2ac\right)+2abC\right)x+\frac{1}{2}B\left(b^2+2ac\right)x^2+\frac{1}{3}\left(2Abc\right.\\ &\qquad \left.+\left(b^2+2ac\right)C\right)x^3+\frac{1}{2}bBcx^4+\frac{1}{5}c(Ac+2bC)x^5+\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7+2abB\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^4}\,dx &= -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{-2aAb-a^2C}{x}\\ &\quad +\left(Ab^2+2aAc+2abC\right)x+\frac{1}{2}B\left(b^2+2ac\right)x^2\\ &\quad +\frac{1}{3}\left(2Abc+b^2C+2acC\right)x^3\\ &\quad +\frac{1}{2}bBcx^4+\frac{1}{5}c(Ac+2bC)x^5\\ &\quad +\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7+2abB\log(x)\end{aligned}$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) + (-2*a*A*b - a^2*C)/x + (A*b^2 + 2*a*A*
c + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^3
)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)
/7 + 2*a*b*B*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\left(\frac{1}{5}A\,c^{2}+\frac{2}{5}Cbc\right)x^{8}+\left(Bac+\frac{1}{2}B\,b^{2}\right)x^{5}+\left(\frac{2}{3}Abc+\frac{2}{3}acC+\frac{1}{3}b^{2}C\right)x^{6}+\left(-2Aab-C\,a^{2}\right)x^{2}+\left(2Aac+A\,b^{2}+2abC\right)x^{4}-\frac{A\,a^{2}}{3}-\frac{B\,a^{2}x}{2}+\frac{B}{}}{x^{3}}$
default	$\frac{c^{2}C\,x^{7}}{7}+\frac{B\,c^{2}x^{6}}{6}+\frac{A\,c^{2}x^{5}}{5}+\frac{2Cbc\,x^{5}}{5}+\frac{bBc\,x^{4}}{2}+\frac{2Abc\,x^{3}}{3}+\frac{2Cac\,x^{3}}{3}+\frac{C\,b^{2}x^{3}}{3}+Bac\,x^{2}+\frac{B\,b^{2}x^{2}}{2}+2Aac$
risch	$\frac{c^{2}C\,x^{7}}{7}+\frac{B\,c^{2}x^{6}}{6}+\frac{A\,c^{2}x^{5}}{5}+\frac{2Cbc\,x^{5}}{5}+\frac{bBc\,x^{4}}{2}+\frac{2Abc\,x^{3}}{3}+\frac{2Cac\,x^{3}}{3}+\frac{C\,b^{2}x^{3}}{3}+Bac\,x^{2}+\frac{B\,b^{2}x^{2}}{2}+2Aac$
parallelrisch	$\frac{30c^{2}C\,x^{10}+35B\,c^{2}x^{9}+42A\,c^{2}x^{8}+84Cbc\,x^{8}+105bBc\,x^{7}+140Abc\,x^{6}+140Cac\,x^{6}+70C\,b^{2}x^{6}+210Bac\,x^{5}+105B\,b^{2}x^{5}+420Aac\,x^{4}}{210x^{3}}$

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] ((1/5*A*c^2+2/5*C*b*c)*x^8+(B*a*c+1/2*B*b^2)*x^5+(2/3*A*b*c+2/3*a*c*C+1/3*b
^2*C)*x^6+(-2*A*a*b-C*a^2)*x^2+(2*A*a*c+A*b^2+2*C*a*b)*x^4-1/3*A*a^2-1/2*B*
a^2*x+1/6*B*c^2*x^9+1/7*c^2*C*x^10+1/2*b*B*c*x^7)/x^3+2*a*b*B*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^4}\,dx$$
$$=\frac{30\,Cc^2x^{10}+35\,Bc^2x^9+105\,Bbcx^7+42\,(2\,Cbc+Ac^2)x^8+70\,(Cb^2+2\,(Ca+Ab)c)x^6+420\,Babx^3\log{(x)}}{210\,x^3}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/210*(30*C*c^2*x^10 + 35*B*c^2*x^9 + 105*B*b*c*x^7 + 42*(2*C*b*c + A*c^2)*
x^8 + 70*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 420*B*a*b*x^3*log(x) + 105*(B*b^2
+ 2*B*a*c)*x^5 + 210*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 105*B*a^2*x - 70*A*a
^2 - 210*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = 2Bab\log(x) + \frac{Bbcx^4}{2} + \frac{Bc^2x^6}{6} + \frac{Cc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Cbc}{5}\right) + x^3\cdot\left(\frac{2Abc}{3} + \frac{2Cac}{3} + \frac{Cb^2}{3}\right) + x^2\left(Bac + \frac{Bb^2}{2}\right) + x(2Aac + Ab^2 + 2Cab) + \frac{-2Aa^2 - 3Ba^2x + x^2(-12Aab - 6Ca^2)}{6x^3}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*B*a*b*log(x) + B*b*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x**5*(A*c**
2/5 + 2*C*b*c/5) + x**3*(2*A*b*c/3 + 2*C*a*c/3 + C*b**2/3) + x**2*(B*a*c +
B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2
*(-12*A*a*b - 6*C*a**2))/(6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}\,Cc^2x^7 + \frac{1}{6}\,Bc^2x^6 + \frac{1}{2}\,Bbcx^4 + \frac{1}{5}\,(2\,Cbc + Ac^2)x^5 + \frac{1}{3}\,(Cb^2 + 2\,(Ca + Ab)c)x^3 + 2\,Bab\log(x) + \frac{1}{2}\,(Bb^2 + 2\,Bac)x^2 + (2\,Cab + Ab^2 + 2\,Aac)x - \frac{3\,Ba^2x + 2\,Aa^2 + 6\,(Ca^2 + 2\,Aab)x^2}{6\,x^3}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/5*(2*C*b*c + A*c^2)*x^5 +
 1/3*(C*b^2 + 2*(C*a + A*b)*c)*x^3 + 2*B*a*b*log(x) + 1/2*(B*b^2 + 2*B*a*c)
*x^2 + (2*C*a*b + A*b^2 + 2*A*a*c)*x - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}\,Cc^2x^7 + \frac{1}{6}\,Bc^2x^6 + \frac{2}{5}\,Cbcx^5 + \frac{1}{5}\,Ac^2x^5 + \frac{1}{2}\,Bbcx^4 + \frac{1}{3}\,Cb^2x^3 + \frac{2}{3}\,Cacx^3 + \frac{2}{3}\,Abcx^3 + \frac{1}{2}\,Bb^2x^2 + Bacx^2 + 2\,Cabx + Ab^2x + 2\,Aacx + 2\,Bab\log(|x|) - \frac{3\,Ba^2x + 2\,Aa^2 + 6\,(Ca^2 + 2\,Aab)x^2}{6\,x^3}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*b*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*b*c*x
^4 + 1/3*C*b^2*x^3 + 2/3*C*a*c*x^3 + 2/3*A*b*c*x^3 + 1/2*B*b^2*x^2 + B*a*c*
x^2 + 2*C*a*b*x + A*b^2*x + 2*A*a*c*x + 2*B*a*b*log(abs(x)) - 1/6*(3*B*a^2*
x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = x^5\left(\frac{A\,c^2}{5} + \frac{2\,C\,b\,c}{5}\right) - \frac{x^2\,(C\,a^2 + 2\,A\,b\,a) + \frac{A\,a^2}{3} + \frac{B\,a^2\,x}{2}}{x^3} + x\,(A\,b^2 + 2\,C\,a\,b + 2\,A\,a\,c) + x^3\left(\frac{C\,b^2}{3} + \frac{2\,A\,c\,b}{3} + \frac{2\,C\,a\,c}{3}\right) + \frac{B\,c^2\,x^6}{6} + \frac{C\,c^2\,x^7}{7} + \frac{B\,x^2\,(b^2 + 2\,a\,c)}{2} + \frac{B\,b\,c\,x^4}{2} + 2\,B\,a\,b\,\ln(x)$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x)
```

```
[Out] x^5*((A*c^2)/5 + (2*C*b*c)/5) - (x^2*(C*a^2 + 2*A*a*b) + (A*a^2)/3 + (B*a^2
*x)/2)/x^3 + x*(A*b^2 + 2*A*a*c + 2*C*a*b) + x^3*((C*b^2)/3 + (2*A*b*c)/3 +
 (2*C*a*c)/3) + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + (B*x^2*(2*a*c + b^2))/2 + (
B*b*c*x^4)/2 + 2*B*a*b*log(x)
```

3.18 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 148

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} \, dx = -\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}-\frac{a(2Ab+aC)}{2x^2}-\frac{2abB}{x}+B(b^2+2ac)\,x+\frac{1}{2}(2Abc+(b^2+2ac)\,C)\,x^2+\frac{2}{3}bBcx^3+\frac{1}{4}c(Ac+2bC)x^4+\frac{1}{5}Bc^2x^5+\frac{1}{6}c^2Cx^6+(A(b^2+2ac)+2abC)\log(x)$$

```
[Out] -1/4*a^2*A/x^4-1/3*a^2*B/x^3-1/2*a*(2*A*b+C*a)/x^2-2*a*b*B/x+B*(2*a*c+b^2)*
x+1/2*(2*A*b*c+(2*a*c+b^2)*C)*x^2+2/3*b*B*c*x^3+1/4*c*(A*c+2*C*b)*x^4+1/5*B
*c^2*x^5+1/6*c^2*C*x^6+(A*(2*a*c+b^2)+2*a*b*C)*ln(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} \, dx = -\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2(C(2ac+b^2)+2Abc)+\log(x)\,(A(2ac+b^2)+2abC)-\frac{a(aC+2Ab)}{2x^2}+Bx(2ac+b^2)-\frac{2abB}{x}+\frac{1}{4}cx^4(Ac+2bC)+\frac{2}{3}bBcx^3+\frac{1}{5}Bc^2x^5+\frac{1}{6}c^2Cx^6$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -1/4*(a^2*A)/x^4 - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) - (2*a*b*B)/
x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^2)/2 + (2*b*B*c*x^3)
/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5 + (c^2*C*x^6)/6 + (A*(b^2 + 2*
a*c) + 2*a*b*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(B\left(b^2+2ac\right)+\frac{a^2A}{x^5}+\frac{a^2B}{x^4}+\frac{a(2Ab+aC)}{x^3}+\frac{2abB}{x^2}+\frac{A(b^2+2ac)+2abC}{x}\right.\\
&\qquad \left.+\left(2Abc+\left(b^2+2ac\right)C\right)x+2bBcx^2+c(Ac+2bC)x^3+Bc^2x^4+c^2Cx^5\right)dx\\
&= -\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}-\frac{a(2Ab+aC)}{2x^2}-\frac{2abB}{x}+B\left(b^2+2ac\right)x+\frac{1}{2}\left(2Abc+\left(b^2+2ac\right)C\right)x^2\\
&\quad +\frac{2}{3}bBcx^3+\frac{1}{4}c(Ac+2bC)x^4+\frac{1}{5}Bc^2x^5+\frac{1}{6}c^2Cx^6+\left(A\left(b^2+2ac\right)+2abC\right)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\begin{aligned}
\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^5}\,dx &= -\frac{a^2(3A+4Bx+6Cx^2)}{12x^4}\\
&\quad+\frac{a(-Ab-2bBx+cx^3(2B+Cx))}{x^2}\\
&\quad+\frac{1}{60}x\left(30b^2(2B+Cx)+10bcx(6A+x(4B+3Cx))\right.\\
&\qquad\left.+c^2x^3(15A+2x(6B+5Cx))\right)\\
&\quad+\left(A\left(b^2+2ac\right)+2abC\right)\log(x)
\end{aligned}$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -1/12*(a^2*(3*A + 4*B*x + 6*C*x^2))/x^4 + (a*(-(A*b) - 2*b*B*x + c*x^3*(2*B
 + C*x)))/x^2 + (x*(30*b^2*(2*B + C*x) + 10*b*c*x*(6*A + x*(4*B + 3*C*x)) +
 c^2*x^3*(15*A + 2*x*(6*B + 5*C*x))))/60 + (A*(b^2 + 2*a*c) + 2*a*b*C)*Log[
x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2C\,x^6}{6}+\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^4}{4}+\frac{Cbc\,x^4}{2}+\frac{2bBc\,x^3}{3}+Abc\,x^2+Cac\,x^2+\frac{C\,b^2x^2}{2}+2Bacx+B\,b^2x+(2A$
norman	$\frac{(\frac{1}{4}A\,c^2+\frac{1}{2}Cbc)x^8+(-Aab-\frac{1}{2}C\,a^2)x^2+(Abc+acC+\frac{1}{2}b^2C)x^6+(2Bac+B\,b^2)x^5-\frac{A\,a^2}{4}-\frac{B\,a^2x}{3}+\frac{B\,c^2x^9}{5}+\frac{c^2C\,x^{10}}{6}-2Bab\,x^3+}{x^4}$
risch	$\frac{c^2C\,x^6}{6}+\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^4}{4}+\frac{Cbc\,x^4}{2}+\frac{2bBc\,x^3}{3}+Abc\,x^2+Cac\,x^2+\frac{C\,b^2x^2}{2}+2Bacx+B\,b^2x+\frac{-2B}{}$
parallelrisch	$\frac{10c^2C\,x^{10}+12B\,c^2x^9+15A\,c^2x^8+30Cbc\,x^8+40bBc\,x^7+60Abc\,x^6+60Cac\,x^6+30C\,b^2x^6+120A\ln(x)x^4ac+60A\ln(x)x^4b^2+120}{60x^4}$

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*C*x^6+1/5*B*c^2*x^5+1/4*A*c^2*x^4+1/2*C*b*c*x^4+2/3*b*B*c*x^3+A*b*c
*x^2+C*a*c*x^2+1/2*C*b^2*x^2+2*B*a*c*x+B*b^2*x+(2*A*a*c+A*b^2+2*C*a*b)*ln(x
)-1/2*a*(2*A*b+C*a)/x^2-2*a*b*B/x-1/4*a^2*A/x^4-1/3*a^2*B/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^5}\,dx$$
$$=\frac{10\,Cc^2x^{10}+12\,Bc^2x^9+40\,Bbcx^7+15\,(2\,Cbc+Ac^2)x^8+30\,(Cb^2+2\,(Ca+Ab)c)x^6-120\,Babx^3+60\,(}{60\,x^4}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*x^10 + 12*B*c^2*x^9 + 40*B*b*c*x^7 + 15*(2*C*b*c + A*c^2)*x^
8 + 30*(C*b^2 + 2*(C*a + A*b)*c)*x^6 - 120*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)
*x^5 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4*log(x) - 20*B*a^2*x - 15*A*a^2 -
30*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^5}\,dx$$
$$=\frac{2Bbcx^3}{3}+\frac{Bc^2x^5}{5}+\frac{Cc^2x^6}{6}+x^4\left(\frac{Ac^2}{4}+\frac{Cbc}{2}\right)+x^2\left(Abc+Cac+\frac{Cb^2}{2}\right)+x\left(2Bac+Bb^2\right)$$
$$+\left(2Aac+Ab^2+2Cab\right)\log(x)+\frac{-3Aa^2-4Ba^2x-24Babx^3+x^2(-12Aab-6Ca^2)}{12x^4}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] 2*B*b*c*x**3/3 + B*c**2*x**5/5 + C*c**2*x**6/6 + x**4*(A*c**2/4 + C*b*c/2)
+ x**2*(A*b*c + C*a*c + C*b**2/2) + x*(2*B*a*c + B*b**2) + (2*A*a*c + A*b**
2 + 2*C*a*b)*log(x) + (-3*A*a**2 - 4*B*a**2*x - 24*B*a*b*x**3 + x**2*(-12*A
*a*b - 6*C*a**2))/(12*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^5}\,dx$$
$$= \frac{1}{6}\,Cc^2x^6 + \frac{1}{5}\,Bc^2x^5 + \frac{2}{3}\,Bbcx^3 + \frac{1}{4}\,\left(2\,Cbc + Ac^2\right)x^4 + \frac{1}{2}\,\left(Cb^2 + 2\,(Ca+Ab)c\right)x^2 + \left(Bb^2 + 2\,Bac\right)x + \left(2\,Cab + Ab^2 + 2\,Aac\right)\log\left(x\right) - \frac{24\,Babx^3 + 4\,Ba^2x + 3\,Aa^2 + 6\,(Ca^2 + 2\,Aab)x^2}{12\,x^4}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/4*(2*C*b*c + A*c^2)*x^4 +
 1/2*(C*b^2 + 2*(C*a + A*b)*c)*x^2 + (B*b^2 + 2*B*a*c)*x + (2*C*a*b + A*b^2
 + 2*A*a*c)*log(x) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2 +
2*A*a*b)*x^2)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^5}\,dx$$
$$= \frac{1}{6}\,Cc^2x^6 + \frac{1}{5}\,Bc^2x^5 + \frac{1}{2}\,Cbcx^4 + \frac{1}{4}\,Ac^2x^4 + \frac{2}{3}\,Bbcx^3 + \frac{1}{2}\,Cb^2x^2 + Cacx^2 + Abcx^2 + Bb^2x + 2\,Bacx + \left(2\,Cab + Ab^2 + 2\,Aac\right)\log\left(|x|\right) - \frac{24\,Babx^3 + 4\,Ba^2x + 3\,Aa^2 + 6\,(Ca^2 + 2\,Aab)x^2}{12\,x^4}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 1/2*C*b*c*x^4 + 1/4*A*c^2*x^4 + 2/3*B*b*c*x
^3 + 1/2*C*b^2*x^2 + C*a*c*x^2 + A*b*c*x^2 + B*b^2*x + 2*B*a*c*x + (2*C*a*b
 + A*b^2 + 2*A*a*c)*log(abs(x)) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2
+ 6*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = x^4\left(\frac{A\,c^2}{4}+\frac{C\,b\,c}{2}\right) - \frac{x^2\left(\frac{C\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{4}+\frac{B\,a^2\,x}{3}+2\,B\,a\,b\,x^3}{x^4} + x^2\left(\frac{C\,b^2}{2}+A\,c\,b+C\,a\,c\right) + \ln(x)\,\left(A\,b^2+2\,C\,a\,b+2\,A\,a\,c\right)+\frac{B\,c^2\,x^5}{5} + \frac{C\,c^2\,x^6}{6}+B\,x\,\left(b^2+2\,a\,c\right)+\frac{2\,B\,b\,c\,x^3}{3}$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x)
```

```
[Out] x^4*((A*c^2)/4 + (C*b*c)/2) - (x^2*((C*a^2)/2 + A*a*b) + (A*a^2)/4 + (B*a^2
*x)/3 + 2*B*a*b*x^3)/x^4 + x^2*((C*b^2)/2 + A*b*c + C*a*c) + log(x)*(A*b^2
+ 2*A*a*c + 2*C*a*b) + (B*c^2*x^5)/5 + (C*c^2*x^6)/6 + B*x*(2*a*c + b^2) +
(2*B*b*c*x^3)/3
```

3.19 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 143

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2}-\frac{A(b^2+2ac)+2abC}{x}+\left(2Abc+\left(b^2+2ac\right)C\right)x+bBcx^2+\frac{1}{3}c(Ac+2bC)x^3+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5+B\left(b^2+2ac\right)\log(x)$$

```
[Out] -1/5*a^2*A/x^5-1/4*a^2*B/x^4-1/3*a*(2*A*b+C*a)/x^3-a*b*B/x^2+(-A*(2*a*c+b^2
)-2*a*b*C)/x+(2*A*b*c+(2*a*c+b^2)*C)*x+b*B*c*x^2+1/3*c*(A*c+2*C*b)*x^3+1/4*
B*c^2*x^4+1/5*c^2*C*x^5+B*(2*a*c+b^2)*ln(x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A(2ac+b^2)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)+bBcx^2+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2*a*c)*C)*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(2Abc\left(1+\frac{b\left(1+\frac{2ac}{b^2}\right)C}{2Ac}\right)+\frac{a^2A}{x^6}+\frac{a^2B}{x^5}+\frac{a(2Ab+aC)}{x^4}+\frac{2abB}{x^3}+\frac{A(b^2+2ac)+2abC}{x^2}+\frac{B(b^2+2ac)}{x}+2bBcx+c(Ac+2bC)x^2+Bc^2x^3+c^2Cx^4\right)dx$$

$$=-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2}-\frac{A(b^2+2ac)+2abC}{x}+\left(2Abc+\left(b^2+2ac\right)C\right)x+bBcx^2+\frac{1}{3}c(Ac+2bC)x^3+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5+B\left(b^2+2ac\right)\log(x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2}-\frac{Ab^2+2aAc+2abC}{x}+2Abcx+\left(b^2+2ac\right)Cx+bBcx^2+\frac{1}{3}c(Ac+2bC)x^3+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5+B\left(b^2+2ac\right)\log(x)$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*b^2 + 2*a*A*c + 2*a*b*C)/x + 2*A*b*c*x + (b^2 + 2*a*c)*C*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2 C x^5}{5}+\frac{B c^2 x^4}{4}+\frac{A c^2 x^3}{3}+\frac{2Cbc x^3}{3}+Bbc x^2+2Abcx+2Cacx+C b^2 x+B(2ac+b^2)\ln(x)-$
risch	$\frac{c^2 C x^5}{5}+\frac{B c^2 x^4}{4}+\frac{A c^2 x^3}{3}+\frac{2Cbc x^3}{3}+Bbc x^2+2Abcx+2Cacx+C b^2 x+\frac{(-2Aac-A b^2-2abC)x^4-Ba}{}$
norman	$\frac{(\frac{1}{3}A c^2+\frac{2}{3}Cbc)x^8+(-\frac{2}{3}Aab-\frac{1}{3}C a^2)x^2+(2Abc+2acC+b^2C)x^6+(-2Aac-A b^2-2abC)x^4+bBc x^7-\frac{A a^2}{5}-\frac{B a^2 x}{4}+\frac{B c^2 x^9}{4}+\frac{c^2}{}}{x^5}$
parallelrisch	$\frac{12c^2 C x^{10}+15B c^2 x^9+20A c^2 x^8+40Cbc x^8+60bBc x^7+120Abc x^6+120B\ln(x)x^5 ac+60B\ln(x)x^5 b^2+120Cac x^6+60C b^2 x^6-12}{60x^5}$

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*c^2*C*x^5+1/4*B*c^2*x^4+1/3*A*c^2*x^3+2/3*C*b*c*x^3+B*b*c*x^2+2*A*b*c*x
+2*C*a*c*x+C*b^2*x+B*(2*a*c+b^2)*ln(x)-a*b*B/x^2-1/5*a^2*A/x^5-(2*A*a*c+A*b
^2+2*C*a*b)/x-1/4*a^2*B/x^4-1/3*a*(2*A*b+C*a)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = \frac{12\,Cc^2x^{10}+15\,Bc^2x^9+60\,Bbcx^7+20\,(2\,Cbc+Ac^2)x^8+60\,(Cb^2+2\,(Ca+Ab)c)x^6+60\,(Bb^2+2\,Bac)x}{60\,x^5}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*x^10 + 15*B*c^2*x^9 + 60*B*b*c*x^7 + 20*(2*C*b*c + A*c^2)*x^
8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 60*(B*b^2 + 2*B*a*c)*x^5*log(x) - 60
*B*a*b*x^3 - 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 15*B*a^2*x - 12*A*a^2 - 2
0*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = Bbcx^2+\frac{Bc^2x^4}{4}+B\left(2ac+b^2\right)\log(x)+\frac{Cc^2x^5}{5}+x^3\left(\frac{Ac^2}{3}+\frac{2Cbc}{3}\right)+x\left(2Abc+2Cac+Cb^2\right) + \frac{-12Aa^2-15Ba^2x-60Babx^3+x^4(-120Aac-60Ab^2-120Cab)+x^2(-40Aab-20Ca^2)}{60x^5}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] B*b*c*x**2 + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**5/5 + x**3
*(A*c**2/3 + 2*C*b*c/3) + x*(2*A*b*c + 2*C*a*c + C*b**2) + (-12*A*a**2 - 15
*B*a**2*x - 60*B*a*b*x**3 + x**4*(-120*A*a*c - 60*A*b**2 - 120*C*a*b) + x**
2*(-40*A*a*b - 20*C*a**2))/(60*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^6}\,dx &= \frac{1}{5}\,Cc^2x^5+\frac{1}{4}\,Bc^2x^4+Bbcx^2+\frac{1}{3}\,\left(2\,Cbc+Ac^2\right)x^3\\&\quad+\left(Cb^2+2\,(Ca+Ab)c\right)x+\left(Bb^2+2\,Bac\right)\log\left(x\right)\\&\quad-\frac{60\,Babx^3+60\,(2\,Cab+Ab^2+2\,Aac)x^4+15\,Ba^2x+12\,Aa^2+20\,(Ca^2+2\,Aab)x^2}{60\,x^5}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/3*(2*C*b*c + A*c^2)*x^3 + (C*
b^2 + 2*(C*a + A*b)*c)*x + (B*b^2 + 2*B*a*c)*log(x) - 1/60*(60*B*a*b*x^3 +
60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 + 2*
A*a*b)*x^2)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^6}\,dx &= \frac{1}{5}\,Cc^2x^5+\frac{1}{4}\,Bc^2x^4+\frac{2}{3}\,Cbcx^3+\frac{1}{3}\,Ac^2x^3+Bbcx^2\\&\quad+Cb^2x+2\,Cacx+2\,Abcx+\left(Bb^2+2\,Bac\right)\log\left(|x|\right)\\&\quad-\frac{60\,Babx^3+60\,(2\,Cab+Ab^2+2\,Aac)x^4+15\,Ba^2x+12\,Aa^2+20\,(Ca^2+2\,Aab)x^2}{60\,x^5}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + 2/3*C*b*c*x^3 + 1/3*A*c^2*x^3 + B*b*c*x^2 +
 C*b^2*x + 2*C*a*c*x + 2*A*b*c*x + (B*b^2 + 2*B*a*c)*log(abs(x)) - 1/60*(60
*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 2
0*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^6}\,dx$$
$$= x^3\left(\frac{A\,c^2}{3}+\frac{2\,C\,b\,c}{3}\right)$$
$$-\frac{x^2\left(\frac{C\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+\frac{A\,a^2}{5}+x^4\left(A\,b^2+2\,C\,a\,b+2\,A\,a\,c\right)+\frac{B\,a^2\,x}{4}+B\,a\,b\,x^3}{x^5}$$
$$+x\left(C\,b^2+2\,A\,c\,b+2\,C\,a\,c\right)+\ln\left(x\right)\,\left(B\,b^2+2\,B\,a\,c\right)+\frac{B\,c^2\,x^4}{4}+\frac{C\,c^2\,x^5}{5}+B\,b\,c\,x^2$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x)
```

```
[Out] x^3*((A*c^2)/3 + (2*C*b*c)/3) - (x^2*((C*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/5
+ x^4*(A*b^2 + 2*A*a*c + 2*C*a*b) + (B*a^2*x)/4 + B*a*b*x^3)/x^5 + x*(C*b^2
 + 2*A*b*c + 2*C*a*c) + log(x)*(B*b^2 + 2*B*a*c) + (B*c^2*x^4)/4 + (C*c^2*x
^5)/5 + B*b*c*x^2
```

3.20 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx = -\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{a(2Ab+aC)}{4x^4}-\frac{2abB}{3x^3}-\frac{A(b^2+2ac)+2abC}{2x^2}-\frac{B(b^2+2ac)}{x}+2bBcx+\frac{1}{2}c(Ac+2bC)x^2+\frac{1}{3}Bc^2x^3+\frac{1}{4}c^2Cx^4+\left(2Abc+\left(b^2+2ac\right)C\right)\log(x)$$

```
[Out] -1/6*a^2*A/x^6-1/5*a^2*B/x^5-1/4*a*(2*A*b+C*a)/x^4-2/3*a*b*B/x^3+1/2*(-A*(2
*a*c+b^2)-2*a*b*C)/x^2-B*(2*a*c+b^2)/x+2*b*B*c*x+1/2*c*(A*c+2*C*b)*x^2+1/3*
B*c^2*x^3+1/4*c^2*C*x^4+(2*A*b*c+(2*a*c+b^2)*C)*ln(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1642}

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx = -\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A(2ac+b^2)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B(2ac+b^2)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2(Ac+2bC)+2bBcx+\frac{1}{3}Bc^2x^3+\frac{1}{4}c^2Cx^4$$

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -1/6*(a^2*A)/x^6 - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) - (2*a*b*B)/
(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2 + 2*a*c))/x + 2*b*B
*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/3 + (c^2*C*x^4)/4 + (2*A*b*c +
 (b^2 + 2*a*c)*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(2bBc + \frac{a^2A}{x^7} + \frac{a^2B}{x^6} + \frac{a(2Ab+aC)}{x^5} + \frac{2abB}{x^4} + \frac{A(b^2+2ac)+2abC}{x^3} \right. \\ &\quad \left. + \frac{B(b^2+2ac)}{x^2} + \frac{2Abc+(b^2+2ac)\,C}{x} + c(Ac+2bC)x + Bc^2x^2 + c^2Cx^3\right) dx \\ &= -\frac{a^2A}{6x^6} - \frac{a^2B}{5x^5} - \frac{a(2Ab+aC)}{4x^4} - \frac{2abB}{3x^3} - \frac{A(b^2+2ac)+2abC}{2x^2} - \frac{B(b^2+2ac)}{x} \\ &\quad + 2bBcx + \frac{1}{2}c(Ac+2bC)x^2 + \frac{1}{3}Bc^2x^3 + \frac{1}{4}c^2Cx^4 + \left(2Abc+\left(b^2+2ac\right)C\right)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^7}\,dx = &-\frac{b^2B}{x} + bcx(2B+Cx) + \frac{1}{12}c^2x^3(4B+3Cx) \\ &+ \frac{A(-b^2+c^2x^4)}{2x^2} - \frac{a^2(10A+3x(4B+5Cx))}{60x^6} \\ &- \frac{a(3A(b+2cx^2)+2x(2bB+3bCx+6Bcx^2))}{6x^4} \\ &+ \left(2Abc+\left(b^2+2ac\right)C\right)\log(x)\end{aligned}$$

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -((b^2*B)/x) + b*c*x*(2*B + C*x) + (c^2*x^3*(4*B + 3*C*x))/12 + (A*(-b^2 +
c^2*x^4))/(2*x^2) - (a^2*(10*A + 3*x*(4*B + 5*C*x)))/(60*x^6) - (a*(3*A*(b
+ 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2)))/(6*x^4) + (2*A*b*c + (b^2
+ 2*a*c)*C)*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

method	result
default	$\frac{c^2 C x^4}{4} + \frac{B c^2 x^3}{3} + \frac{A c^2 x^2}{2} + Cbc x^2 + 2Bbcx + (2Abc + 2acC + b^2 C) \ln(x) - \frac{2Aac + A b^2 + 2abC}{2x^2} -$
norman	$\frac{\left(\frac{1}{2} A c^2 + Cbc\right) x^8 + \left(-\frac{1}{2} Aab - \frac{1}{4} C a^2\right) x^2 + \left(-Aac - \frac{1}{2} A b^2 - abC\right) x^4 + \left(-2Bac - B b^2\right) x^5 - \frac{A a^2}{6} - \frac{B a^2 x}{5} + \frac{B c^2 x^9}{3} + \frac{c^2 C x^{10}}{4} - \frac{2Bab x}{3}}{x^6}$
risch	$\frac{c^2 C x^4}{4} + \frac{B c^2 x^3}{3} + \frac{A c^2 x^2}{2} + Cbc x^2 + 2Bbcx + \frac{(-2Bac - B b^2) x^5 + \left(-Aac - \frac{1}{2} A b^2 - abC\right) x^4 - \frac{2Bab x^3}{3} + \left(-\frac{1}{2} Aab\right.}{x^6}$
parallelrisch	$\frac{15c^2 C x^{10} + 20B c^2 x^9 + 30A c^2 x^8 + 60Cbc x^8 + 120A \ln(x) x^6 bc + 120bBc x^7 + 120C \ln(x) x^6 ac + 60C \ln(x) x^6 b^2 - 120Bac x^5 - 60B}{60x^6}$

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c^2*C*x^4+1/3*B*c^2*x^3+1/2*A*c^2*x^2+C*b*c*x^2+2*B*b*c*x+(2*A*b*c+2*C*
a*c+C*b^2)*ln(x)-1/2*(2*A*a*c+A*b^2+2*C*a*b)/x^2-1/6*a^2*A/x^6-1/5*a^2*B/x^
5-B*(2*a*c+b^2)/x-1/4*a*(2*A*b+C*a)/x^4-2/3*a*b*B/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^7} dx$$
$$= \frac{15 Cc^2x^{10} + 20 Bc^2x^9 + 120 Bbcx^7 + 30 (2 Cbc + Ac^2)x^8 + 60 (Cb^2 + 2 (Ca + Ab)c)x^6 \log(x) - 40 Babx^3}{60 x^6}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*C*c^2*x^10 + 20*B*c^2*x^9 + 120*B*b*c*x^7 + 30*(2*C*b*c + A*c^2)*x
^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6*log(x) - 40*B*a*b*x^3 - 60*(B*b^2 + 2
*B*a*c)*x^5 - 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 12*B*a^2*x - 10*A*a^2 -
15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 28.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.06

$$\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)^2}{x^7} dx$$
$$= 2Bbcx + \frac{Bc^2x^3}{3} + \frac{Cc^2x^4}{4} + x^2\left(\frac{Ac^2}{2} + Cbc\right) + \left(2Abc + 2Cac + Cb^2\right) \log(x)$$
$$+ \frac{-10Aa^2 - 12Ba^2x - 40Babx^3 + x^5(-120Bac - 60Bb^2) + x^4(-60Aac - 30Ab^2 - 60Cab) + x^2(-30A}{60x^6}$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*B*b*c*x + B*c**2*x**3/3 + C*c**2*x**4/4 + x**2*(A*c**2/2 + C*b*c) + (2*A*
b*c + 2*C*a*c + C*b**2)*log(x) + (-10*A*a**2 - 12*B*a**2*x - 40*B*a*b*x**3
+ x**5*(-120*B*a*c - 60*B*b**2) + x**4*(-60*A*a*c - 30*A*b**2 - 60*C*a*b) +
 x**2*(-30*A*a*b - 15*C*a**2))/(60*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^7}\,dx$$
$$= \frac{1}{4}\,Cc^2x^4 + \frac{1}{3}\,Bc^2x^3 + 2\,Bbcx + \frac{1}{2}\left(2\,Cbc + Ac^2\right)x^2 + \left(Cb^2 + 2\,(Ca + Ab)c\right)\log\left(x\right)$$
$$- \frac{40\,Babx^3 + 60\,(Bb^2 + 2\,Bac)x^5 + 30\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + 12\,Ba^2x + 10\,Aa^2 + 15\,(Ca^2 + 2\,Aab)x}{60\,x^6}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + 2*B*b*c*x + 1/2*(2*C*b*c + A*c^2)*x^2 + (C*
b^2 + 2*(C*a + A*b)*c)*log(x) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)*x
^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15*(C*a^2
 + 2*A*a*b)*x^2)/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^7}\,dx$$
$$= \frac{1}{4}\,Cc^2x^4 + \frac{1}{3}\,Bc^2x^3 + Cbcx^2 + \frac{1}{2}\,Ac^2x^2 + 2\,Bbcx + \left(Cb^2 + 2\,Cac + 2\,Abc\right)\log\left(|x|\right)$$
$$- \frac{40\,Babx^3 + 60\,(Bb^2 + 2\,Bac)x^5 + 30\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + 12\,Ba^2x + 10\,Aa^2 + 15\,(Ca^2 + 2\,Aab)x}{60\,x^6}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + C*b*c*x^2 + 1/2*A*c^2*x^2 + 2*B*b*c*x + (C*
b^2 + 2*C*a*c + 2*A*b*c)*log(abs(x)) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B
*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15
*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^7}\,dx = x^2\left(\frac{A\,c^2}{2}+C\,b\,c\right) - \frac{x^2\left(\frac{C\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^5\,(B\,b^2+2\,B\,a\,c)+\frac{A\,a^2}{6}+x^4\left(\frac{A\,b^2}{2}+C\,a\,b+A\,a\,c\right)+\frac{B\,a^2\,x}{5}+\frac{2\,B\,a\,b\,x^3}{3}}{x^6} + \ln(x)\,\left(C\,b^2+2\,A\,c\,b+2\,C\,a\,c\right)+\frac{B\,c^2\,x^3}{3}+\frac{C\,c^2\,x^4}{4}+2\,B\,b\,c\,x$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x)
```

```
[Out] x^2*((A*c^2)/2 + C*b*c) - (x^2*((C*a^2)/4 + (A*a*b)/2) + x^5*(B*b^2 + 2*B*a
*c) + (A*a^2)/6 + x^4*((A*b^2)/2 + A*a*c + C*a*b) + (B*a^2*x)/5 + (2*B*a*b*
x^3)/3)/x^6 + log(x)*(C*b^2 + 2*A*b*c + 2*C*a*c) + (B*c^2*x^3)/3 + (C*c^2*x
^4)/4 + 2*B*b*c*x
```

3.21 $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 339

$$\begin{aligned}
&\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx \\
&= \frac{(Ac-bC)x}{c^2}+\frac{Bx^2}{2c}+\frac{Cx^3}{3c} \\
&\quad -\frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad -\frac{\left(Abc-b^2C+acC+\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad -\frac{B(b^2-2ac)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{bB\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

```
[Out] (A*c-C*b)*x/c^2+1/2*B*x^2/c+1/3*C*x^3/c-1/4*b*B*ln(c*x^4+b*x^2+a)/c^2-1/2*B
*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2
)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(A*b*c-b^2*C+a
*c*C+(-A*c*(-2*a*c+b^2)+b*(-3*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*(A*b*c-b^2*C+a*c*C+(A*c*(-2*a*c+b^2)-b*(-3*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1676, 1293, 1180, 211, 12, 1128, 717, 648, 632, 212, 642}

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$= -\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{B(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{bB\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x(Ac-bC)}{c^2}+\frac{Bx^2}{2c}+\frac{Cx^3}{3c}$$

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((A*c - b*C)*x)/c^2 + (B*x^2)/(2*c) + (C*x^3)/(3*c) - ((A*b*c - b^2*C + a*c
*C - (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((A*b*c - b^2*C + a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 -
3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*(b^2 - 2*a*c)
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*B
*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{Bx^5}{a+bx^2+cx^4}\,dx + \int \frac{x^4(A+Cx^2)}{a+bx^2+cx^4}\,dx\\
&= \frac{Cx^3}{3c} + B\int \frac{x^5}{a+bx^2+cx^4}\,dx - \frac{\int \frac{x^2\left(3aC-3(Ac-bC)x^2\right)}{a+bx^2+cx^4}\,dx}{3c}\\
&= \frac{(Ac-bC)x}{c^2} + \frac{Cx^3}{3c} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&\quad + \frac{\int \frac{-3a(Ac-bC)-3\left(Abc-b^2C+acC\right)x^2}{a+bx^2+cx^4}\,dx}{3c^2}\\
&= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} + \frac{B\,\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&\quad - \frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}\\
&\quad - \frac{\left(Abc-b^2C+acC+\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}\\
&= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c}\\
&\quad - \frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad - \frac{\left(Abc-b^2C+acC+\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\\
&\quad - \frac{(bB)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} + \frac{\left(B(b^2-2ac)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}
\end{aligned}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(Abc-b^2C+acC+\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\log\left(a+bx^2+cx^4\right)}{4c^2} - \frac{(B(b^2-2ac))\,\mathrm{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(Abc-b^2C+acC+\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{B(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{bB\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.36

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{12\sqrt{c}(Ac-bC)x + 6Bc^{3/2}x^2 + 4c^{3/2}Cx^3 + \frac{6\sqrt{2}\left(Ac\left(b^2-2ac-b\sqrt{b^2-4ac}\right)+\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)C\right)\arctan\left(\cdots\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} \cdots}{\cdots}$$

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (12*Sqrt[c]*(A*c - b*C)*x + 6*B*c^(3/2)*x^2 + 4*c^(3/2)*C*x^3 + (6*Sqrt[2]*
(A*c*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]) + (-b^3 + 3*a*b*c + b^2*Sqrt[b^2 -
 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (6*Sqrt
[2]*(-(A*c*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])) + (b^3 - 3*a*b*c + b^2*Sqrt
[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
3*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (3*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4
*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(12*c^(5/2)
)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.35

method	result
risch	$\frac{C x^3}{3c} + \frac{B x^2}{2c} + \frac{Ax}{c} - \frac{Cbx}{c^2} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(-bBc\_R^3+\left(-Abc-acC+b^2C\right)\_R^2-Bac\_R-Aac+abC\right)\ln\left(x-\right.}{2c\_R^3+\_Rb}}{2c^2}$
default	$\frac{\frac{1}{3}cC x^3+\frac{1}{2}B x^2 c+Acx-Cbx}{c^2} + \frac{-\left(2ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}+4abc-b^3\right)\left(\frac{B\ln\left(2c x^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^2}-Cb\right)\sqrt{2}\ \mathrm{arc}}{2\sqrt{\left(b+\sqrt{-4\right.}}}\right.}{2c\left(4ac-b^2\right)}$

```
[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*C*x^3/c+1/2*B*x^2/c+1/c*A*x-1/c^2*C*b*x+1/2/c^2*sum((-b*B*c*_R^3+(-A*b*
c-C*a*c+C*b^2)*_R^2-B*a*c*_R-A*a*c+a*b*C)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootO
f(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*C*c*x^3 + 3*B*c*x^2 - 6*(C*b - A*c)*x)/c^2 - integrate((B*b*c*x^3 +
B*a*c*x - C*a*b + A*a*c - (C*b^2 - (C*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a),
 x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5304 vs. 2(294) = 588.

Time = 1.42 (sec) , antiderivative size = 5304, normalized size of antiderivative = 15.65

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*B*b*log(abs(c*x^4 + b*x^2 + a))/c^2 - 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 +
 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^
3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c -
 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2
 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*C*c^2 + 2*(sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*s
```

qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a
*c)*a^2*c^5)*A*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^
2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^
3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5
+ 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(c) - (2*b^5*
c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6
 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a
*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4
 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 -
2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*C)*arctan(2*sqrt(1/2)*
x/sqrt((b*c^7 + sqrt(b^2*c^14 - 4*a*c^15))/c^8))/((a*b^4*c^4 - 8*a^2*b^2*c^
5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) +
1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2
*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c

```
)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*
a*c)*a^2*c^4)*C*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5
- 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 -
 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(c) + 2*(sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*
b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*
c)*a^2*b*c^4)*C*abs(c) - (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c
)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a
*b^2*c^5)*C)*arctan(2*sqrt(1/2)*x/sqrt((b*c^7 - sqrt(b^2*c^14 - 4*a*c^15))/
c^8))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6
+ a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/6*(2*C*c^2*x^3 + 3*B*c^2*x^2 - 6*C*b*c*x
+ 6*A*c^2*x)/c^3 + 1/16*((b^7 - 10*a*b^5*c - 2*b^6*c + 32*a^2*b^3*c^2 + 12*
a*b^4*c^2 + b^5*c^2 - 32*a^3*b*c^3 - 16*a^2*b^2*c^3 - 6*a*b^3*c^3 + 8*a^2*b
*c^4 + (b^6 - 10*a*b^4*c - 2*b^5*c + 32*a^2*b^2*c^2 + 12*a*b^3*c^2 + b^4*c^
2 - 32*a^3*c^3 - 16*a^2*b*c^3 - 6*a*b^2*c^3 + 8*a^2*c^4)*sqrt(b^2 - 4*a*c))
*B*abs(c) - (b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c
^3 + b^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 +
(b^6*c - 6*a*b^4*c^2 - 2*b^5*c^2 + 8*a^2*b^2*c^3 + 4*a*b^3*c^3 + b^4*c^3 -
2*a*b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b*c^7 + sqrt(b^2*c^14 - 4
*a*c^15))/c^8)/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2
*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2*abs(c)) + 1/16*((b^7 - 10*a*b^5*c - 2*b
^6*c + 32*a^2*b^3*c^2 + 12*a*b^4*c^2 + b^5*c^2 - 32*a^3*b*c^3 - 16*a^2*b^2*
```

```
c^3 - 6*a*b^3*c^3 + 8*a^2*b*c^4 + (b^6 - 10*a*b^4*c - 2*b^5*c + 32*a^2*b^2*
c^2 + 12*a*b^3*c^2 + b^4*c^2 - 32*a^3*c^3 - 16*a^2*b*c^3 - 6*a*b^2*c^3 + 8*
a^2*c^4)*sqrt(b^2 - 4*a*c))*B*abs(c) - (b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 +
32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6
*a*b^3*c^4 + 8*a^2*b*c^5 + (b^6*c - 6*a*b^4*c^2 - 2*b^5*c^2 + 8*a^2*b^2*c^3
 + 4*a*b^3*c^3 + b^4*c^3 - 2*a*b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2
*(b*c^7 - sqrt(b^2*c^14 - 4*a*c^15))/c^8)/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b
^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 2588, normalized size of antiderivative = 7.63

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (C*b)/c^2) + symsum(log((C^3*a^4*c - C^3*a^3*b^2 - A*B^2*a^3*c^2 +
 A*C^2*a^2*b^3 + A^2*C*a^3*c^2 + A^3*a^2*b*c^2 + A*B^2*a^2*b^2*c - 2*A^2*C*
a^2*b^2*c - B^2*C*a^3*b*c)/c^3 - root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 -
256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*
z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80
*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a
*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^
2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A
^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z
 - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^
3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z
 + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^
2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c -
 C^4*a^5, z, k)*(root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4
- 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b
^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z
^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 3
6*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a
^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 -
 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*
b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z +
16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b
*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c -
B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z, k)*
((x*(16*B*a^2*c^5 + 8*B*b^4*c^3 - 36*B*a*b^2*c^4))/c^3 - (16*A*a^2*c^5 - 4*
A*a*b^2*c^4 + 4*C*a*b^3*c^3 - 16*C*a^2*b*c^4)/c^3 + (root(128*a*b^2*c^6*z^4
 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4
```

```
*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 +
8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*
b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2
- 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B
^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z -
 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^
3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z
- 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c
+ 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a
^3*c^2 - B^4*a^4*c - C^4*a^5, z, k)*x*(8*b^3*c^5 - 32*a*b*c^6))/c^3) + (8*B
*C*a^3*c^3 - 4*A*B*a^2*b*c^3)/c^3 + (x*(2*C^2*b^6 + 2*B^2*b^5*c + 4*A^2*a^2
*c^4 + 2*A^2*b^4*c^2 - 4*C^2*a^3*c^3 - 4*A*C*b^5*c + 18*C^2*a^2*b^2*c^2 - 1
2*C^2*a*b^4*c - 8*A^2*a*b^2*c^3 - 10*B^2*a*b^3*c^2 + 6*B^2*a^2*b*c^3 + 20*A
*C*a*b^3*c^2 - 20*A*C*a^2*b*c^3))/c^3) + (x*(B*C^2*a^2*b^3 - B^3*a^3*c^2 +
B^3*a^2*b^2*c + A^2*B*a^2*b*c^2 + 2*A*B*C*a^3*c^2 - 2*B*C^2*a^3*b*c - 2*A*B
*C*a^2*b^2*c))/c^3)*root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z
^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*
a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^
3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2
+ 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^
2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^
2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a
^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z
 + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^
3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c
 - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z,
k), k, 1, 4) + (B*x^2)/(2*c) + (C*x^3)/(3*c)
```

3.22 $\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 278

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx}{c}+\frac{Cx^2}{2c}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{(Abc-b^2C+2acC)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] B*x/c+1/2*C*x^2/c+1/4*(A*c-C*b)*ln(c*x^4+b*x^2+a)/c^2+1/2*(A*b*c+2*C*a*c-C*
b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)-1/2*B*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*
c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(
1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1676, 1265, 787, 648, 632, 212, 642, 12, 1136, 1180, 211}

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{(2acC+Abc+b^2(-C))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} + \frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2} - \frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{Bx}{c} + \frac{Cx^2}{2c}$$

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c + (C*x^2)/(2*c) - (B*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (B*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt
[b^2 - 4*a*c]]) + ((A*b*c - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4]
)/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{Bx^4}{a+bx^2+cx^4}\,dx + \int \frac{x^3(A+Cx^2)}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Cx)}{a+bx+cx^2}\,dx, x, x^2\right) + B\int \frac{x^4}{a+bx^2+cx^4}\,dx \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} + \frac{\text{Subst}\left(\int \frac{-aC+(Ac-bC)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} - \frac{B\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{\left(B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&\quad - \frac{\left(B\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} + \frac{(Ac-bC)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} \\
&\quad - \frac{(Abc-b^2C+2acC)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad + \frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{(Abc-b^2C+2acC)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad - \frac{B\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Abc-b^2C+2acC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} \\
&\quad + \frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.36

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$= \frac{4Bcx+2cCx^2-\frac{2\sqrt{2}B\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}B\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{}$$

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*B*c*x + 2*c*C*x^2 - (2*Sqrt[2]*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*B*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*c*(-b + Sqrt[b^2 -
4*a*c]) + (b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((-(A*c*(b + Sqrt[b^2 - 4*a*c])) + (b^2 -
2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[
b^2 - 4*a*c])/(4*c^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.31

method	result
risch	$\frac{C\,x^2}{2c}+\frac{Bx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^3(Ac-Cb)-bB\_R^2-Ca\_R-Ba\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{\frac{1}{2}C\,x^2+Bx}{c}+\frac{\frac{\left(-Abc\sqrt{-4ac+b^2}+4Aa\,c^2-A\,b^2c-2C\sqrt{-4ac+b^2}\,ac+C\sqrt{-4ac+b^2}\,b^2-4Cabc+C\,b^3\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{\left(-2Bac\sqrt{-4a\right.}{}}{c(4ac-b^2)}$

```
[In] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*C*x^2/c+B*x/c+1/2/c*sum((_R^3*(A*c-C*b)-b*B*_R^2-C*a*_R-B*a)/(2*_R^3*c+
_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 61.66 (sec) , antiderivative size = 1329593, normalized size of antiderivative = 4782.71

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/2*(C*x^2 + 2*B*x)/c + integrate(-(B*b*x^2 + (C*b - A*c)*x^3 + C*a*x + B*a
)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3519 vs. 2(232) = 464.

Time = 1.32 (sec) , antiderivative size = 3519, normalized size of antiderivative = 12.66

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -1/4*(C*b - A*c)*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/2*(C*c*x^2 + 2*B*c*x)/
c^2 + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*
```

```
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 - 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c
^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b
^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^5 + sqrt(b^2*c^10 - 4*a
*c^11))/c^6))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^
2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32
*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5
+ 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8
*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^
2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5
- 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^5*c^
4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 -
```

```
 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x
/sqrt((b*c^5 - sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^3 - 8*a^2*b^2*c^4
 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1
/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*
c^3 - 4*a*b^2*c^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b
^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - (b^7 - 10*a*b^5
*c - 2*b^6*c + 32*a^2*b^3*c^2 + 12*a*b^4*c^2 + b^5*c^2 - 32*a^3*b*c^3 - 16*
a^2*b^2*c^3 - 6*a*b^3*c^3 + 8*a^2*b*c^4 + (b^6 - 10*a*b^4*c - 2*b^5*c + 32*
a^2*b^2*c^2 + 12*a*b^3*c^2 + b^4*c^2 - 32*a^3*c^3 - 16*a^2*b*c^3 - 6*a*b^2*
c^3 + 8*a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) - (b^6*c^2 - 8*a*b^4*c^3 - 2*b
^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 + (b^5*c^2 -
4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A + (b^7*c - 10*a*b^5
*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 -
 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 + (b^6*c - 6*a*b^4*c^2 - 2*b^5*
c^2 + 8*a^2*b^2*c^3 + 4*a*b^3*c^3 + b^4*c^3 - 2*a*b^2*c^4)*sqrt(b^2 - 4*a*c
))*C)*log(x^2 + 1/2*(b*c^5 + sqrt(b^2*c^10 - 4*a*c^11))/c^6)/((a*b^4*c - 8*
a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^
4)*c^2*abs(c)) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 +
8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16
*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c)
 - (b^7 - 10*a*b^5*c - 2*b^6*c + 32*a^2*b^3*c^2 + 12*a*b^4*c^2 + b^5*c^2 -
32*a^3*b*c^3 - 16*a^2*b^2*c^3 - 6*a*b^3*c^3 + 8*a^2*b*c^4 - (b^6 - 10*a*b^4
*c - 2*b^5*c + 32*a^2*b^2*c^2 + 12*a*b^3*c^2 + b^4*c^2 - 32*a^3*c^3 - 16*a^
2*b*c^3 - 6*a*b^2*c^3 + 8*a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) + (b^6*c^2 -
 8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2
*c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A -
 (b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^
3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 - (b^6*c - 6*
a*b^4*c^2 - 2*b^5*c^2 + 8*a^2*b^2*c^3 + 4*a*b^3*c^3 + b^4*c^3 - 2*a*b^2*c^4
)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b*c^5 - sqrt(b^2*c^10 - 4*a*c^11))/c
^6)/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*
b^2*c^3 - 4*a^2*c^4)*c^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 2696, normalized size of antiderivative = 9.70

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log((B^3*a^2*b*c - B*C^2*a^3*c + A^2*B*a^2*c^2 + B*C^2*a^2*b^2 - 2*A
*B*C*a^2*b*c)/c^2 - root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z
^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C
```

*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2
 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a
*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c
^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*
a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z +
12*A*C^2*a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a
*b^3*c*z - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*
C^2*a^3*c^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A
^3*C*a^2*b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*
a^2*b*c - B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a
^4, z, k)*(root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*
C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*
z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C
*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*
z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 -
4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z
^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*
a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z
 - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c
^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*
b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c -
 B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k)
((x(16*C*a^2*c^4 - 8*A*b^3*c^3 + 8*C*b^4*c^2 + 32*A*a*b*c^4 - 36*C*a*b^2*
c^3))/c^2 - (16*B*a^2*c^4 - 4*B*a*b^2*c^3)/c^2 + (root(128*a*b^2*c^5*z^4 -
16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^
3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c
^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 4
8*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*
a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2
 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2
*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z +
 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*
z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*
b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^
3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c - B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A
^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k)*x*(8*b^3*c^4 - 32*a*b*c^5))/c^2) +
(8*A*B*a^2*c^3 - 4*B*C*a^2*b*c^2)/c^2 + (x*(2*C^2*b^5 + 2*B^2*b^4*c + 2*A^2
*b^3*c^2 + 4*B^2*a^2*c^3 - 4*A*C*b^4*c - 8*A*C*a^2*c^3 - 10*A^2*a*b*c^3 - 1
0*C^2*a*b^3*c - 8*B^2*a*b^2*c^2 + 6*C^2*a^2*b*c^2 + 20*A*C*a*b^2*c^2))/c^2)
 - (x*(C^3*a^3*c - C^3*a^2*b^2 + A*C^2*a*b^3 + A^3*a*b*c^2 - A*B^2*a^2*c^2
+ A^2*C*a^2*c^2 + A*B^2*a*b^2*c - 2*A^2*C*a*b^2*c - B^2*C*a^2*b*c))/c^2)*ro
ot(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*C*a^2*b*c^4*z
^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*z^3 + 16*A*b^
4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C*a*b^3*c^2*z^
2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*z^2 + 40*A^2*

```
a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z
^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^
6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*a^2*b^2*c*z +
 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b
^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c^2*z - 16*C^3
*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2
*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c - B^2*C^2*a^3*
b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k), k, 1, 4) +
(C*x^2)/(2*c) + (B*x)/c
```

3.23 $\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 270

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4} \, dx = \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log(a+bx^2+cx^4)}{4c}$$

```
[Out] C*x/c+1/4*B*ln(c*x^4+b*x^2+a)/c+1/2*b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1
/2))/c/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(A*c-C*b+(-A*b*c+(-2*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*(A*c-C*b+(A*b*c+2*C*a*c-C*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)
*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules

used = {1676, 1293, 1180, 211, 12, 1128, 648, 632, 212, 642}

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}}+Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c} + \frac{Cx}{c}$$

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (C*x)/c + ((A*c - b*C - (A*b*c - (b^2 - 2*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + ((A*c - b*C + (A*b*c - b^2*C + 2*a*c*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{Bx^3}{a+bx^2+cx^4}\,dx + \int \frac{x^2(A+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{Cx}{c} + B\int \frac{x^3}{a+bx^2+cx^4}\,dx - \frac{\int \frac{aC+(-Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{Cx}{c} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^2\right) - \frac{\left(-Ac+bC+\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(-Ac+bC-\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{B\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} - \frac{(bB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c} + \frac{(bB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.33

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{4\sqrt{c}Cx - \frac{2\sqrt{2}\left(Ac\left(b-\sqrt{b^2-4ac}\right)+\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\left(-Ac\left(b+\sqrt{b^2-4ac}\right)+\left(b^2-2ac+b\sqrt{b^2-4ac}\right.\right.}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4\right.}}}{4c^{3/2}}$$

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*Sqrt[c]*C*x - (2*Sqrt[2]*(A*c*(b - Sqrt[b^2 - 4*a*c]) + (-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*(-(A*c*(
b + Sqrt[b^2 - 4*a*c])) + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) + (B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c]
)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.26

method	result
risch	$\frac{Cx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(Bc\_R^3+\_R^2(Ac-Cb)-Ca\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{Cx}{c}-\frac{\left(b^2-4ac+b\sqrt{-4ac+b^2}\right)\left(\frac{B\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^2}-Cb\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c(4ac-b^2)}+\frac{\left(b\sqrt{-4ac\right.}}{}$

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*x/c+1/2/c*sum((B*c*_R^3+_R^2*(A*c-C*b)-C*a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=R
ootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 57.38 (sec) , antiderivative size = 861800, normalized size of antiderivative = 3191.85

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] C*x/c + integrate((B*c*x^3 - (C*b - A*c)*x^2 - C*a)/(c*x^4 + b*x^2 + a), x)
/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3843 vs. $2(227) = 454$.

Time = 1.47 (sec) , antiderivative size = 3843, normalized size of antiderivative = 14.23

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] C*x/c + 1/4*B*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^
4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
```

```
b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3
 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a
*c)*a*b*c^3)*C*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 -
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 -
4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*C*abs(c) - (2*b^4*c^5 - 8*a*b^2
*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)
*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*
C)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^
4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5
- 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*
A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

```
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*C*c^2 + 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4
+ 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2
- 4*a*c)*a^2*c^4)*C*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 -
 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*
(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*C)*arctan(2*sqrt(1/2)*x/sq
rt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*
a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/16*(
(b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b
^2*c^3 + (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2
- 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2
 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*
c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b*c^3 + sqr
t(b^2*c^6 - 4*a*c^7))/c^4)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 +
 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c)) - 1/16*((b^6 - 8*a*b^4*c
- 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 + (b^5 - 8
*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*sqrt
(b^2 - 4*a*c))*B*abs(c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3
 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 +
 b^3*c^3)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b*c^3 - sqrt(b^2*c^6 - 4*a*c
^7))/c^4)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*
b^2*c^2 - 4*a^2*c^3)*c^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 1890, normalized size of antiderivative = 7.00

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(- root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 12
8*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2
*z^2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C
^2*a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2
- 96*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*
z + 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2
*c*z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c -
 2*A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*
c - A^4*a*c^2 - C^4*a^3, z, k)*((8*B*C*a^2*c^2 - 4*A*B*a*b*c^2)/c - root(12
8*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a*b^2*c^3*z^3 +
16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2 + 8*A*C*b^4*c*z
^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*b^3*c*z^2 + 16*
A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*B^2*a^2*c^3*z^2
 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 16*B*C^2*a^2*b*c
*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z + 16*B^3*a^2*c^
2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A^2*C^2*a^2*c + 2
*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c - A^4*a*c^2 - C^4*
a^3, z, k)*((16*C*a^2*c^3 - 4*C*a*b^2*c^2)/c + (x*(8*B*b^3*c^2 - 32*B*a*b*c
^3))/c - (root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B
*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^
2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*
a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 9
6*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z +
 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*
z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*
A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c -
 A^4*a*c^2 - C^4*a^3, z, k)*x*(8*b^3*c^3 - 32*a*b*c^4))/c) + (x*(2*C^2*b^4
- 4*A^2*a*c^3 + 2*B^2*b^3*c + 2*A^2*b^2*c^2 + 4*C^2*a^2*c^2 - 4*A*C*b^3*c -
 10*B^2*a*b*c^2 - 8*C^2*a*b^2*c + 12*A*C*a*b*c^2))/c) - (A^3*a*c^2 - C^3*a^
2*b + A*C^2*a*b^2 + A*C^2*a^2*c - B^2*C*a^2*c + A*B^2*a*b*c - 2*A^2*C*a*b*c
)/c - (x*(B^3*a*b*c + A^2*B*a*c^2 + B*C^2*a*b^2 - B*C^2*a^2*c - 2*A*B*C*a*b
*c))/c)*root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a
*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2
+ 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*
b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*
B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 1
```

```
6*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z
+ 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A^
2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c - A
^4*a*c^2 - C^4*a^3, z, k), k, 1, 4) + (C*x)/c
```

3.24 $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{C\log(a+bx^2+cx^4)}{4c}$$

```
[Out] 1/4*C*ln(c*x^4+b*x^2+a)/c-1/2*(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^
(1/2))/c/(-4*a*c+b^2)^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/
2)+1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used

= {1676, 1261, 648, 632, 212, 642, 12, 1144, 211}

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = -\frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} - \frac{B\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{\sqrt{b^2-4ac}+b}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{C\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((B*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (B*Sqrt[b + Sqrt[b^2
- 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - ((2*A*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (C*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{Bx^2}{a+bx^2+cx^4}\,dx+\int \frac{x(A+Cx^2)}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{a+bx+cx^2}\,dx,x,x^2\right)+B\int \frac{x^2}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\left(B\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\
&\quad +\frac{1}{2}\left(B\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\
&\quad +\frac{C\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}+\frac{(2Ac-bC)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}
\end{aligned}$$

$$= -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{C\log(a+bx^2+cx^4)}{4c} - \frac{(2Ac-bC)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c}$$

$$= -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{C\log(a+bx^2+cx^4)}{4c}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{-2\sqrt{2}B\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + 2\sqrt{2}B\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right) + (2A}{4c}$$

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*Sqrt[2]*B*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + 2*Sqrt[2]*B*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]] + (2*A*c + (-
b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - (2*A*c -
(b + Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[
b^2 - 4*a*c])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(C\_R^3+B\_R^2+A\_R\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(\frac{\frac{\left(2Ac\sqrt{-4ac+b^2}-C\sqrt{-4ac+b^2}\,b+4acC-b^2C\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{\left(-Bb\sqrt{-4ac+b^2}+4Bac-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)}\right.$

```
[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((C*_R^3+B*_R^2+A*_R)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2
*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.96 (sec) , antiderivative size = 845032, normalized size of antiderivative = 3789.38

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2368 vs. $2(179) = 358$.

Time = 1.46 (sec) , antiderivative size = 2368, normalized size of antiderivative = 10.62

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/4*C*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a
^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*
a*c)*a*c^3)*B*c^2 - (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/s
qrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a
*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*c^2) - 1/8*((2
*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2
- 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - (2*b^4*c^4 - 8*a*b^2*c^5
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2
```

```
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*
c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a
*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c
^4 - 4*a^2*c^5)*c^2) + 1/16*(2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*
c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 - (b^4*c - 8*a*b^2*c^2 - 2*b^3*c^2
+ 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^3 - 4*a*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) -
 (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*
b^2*c^3 + (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2
 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*C*abs(c) - 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^
4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b
^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*A + (b^6*c - 8*a*b^4*c^2 -
 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c
- 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b
*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a
^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c)) + 1/16*(2*(b^5*c
- 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^
4 + (b^4*c - 8*a*b^2*c^2 - 2*b^3*c^2 + 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^3 - 4
*a*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b
^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 - (b^5 - 8*a*b^3*c - 2*b^4*c +
 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*C*abs
(c) + 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b
^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2
 - 4*a*c))*A - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*
c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*s
qrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a
^2*c^3)*c^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 5594, normalized size of antiderivative = 25.09

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(A^3*c^2*x - B^3*a*c - B*C^2*a*b - 8*root(128*a*b^2*c^3*z^4 - 16*
b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 1
6*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 +
 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z
^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z
 + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z -
 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*
```

c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a
^2 - A^4*c^2, z, k)^3*b^3*c^2*x - C^3*a*b*x + A*C^2*b^2*x - 2*C^2*root(128*
a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 25
6*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 4
0*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2
 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*
z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z
 + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C
^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c -
A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^3*x + 32*root(128*a*b^2*c^3*z^4 -
16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3
+ 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^
2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^
2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*
c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*
z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C
*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^
4*a^2 - A^4*c^2, z, k)^3*a*b*c^3*x - 4*A*root(128*a*b^2*c^3*z^4 - 16*b^4*c^
2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^
4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^
2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8
*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*
A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3
*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*
A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A
^4*c^2, z, k)^2*b^2*c^2*x - 8*A*B*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 -
 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3
 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c
^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^
2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a
*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*
z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a
*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2,
 z, k)*a*c^2 + A*B^2*b*c*x + A*C^2*a*c*x - 2*A^2*C*b*c*x - B^2*C*a*c*x + 2*
A*B*C*a*c + 16*A*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4
- 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2
*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^
3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2
*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*
b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z
 + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b
- A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^2*a*c^3*x
+ 2*A^2*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a
*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*
A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 -

```
 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2
- 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z -
 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^
2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2
*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b*c^2*x + 4*B^2*roo
t(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^
3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z
^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c
^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*
a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a
*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2
*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*
a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*c^2*x - 2*B^2*root(128*a*b^2
*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a
^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2
*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96
*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4
*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16
*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*
c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C
^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^2*c*x + 8*C*root(128*a*b^2*c^3*z^4 - 16
*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 +
16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2
+ 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*
z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*
z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z
- 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b
*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*
a^2 - A^4*c^2, z, k)^2*b^3*c*x - 32*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z
^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c
*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a
*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^
2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2
*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*
b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C
^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*
c^2, z, k)^2*a*b*c^2*x + 4*B*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 25
6*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 +
32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*
z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c
^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^
2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z +
 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b
- B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z,
 k)*a*b*c - 8*A*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4
```

```
 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^
2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b
^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^
2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2
*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*
z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b
 - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*c^2*x +
 10*C^2*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a
*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*
A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 -
 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2
- 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z -
 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^
2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2
*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*b*c*x)*root(128*a
*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256
*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40
*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2
- 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z
 - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z
+ 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^
2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A
^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k), k, 1, 4)
```

3.25 $\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx=\frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{B\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(C+(2*A*c-C*b)/(-4*a*c+b^2)^(
1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(C+(-2*A*c+C*b)/(-4*a*c+b^2)^(1/2))*2^(1
/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1687, 1180, 211, 12, 1121, 632, 212}

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx=\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((C -
(2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{Bx}{a+bx^2+cx^4}\,dx + \int \frac{A+Cx^2}{a+bx^2+cx^4}\,dx \\
&= B\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\
&\quad + \frac{1}{2}\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\
&= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad + \frac{1}{2}B\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad - B\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right) \\
&= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11

$$
\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \frac{\frac{\sqrt{2}\left(2Ac+\left(-b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac+\left(b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + B\log\left(-b+\sqrt{b^2-\right.}{2\sqrt{b^2-4ac}}
$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*A*c + (-b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
```

```
2]*(-2*A*c + (b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + B*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2] - B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqr
t[b^2 - 4*a*c])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(C\_R^2+B\_R+A\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(\frac{\sqrt{-4ac+b^2}\left(\frac{B\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^2}-Cb\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c(4ac-b^2)}-\frac{\sqrt{-4ac+b^2}\left(\frac{B\ln\left(-2c\,x^2\right.}{}\right.}{}\right.$

```
[In] int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*sum((C*_R^2+B*_R+A)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.86 (sec) , antiderivative size = 578003, normalized size of antiderivative = 2739.35

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} \, dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} \, dx = \int \frac{Cx^2 + Bx + A}{cx^4 + bx^2 + a} \, dx$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(171) = 342$.

Time = 1.29 (sec) , antiderivative size = 1714, normalized size of antiderivative = 8.12

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*B*log(x^2 + 1/2*
(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b
*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^
2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^
3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2
 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^
```

```
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*C)*arctan(
2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*
b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b
^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c
^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)
*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2
- 2*(b^2 - 4*a*c)*a*c^2)*C)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c
))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2
*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b
*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 6*a*b^2*c^2 - 2*b^3*c^2
 + 8*a^2*c^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c))*B*log(x^2
+ 1/2*(b + sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3
*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 3942, normalized size of antiderivative = 18.68

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(A*B^2*c^2 - A^2*C*c^2 + B^3*c^2*x - C^3*a*c + A*C^2*b*c - 8*root
(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^
2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^
2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C
^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2
*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*
b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*b^3*c^2*x - 16*A*r
oot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c
*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C
*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*
```

B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*
B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B
^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*a*c^3 - 4*A^2*r
oot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c
*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C
*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*
B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*
B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B
^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*c^3*x + 4*A*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2
 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2
*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^
2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*
C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b
*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*b^2*c^2 + 32*root(1
6*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2
- 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*
c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2
*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C
*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b*
c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*a*b*c^3*x - 4*B*root
(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^
2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^
2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C
^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2
*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*
b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*b^2*c^2*x + 4*A*B*
root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*
c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*
C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16
*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A
*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*
B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*b*c^2 - 8*B*C*ro
ot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*
z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*
a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B
*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B
^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^
2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*a*c^2 - 2*A*B*C*c^
2*x + B*C^2*b*c*x + 16*B*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^
3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 1
6*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2
+ 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z
 - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3
*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^

```
2, z, k)^2*a*c^3*x + 2*B^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*b*c^2*x + 4*C^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*a*c^2*x - 2*C^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*b^2*c*x + 4*A*C*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*b*c^2*x)*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3
*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2
*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*
B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16
*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b
+ B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z,
 k), k, 1, 4)
```

3.26 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{A\log(x)}{a} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a}$$

```
[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a+1/2*(A*b-2*C*a)*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c
^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1676, 1265, 814, 648, 632, 212, 642, 12, 1107, 211}

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a} + \frac{A\log(x)}{a} + \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4]
)/(4*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{B}{a+bx^2+cx^4}\,dx + \int \frac{A+Cx^2}{x\,(a+bx^2+cx^4)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x\,(a+bx+cx^2)}\,dx, x, x^2\right) + B\int \frac{1}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax} + \frac{-Ab+aC-Acx}{a\,(a+bx+cx^2)}\right)dx, x, x^2\right) \\
&\quad + \frac{(Bc)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(Bc)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad + \frac{A\log(x)}{a} + \frac{\text{Subst}\left(\int \frac{-Ab+aC-Acx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a}
\end{aligned}$$

$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{A\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a} + \frac{(-Ab+2aC)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}$$

$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a} - \frac{(-Ab+2aC)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a}$$

$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{A\log(x)}{a} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{\left(A\left(b+\sqrt{b^2-4ac}\right)-2aC\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{4a\sqrt{b^2-4ac}} - \frac{\left(A\left(-b+\sqrt{b^2-4ac}\right)+2aC\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4a\sqrt{b^2-4ac}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + (A*Log[x])/a - ((A*(b + Sqrt[b^2 - 4*a*c]) - 2*a
*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c]) - ((A*(-
b + Sqrt[b^2 - 4*a*c]) + 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*
Sqrt[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

method	result
default	$\frac{A\ln(x)}{a}+\frac{4c\left(\frac{\sqrt{-4ac+b^2}\left(\frac{\left(A\sqrt{-4ac+b^2}-Ab+2Ca\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{Ba\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac-4b^2}+\frac{\sqrt{-4ac+b^2}\left(-\frac{(-\cdots}{\cdots}\right.}{\cdots}\right.}{a}$
risch	$\frac{A\ln(x)}{a}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(16a^4c^2-8a^3b^2c+a^2b^4\right)\_Z^4+\left(32A\,a^3c^2-16A\,a^2b^2c+2Aa\,b^4\right)\_Z^3+\left(24a^2c^2A^2-10a\,b^2c\,A^2+b^4A^2-8AC\,a^2bc+2A\cdots\right.\right.\right.}{\cdots}$

[In] int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] A*ln(x)/a+4/a*c*((-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(1/4*(A*(-4*a*c+b^2)^(1/2)-A*b+2*C*a)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+B*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+(-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(-1/4*(-A*(-4*a*c+b^2)^(1/2)-A*b+2*C*a)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx = \int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] A*log(x)/a - integrate((A*c*x^3 - B*a - (C*a - A*b)*x)/(c*x^4 + b*x^2 + a),
 x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2339 vs. 2(186) = 372.

Time = 1.43 (sec) , antiderivative size = 2339, normalized size of antiderivative = 10.21

$$\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -1/4*A*log(abs(c*x^4 + b*x^2 + a))/a + A*log(abs(x))/a + 1/4*((sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 +
 16*a*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^
3 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) - (2*b^3*c^3 -
8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*
b*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c + sqrt(a^4*b^2*c^2 - 4*a^5*c^3
))/(a^2*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
```

```
*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 4*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 2*(b^2 - 4*a*c)*b^2*
c + 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*B)*arctan(2*sqrt(
1/2)*x/sqrt((a^2*b*c - sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2)))/((a*b^4 -
 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3
)*c^2) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*
c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c
^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - 2*(a*
b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*
c^3 - 4*a^2*b*c^4 + (a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8
*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) - (b^6*c^2
- 8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^
2*c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A
+ 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4
 + a*b^3*c^4 - 4*a^2*b*c^5 + (a*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b
^2*c^4)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^2*b*c + sqrt(a^4*b^2*c^2 - 4
*a^5*c^3))/(a^2*c^2))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 +
8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*c^2*abs(c)) - 1/16*((b^6*c - 8*a*b^4
*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (
b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*
a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4
*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 - (a*b^4*c -
8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*
c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) - (b^6*c^2 - 8*a*b^4*c^3 - 2*b^5*c^3 + 16*
a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 + (b^5*c^2 - 4*a*b^3*c^3
- 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3
- 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 - (a
*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b^2*c^4)*sqrt(b^2 - 4*a*c))*C)*l
og(x^2 + 1/2*(a^2*b*c - sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2))/((a^2*b^4
 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a
^3*c^3)*c^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 2258, normalized size of antiderivative = 9.86

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(x*(B^4*c^3 + C^4*a*c^2 + A^2*C^2*c^3 - 3*A*B^2*C*c^3 - A*C^3*b*c
^2 + B^2*C^2*b*c^2) - root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4
*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^
2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*
C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 -
4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*
B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*
b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2
*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(A
*B^2*b*c^3 - 5*A^3*c^4 - 13*A*C^2*a*c^3 + 6*A^2*C*b*c^3 + 17*B^2*C*a*c^3 +
C^3*a*b*c^2 + A*C^2*b^2*c^2 - 4*B^2*C*b^2*c^2) - root(128*a^3*b^2*c*z^4 - 2
56*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 -
 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2
 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*
z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*
c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z
- 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*
b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4
*a^2 - A^4*c^2, z, k)*(x*(60*A^2*a*c^4 - 16*A^2*b^2*c^3 + 4*B^2*b^3*c^2 + 3
6*C^2*a^2*c^3 + 8*A*C*b^3*c^2 - 14*B^2*a*b*c^3 - 10*C^2*a*b^2*c^2 - 28*A*C*
a*b*c^3) + root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*
A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 -
 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z
^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z
^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2
*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A
*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*
B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(240*A*a^2*c^
4 + 12*A*b^4*c^2 - 108*A*a*b^2*c^3 + 4*C*a*b^3*c^2 - 16*C*a^2*b*c^3) + root
(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3
 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^
2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^
3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a
*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*
b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*
A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a
```

```
*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*x*(320*a^3*c^4 + 24*a*b^4*c^2 -
 176*a^2*b^2*c^3) - 4*B*a*b^3*c^2 + 16*B*a^2*b*c^3) + 4*A*B*b^3*c^2 + 8*B*C
*a^2*c^3 - 12*A*B*a*b*c^3 - 4*B*C*a*b^2*c^2) + B^3*a*c^3 + 4*A^2*B*b*c^3 +
6*A*B*C*a*c^3 - 4*A*B*C*b^2*c^2 + B*C^2*a*b*c^2) + A*B^3*c^3 - 2*A^2*B*C*c^
3 + A*B*C^2*b*c^2)*root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^
4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b
*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2
*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A
^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2
*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3
*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*
b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k), k, 1, 4
) + (A*log(x))/a
```

3.27 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 260

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx = -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{B\log(x)}{a} - \frac{B\log(a+bx^2+cx^4)}{4a}$$

```
[Out] -A/a/x+B*ln(x)/a-1/4*B*ln(c*x^4+b*x^2+a)/a+1/2*b*B*arctanh((2*c*x^2+b)/(-4*
a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*c^(1/2)*(A+(A*b-2*C*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(A+(-A*b+2*C*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules

used = {1676, 1295, 1180, 211, 12, 1128, 719, 29, 648, 632, 212, 642}

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}} - \frac{A}{ax} + \frac{bB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{B\log\left(a+bx^2+cx^4\right)}{4a} + \frac{B\log(x)}{a}$$

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c])
 + (B*Log[x])/a - (B*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
```

```
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{B}{x\left(a+bx^2+cx^4\right)}\,dx+\int \frac{A+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx\\
&=-\frac{A}{ax}-\frac{\int \frac{Ab-aC+Acx^2}{a+bx^2+cx^4}\,dx}{a}+B\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx\\
&=-\frac{A}{ax}+\frac{1}{2}B\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&\quad-\frac{\left(c\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}\\
&\quad-\frac{\left(c\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}\\
&=-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}\\
&\quad+\frac{B\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}+\frac{B\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&=-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}\\
&\quad+\frac{B\log(x)}{a}-\frac{B\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}-\frac{(bB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}\\
&=-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}\\
&\quad+\frac{B\log(x)}{a}-\frac{B\log\left(a+bx^2+cx^4\right)}{4a}+\frac{(bB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a}
\end{aligned}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A + \frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt{c}\left(A - \frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{B\log(x)}{a} - \frac{B\log\left(a+bx^2+cx^4\right)}{4a}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx =$$
$$-\frac{\frac{4A}{x} + \frac{2\sqrt{2}\sqrt{c}\left(A\left(b+\sqrt{b^2-4ac}\right)-2aC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{2\sqrt{2}\sqrt{c}\left(A\left(-b+\sqrt{b^2-4ac}\right)+2aC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} - 4B\log(}{4a}$$

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*((4*A)/x + (2*Sqrt[2]*Sqrt[c]*(A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C)*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (2*Sqrt[2]*Sqrt[c]*(A*(-b + Sqrt[b^2 - 4*a*c])
+ 2*a*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - 4*B*Log[x] + (B*(b + Sqrt[b^2 - 4*
a*c])*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*(-b + S
qrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/
a
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.25

method	result
default	$-\frac{A}{ax}+\frac{B\ln(x)}{a}+\frac{4c\left(\frac{\frac{\left(-Bb\sqrt{-4ac+b^2}-4Bac+Bb^2\right)\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{\left(-Ab\sqrt{-4ac+b^2}-4Aac+Ab^2+2C\sqrt{-4ac+b^2}\,a\right)\sqrt{2}\,\text{arcta}}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{16ac-4b^2}\right.}{}$
risch	$-\frac{A}{ax}+\frac{\left(\_R=\text{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(32Ba^4c^2-16Ba^3b^2c+2Ba^2b^4\right)\_Z^3+\left(12A^2a^2bc^2-7A^2ab^3c+A^2b^5-16ACa^3c^2+12\right.\right.\right.}{}$

```
[In] int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -A/a/x+B*ln(x)/a+4/a*c*(1/(16*a*c-4*b^2)*(1/4*(-B*b*(-4*a*c+b^2)^(1/2)-4*B*
a*c+B*b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*(-A*b*(-4*a*c+b^2)^(1/2)-
4*A*a*c+A*b^2+2*C*(-4*a*c+b^2)^(1/2)*a)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2
)*(-1/4*(-B*b*(-4*a*c+b^2)^(1/2)+4*B*a*c-B*b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^
(1/2)-b)+1/2*(-A*b*(-4*a*c+b^2)^(1/2)+4*A*a*c-A*b^2+2*C*(-4*a*c+b^2)^(1/2)*
a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int\frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x^2}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*log(x)/a - integrate((B*c*x^3 + A*c*x^2 + B*b*x - C*a + A*b)/(c*x^4 + b*x
^2 + a), x)/a - A/(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3505 vs. $2(218) = 436$.

Time = 1.57 (sec) , antiderivative size = 3505, normalized size of antiderivative = 13.48

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*B*log(abs(c*x^4 + b*x^2 + a))/a + B*log(abs(x))/a - A/(a*x) - 1/8*((2*
b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 -
 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3
+ 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2
```

```
*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(c) - 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 +
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 -
 4*a*c)*a^2*c^3)*C*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*A - 2*(2*a*b^3*c^
4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 -
2*(b^2 - 4*a*c)*a*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c + sqrt(a^4*b
^2*c^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^
2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) + 1/8*((2*b^4*
c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a
*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16
*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c
^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(c) + 2*(sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c
^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a
*c)*a^2*c^3)*C*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*A - 2*(2*a*b^3*c^4 -
8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
```

```
^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b
^2 - 4*a*c)*a*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c - sqrt(a^4*b^2*c
^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^2 +
16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) - 1/16*((b^6*c - 8
*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c
^4 + (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^
3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*B*abs(c) + (b^6*c^2 - 8*a*b^4*c^3 - 2*b^5
*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 + (b^5*c^2 - 4*
a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(a^2*b
*c + sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2))/((a^2*b^4 - 8*a^3*b^2*c - 2*
a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*c^2*abs(c))
 - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 +
b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8
*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*B*abs(c) + (b^6*c^2 -
8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*
c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*B)*l
og(x^2 + 1/2*(a^2*b*c - sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2))/((a^2*b^4
 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a
^3*c^3)*c^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 2588, normalized size of antiderivative = 9.95

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*
B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z
^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2
*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 -
96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z
- 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c
*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2
*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c
- B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16
*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 -
 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2
*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 -
 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 -
 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*
```

c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c
- B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2
*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 2
56*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 -
16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2
*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 6
4*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^
2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c
^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c
^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*
C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*((16*A*a
^3*c^4 + 4*A*a*b^4*c^2 + 16*C*a^3*b*c^3 - 20*A*a^2*b^2*c^3 - 4*C*a^2*b^3*c^
2)/a + (x*(240*B*a^4*c^4 + 12*B*a^2*b^4*c^2 - 108*B*a^3*b^2*c^3))/a^2 + (ro
ot(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z
^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*
b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2
+ 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^
2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*
b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a
^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^
2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 -
A^4*c^3, z, k)*x*(320*a^5*c^4 + 24*a^3*b^4*c^2 - 176*a^4*b^2*c^3))/a^2) -
(8*A*B*a^2*c^4 + 4*A*B*b^4*c^2 - 16*A*B*a*b^2*c^3 - 4*B*C*a*b^3*c^2 + 12*B*
C*a^2*b*c^3)/a + (x*(4*A^2*b^5*c^2 + 60*B^2*a^3*c^4 - 16*B^2*a^2*b^2*c^3 +
4*C^2*a^2*b^3*c^2 - 72*A*C*a^3*c^4 - 28*A^2*a*b^3*c^3 + 50*A^2*a^2*b*c^4 -
14*C^2*a^3*b*c^3 + 48*A*C*a^2*b^2*c^3 - 8*A*C*a*b^4*c^2))/a^2) - (C^3*a^2*c
^3 + 7*A*B^2*a*c^4 + A^2*C*a*c^4 - 4*A*B^2*b^2*c^3 - A*C^2*a*b*c^3 + 4*B^2*
C*a*b*c^3)/a + (x*(5*B^3*a^2*c^4 - 4*A^2*B*b^3*c^3 - B*C^2*a^2*b*c^3 - 26*A
*B*C*a^2*c^4 + 14*A^2*B*a*b*c^4 + 8*A*B*C*a*b^2*c^3))/a^2) - (A*B^3*c^4 - A
^2*B*C*c^4 - B*C^3*a*c^3 + A*B*C^2*b*c^3)/a + (x*(A^4*c^5 + C^4*a^2*c^3 + A
^2*C^2*b^2*c^3 - 2*A^3*C*b*c^4 + A^2*B^2*b*c^4 + 2*A^2*C^2*a*c^4 - 2*A*B^2*
C*a*c^4 - 2*A*C^3*a*b*c^3))/a^2)*root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 -
16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^
3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*
a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^
2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^
2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b
^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b
*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*
B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k), k, 1, 4) - A/(a*x) + (B
*log(x))/a

3.28 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 288

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx = -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}-\frac{(A(b^2-2ac)-abC)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] -1/2*A/a/x^2-B/a/x-(A*b-C*a)*ln(x)/a^2+1/4*(A*b-C*a)*ln(c*x^4+b*x^2+a)/a^2-
1/2*(A*(-2*a*c+b^2)-a*b*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/
2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4
*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1676, 1265, 814, 648, 632, 212, 642, 12, 1137, 1180, 211}

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=-\frac{(A(b^2-2ac)-abC)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B}{ax}$$

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*A/(a*x^2) - B/(a*x) - (B*Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2
- 4*a*c]]) - (B*Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
(A*(b^2 - 2*a*c) - a*b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*
Sqrt[b^2 - 4*a*c]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2
+ c*x^4])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{B}{x^2\left(a+bx^2+cx^4\right)}\,dx+\int \frac{A+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)+B\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx\\
&= -\frac{B}{ax}\\
&\quad +\frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aC}{a^2x}+\frac{A(b^2-ac)-abC+c(Ab-aC)x}{a^2\left(a+bx+cx^2\right)}\right)\,dx,x,x^2\right)\\
&\quad +\frac{B\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}\\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{A(b^2-ac)-abC+c(Ab-aC)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}\\
&\quad -\frac{\left(Bc\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(Bc\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}\\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad -\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}\\
&\quad -\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2}\\
&\quad +\frac{\left(A(b^2-2ac)-abC\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2}
\end{aligned}$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(Ab-aC)\log(x)}{a^2} + \frac{(Ab-aC)\log(a+bx^2+cx^4)}{4a^2} - \frac{(A(b^2-2ac)-abC)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^2}$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(A(b^2-2ac)-abC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{(Ab-aC)\log(x)}{a^2} + \frac{(Ab-aC)\log(a+bx^2+cx^4)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$$

$$= \frac{-\frac{2aA}{x^2} - \frac{4aB}{x} - \frac{2\sqrt{2}aB\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}aB\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + 4(-Ab+a\mathrm{C}$$

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*Sqrt[2]*a*B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*a*B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*(-(A*b) + a*C)*Log[x] + ((A*(b^2 - 2
*a*c + b*Sqrt[b^2 - 4*a*c]) - a*(b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^
2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((A*(-b^2 + 2*a*c + b*Sqrt[b^2 -
 4*a*c]) + a*(b - Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.04

method	result
default	$-\frac{A}{2a\,x^2}-\frac{B}{ax}+\frac{(-Ab+Ca)\ln(x)}{a^2}+ 4c\left(-\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)\left(\frac{\left(A\sqrt{-4ac+b^2}-Ab+2Ca\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{Ba\sqrt{2}\,\arctan}{\sqrt{(b}}\right.}{32ac-8b^2}\right.$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/a/x^2-B/a/x+1/a^2*(-A*b+C*a)*ln(x)+4/a^2*c*(-(b*(-4*a*c+b^2)^(1/2)+4
*a*c-b^2)/(32*a*c-8*b^2)*(1/4*(A*(-4*a*c+b^2)^(1/2)-A*b+2*C*a)/c*ln(2*c*x^2
+(-4*a*c+b^2)^(1/2)+b)+B*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-(b^2-4*a*c+b*(-4*a*c+b^2)^(1
/2))/(32*a*c-8*b^2)*(-1/4*(-A*(-4*a*c+b^2)^(1/2)-A*b+2*C*a)/c*ln(-2*c*x^2+(
-4*a*c+b^2)^(1/2)-b)+B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x^3}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] (C*a - A*b)*log(x)/a^2 + integrate(-(B*a*c*x^2 + (C*a - A*b)*c*x^3 + B*a*b
+ (C*a*b - A*b^2 + A*a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2*(2*B*x + A)/
(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3353 vs. $2(240) = 480$.

Time = 1.49 (sec) , antiderivative size = 3353, normalized size of antiderivative = 11.64

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(C*a - A*b)*log(abs(c*x^4 + b*x^2 + a))/a^2 + (C*a - A*b)*log(abs(x))/
a^2 + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - 2*(sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5
*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*B
*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b
*c + sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2
 - 2*a^2*b^3*c^2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2)
 + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
```

sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3
 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - 2*(sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^
2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c
^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*ab
s(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b*c
- sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 -
2*a^2*b^3*c^2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) +
1/16*((b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b
^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 - (b^6*c
 - 10*a*b^4*c^2 - 2*b^5*c^2 + 32*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 32*
a^3*c^4 - 16*a^2*b*c^4 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(b^2 - 4*a*c))*A*abs(
c) - (a*b^6*c - 8*a^2*b^4*c^2 - 2*a*b^5*c^2 + 16*a^3*b^2*c^3 + 8*a^2*b^3*c^
3 + a*b^4*c^3 - 4*a^2*b^2*c^4 + (a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16
*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(b^2 - 4*a*c))*C*
abs(c) - (b^7*c^2 - 10*a*b^5*c^3 - 2*b^6*c^3 + 32*a^2*b^3*c^4 + 12*a*b^4*c^
4 + b^5*c^4 - 32*a^3*b*c^5 - 16*a^2*b^2*c^5 - 6*a*b^3*c^5 + 8*a^2*b*c^6 + (
b^6*c^2 - 6*a*b^4*c^3 - 2*b^5*c^3 + 8*a^2*b^2*c^4 + 4*a*b^3*c^4 + b^4*c^4 -
 2*a*b^2*c^5)*sqrt(b^2 - 4*a*c))*A + (a*b^6*c^2 - 8*a^2*b^4*c^3 - 2*a*b^5*c
^3 + 16*a^3*b^2*c^4 + 8*a^2*b^3*c^4 + a*b^4*c^4 - 4*a^2*b^2*c^5 + (a*b^5*c^
2 - 4*a^2*b^3*c^3 - 2*a*b^4*c^3 + a*b^3*c^4)*sqrt(b^2 - 4*a*c))*C)*log(x^2
+ 1/2*(a^4*b*c + sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2))/((a^3*b^4 - 8*a^
4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)
*c^2*abs(c)) + 1/16*((b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 1
2*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2
*b*c^5 + (b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 32*a^2*b^2*c^3 + 12*a*b^3*c^3
+ b^4*c^3 - 32*a^3*c^4 - 16*a^2*b*c^4 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(b^2 -
 4*a*c))*A*abs(c) - (a*b^6*c - 8*a^2*b^4*c^2 - 2*a*b^5*c^2 + 16*a^3*b^2*c^3
 + 8*a^2*b^3*c^3 + a*b^4*c^3 - 4*a^2*b^2*c^4 - (a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(b
^2 - 4*a*c))*C*abs(c) - (b^7*c^2 - 10*a*b^5*c^3 - 2*b^6*c^3 + 32*a^2*b^3*c^
4 + 12*a*b^4*c^4 + b^5*c^4 - 32*a^3*b*c^5 - 16*a^2*b^2*c^5 - 6*a*b^3*c^5 +
8*a^2*b*c^6 + (b^6*c^2 - 6*a*b^4*c^3 - 2*b^5*c^3 + 8*a^2*b^2*c^4 + 4*a*b^3*

```
c^4 + b^4*c^4 - 2*a*b^2*c^5)*sqrt(b^2 - 4*a*c))*A + (a*b^6*c^2 - 8*a^2*b^4*
c^3 - 2*a*b^5*c^3 + 16*a^3*b^2*c^4 + 8*a^2*b^3*c^4 + a*b^4*c^4 - 4*a^2*b^2*
c^5 - (a*b^5*c^2 - 4*a^2*b^3*c^3 - 2*a*b^4*c^3 + a*b^3*c^4)*sqrt(b^2 - 4*a*
c))*C)*log(x^2 + 1/2*(a^4*b*c - sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2))/(
(a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c
^2 - 4*a^4*c^3)*c^2*abs(c)) - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 3563, normalized size of antiderivative = 12.37

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*
C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2
*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C
*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*
z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 -
4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z
^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C
*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z
+ 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3
*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*
c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 -
 A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)
*(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2
*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16
*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*
c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*
B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b
^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^
2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^
2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*
a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A
*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A
*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*
b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)*(root(12
8*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 +
256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^
4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8
*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b
^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b^5*z^2 -
```

```
96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^2*b^6*z^2
 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^2*z - 4*A
*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*a*b*c^3*z
 + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A*B^2*a^2*
c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A*B^2*C*a*
c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*b*c^3 - A
^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)*((16*B*a^5*c^4 +
4*B*a^3*b^4*c^2 - 20*B*a^4*b^2*c^3)/a^3 + (x*(240*C*a^5*c^4 - 224*A*a^4*b*c
^4 - 12*A*a^2*b^5*c^2 + 104*A*a^3*b^3*c^3 + 12*C*a^3*b^4*c^2 - 108*C*a^4*b^
2*c^3))/a^3 + (root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 +
128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5
*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72
*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*
c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^
2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c
^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A
^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*
c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2
*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*
a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c
^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z
, k)*x*(320*a^6*c^4 + 24*a^4*b^4*c^2 - 176*a^5*b^2*c^3))/a^3) - (8*B*C*a^4*
c^4 + 20*A*B*a^2*b^3*c^3 + 4*B*C*a^2*b^4*c^2 - 16*B*C*a^3*b^2*c^3 - 4*A*B*a
*b^5*c^2 - 20*A*B*a^3*b*c^4)/a^3 + (x*(36*A^2*a^3*c^5 + 60*C^2*a^4*c^4 + 22
*A^2*a^2*b^2*c^4 - 28*B^2*a^2*b^3*c^3 - 16*C^2*a^3*b^2*c^3 - 8*A^2*a*b^4*c^
3 + 4*B^2*a*b^5*c^2 + 50*B^2*a^3*b*c^4 + 24*A*C*a^2*b^3*c^3 - 92*A*C*a^3*b*
c^4))/a^3) - (A^2*B*a^2*c^5 + 7*B*C^2*a^3*c^4 - 4*A^2*B*a*b^2*c^4 - 4*B*C^2
*a^2*b^2*c^3 + 4*A*B*C*a*b^3*c^3 - 4*A*B*C*a^2*b*c^4)/a^3 + (x*(2*A^3*b^3*c
^4 + 5*C^3*a^3*c^4 - 12*A^3*a*b*c^5 - 17*A*B^2*a^2*c^5 + 13*A^2*C*a^2*c^5 +
 6*A*B^2*a*b^2*c^4 - 9*A*C^2*a^2*b*c^4 + 2*A^2*C*a*b^2*c^4 - 4*B^2*C*a*b^3*
c^3 + 14*B^2*C*a^2*b*c^4))/a^3) - (A^3*B*b*c^5 + B*C^3*a^2*c^4 - A^2*B*C*a*
c^5 - A*B*C^2*a*b*c^4)/a^3 + (x*(A^4*c^6 + B^4*a*c^5 - A^3*C*b*c^5 + A^2*C^
2*a*c^5 + B^2*C^2*a*b*c^4 - 3*A*B^2*C*a*c^5))/a^3)*root(128*a^5*b^2*c*z^4 -
 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z
^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*
b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 +
40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2
*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^
2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b
*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z
+ 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*
z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^
3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a
*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 -
C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k), k, 1, 4) - (A/(2*a) + (B*x)/a)/x^
```

```
2 - (log(x)*(A*b - C*a))/a^2
```

3.29 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 412

$$\begin{aligned}
&\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{(2Ac-bC)x}{2c(b^2-4ac)}+\frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&+\frac{\left(Abc+(b^2-6ac)C-\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\
&+\frac{\left(Abc+(b^2-6ac)C+\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} \\
&+\frac{2aB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

```
[Out] 1/2*(2*A*c-C*b)*x/c/(-4*a*c+b^2)+1/2*B*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)-1/2*x^3*(A*b-2*C*a+(2*A*c-C*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2
*a*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(
x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(A*b*c+(-6*a*c+b^2)*C+(-A*c
*(4*a*c+b^2)-b*(-8*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*(A*b*c+(-6*a*c+b^2)*C+(A*c*(4*a*c+b^2)+b*(-8*a*c+b^2)*C)/
(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1676, 1289, 1293, 1180, 211, 12, 1128, 736, 632, 212}

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C(b^2-6ac)+Abc\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C(b^2-6ac)+Abc\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{x^3(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{x(2Ac-bC)}{2c(b^2-4ac)}$$
$$+\frac{2aB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*A*c - b*C)*x)/(2*c*(b^2 - 4*a*c)) + (B*x^2*(2*a + b*x^2))/(2*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) - (x^3*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + ((A*b*c + (b^2 - 6*a*c)*C - (A*c*(b^2 + 4*a
*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + ((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*
a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2
*a*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
```

```
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{Bx^5}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^4(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + B\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{x^2\left(3(Ab-2aC)+(2Ac-bC)x^2\right)}{a+bx^2+cx^4}\,dx}{2\,(b^2-4ac)}$$

$$= \frac{(2Ac-bC)x}{2c\,(b^2-4ac)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right) - \frac{\int \frac{a(2Ac-bC)+\left(-Abc-\left(b^2-6ac\right)C\right)x^2}{a+bx^2+cx^4}\,dx}{2c\,(b^2-4ac)}$$

$$= \frac{(2Ac-bC)x}{2c\,(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(aB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac} + \frac{\left(Abc+(b^2-6ac)\,C-\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c\,(b^2-4ac)} + \frac{\left(Abc+(b^2-6ac)\,C+\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c\,(b^2-4ac)}$$

$$= \frac{(2Ac-bC)x}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$
$$+ \frac{\left(Abc+(b^2-6ac)C-\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(Abc+(b^2-6ac)C+\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{(2aB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= \frac{(2Ac-bC)x}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$
$$+ \frac{\left(Abc+(b^2-6ac)C-\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(Abc+(b^2-6ac)C+\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{2aB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.08

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{2(bx^2(-Acx+b(B+Cx))+a(b(B+Cx)-2cx(A+x(B+Cx))))}{c(-b^2+4ac)(a+bx^2+cx^4)}\right.$$
$$+ \frac{\sqrt{2}\left(-Ac\left(b^2+4ac-b\sqrt{b^2-4ac}\right)+\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-}}}\right.}{c^{3/2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt{2}\left(Ac\left(b^2+4ac+b\sqrt{b^2-4ac}\right)+\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^{3/2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left. - \frac{4aB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{4aB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(b*x^2*(-(A*c*x) + b*(B + C*x)) + a*(b*(B + C*x) - 2*c*x*(A + x*(B + C*
x)))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(A*c*(b^2 + 4*a*
c - b*Sqrt[b^2 - 4*a*c])) + (-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c
*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(
A*c*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4
*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 - (4*a*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*a
*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{\left(Abc+2acC-b^2C\right)x^3}{2c\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)Bx^2}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-Cb)x}{2\left(4ac-b^2\right)c}+\frac{aBb}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\left(Abc-6acC+b^2C\right)\_R^2}{c\left(4ac-b^2\right)}+\frac{4\ldots}{4a\ldots}\right.}{2c\_R^3\ldots}\right.}{4}$
default	$\frac{-\frac{\left(Abc+2acC-b^2C\right)x^3}{2c\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)Bx^2}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-Cb)x}{2\left(4ac-b^2\right)c}+\frac{aBb}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{\frac{2B\sqrt{-4ac+b^2}\,ac\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(4Aac^2\sqrt{-4ac+b^2}+Ab^2c\ldots\right.}{\ldots}}{\ldots}}{\ldots}$

```
[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(A*b*c+2*C*a*c-C*b^2)/c/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)*B/c/(4*a*c-b^
2)*x^2-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c*x+1/2*a/c/(4*a*c-b^2)*B*b)/(c*x^4+b*
x^2+a)+1/4*sum((-(A*b*c-6*C*a*c+C*b^2)/c/(4*a*c-b^2)*_R^2+4/(4*a*c-b^2)*_R*
B*a+a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+
_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*((C*b^2 - (2*C*a + A*b)*c)*x^3 + B*a*b + (B*b^2 - 2*B*a*c)*x^2 + (C*a*
b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4
*a*b*c^2)*x^2) + 1/2*integrate(-(4*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (6*
C*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5217 vs. 2(361) = 722.

Time = 1.86 (sec) , antiderivative size = 5217, normalized size of antiderivative = 12.66

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(C*b^2*x^3 - 2*C*a*c*x^3 - A*b*c*x^3 + B*b^2*x^2 - 2*B*a*c*x^2 + C*a*b
*x - 2*A*a*c*x + B*a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*
```

b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2
 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4
*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 +
16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3
c^6 + 2(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4
*a*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a
*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(b^2*c - 4*a*c^2) - (2*b^7*
c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b
*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a

*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4*a*c^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^

```
3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2
 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c
^6)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c
^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))
/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c
^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*ab
s(b^2*c - 4*a*c^2)*abs(c)) - 1/4*((b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3 +
(b^2*c - 4*a*c^2 - 2*b*c^2 + c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2*c - 4*a*c^2)
 + (b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^
4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*
a*c))*B)*log(x^2 + 1/2*(b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*
(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3))/((b^4 - 8*
a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2*abs(b^2
*c - 4*a*c^2)) - 1/4*((b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3 + (b^2*c - 4*a
*c^2 - 2*b*c^2 + c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2*c - 4*a*c^2) - (b^5*c^2
- 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^
5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log
(x^2 + 1/2*(b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4
*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3))/((b^4 - 8*a*b^2*c - 2*
b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2*abs(b^2*c - 4*a*c^2
))
```

Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 4754, normalized size of antiderivative = 11.54

$$\int \frac{x^4(A+Bx+Cx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log(- root(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680
*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c
^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^
5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c
*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*
z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5
*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*
a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536
```

*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*
b^11*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*
a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c
^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^
3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*
c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4
*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c -
960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144
*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C
^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*
c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c
^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*
a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*c^4, z, k)*(root(1572864*a^5*b^2*c^8*
z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z
^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C
*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 204
8*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288
*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^
2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 +
24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*
z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c
^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b^3*c^3*z +
768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672
*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 499
2*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z
- 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B
^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*
B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^
3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b
*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 7
68*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5
 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*
a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*
c^4, z, k)*((x*(1024*B*a^4*c^6 - 16*B*a*b^6*c^3 + 192*B*a^2*b^4*c^4 - 768*B
*a^3*b^2*c^5))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (
2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*C*a*b^7*c^2 - 1024*C*a^4*b*c^5 + 384*A
*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*C*a^2*b^5*c^3 + 768*C*a^3*b^3*c^4)/
(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (root(1572864*a^
5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2
*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c^3*z^4 - 1048576*a^6*c^9*z^4
 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3
*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c*z^2 + 61440*C^2*a^5*b*c^5*z
^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2
- 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*
c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^

2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768
*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b^
3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7
*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c
^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*
b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z
 + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2
 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*
B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2
*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3
*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C
^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 1
6*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 1
6*A^4*a^3*c^4, z, k)*x*(16*b^9*c^3 - 256*a*b^7*c^4 + 4096*a^4*b*c^7 + 1536*
a^2*b^5*c^5 - 4096*a^3*b^3*c^6))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48
*a^2*b^2*c^3))) - (1536*B*C*a^4*c^4 + 128*A*B*a^2*b^3*c^3 + 32*B*C*a^2*b^4*
c^2 - 512*B*C*a^3*b^2*c^3 - 512*A*B*a^3*b*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*
a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(C^2*b^8 - 32*A^2*a^3*c^5 + A^2*b^6*c^2 +
 288*C^2*a^4*c^4 + 2*A*C*b^7*c - 16*B^2*a^2*b^3*c^3 + 138*C^2*a^2*b^4*c^2 -
 368*C^2*a^3*b^2*c^3 - 20*C^2*a*b^6*c - 2*A^2*a*b^4*c^3 + 64*B^2*a^3*b*c^4
+ 48*A*C*a^2*b^3*c^3 - 22*A*C*a*b^5*c^2 + 32*A*C*a^3*b*c^4))/(2*(b^6*c - 64
*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))) - (3*A*C^2*a*b^5 - 216*C^3*a^4*
c^2 - 5*C^3*a^2*b^4 + 32*A*B^2*a^3*c^3 - 24*A^2*C*a^3*c^3 + 3*A^3*a*b^3*c^2
 + 4*A^3*a^2*b*c^3 + 66*C^3*a^3*b^2*c - 51*A*C^2*a^2*b^3*c + 204*A*C^2*a^3*
b*c^2 - 16*B^2*C*a^3*b*c^2 - 42*A^2*C*a^2*b^2*c^2 + 6*A^2*C*a*b^4*c)/(8*(b^
6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(16*B^3*a^3*c^3 + B
*C^2*a*b^5 + A^2*B*a*b^3*c^2 + 4*A^2*B*a^2*b*c^3 - 14*B*C^2*a^2*b^3*c + 48*
B*C^2*a^3*b*c^2 - 24*A*B*C*a^3*c^3 - 10*A*B*C*a^2*b^2*c^2 + 2*A*B*C*a*b^4*c
))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))*root(1572864*a
^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^
2*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c^3*z^4 - 1048576*a^6*c^9*z^
4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^
3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c*z^2 + 61440*C^2*a^5*b*c^5*
z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2
 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7
*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a
^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 3276
8*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b
^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^
7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*
c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2
*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*
z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^
2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240
*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^

```
2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^
3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*
C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 -
16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 -
16*A^4*a^3*c^4, z, k), k, 1, 4) - ((x^3*(A*b*c - C*b^2 + 2*C*a*c))/(2*c*(4*
a*c - b^2)) + (x*(2*A*a*c - C*a*b))/(2*c*(4*a*c - b^2)) - (B*a*b)/(2*c*(4*a
*c - b^2)) + (B*x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.30 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 347

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*B*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(a*(2*A*c-C*b)+(A*b*c+
2*C*a*c-C*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(A*b-2*C*a)*arctanh((2*c
*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*B*arctan(x*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*B*arctan(x*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1676, 1265, 791, 632, 212, 12, 1134, 1180, 211}

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{x^2(2acC+Abc+b^2(-C))+a(2Ac-bC)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{B\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{B\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{Bx(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (a*(2*A*c - b*C
) + (A*b*c - b^2*C + 2*a*c*C)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ (B*(b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (B*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]) - ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
```

```
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{Bx^4}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^3(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Cx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right) + B\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad - \frac{B\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)} + \frac{(Ab-2aC)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)} \\
&= \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad - \frac{\left(B\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)^{3/2}} \\
&\quad + \frac{\left(B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)^{3/2}} \\
&\quad - \frac{(Ab-2aC)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac} \\
&= \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad - \frac{B\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.03

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(-\frac{2(bx^2(Ac-bC+Bcx)+a(2Ac-bC+2cx(B+Cx)))}{c(-b^2+4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}B\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2(Ab-2aC)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2(Ab-2aC)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(b*x^2*(A*c - b*C + B*c*x) + a*(2*A*c - b*C + 2*c*x*(B + C*x))))/(c*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*B*(-b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*B*(b^2 + 4*a*c
+ b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(A*b -
2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*(A*b
 - 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.61

method	result
risch	$\frac{-\frac{bB\,x^{3}}{2(4ac-b^{2})}-\frac{(Abc+2acC-b^{2}C)x^{2}}{2c(4ac-b^{2})}-\frac{xBa}{4ac-b^{2}}-\frac{a(2Ac-Cb)}{2(4ac-b^{2})c}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(-\frac{\_R^{2}Bb}{4ac-b^{2}}-\frac{2(Ab-2Ca)\_R}{4ac-b^{2}}+\frac{2Ba}{4ac-b^{2}}\right.}{2c\_R^{3}+\_Rb}\right.}{4}$
default	$\frac{-\frac{bB\,x^{3}}{2(4ac-b^{2})}-\frac{(Abc+2acC-b^{2}C)x^{2}}{2c(4ac-b^{2})}-\frac{xBa}{4ac-b^{2}}-\frac{a(2Ac-Cb)}{2(4ac-b^{2})c}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(\frac{\frac{\left(-4Abc\sqrt{-4ac+b^{2}}+8C\sqrt{-4ac+b^{2}}\,ac\right)\ln\left(2c\,x^{2}+\sqrt{-4ac+b^{2}}+b\right)}{4c}+\frac{(4Ba}{}}{4c}\right.}{}$

```
[In] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/(4*a*c-b^2)*b*B*x^3-1/2*(A*b*c+2*C*a*c-C*b^2)/c/(4*a*c-b^2)*x^2-1/(4*
a*c-b^2)*x*B*a-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(c*x^4+b*x^2+a)+1/4*sum((-1
/(4*a*c-b^2)*_R^2*B*b-2*(A*b-2*C*a)/(4*a*c-b^2)*_R+2/(4*a*c-b^2)*B*a)/(2*_R
^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx=\int \frac{(Cx^2+Bx+A)x^3}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c*x^3 + 2*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (2*C*a + A*b)*c)*x^
2)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2
) + 1/2*integrate((B*b*x^2 - 2*B*a - 2*(2*C*a - A*b)*x)/(c*x^4 + b*x^2 + a)
, x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3227 vs. $2(296) = 592$.

Time = 1.22 (sec) , antiderivative size = 3227, normalized size of antiderivative = 9.30

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*c*x^3 - C*b^2*x^2 + 2*C*a*c*x^2 + A*b*c*x^2 + 2*B*a*c*x - C*a*b +
2*A*a*c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^2 - 8*a*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2
 - 4*a*c)^2*B - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*
a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2*b^7*c^2 - 8*a*
b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
```

```
*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4)*B)*arctan(2*
sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*
a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a
^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) - 1/16*((2*
b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a
*c)*b*c^2)*(b^2 - 4*a*c)^2*B + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b
^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2
*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2
*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^
4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4
*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a
^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64
*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(
c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c - 4*a*b*c^2
- 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - 2*(a*b^3*c - 4
*a^2*b*c^2 - 2*a*b^2*c^2 + a*b*c^3 + (a*b^2*c - 4*a^2*c^2 - 2*a*b*c^2 + a*c
^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^
2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3
*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*A + 2*(a*b^5*c - 8*a^2*b^3*c
^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 +
 (a*b^4*c - 4*a^2*b^2*c^2 - 2*a*b^3*c^2 + a*b^2*c^3)*sqrt(b^2 - 4*a*c))*C)*
log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)
*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c +
 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c)) +
1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c - 4*a*b*c^2 - 2*b^
2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - 2*(a*b^3*c - 4*a^2*b
*c^2 - 2*a*b^2*c^2 + a*b*c^3 - (a*b^2*c - 4*a^2*c^2 - 2*a*b*c^2 + a*c^3)*sq
rt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16
*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 -
```

```
 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*A + 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 - (a*b^
4*c - 4*a^2*b^2*c^2 - 2*a*b^3*c^2 + a*b^2*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^
2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*
c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^
3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 3278, normalized size of antiderivative = 9.45

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log(root(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a
^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*
c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 -
24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^
2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^
3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*
b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2
*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z
 - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z
+ 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*
B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c +
 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^
2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*
B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*c - 9*B^4*a*b^4, z, k)*((256*A*B*a^2*b
^2*c^3 + 128*B*C*a^2*b^3*c^2 - 64*A*B*a*b^4*c^2 - 512*B*C*a^3*b*c^3)/(8*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(1572864*a^5*b^2*c^6*z
^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^
4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*
a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*
a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^
2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 81
92*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^
2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 -
16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2
*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*
a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 5
12*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b
^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2
*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*
```

c - 9*B^4*a*b^4, z, k)*((x*(16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3
 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^
3 - 1536*C*a^3*b^2*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)) - (2048*B*a^4*c^5 - 32*B*a*b^6*c^2 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*
c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(1572864*a
^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 61440*a^
2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*c*z^4
+ 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2
 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z
^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z^2 - 6144*C^2*a^3*b^4*
c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 - 8192*A^2*a^
3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^8*c*z^2 - 32768*C^2*a^
5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z
 - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z
 - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a
^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A
^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^
2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c -
 16*A^4*b^4*c - 9*B^4*a*b^4, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4
*b*c^6 + 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c))) + (x*(8*A^2*b^5*c^2 - 2*B^2*b^6*c + 64*B^2*a^3*c^
4 + 32*C^2*a^2*b^3*c^2 - 32*A^2*a*b^3*c^3 + 4*B^2*a*b^4*c^2 - 128*C^2*a^3*b
*c^3 + 128*A*C*a^2*b^2*c^3 - 32*A*C*a*b^4*c^2))/(4*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c))) - (3*B^3*a*b^3*c + 32*B*C^2*a^3*c^2 + 4*B^3*a^2*
b*c^2 + 8*A^2*B*a*b^2*c^2 - 32*A*B*C*a^2*b*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)) + (x*(4*A^3*b^3*c^2 - 32*C^3*a^3*c^2 + A*B^2*b^4*
c + 4*A*B^2*a*b^2*c^2 + 48*A*C^2*a^2*b*c^2 - 24*A^2*C*a*b^2*c^2 - 8*B^2*C*a
^2*b*c^2 - 2*B^2*C*a*b^3*c))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)))*root(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*
b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7
*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 245
76*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 +
 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z
^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5
*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^
8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z -
1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 7
68*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2
*b^7*z + 192*A*B^2*C*a^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16
*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a
*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2
*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*c - 9*B^4*a*b^4, z, k), k, 1, 4) - ((B*a*
x)/(4*a*c - b^2) + (x^2*(A*b*c - C*b^2 + 2*C*a*c))/(2*c*(4*a*c - b^2)) + (B
*b*x^3)/(2*(4*a*c - b^2)) + (a*(2*A*c - C*b))/(2*c*(4*a*c - b^2)))/(a + b*x
^2 + c*x^4)

3.31 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{B(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(bB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(bB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)\,(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln(x-\_\ldots}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{cx^4+bx^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aac^2-2Ab^2c+4C\ldots\right.}{\ldots}}{4c(4ac\ldots}\right.}{\ldots}$

[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. $2(306) = 612$.

Time = 1.59 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
```

^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt

```
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
```

^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153

```
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.32 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 317

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\sqrt{c}(2b-\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}(2b+\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] -1/2*B*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(-A*b+2*C*a-(2*A*c-C*
b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)*2^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)*2^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1676, 1261, 652, 632, 212, 12, 1133, 1180, 211}

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{-2aC+x^2(2Ac-bC)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(B*x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (A*b - 2*a*C
 + (2*A*c - b*C)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (B*Sqrt[c]*(2
*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c
]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*A
*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{Bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x(A+Cx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{(a+bx+cx^2)^2}\,dx, x, x^2\right) + B\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)} - \frac{(2Ac-bC)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$

$$= -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(Bc\left(2b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(b^2-4ac)^{3/2}} - \frac{\left(Bc\left(2b+\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(b^2-4ac)^{3/2}} + \frac{(2Ac-bC)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\left(\frac{2aC-A(b+2cx^2)+x(-bB+bCx-2Bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}B\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(-2Ac+bC)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{(2Ac-bC)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*C - A*(b + 2*c*x^2) + x*(-(b*B) + b*C*x - 2*B*c*x^2))/((b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) - (Sqrt[2]*B*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-2*A*c + b*C)*Log[-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*c - b*C)*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\frac{Bc\,x^3}{4ac-b^2}+\frac{(2Ac-Cb)x^2}{8ac-2b^2}+\frac{Bbx}{8ac-2b^2}+\frac{Ab-2Ca}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{2c\_R^2B}{4ac-b^2}+\frac{2(2Ac-Cb)\_R}{4ac-b^2}-\frac{bB}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right.}{4}$
default	$16c^2\left(-\frac{\frac{-\frac{B\left(4ac-b^2\right)x}{8c}-\frac{8Aa\,c^2-2A\,b^2c+4C\sqrt{-4ac+b^2}\,ac-C\sqrt{-4ac+b^2}\,b^2-4Cabc+C\,b^3}{16c^2}}{x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}}-\frac{\left(-4Ac\sqrt{-4ac+b^2}+2C\sqrt{-4ac+b^2}\,b\right)\ln\left(-2c\,x^2\right.}{16c}}{4c\left(4ac-b^2\right)^2}\right.$

```
[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (B*c/(4*a*c-b^2)*x^3+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^2+1/2/(4*a*c-b^2)*x*B*b+
1/2*(A*b-2*C*a)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum((2*c/(4*a*c-b^2)*_R^2*
B+2*(2*A*c-C*b)/(4*a*c-b^2)*_R-1/(4*a*c-b^2)*b*B)/(2*_R^3*c+_R*b)*ln(x-_R),
_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*B*c*x^3 + B*b*x - (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/((b^2*c - 4*a*c^
2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate((2*B*c*x^2
- B*b - 2*(C*b - 2*A*c)*x)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3014 vs. $2(270) = 540$.

Time = 1.24 (sec) , antiderivative size = 3014, normalized size of antiderivative = 9.51

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*c*x^3 - C*b*x^2 + 2*A*c*x^2 + B*b*x - 2*C*a + A*b)/((c*x^4 + b*x^
2 + a)*(b^2 - 4*a*c)) - 1/8*((2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
```

```
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2*B - (sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a
*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3
 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*B*abs(b^2 - 4*a*c) - 2*
(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^2 - 4*a*c)*a*b^2*c^3)*B)*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5
*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^
3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((2*b^2*c^2
 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 -
4*a*c)^2*B + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*
c)*a*b*c^2)*B*abs(b^2 - 4*a*c) - 2*(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c
^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^
2 - 4*a*c)*a*b^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b
^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)
))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a
*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2
 - 4*a*c)*abs(c)) - 1/8*(2*(b^3*c^2 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 - (b^2*c
```

```
^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - (b^4*
c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*
c^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) + 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b
^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*
b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*A - (b^6*c - 8*a*b^4*c^2
- 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c
 - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(
b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c
^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8
*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c)) - 1/8*(2*(b^3*c^2
 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt
(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - (b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 - (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2
- 4*a*c) + 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^
4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqr
t(b^2 - 4*a*c))*A - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a
*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c
^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*
c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 -
 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3
)*c^2*abs(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 3198, normalized size of antiderivative = 10.09

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((4*B^3*a*c^4 + 3*B^3*b^2*c^3 + 8*A^2*B*b*c^4 + 2*B*C^2*b^3*c^2 -
 8*A*B*C*b^2*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ro
ot(1572864*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^
4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256
*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3
*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^
2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C
^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 +
24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 -
32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2
*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a
^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 1
92*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b
^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 2
```

4*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B
^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^4, z, k)*(root(1572864*a^6*b^2*c^5*z^
4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4
 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a
^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a
^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2
*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192
*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2
 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 1
6*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*
a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b
^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 51
2*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*
c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*
B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 -
 256*A^4*a*c^4, z, k)*((x*(512*A*a^3*c^6 - 8*A*b^6*c^3 + 4*C*b^7*c^2 + 96*A
*a*b^4*c^4 - 48*C*a*b^5*c^3 - 256*C*a^3*b*c^5 - 384*A*a^2*b^2*c^5 + 192*C*a
^2*b^3*c^4))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (8*B*b^7*c^
2 - 96*B*a*b^5*c^3 - 512*B*a^3*b*c^5 + 384*B*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(1572864*a^6*b^2*c^5*z^4 - 98304
0*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a
^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*
z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^
2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^
2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*
b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A
^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9
*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z
 - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z -
2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a
*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*
B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c
^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*
a*c^4, z, k)*x*(8*b^9*c^2 - 128*a*b^7*c^3 + 2048*a^4*b*c^6 + 768*a^2*b^5*c^
4 - 2048*a^3*b^3*c^5))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) -
(256*A*B*a^2*c^5 - 16*A*B*b^4*c^3 + 8*B*C*b^5*c^2 - 128*B*C*a^2*b*c^4)/(4*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*B^2*a^2*c^5 - 8*A
^2*b^3*c^4 + 5*B^2*b^4*c^3 - 2*C^2*b^5*c^2 + 8*A*C*b^4*c^3 + 32*A^2*a*b*c^5
 - 24*B^2*a*b^2*c^4 + 8*C^2*a*b^3*c^3 - 32*A*C*a*b^2*c^4))/(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(8*A^3*c^5 - C^3*b^3*c^2 + 4*A*B^2*b
*c^4 - 12*A^2*C*b*c^4 + 6*A*C^2*b^2*c^3 - 2*B^2*C*b^2*c^3))/(b^6 - 64*a^3*c
^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))*root(1572864*a^6*b^2*c^5*z^4 - 983040*a^
5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b
^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2
- 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^

```
2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^
2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*
c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a
^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2
 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 7
68*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048
*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c
^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*
C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 -
 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^
4, z, k), k, 1, 4) + ((A*b - 2*C*a)/(2*(4*a*c - b^2)) + (x^2*(2*A*c - C*b))
/(2*(4*a*c - b^2)) + (B*b*x)/(2*(4*a*c - b^2)) + (B*c*x^3)/(4*a*c - b^2))/(
a + b*x^2 + c*x^4)
```

3.33 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 368

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2} dx = &-\frac{B(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\ &+\frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\ &+\frac{\sqrt{c}\left(Ab-2aC+\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} \\ &+\frac{\sqrt{c}\left(Ab-2aC-\frac{Ab^2-12aAc+4abC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}} \\ &+\frac{2Bc\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

```
[Out] -1/2*B*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(A*b^2-2*a*A*c-a*b*C+
c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*B*c*arctanh((2*c*x^2+b)
/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2*C*a+(A*(-12*a*c+b^2)+4*a*b*C)/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2*C*a+(12*A*
a*c-A*b^2-4*C*a*b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1687, 1192, 1180, 211, 12, 1121, 628, 632, 212}

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \frac{\sqrt{c}\left(\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}-2aC+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(cx^2(Ab-2aC)-2aAc-abC+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{2Bc\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{B(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(B*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(A*b^2 - 2*
a*A*c - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c) + 4*a*b*C)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 -
12*a*A*c + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (2*B*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{Bx}{(a+bx^2+cx^4)^2}\,dx + \int \frac{A+Cx^2}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x}{(a+bx^2+cx^4)^2}\,dx - \frac{\int \frac{-Ab^2+6aAc-abC-c(Ab-2aC)x^2}{a+bx^2+cx^4}\,dx}{2a(b^2-4ac)}$$

$$= \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, x^2\right) + \frac{\left(c\left(A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)+2a\left(2b-\sqrt{b^2-4ac}\right)C\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a(b^2-4ac)^{3/2}} + \frac{\left(c\left(Ab-2aC-\frac{Ab^2-12aAc+4abC}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a(b^2-4ac)}$$

$$= -\frac{B(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)+2a\left(2b-\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(Ab-2aC-\frac{Ab^2-12aAc+4abC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(Bc)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= -\frac{B(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)+2a\left(2b-\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(Ab-2aC-\frac{Ab^2-12aAc+4abC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(2Bc)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= -\frac{B(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$
$$+ \frac{\sqrt{c}\left(A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)+2a\left(2b-\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt{c}\left(Ab-2aC-\frac{Ab^2-12aAc+4abC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2Bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{2ab(B+Cx)-2Abx(b+cx^2)+4acx(A+x(B+Cx))}{a(-b^2+4ac)(a+bx^2+cx^4)}\right.$$
$$+ \frac{\sqrt{2}\sqrt{c}\left(A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-2a\left(-2b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt{2}\sqrt{c}\left(A\left(b^2-12ac-b\sqrt{b^2-4ac}\right)+2a\left(2b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.- \frac{4Bc\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{4Bc\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*(B + C*x) - 2*A*b*x*(b + c*x^2) + 4*a*c*x*(A + x*(B + C*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c + b*Sq
rt[b^2 - 4*a*c]) - 2*a*(-2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c]) + 2*
a*(2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*B*c*L
og[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*B*c*Log[b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.63

method	result
risch	$\frac{-\frac{c(Ab-2Ca)x^3}{2a(4ac-b^2)}+\frac{c\,x^2 B}{4ac-b^2}+\frac{(2Aac-A\,b^2+abC)x}{2a(4ac-b^2)}+\frac{Bb}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2 b+a)}\frac{\left(-\frac{c(Ab-2Ca)\_R^2}{a(4ac-b^2)}+\frac{4\_R Bc}{4ac-b^2}+\frac{6Aac-A\,b}{a(4ac-}\right.}{2c\_R^3+\_R b}\right.}{4}$
default	$16c^2\left(-\frac{\frac{-\frac{\left(-4A\sqrt{-4ac+b^2}\,ac+A\sqrt{-4ac+b^2}\,b^2-4Aabc+A\,b^3+8a^2cC-2Ca\,b^2\right)x}{16ac}-\frac{B(4ac-b^2)}{8c}}{x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}}+\frac{2Ba\sqrt{-4ac+b^2}\,\ln\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)+}{}}{4c(4ac-b^2}\right.$

```
[In] int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*c*(A*b-2*C*a)/a/(4*a*c-b^2)*x^3+c/(4*a*c-b^2)*x^2*B+1/2*(2*A*a*c-A*b^
2+C*a*b)/a/(4*a*c-b^2)*x+1/2/(4*a*c-b^2)*b*B)/(c*x^4+b*x^2+a)+1/4*sum((-c*(
A*b-2*C*a)/a/(4*a*c-b^2)*_R^2+4/(4*a*c-b^2)*_R*B*c+(6*A*a*c-A*b^2-C*a*b)/a/
(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Cx^2+Bx+A}{\left(cx^4+bx^2+a\right)^2}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*(2*B*a*c*x^2 + (2*C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2 + 2*A*a*c)
*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^
2) + 1/2*integrate(-(4*B*a*c*x + (2*C*a - A*b)*c*x^2 - C*a*b - A*b^2 + 6*A*
a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5156 vs. $2(323) = 646$.

Time = 1.59 (sec) , antiderivative size = 5156, normalized size of antiderivative = 14.01

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(2*C*a*c*x^3 - A*b*c*x^3 + 2*B*a*c*x^2 + C*a*b*x - A*b^2*x + 2*A*a*c*x
 + B*a*b)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*
b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*
b^2 - 4*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*C + 2*(sqrt(2)
```

```
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*
a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 -
 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 +
 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2
- 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)
*A*abs(a*b^2 - 4*a^2*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^
3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3
+ 2*(b^2 - 4*a*c)*a^2*b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*C*abs(a*b^2 - 4*a^
2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a
^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A +
 4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 -
4*a*c)*a^4*b^2*c^3)*C)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt(
(a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^
2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2
+ 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3
+ 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4
```

```
*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*C - 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c
 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c
)*a*b^4*c + 20*(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(
a*b^2 - 4*a^2*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 16*a^3*b^3*c^2 -
 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 32*a^4*b*c^3 - 2*(b^
2 - 4*a*c)*a^2*b^3*c + 8*(b^2 - 4*a*c)*a^3*b*c^2)*C*abs(a*b^2 - 4*a^2*c) +
(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*
c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a
^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*
a^4*b^2*c^3)*C)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3
- 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4
*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^
4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^
```

```
5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/4*((b^3*c^2 - 4*a*b*c^3 - 2*b^2*c^3
 + b*c^4 + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c))*B*abs(a*b
^2 - 4*a^2*c) - (a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8
*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 + (a*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b
^3*c^3 + a*b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(a*b^3 - 4*a^2*b*c
+ sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2))
)/(a*b^2*c - 4*a^2*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8
*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(a*b^2 - 4*a^2*c)) - 1/4*((b^3*c
^2 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sq
rt(b^2 - 4*a*c))*B*abs(a*b^2 - 4*a^2*c) - (a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*
b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 + (a*b^4*c
^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2
 + 1/2*(a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3
*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2))/((a*b^4 - 8*a^2*b^2*c -
2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(a*b^2
 - 4*a^2*c))
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 4707, normalized size of antiderivative = 12.79

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((B*b)/(2*(4*a*c - b^2)) + (x*(2*A*a*c - A*b^2 + C*a*b))/(2*a*(4*a*c - b^2)
) + (B*c*x^2)/(4*a*c - b^2) - (c*x^3*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) + symsum(log((5*A^3*b^3*c^4 + 8*C^3*a^3*c^4 + 6*C^3*a^2*b
^2*c^3 - 36*A^3*a*b*c^5 - 96*A*B^2*a^2*c^5 + 72*A^2*C*a^2*c^5 - 3*A^2*C*b^4
*c^3 + 16*A*B^2*a*b^2*c^4 + 3*A*C^2*a*b^3*c^3 - 60*A*C^2*a^2*b*c^4 + 18*A^2
*C*a*b^2*c^4 + 16*B^2*C*a^2*b*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4
+ 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 10
48576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^
5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*
C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*
a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a
^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512
*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^
2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 -
 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 409
6*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B
*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A
^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C
```

^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*C
*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a*
b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*c
^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4 -
496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a^
2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*a
^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 25
6*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k)*(root(1572864*a^8*
b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b
^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 +
576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z
^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2
+ 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 -
8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4
*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b
^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B
^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 - 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*
c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 4096*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c
*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*
z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^
4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z +
192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 -
960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^
2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B
^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4 - 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b
*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C
*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*
A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 256*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*
A^4*a^2*c^5, z, k)*((x*(1024*B*a^5*c^6 - 16*B*a^2*b^6*c^3 + 192*B*a^3*b^4*c
^4 - 768*B*a^4*b^2*c^5))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2)) - (6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*C*a^5*b*c^5 - 288*A*a^2*
b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*C*a^2*b^7*c^2 - 192*
C*a^3*b^5*c^3 + 768*C*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)) + (root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4
+ 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1
048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a
^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A
*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2
*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*
a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 51
2*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z
^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2
- 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 40
96*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*
B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*

A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*
C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*
C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a
*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*
c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4
- 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a
^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*
a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 2
56*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k)*x*(4096*a^6*b*c^6
 + 16*a^2*b^9*c^2 - 256*a^3*b^7*c^3 + 1536*a^4*b^5*c^4 - 4096*a^5*b^3*c^5))
/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) + (32*B*C*a^2*
b^4*c^3 - 384*A*B*a^2*b^3*c^4 - 512*B*C*a^4*c^5 + 32*A*B*a*b^5*c^3 + 1024*A
*B*a^3*b*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) +
(x*(A^2*b^6*c^3 - 288*A^2*a^3*c^6 + 32*C^2*a^4*c^5 + 128*A^2*a^2*b^2*c^5 -
16*B^2*a^2*b^3*c^4 + 10*C^2*a^2*b^4*c^3 - 48*C^2*a^3*b^2*c^4 - 18*A^2*a*b^4
*c^4 + 64*B^2*a^3*b*c^5 - 48*A*C*a^2*b^3*c^4 + 2*A*C*a*b^5*c^3 + 160*A*C*a^
3*b*c^5))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) - (x*
(16*B^3*a^2*c^5 - A^2*B*b^3*c^4 + 8*B*C^2*a^2*b*c^4 - 24*A*B*C*a^2*c^5 + 12
*A^2*B*a*b*c^5 - 2*A*B*C*a*b^2*c^4))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2)))*root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4
 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1
048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a
^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A
*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2
*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*
a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 51
2*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z
^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2
- 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 40
96*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*
B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*
A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*
C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*
C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a
*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*
c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4
- 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a
^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*
a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 2
56*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k), k, 1, 4)

3.34 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 403

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{\left(A(b^3-6abc)+4a^2cC\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] 1/2*B*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+b
^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-6*a*b*
c+b^3)+4*a^2*c*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^
(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2+1/4*B*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a
/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*B*arctan(x*2^(
1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^
2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1265, 836, 814, 648, 632, 212, 642, 12, 1106, 1180, 211}

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{(4a^2cC+A(b^3-6abc))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2} + \frac{A(b^2-2ac)+cx^2(Ab-2aC)-abC}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{Bx(-2ac+b^2+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*
(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) + (B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*(b^3 - 6*a*b*c) + 4*a
^2*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{B}{(a+bx^2+cx^4)^2}\,dx + \int \frac{A+Cx^2}{x\,(a+bx^2+cx^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x\,(a+bx+cx^2)^2}\,dx, x, x^2\right) + B\int \frac{1}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{Bx(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-A(b^2-4ac)-c(Ab-2aC)x}{x(a+bx+cx^2)}\,dx, x, x^2\right)}{2a\,(b^2-4ac)} - \frac{B\int \frac{b^2-2ac-2(b^2-4ac)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)}
\end{aligned}
$$

$$= \frac{Bx(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \left(\frac{A(-b^2+4ac)}{ax} + \frac{A(b^3-5abc)+2a^2cC+Ac(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx, x, x^2\right)}{2a(b^2-4ac)} - \frac{\left(Bc\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a(b^2-4ac)^{3/2}} + \frac{\left(Bc\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a(b^2-4ac)^{3/2}}$$

$$= \frac{Bx(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{A(b^3-5abc)+2a^2cC+Ac(b^2-4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2(b^2-4ac)}$$

$$= \frac{Bx(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a^2} - \frac{A\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{(A(b^3-6abc)+4a^2cC)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2(b^2-4ac)}$$

$$= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+ \frac{B\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{B\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}$$
$$+ \frac{(A(b^3-6abc)+4a^2cC)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+ \frac{B\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{B\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{(A(b^3-6abc)+4a^2cC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{2a\left(abC+2acx(B+Cx)-bBx(b+cx^2)-A\left(b^2-2ac+bcx^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}aB\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}aB\sqrt{c}\left(-b^2+12a\ldots\right.}{(b^2\ldots}}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*b*C + 2*a*c*x*(B + C*x) - b*B*x*(b + c*x^2) - A*(b^2 - 2*a*c + b*
c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(b^2 -
12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*
B*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + 4*A*Log[x] - ((A*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt
```

```
[b^2 - 4*a*c]) + 4*a^2*c*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4
*a*c)^(3/2) - ((A*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2
- 4*a*c]) - 4*a^2*c*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^
(3/2))/(4*a^2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.40

method	result
default	$\frac{A\ln(x)}{a^2} - \frac{\frac{\frac{Babc\,x^3}{8ac-2b^2}+\frac{ac(Ab-2Ca)x^2}{8ac-2b^2}-\frac{aB\left(2ac-b^2\right)x}{2\left(4ac-b^2\right)}-\frac{a\left(2Aac-A\,b^2+abC\right)}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(12Aabc\sqrt{-4ac+b^2}-2A\,b^3\sqrt{-4ac+b^2}+32A\,a^2c^2-16Aa\,b^2 \cdots\right.}{4c \cdots}\cdots\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] A*ln(x)/a^2-1/a^2*((1/2*B*a*b*c/(4*a*c-b^2)*x^3+1/2*a*c*(A*b-2*C*a)/(4*a*c-
b^2)*x^2-1/2*a*B*(2*a*c-b^2)/(4*a*c-b^2)*x-1/2*a*(2*A*a*c-A*b^2+C*a*b)/(4*a
*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(1/4*(12*A*a*b*c
*(-4*a*c+b^2)^(1/2)-2*A*b^3*(-4*a*c+b^2)^(1/2)+32*A*a^2*c^2-16*A*a*b^2*c+2*
A*b^4-8*C*(-4*a*c+b^2)^(1/2)*a^2*c)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*
(-12*a^2*B*c*(-4*a*c+b^2)^(1/2)+B*a*b^2*(-4*a*c+b^2)^(1/2)+4*a^2*b*B*c-B*a*
b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(-1/4*(12*A*a*b*c*(-4*a*c+b^2)^(1
/2)-2*A*b^3*(-4*a*c+b^2)^(1/2)-32*A*a^2*c^2+16*A*a*b^2*c-2*A*b^4-8*C*(-4*a*
c+b^2)^(1/2)*a^2*c)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(-12*a^2*B*c*(-
4*a*c+b^2)^(1/2)+B*a*b^2*(-4*a*c+b^2)^(1/2)-4*a^2*b*B*c+B*a*b^3)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{Cx^2+Bx+A}{\left(cx^4+bx^2+a\right)^2x}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*(B*b*c*x^3 - (2*C*a - A*b)*c*x^2 - C*a*b + A*b^2 - 2*A*a*c + (B*b^2 - 2
*B*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*
b*c)*x^2) + 1/2*integrate((B*a*b*c*x^2 + B*a*b^2 - 6*B*a^2*c - 2*(A*b^2*c -
 4*A*a*c^2)*x^3 - 2*(A*b^3 + (2*C*a^2 - 5*A*a*b)*c)*x)/(c*x^4 + b*x^2 + a),
 x)/(a^2*b^2 - 4*a^3*c) + A*log(x)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6021 vs. $2(348) = 696$.

Time = 1.65 (sec) , antiderivative size = 6021, normalized size of antiderivative = 14.94

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] -1/4*A*log(abs(c*x^4 + b*x^2 + a))/a^2 + A*log(abs(x))/a^2 - 1/16*((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*B - 2*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c - 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^6*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 - 2*a^
4*b^8*c^2 + 120*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 28*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + sqrt(2)*sqrt(b*c + sqrt
```

(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 36*a^5*b^6*c^3 - 352*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^6*b^3*c^4 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 240*
a^6*b^4*c^4 + 384*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^5 + 192*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 + 64*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 704*a^7*b^2*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^7*c^6 - 768*a^8*c^6 + 2*(b^2 - 4*a*c)*a^4*b^6*c^2 - 28*(
b^2 - 4*a*c)*a^5*b^4*c^3 + 128*(b^2 - 4*a*c)*a^6*b^2*c^4 - 192*(b^2 - 4*a*c
)*a^7*c^5)*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (2*a^8*b^11*c^4
- 56*a^9*b^9*c^5 + 576*a^10*b^7*c^6 - 2816*a^11*b^5*c^7 + 6656*a^12*b^3*c^8
- 6144*a^13*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^11*c^2 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^9*b^9*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^10*c^3 - 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^10*b^7*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^9*b^8*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^9*c^4 + 1408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^11*b^5*c^5 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^10*b^6*c^5 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^9*b^7*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^12*b^3*c^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^11*b^4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^10*b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^13*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^12*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^12*b*c^8 - 2*(b^2 - 4*a*c)*a^8*b^9*c^4 + 48*(b^2
- 4*a*c)*a^9*b^7*c^5 - 384*(b^2 - 4*a*c)*a^10*b^5*c^6 + 1280*(b^2 - 4*a*c)*
a^11*b^3*c^7 - 1536*(b^2 - 4*a*c)*a^12*b*c^8)*B)*arctan(2*sqrt(1/2)*x/sqrt(
(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 + sqrt((a^4*b^5*c - 8*a^5*b^3*c^2
+ 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^
2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4
)))/((a^6*b^8*c - 16*a^7*b^6*c^2 - 2*a^6*b^7*c^2 + 96*a^8*b^4*c^3 + 24*a^7*
b^5*c^3 + a^6*b^6*c^3 - 256*a^9*b^2*c^4 - 96*a^8*b^3*c^4 - 12*a^7*b^4*c^4 +
256*a^10*c^5 + 128*a^9*b*c^5 + 48*a^8*b^2*c^5 - 64*a^9*c^6)*abs(a^4*b^4*c
- 8*a^5*b^2*c^2 + 16*a^6*c^3)*abs(c)) + 1/16*((a^4*b^4*c - 8*a^5*b^2*c^2 +
16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c
^2 - 2*(b^2 - 4*a*c)*b*c^2)*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^8*c - 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 - 2*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 + 2*a^4*b^8*c^2 + 120*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 28*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*

b^6*c^3 - 36*a^5*b^6*c^3 - 352*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*
b^2*c^4 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 - 14*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 + 240*a^6*b^4*c^4 + 384*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^5 + 192*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b*c^5 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*
b^2*c^5 - 704*a^7*b^2*c^5 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*
c^6 + 768*a^8*c^6 - 2*(b^2 - 4*a*c)*a^4*b^6*c^2 + 28*(b^2 - 4*a*c)*a^5*b^4*
c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + 192*(b^2 - 4*a*c)*a^7*c^5)*B*abs(a^4*
b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (2*a^8*b^11*c^4 - 56*a^9*b^9*c^5 + 57
6*a^10*b^7*c^6 - 2816*a^11*b^5*c^7 + 6656*a^12*b^3*c^8 - 6144*a^13*b*c^9 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^11*c^2 + 28
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^9*c^3 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^3 - 28
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^7*c^4 -
48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c^4 + 140
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^5*c^5 +
384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^6*c^5
+ 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^7*c^5
- 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^3*c
^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^
4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*
b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
13*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^12*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^11*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^12*b*c^8 - 2*(b^2 - 4*a*c)*a^8*b^9*c^4 + 48*(b^2 - 4*a*c)*a^9*b^7*c^5
- 384*(b^2 - 4*a*c)*a^10*b^5*c^6 + 1280*(b^2 - 4*a*c)*a^11*b^3*c^7 - 1536*(
b^2 - 4*a*c)*a^12*b*c^8)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b^5*c - 8*a^5*b^
3*c^2 + 16*a^6*b*c^3 - sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 -
4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 1
6*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/((a^6*b^8*c - 16*
a^7*b^6*c^2 - 2*a^6*b^7*c^2 + 96*a^8*b^4*c^3 + 24*a^7*b^5*c^3 + a^6*b^6*c^3
- 256*a^9*b^2*c^4 - 96*a^8*b^3*c^4 - 12*a^7*b^4*c^4 + 256*a^10*c^5 + 128*a
^9*b*c^5 + 48*a^8*b^2*c^5 - 64*a^9*c^6)*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*
a^6*c^3)*abs(c)) - 1/16*((b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 24*a^2*b^2*c^3
+ 12*a*b^3*c^3 + b^4*c^3 - 6*a*b^2*c^4 - (b^5*c - 10*a*b^3*c^2 - 2*b^4*c^2
+ 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*
abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + 4*(a^2*b^3*c^2 - 4*a^3*b*c^3
- 2*a^2*b^2*c^3 + a^2*b*c^4 + (a^2*b^2*c^2 - 4*a^3*c^3 - 2*a^2*b*c^3 + a^2*
c^4)*sqrt(b^2 - 4*a*c))*C*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) - (a^
4*b^10*c^2 - 18*a^5*b^8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 + 28*a^5*b^7*
c^4 + a^4*b^8*c^4 - 352*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*b^6*c^5 + 38
4*a^8*b^2*c^6 + 192*a^7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c^7 + (a^4*b^
9*c^2 - 14*a^5*b^7*c^3 - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^5*b^6*c^4 +

```
a^4*b^7*c^4 - 96*a^7*b^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5 + 24*a^6*b^3
*c^6)*sqrt(b^2 - 4*a*c))*A - 4*(a^6*b^7*c^3 - 12*a^7*b^5*c^4 - 2*a^6*b^6*c^
4 + 48*a^8*b^3*c^5 + 16*a^7*b^4*c^5 + a^6*b^5*c^5 - 64*a^9*b*c^6 - 32*a^8*b
^2*c^6 - 8*a^7*b^3*c^6 + 16*a^8*b*c^7 + (a^6*b^6*c^3 - 8*a^7*b^4*c^4 - 2*a^
6*b^5*c^4 + 16*a^8*b^2*c^5 + 8*a^7*b^3*c^5 + a^6*b^4*c^5 - 4*a^7*b^2*c^6)*s
qrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^
3 + sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^
6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b
^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c
 + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*c^2*abs(a^4*b^4*c -
8*a^5*b^2*c^2 + 16*a^6*c^3)) - 1/16*((b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 24
*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 6*a*b^2*c^4 + (b^5*c - 10*a*b^3*c^2
 - 2*b^4*c^2 + 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*sqrt(b^2
- 4*a*c))*A*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + 4*(a^2*b^3*c^2 -
4*a^3*b*c^3 - 2*a^2*b^2*c^3 + a^2*b*c^4 - (a^2*b^2*c^2 - 4*a^3*c^3 - 2*a^2*
b*c^3 + a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^
6*c^3) + (a^4*b^10*c^2 - 18*a^5*b^8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 +
 28*a^5*b^7*c^4 + a^4*b^8*c^4 - 352*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*
b^6*c^5 + 384*a^8*b^2*c^6 + 192*a^7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c
^7 + (a^4*b^9*c^2 - 14*a^5*b^7*c^3 - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^
5*b^6*c^4 + a^4*b^7*c^4 - 96*a^7*b^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5
+ 24*a^6*b^3*c^6)*sqrt(b^2 - 4*a*c))*A + 4*(a^6*b^7*c^3 - 12*a^7*b^5*c^4 -
2*a^6*b^6*c^4 + 48*a^8*b^3*c^5 + 16*a^7*b^4*c^5 + a^6*b^5*c^5 - 64*a^9*b*c^
6 - 32*a^8*b^2*c^6 - 8*a^7*b^3*c^6 + 16*a^8*b*c^7 - (a^6*b^6*c^3 - 8*a^7*b^
4*c^4 - 2*a^6*b^5*c^4 + 16*a^8*b^2*c^5 + 8*a^7*b^3*c^5 + a^6*b^4*c^5 - 4*a^
7*b^2*c^6)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 +
 16*a^6*b*c^3 - sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*
b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c
^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4))/((a^3*b^4 - 8*a^4*b^2*c -
 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*c^2*abs(
a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)) + 1/2*(B*a*b*c*x^3 - C*a^2*b + A*a
*b^2 - 2*A*a^2*c - (2*C*a^2*c - A*a*b*c)*x^2 + (B*a*b^2 - 2*B*a^2*c)*x)/((c
*x^4 + b*x^2 + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 8129, normalized size of antiderivative = 20.17

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*A*a*c - A*b^2 + C*a*b)/(2*a*(4*a*c - b^2)) + (B*x*(2*a*c - b^2))/(2*a*(
4*a*c - b^2)) - (c*x^2*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)) - (B*b*c*x^3)/(2*
```

a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log(root(1572864*a^9*b^2*c^5
*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*
z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^10*c^6*z^4 - 256*a^4*b^12*z^4 + 15728
64*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3
- 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 -
256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 901
12*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z
^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^
2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*
c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 + 24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*
a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 +
88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2
- 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*
A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*
B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z - 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2
*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15
360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^
3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^
5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c
^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z
- 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*
A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 96
0*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*
C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^
3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 + 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*
a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B
^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 - 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^
5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2
c^6, z, k)(root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680
*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^
10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^
4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3
*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*
c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4
*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2
*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C
^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 +
24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c
^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2
*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a
^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c
^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z -
15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c
^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*
a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A

*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^
3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^
2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c
^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^
3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5
*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c
^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 +
 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*
a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 -
2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3
*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*((1032*A*B*a^3*b^5*c^4 - 1
52*A*B*a^2*b^7*c^3 - 768*B*C*a^6*c^6 - 2944*A*B*a^4*b^3*c^5 + 16*B*C*a^3*b^
6*c^3 - 208*B*C*a^4*b^4*c^4 + 768*B*C*a^5*b^2*c^5 + 8*A*B*a*b^9*c^2 + 2944*
A*B*a^5*b*c^6)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) +
 root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680*a^7*b^6*c^3
*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^10*c^6*z^4 -
 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^4*c^4*z^3 +
327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3*b^10*c*z^3
- 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*c^5*z^2 + 25
6*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4*b^5*c^3*z^2
 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2*a^2*b^9*c*z
^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C^2*a^5*b^4*c
^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 + 24064*B^2*a^
4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c^5*z^2 - 288
768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2*a^2*b^8*c^2
*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a^6*c^6*z^2 -
 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c^2*z + 3072*
A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z - 15872*B^2*C
*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c^2*z - 45056
*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*a^4*b^2*c^4*
z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A*B^2*a^3*b^3
*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^3*a^3*b^2*c^
5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^2*C*a^5*c^5*
z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c^6*z - 1088*
A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^3*c^3 - 1920
*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5*c^2 + 768*A
^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c^5 + 1104*A^
2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 + 1536*A*C^3*
a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*a*b^2*c^5 +
192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 - 2048*A^2*C^2
*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3*c^5 - 144*A
^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*((x*(983040*A*a^8*c^8 - 32768*C*a^8*b*
c^7 + 192*A*a^2*b^12*c^2 - 4736*A*a^3*b^10*c^3 + 48896*A*a^4*b^8*c^4 - 2703
36*A*a^5*b^6*c^5 + 843776*A*a^6*b^4*c^6 - 1409024*A*a^7*b^2*c^7 - 128*C*a^4
*b^9*c^3 + 2048*C*a^5*b^7*c^4 - 12288*C*a^6*b^5*c^5 + 32768*C*a^7*b^3*c^6))

/(16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c ^3)) - (3584*B*a^7*b*c^6 + 8*B*a^3*b^9*c^2 - 152*B*a^4*b^7*c^3 + 1056*B*a^5 *b^5*c^4 - 3200*B*a^6*b^3*c^5)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48 *a^5*b^2*c^2)) + (root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 3 27680*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 10485 76*a^10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a ^6*b^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144* A*a^3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a ^6*b*c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A* C*a^4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 43 2*B^2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6 144*C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z ^2 + 24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5* b^2*c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 1574 4*A^2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216* A^2*a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a* b^7*c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8* c*z - 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2* b^6*c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A *C^2*a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2 304*A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 614 40*A^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 184 32*B^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3* a^4*c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a ^2*b^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2* a*b^5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^ 2*b*c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3* c^4 + 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536 *A^4*a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c ^2 - 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^ 4*a^3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*x*(1310720*a^10*c^8 + 384*a^4*b^12*c^2 - 8960*a^5*b^10*c^3 + 87040*a^6*b^8*c^4 - 450560*a^7*b^6* c^5 + 1310720*a^8*b^4*c^6 - 2031616*a^9*b^2*c^7))/(16*(a^3*b^8 + 256*a^7*c^ 4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))) - (x*(26560*A^2*a^3* b^6*c^5 - 36864*C^2*a^7*c^7 - 2912*A^2*a^2*b^8*c^4 - 245760*A^2*a^6*c^8 - 1 20832*A^2*a^4*b^4*c^6 + 273408*A^2*a^5*b^2*c^7 + 432*B^2*a^2*b^9*c^3 - 4616 *B^2*a^3*b^7*c^4 + 24032*B^2*a^4*b^5*c^5 - 60800*B^2*a^5*b^3*c^6 + 640*C^2* a^4*b^6*c^4 - 7424*C^2*a^5*b^4*c^5 + 28672*C^2*a^6*b^2*c^6 + 128*A^2*a*b^10 *c^3 - 16*B^2*a*b^11*c^2 + 59904*B^2*a^6*b*c^7 + 256*A*C*a^2*b^9*c^3 - 4608 *A*C*a^3*b^7*c^4 + 30464*A*C*a^4*b^5*c^5 - 88064*A*C*a^5*b^3*c^6 + 94208*A* C*a^6*b*c^7))/(16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))) + (108*B^3*a^4*c^6 - 15*B^3*a^3*b^2*c^5 + 24*A^2*B*a*b^5 *c^4 + 704*A^2*B*a^3*b*c^6 + 56*B*C^2*a^4*b*c^5 - 266*A^2*B*a^2*b^3*c^5 - 8 *B*C^2*a^3*b^3*c^4 + 576*A*B*C*a^4*c^6 - 16*A*B*C*a*b^6*c^3 + 208*A*B*C*a^2 *b^4*c^4 - 744*A*B*C*a^3*b^2*c^5)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +

48*a^5*b^2*c^2)) + (x*(20480*A^3*a^4*c^8 - 32*A^3*b^8*c^4 + 1216*A^3*a^2*b
^4*c^6 - 11008*A^3*a^3*b^2*c^7 + 128*C^3*a^4*b^3*c^5 + 13312*A*C^2*a^5*c^7
- 19584*B^2*C*a^5*c^7 + 192*A^3*a*b^6*c^5 - 512*C^3*a^5*b*c^6 + 40*A*B^2*a*
b^7*c^4 - 2496*A*B^2*a^4*b*c^7 + 256*A^2*C*a*b^7*c^4 - 25600*A^2*C*a^4*b*c^
7 - 32*B^2*C*a*b^8*c^3 - 508*A*B^2*a^2*b^5*c^5 + 2016*A*B^2*a^3*b^3*c^6 - 6
4*A*C^2*a^2*b^6*c^4 + 1152*A*C^2*a^3*b^4*c^5 - 6912*A*C^2*a^4*b^2*c^6 - 355
2*A^2*C*a^2*b^5*c^5 + 16512*A^2*C*a^3*b^3*c^6 + 672*B^2*C*a^2*b^6*c^4 - 500
0*B^2*C*a^3*b^4*c^5 + 16192*B^2*C*a^4*b^2*c^6))/(16*(a^3*b^8 + 256*a^7*c^4
- 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))) - (108*A*B^3*a^2*c^6 -
10*A^3*B*b^3*c^5 - 192*A^2*B*C*a^2*c^6 - 15*A*B^3*a*b^2*c^5 + 64*A^3*B*a*b
*c^6 - 8*A*B*C^2*a*b^3*c^4 + 56*A*B*C^2*a^2*b*c^5 + 24*A^2*B*C*a*b^2*c^5)/(
4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) + (x*(1296*B^4*a^
3*c^7 - 48*A^4*b^4*c^6 + 256*C^4*a^4*c^6 + 1024*A^2*C^2*a^3*c^7 - 360*B^4*a
^2*b^2*c^6 + 32*A^3*C*b^5*c^5 + 256*A^4*a*b^2*c^7 + 25*B^4*a*b^4*c^5 - 3456
*A*B^2*C*a^3*c^7 - 1024*A*C^3*a^3*b*c^6 - 1024*A^3*C*a^2*b*c^7 - 176*A^2*B^
2*a*b^3*c^6 + 960*A^2*B^2*a^2*b*c^7 + 128*A*C^3*a^2*b^3*c^5 - 128*A^2*C^2*a
*b^4*c^5 + 16*B^2*C^2*a*b^5*c^4 + 960*B^2*C^2*a^3*b*c^6 + 640*A^2*C^2*a^2*b
^2*c^6 - 240*B^2*C^2*a^2*b^3*c^5 - 40*A*B^2*C*a*b^4*c^5 + 768*A*B^2*C*a^2*b
^2*c^6))/(16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3)))*root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 32768
0*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a
^10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b
^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^
3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b
*c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^
4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^
2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*
C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 +
24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*
c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^
2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*
a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*
c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z
- 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*
c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2
*a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*
A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A
^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B
^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*
c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b
^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^
5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*
c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4
+ 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4
*a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 -

```
 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^
3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k), k, 1, 4) + (A*log(x))/a^
2
```

3.35 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 514

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx = -\frac{3Ab^2-10aAc-abC}{2a^2(b^2-4ac)x} + \frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} - \frac{\sqrt{c}\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{b-\sqrt{b\cdots}}}\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(3Ab^2-10aAc-abC-\frac{A(3b^3-16abc)-a(b^2-12ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB(b^2-6ac)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}} + \frac{B\log(x)}{a^2} - \frac{B\log(a+bx^2+cx^4)}{4a^2}$$

```
[Out] 1/2*(10*A*a*c-3*A*b^2+C*a*b)/a^2/(-4*a*c+b^2)/x+1/2*B*(b*c*x^2-2*a*c+b^2)/a
/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/
a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1/2*b*B*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(
-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+B*ln(x)/a^2-1/4*B*ln(c*x^4+b*x^2+
a)/a^2-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
-a*C*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))+A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)
^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(3*A*b^2-10*a*A*c-a*b*C+(-A*(-16*a*b*c+3*b^3)+a*(-12*a*c+b^2)*C
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1676, 1291, 1295, 1180, 211, 12, 1128, 754, 814, 648, 632, 212, 642}

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx = -\frac{\sqrt{c}\left(A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(-\frac{A(3b^3-16abc)-aC(b^2-12ac)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{-10aAc-abC+3Ab^2}{2a^2x(b^2-4ac)} + \frac{bB(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}} - \frac{B\log(a+bx^2+cx^4)}{4a^2} + \frac{B\log(x)}{a^2} + \frac{A(b^2-2ac)+cx^2(Ab-2aC)-abC}{2ax(b^2-4ac)(a+bx^2+cx^4)} + \frac{B(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(3*A*b^2 - 10*a*A*c - a*b*C)/(a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c +
 b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b
*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt
[c]*(A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*
c]) - a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - 10*a*A*c - a*b*C - (A*(3*b^3 - 16*
a*b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^
2 - 4*a*c]]) + (b*B*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])
/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[x])/a^2 - (B*Log[a + b*x^2 + c*x^4])/
(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{B}{x\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}$$

$$+ B\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx - \frac{\int \frac{-3Ab^2+10aAc+abC-3c(Ab-2aC)x^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}$$

$$+ \frac{1}{2}B\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$+ \frac{\int \frac{-A\left(3b^3-13abc\right)+a\left(b^2-6ac\right)C-c\left(3Ab^2-10aAc-abC\right)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$+ \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{B\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$- \frac{\left(c\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}\sqrt{}}}{4a^2\left(b^2-4ac\right)^{3/2}}$$

$$- \frac{\left(c\left(3Ab^2-10aAc-abC-\frac{A\left(3b^3-16abc\right)-a\left(b^2-12ac\right)C}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a^2\left(b^2-4ac\right)}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$+ \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}$$

$$- \frac{\sqrt{c}\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt{c}\left(3Ab^2-10aAc-abC-\frac{A\left(3b^3-16abc\right)-a\left(b^2-12ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

$$- \frac{B\text{Subst}\left(\int\left(\frac{-b^2+4ac}{ax}+\frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2-4ac)x} + \frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$
$$+ \frac{A(b^2-2ac) - abC + c(Ab-2aC)x^2}{2a(b^2-4ac)x(a+bx^2+cx^4)}$$
$$- \frac{\sqrt{c}\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt{c}\left(3Ab^2-10aAc-abC-\frac{A(3b^3-16abc)-a(b^2-12ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{B\log(x)}{a^2} - \frac{B\text{Subst}\left(\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2(b^2-4ac)}$$

$$= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2-4ac)x} + \frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$
$$+ \frac{A(b^2-2ac) - abC + c(Ab-2aC)x^2}{2a(b^2-4ac)x(a+bx^2+cx^4)}$$
$$- \frac{\sqrt{c}\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt{c}\left(3Ab^2-10aAc-abC-\frac{A(3b^3-16abc)-a(b^2-12ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{B\log(x)}{a^2} - \frac{B\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}$$
$$- \frac{(bB(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2(b^2-4ac)}$$

$$= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2-4ac)x} + \frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$
$$+ \frac{A(b^2-2ac) - abC + c(Ab-2aC)x^2}{2a(b^2-4ac)x(a+bx^2+cx^4)}$$
$$- \frac{\sqrt{c}\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt{c}\left(3Ab^2-10aAc-abC-\frac{A(3b^3-16abc)-a(b^2-12ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{B\log(x)}{a^2} - \frac{B\log\left(a+bx^2+cx^4\right)}{4a^2}$$
$$+ \frac{(bB(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^2(b^2-4ac)}$$

$$= -\frac{3Ab^2 - 10aAc - abC}{2a^2(b^2-4ac)x} + \frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{A(b^2-2ac) - abC + c(Ab-2aC)x^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} - \frac{\sqrt{c}\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(3Ab^2-10aAc-abC-\frac{A(3b^3-16abc)-a(b^2-12ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}} + \frac{B\log(x)}{a^2} - \frac{B\log(a+bx^2+cx^4)}{4a^2}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{4A}{x} + \frac{-4a^2c(B+Cx)-2Ab^2x(b+cx^2)+2a(2Ac^2x^3+b^2(B+Cx)+bcx(3A+x(B+Cx)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-}\right.\right.}{(b^2-4ac}}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (-4*a^2*c*(B + C*x) - 2*A*b^2*x*(b + c*x^2) + 2*a*(2*A*c^2*x^3
+ b^2*(B + C*x) + b*c*x*(3*A + x*(B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] +
 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(A*(3*b^3 - 16*a*b*c - 3*b^2
*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 12*a*c + b*Sqrt[
b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*B*Log[x] - (B*(b^3 - 6
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (B*(-b^3 + 6*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.30

method	result
default	$-\frac{A}{a^2 x}+\frac{\ln(x)B}{a^2}-\frac{\frac{c(2Aac-A\,b^2+abC)x^3}{8ac-2b^2}+\frac{x^2 Babc}{8ac-2b^2}+\frac{(3Aabc-A\,b^3-2a^2cC+Ca\,b^2)x}{8ac-2b^2}-\frac{aB(2ac-b^2)}{2(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{(12Babc\sqrt{-4ac+b^2}-2B\,b^3\sqrt{-}}{}\right.}{}$
risch	Expression too large to display

[In] int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -A/a^2/x+ln(x)/a^2*B-1/a^2*((1/2*c*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2)*x^3+1/
2/(4*a*c-b^2)*x^2*B*a*b*c+1/2*(3*A*a*b*c-A*b^3-2*C*a^2*c+C*a*b^2)/(4*a*c-b^
2)*x-1/2*a*B*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(1
6*a*c-4*b^2)*(1/4*(12*B*a*b*c*(-4*a*c+b^2)^(1/2)-2*B*b^3*(-4*a*c+b^2)^(1/2)
+32*B*a^2*c^2-16*B*a*b^2*c+2*B*b^4)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*
(16*A*a*b*c*(-4*a*c+b^2)^(1/2)-3*A*b^3*(-4*a*c+b^2)^(1/2)+40*A*a^2*c^2-22*A
*a*b^2*c+3*A*b^4-12*C*(-4*a*c+b^2)^(1/2)*a^2*c+C*(-4*a*c+b^2)^(1/2)*a*b^2+4
*C*a^2*b*c-C*a*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(-1/4*(12*B*a*b*c*
(-4*a*c+b^2)^(1/2)-2*B*b^3*(-4*a*c+b^2)^(1/2)-32*B*a^2*c^2+16*B*a*b^2*c-2*B
*b^4)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(16*A*a*b*c*(-4*a*c+b^2)^(1/2
)-3*A*b^3*(-4*a*c+b^2)^(1/2)-40*A*a^2*c^2+22*A*a*b^2*c-3*A*b^4-12*C*(-4*a*c
+b^2)^(1/2)*a^2*c+C*(-4*a*c+b^2)^(1/2)*a*b^2-4*C*a^2*b*c+C*a*b^3)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)))))

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Cx^2+Bx+A}{\left(cx^4+bx^2+a\right)^2x^2}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*(B*a*b*c*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A
*a^2*c + (C*a*b^2 - 3*A*b^3 - (2*C*a^2 - 11*A*a*b)*c)*x^2 + (B*a*b^2 - 2*B*
a^2*c)*x)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b
^2 - 4*a^4*c)*x) + 1/2*integrate((C*a*b^2 - 3*A*b^3 - 2*(B*b^2*c - 4*B*a*c^
2)*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^2 - (6*C*a^2 - 13*A*a*b)*c -
2*(B*b^3 - 5*B*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + B*lo
g(x)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9013 vs. $2(453) = 906$.

Time = 2.08 (sec) , antiderivative size = 9013, normalized size of antiderivative = 17.54

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] -1/4*B*log(abs(c*x^4 + b*x^2 + a))/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a*b*c*x
^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b*c*x^3 + C*a*b^2*x^2 - 3*A*b^3*x
^2 - 2*C*a^2*c*x^2 + 11*A*a*b*c*x^2 + B*a*b^2*x - 2*B*a^2*c*x - 2*A*a*b^2 +
 8*A*a^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) + 1/16*((a^4*b^4*c
- 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 -
```

```
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c
)*a*c^3)*A - (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b^3*c^2 - 8*a^
2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*
a*b*c^2)*C - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 49*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 6*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 6*a^4*b^9*c^2 + 300*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^6*b^5*c^3 + 74*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 98*a^
5*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 304*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 37*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 600*a^6*b^5*c^4 + 832*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5 + 416*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^7*b^2*c^5 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 +
1632*a^7*b^3*c^5 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 -
1664*a^8*b*c^6 + 6*(b^2 - 4*a*c)*a^4*b^7*c^2 - 74*(b^2 - 4*a*c)*a^5*b^5*c^3
 + 304*(b^2 - 4*a*c)*a^6*b^3*c^4 - 416*(b^2 - 4*a*c)*a^7*b*c^5)*A*abs(a^4*b
^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^8*c - 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^2 - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 2*a^5*b^8*c^2 + 120*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^3 + 28*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^6*c^3 + 36*a^6*b^6*c^3 - 352*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^8*b^2*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 14
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 240*a^7*b^4*c^4 + 38
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*c^5 + 192*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^7*b^2*c^5 + 704*a^8*b^2*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^8*c^6 - 768*a^9*c^6 + 2*(b^2 - 4*a*c)*a^5*b^6*c^2 - 28*(b^2 - 4*a*c)*a^6
*b^4*c^3 + 128*(b^2 - 4*a*c)*a^7*b^2*c^4 - 192*(b^2 - 4*a*c)*a^8*c^5)*C*abs
(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (6*a^8*b^12*c^4 - 128*a^9*b^10*c
^5 + 1088*a^10*b^8*c^6 - 4608*a^11*b^6*c^7 + 9728*a^12*b^4*c^8 - 8192*a^13*
b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b
^12*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*
b^10*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*
b^11*c^3 - 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
10*b^8*c^4 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^9*b^9*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^8*b^10*c^4 + 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^11*b^6*c^5 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
```

```
*c)*a^10*b^7*c^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^9*b^8*c^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^12*b^4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^11*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^10*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^13*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^12*b^3*c^7 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^11*b^4*c^7 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^8 - 6*(b^2 - 4*a*c)*a^8*b^10*c^4 + 104*(b
^2 - 4*a*c)*a^9*b^8*c^5 - 672*(b^2 - 4*a*c)*a^10*b^6*c^6 + 1920*(b^2 - 4*a*
c)*a^11*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^12*b^2*c^8)*A - (2*a^9*b^11*c^4 - 56
*a^10*b^9*c^5 + 576*a^11*b^7*c^6 - 2816*a^12*b^5*c^7 + 6656*a^13*b^3*c^8 -
6144*a^14*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^9*b^11*c^2 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^10*b^9*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^9*b^10*c^3 - 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^11*b^7*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^10*b^8*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^9*b^9*c^4 + 1408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^12*b^5*c^5 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^11*b^6*c^5 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^10*b^7*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^13*b^3*c^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^12*b^4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^11*b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^14*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^13*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^12*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^13*b*c^8 - 2*(b^2 - 4*a*c)*a^9*b^9*c^4 + 48*(b^2
- 4*a*c)*a^10*b^7*c^5 - 384*(b^2 - 4*a*c)*a^11*b^5*c^6 + 1280*(b^2 - 4*a*c)
*a^12*b^3*c^7 - 1536*(b^2 - 4*a*c)*a^13*b*c^8)*C)*arctan(2*sqrt(1/2)*x/sqrt
((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 + sqrt((a^4*b^5*c - 8*a^5*b^3*c^
2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c
^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^
4)))/((a^7*b^8*c - 16*a^8*b^6*c^2 - 2*a^7*b^7*c^2 + 96*a^9*b^4*c^3 + 24*a^8
*b^5*c^3 + a^7*b^6*c^3 - 256*a^10*b^2*c^4 - 96*a^9*b^3*c^4 - 12*a^8*b^4*c^4
 + 256*a^11*c^5 + 128*a^10*b*c^5 + 48*a^9*b^2*c^5 - 64*a^10*c^6)*abs(a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*abs(c)) + 1/16*((a^4*b^4*c - 8*a^5*b^2*c^
2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
```

- 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*A - (
a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*C - 2
*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 49*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^8*c^2 + 6*a^4*b^9*c^2 + 300*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b^5*c^3 + 74*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 +
3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 - 98*a^5*b^7*c^3 - 81
6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 304*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^5*b^5*c^4 + 600*a^6*b^5*c^4 + 832*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^8*b*c^5 + 416*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5
 + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 - 1632*a^7*b^3*c
^5 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 + 1664*a^8*b*c^6
 - 6*(b^2 - 4*a*c)*a^4*b^7*c^2 + 74*(b^2 - 4*a*c)*a^5*b^5*c^3 - 304*(b^2 -
4*a*c)*a^6*b^3*c^4 + 416*(b^2 - 4*a*c)*a^7*b*c^5)*A*abs(a^4*b^4*c - 8*a^5*b
^2*c^2 + 16*a^6*c^3) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^8*c
 - 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^2 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 + 2*a^5*b^8*c^2 + 120*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^3 + 28*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^6*b^5*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 -
 36*a^6*b^6*c^3 - 352*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^4 -
 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 14*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 + 240*a^7*b^4*c^4 + 384*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*c^5 + 192*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b*c^5 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 -
 704*a^8*b^2*c^5 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^6 + 768
*a^9*c^6 - 2*(b^2 - 4*a*c)*a^5*b^6*c^2 + 28*(b^2 - 4*a*c)*a^6*b^4*c^3 - 128
*(b^2 - 4*a*c)*a^7*b^2*c^4 + 192*(b^2 - 4*a*c)*a^8*c^5)*C*abs(a^4*b^4*c - 8
*a^5*b^2*c^2 + 16*a^6*c^3) + (6*a^8*b^12*c^4 - 128*a^9*b^10*c^5 + 1088*a^10
*b^8*c^6 - 4608*a^11*b^6*c^7 + 9728*a^12*b^4*c^8 - 8192*a^13*b^2*c^9 - 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^12*c^2 + 64*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^10*c^3 + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^11*c^3 - 544
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^8*c^4 - 1
04*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^9*c^4 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^4 +
2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^6*c^5
 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^7*c
^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c
^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^
4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11

*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^13*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^3*c^7 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^4*c^7 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^8 - 6*(b^2 - 4*a*c)*a^8*b^10*c^4 + 104*(b^2 - 4*a*c)*a^9*b^8*c^5 - 672*(b^2 - 4*a*c)*a^10*b^6*c^6 + 1920*(b^2 - 4*a*c)*a^11*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^12*b^2*c^8)*A - (2*a^9*b^11*c^4 - 56*a^10*b^9*c^5 + 576*a^11*b^7*c^6 - 2816*a^12*b^5*c^7 + 6656*a^13*b^3*c^8 - 6144*a^14*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^11*c^2 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^9*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^10*c^3 - 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^7*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^8*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^9*c^4 + 1408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^5*c^5 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^6*c^5 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^7*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^13*b^3*c^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^14*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^13*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^13*b*c^8 - 2*(b^2 - 4*a*c)*a^9*b^9*c^4 + 48*(b^2 - 4*a*c)*a^10*b^7*c^5 - 384*(b^2 - 4*a*c)*a^11*b^5*c^6 + 1280*(b^2 - 4*a*c)*a^12*b^3*c^7 - 1536*(b^2 - 4*a*c)*a^13*b*c^8)*C)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 - sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/((a^7*b^8*c - 16*a^8*b^6*c^2 - 2*a^7*b^7*c^2 + 96*a^9*b^4*c^3 + 24*a^8*b^5*c^3 + a^7*b^6*c^3 - 256*a^10*b^2*c^4 - 96*a^9*b^3*c^4 - 12*a^8*b^4*c^4 + 256*a^11*c^5 + 128*a^10*b*c^5 + 48*a^9*b^2*c^5 - 64*a^10*c^6)*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*abs(c)) - 1/16*((b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 24*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 6*a*b^2*c^4 + (b^5*c - 10*a*b^3*c^2 - 2*b^4*c^2 + 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*sqrt(b^2 - 4*a*c))*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) - (a^4*b^10*c^2 - 18*a^5*b^8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 + 28*a^5*b^7*c^4 + a^4*b^8*c^4 - 352*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*b^6*c^5 + 384*a^8*b^2*c^6 + 192*a^7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c^7 + (a^4*b^9*c^2 - 14*a^5*b^7*c^3 - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^5*b^6*c^4 + a^4*b^7*c^4 - 96*a^7*b^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5 + 24*a^6*b^3*c^6)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 + sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^

```
2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 -
 8*a^5*b^2*c^3 + 16*a^6*c^4))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^
5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*c^2*abs(a^4*b^4*c - 8*a^5*b^
2*c^2 + 16*a^6*c^3)) - 1/16*((b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 24*a^2*b^2
*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 6*a*b^2*c^4 + (b^5*c - 10*a*b^3*c^2 - 2*b^4
*c^2 + 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*sqrt(b^2 - 4*a*c)
)*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) - (a^4*b^10*c^2 - 18*a^5*b^
8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 + 28*a^5*b^7*c^4 + a^4*b^8*c^4 - 35
2*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*b^6*c^5 + 384*a^8*b^2*c^6 + 192*a^
7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c^7 + (a^4*b^9*c^2 - 14*a^5*b^7*c^3
 - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^5*b^6*c^4 + a^4*b^7*c^4 - 96*a^7*b
^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5 + 24*a^6*b^3*c^6)*sqrt(b^2 - 4*a*c
))*B)*log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 - sqrt((a^4*b
^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*
a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*
b^2*c^3 + 16*a^6*c^4))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 +
 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*c^2*abs(a^4*b^4*c - 8*a^5*b^2*c^2 +
 16*a^6*c^3))
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 8684, normalized size of antiderivative = 16.89

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680*
a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*z^4 - 1048576*a^1
1*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4
*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*
b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10
*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 129024*A
*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 + 61440*
C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2 - 43008
0*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z^2 - 6144
0*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^
2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4
*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c^5*z^2 - 483
840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^
2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 -
144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*a^3*b^4*c^4*z -
 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z
- 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 122880*A*B*C*a^5*c^6*
```

z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c
^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a
^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*c^3*z - 192*B^3*a
*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b
^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^
2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a
*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a
*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*
c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^3*c^6 - 144*B^4*a*b^4*c^4 + 420
0*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*
c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 3
24*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 - 441*A^4*b^4*c^5
- 10000*A^4*a^2*c^7, z, k)*(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*
c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*
z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 -
983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*
z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 -
2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c
^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*
b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b
^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a
^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C
^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^
2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*
b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 332
32*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^
2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*
C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B
*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 1228
80*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z - 4
8*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*c^
4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*
c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3*a^5*c^6*z - 5
568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4
- 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3
+ 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6
+ 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*C*a*b^3*c^5 +
5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^3*c^6 - 144*B^
4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4
- 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6*c^3 - 7200*A^
2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 -
441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k)*(root(1572864*a^10*b^2*c^5*z^4
- 983040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 +
6144*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*
a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 614

40*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*
B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358
400*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^
2*z^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2
 + 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z
^2 + 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5
*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B
^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2
+ 716800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*
b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^
7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c
^5*z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b
^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a
*b^7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*
a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656
*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z +
3328*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*
B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2
*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*
B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*
A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A
^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C
*a^3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360
*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*
b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4
096*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k)*((x*(983040*B*
a^9*c^8 + 192*B*a^3*b^12*c^2 - 4736*B*a^4*b^10*c^3 + 48896*B*a^5*b^8*c^4 -
270336*B*a^6*b^6*c^5 + 843776*B*a^7*b^4*c^6 - 1409024*B*a^8*b^2*c^7))/(16*(
a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) -
 (10240*A*a^8*c^7 + 7168*C*a^8*b*c^6 - 48*A*a^3*b^10*c^2 + 832*A*a^4*b^8*c^
3 - 5536*A*a^5*b^6*c^4 + 17280*A*a^6*b^4*c^5 - 24064*A*a^7*b^2*c^6 + 16*C*a
^4*b^9*c^2 - 304*C*a^5*b^7*c^3 + 2112*C*a^6*b^5*c^4 - 6400*C*a^7*b^3*c^5)/(
8*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (root(1572864*a
^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a
^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*
z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b
^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^
9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b
^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 2476
8*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C
^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 340
8*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 2
4064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^
5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*
a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 +

170496*A^2*a^3*b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2
- 393216*B^2*a^7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592
*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z
+ 288*A*B*C*a*b^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*
z + 7344*A^2*B*a*b^7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^
4*z + 576*B*C^2*a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b
^3*c^5*z - 46656*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*
a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^
9*c^2*z - 65536*B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^
4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*
a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2
*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^
2*b*c^6 - 6192*A^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3
+ 15360*A*B^2*C*a^3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3
*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^
5 - 225*A^2*C^2*b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296
*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k
)*x*(1310720*a^11*c^8 + 384*a^5*b^12*c^2 - 8960*a^6*b^10*c^3 + 87040*a^7*b^
8*c^4 - 450560*a^8*b^6*c^5 + 1310720*a^9*b^4*c^6 - 2031616*a^10*b^2*c^7))/(
16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3
))) + (5120*A*B*a^6*c^7 + 832*A*B*a^2*b^8*c^3 - 5392*A*B*a^3*b^6*c^4 + 1574
4*A*B*a^4*b^4*c^5 - 18944*A*B*a^5*b^2*c^6 + 16*B*C*a^2*b^9*c^2 - 304*B*C*a^
3*b^7*c^3 + 2064*B*C*a^4*b^5*c^4 - 5888*B*C*a^5*b^3*c^5 - 48*A*B*a*b^10*c^2
+ 5888*B*C*a^6*b*c^6)/(8*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2
c^2)) + (x(144*A^2*b^13*c^2 + 245760*B^2*a^7*c^8 + 33304*A^2*a^2*b^9*c^4
- 171768*A^2*a^3*b^7*c^5 + 492320*A^2*a^4*b^5*c^6 - 742016*A^2*a^5*b^3*c^7
- 128*B^2*a^2*b^10*c^3 + 2912*B^2*a^3*b^8*c^4 - 26560*B^2*a^4*b^6*c^5 + 120
832*B^2*a^5*b^4*c^6 - 273408*B^2*a^6*b^2*c^7 + 16*C^2*a^2*b^11*c^2 - 432*C^
2*a^3*b^9*c^3 + 4616*C^2*a^4*b^7*c^4 - 24032*C^2*a^5*b^5*c^5 + 60800*C^2*a^
6*b^3*c^6 - 276480*A*C*a^7*c^8 - 3408*A^2*a*b^11*c^3 + 458240*A^2*a^6*b*c^8
- 59904*C^2*a^7*b*c^7 + 2432*A*C*a^2*b^10*c^3 - 24816*A*C*a^3*b^8*c^4 + 12
9952*A*C*a^4*b^6*c^5 - 365440*A*C*a^5*b^4*c^6 + 515584*A*C*a^6*b^2*c^7 - 96
*A*C*a*b^12*c^2))/(16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^
2 - 256*a^7*b^2*c^3))) + (216*C^3*a^5*c^6 + 63*A^3*a^2*b^3*c^6 - 30*C^3*a^4
*b^2*c^5 + 4480*A*B^2*a^4*c^7 + 600*A^2*C*a^4*c^7 - 300*A^3*a^3*b*c^7 - 144
*A*B^2*a*b^6*c^4 - 564*A*C^2*a^4*b*c^6 + 1408*B^2*C*a^4*b*c^6 + 1536*A*B^2*
a^2*b^4*c^5 - 4984*A*B^2*a^3*b^2*c^6 + 105*A*C^2*a^3*b^3*c^5 - 45*A^2*C*a^2
*b^4*c^5 + 102*A^2*C*a^3*b^2*c^6 + 48*B^2*C*a^2*b^5*c^4 - 532*B^2*C*a^3*b^3
c^5)/(8(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (x*(2048
0*B^3*a^5*c^8 + 192*B^3*a^2*b^6*c^5 + 1216*B^3*a^3*b^4*c^6 - 11008*B^3*a^4*
b^2*c^7 + 360*A^2*B*b^9*c^4 - 32*B^3*a*b^8*c^4 - 6072*A^2*B*a*b^7*c^5 + 112
320*A^2*B*a^4*b*c^8 - 2496*B*C^2*a^5*b*c^7 + 38284*A^2*B*a^2*b^5*c^6 - 1071
04*A^2*B*a^3*b^3*c^7 + 40*B*C^2*a^2*b^7*c^4 - 508*B*C^2*a^3*b^5*c^5 + 2016*
B*C^2*a^4*b^3*c^6 - 99840*A*B*C*a^5*c^8 - 240*A*B*C*a*b^8*c^4 + 4448*A*B*C*
a^2*b^6*c^5 - 30176*A*B*C*a^3*b^4*c^6 + 89856*A*B*C*a^4*b^2*c^7))/(16*(a^4*

b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) - (6
3*A^3*B*b^3*c^6 - 640*A*B^3*a^2*c^7 + 216*B*C^3*a^3*c^6 + 600*A^2*B*C*a^2*c
^7 - 45*A^2*B*C*b^4*c^5 + 136*A*B^3*a*b^2*c^6 - 20*B^3*C*a*b^3*c^5 + 128*B^
3*C*a^2*b*c^6 - 30*B*C^3*a^2*b^2*c^5 - 300*A^3*B*a*b*c^7 + 105*A*B*C^2*a*b^
3*c^5 - 564*A*B*C^2*a^2*b*c^6 + 102*A^2*B*C*a*b^2*c^6)/(8*(a^4*b^6 - 64*a^7
*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (x*(10000*A^4*a^2*c^9 + 441*A^4*b^
4*c^7 + 1296*C^4*a^4*c^7 + 216*A^2*B^2*b^5*c^6 + 7200*A^2*C^2*a^3*c^8 + 225
*A^2*C^2*b^6*c^5 + 256*B^4*a^2*b^2*c^7 + 25*C^4*a^2*b^4*c^5 - 360*C^4*a^3*b
^2*c^6 - 630*A^3*C*b^5*c^6 - 4200*A^4*a*b^2*c^8 - 48*B^4*a*b^4*c^6 - 7680*A
*B^2*C*a^3*c^8 - 150*A*C^3*a*b^5*c^5 - 5472*A*C^3*a^3*b*c^7 + 6192*A^3*C*a*
b^3*c^7 - 15200*A^3*C*a^2*b*c^8 - 2160*A^2*B^2*a*b^3*c^7 + 5440*A^2*B^2*a^2
*b*c^8 + 1840*A*C^3*a^2*b^3*c^6 - 2070*A^2*C^2*a*b^4*c^6 + 960*B^2*C^2*a^3*
b*c^7 + 3264*A^2*C^2*a^2*b^2*c^7 - 176*B^2*C^2*a^2*b^3*c^6 - 144*A*B^2*C*a*
b^4*c^6 + 2240*A*B^2*C*a^2*b^2*c^7))/(16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^
6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*root(1572864*a^10*b^2*c^5*z^4 - 9
83040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 61
44*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8
*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*
B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a
^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400
*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z
^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 +
1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2
+ 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^
3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*
a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 7
16800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7
*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c
^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*
z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*
c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^
7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3
*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^
2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 332
8*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3
*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^
2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2
*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2
*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*
C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^
3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^
4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6
*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096
*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k), k, 1, 4) - (A/a
- (x^2*(3*A*b^3 - C*a*b^2 + 2*C*a^2*c - 11*A*a*b*c))/(2*a^2*(4*a*c - b^2))

```
+ (x^4*(10*A*a*c^2 - 3*A*b^2*c + C*a*b*c))/(2*a^2*(4*a*c - b^2)) - (B*x*(2*
a*c - b^2))/(2*a*(4*a*c - b^2)) + (B*b*c*x^3)/(2*a*(4*a*c - b^2)))/(a*x + b
*x^3 + c*x^5) + (B*log(x))/a^2
```

3.36 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 534

$$
\begin{aligned}
&\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx \\
&= -\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x} \\
&\quad+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} \\
&\quad-\frac{B\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{B\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad-\frac{\left(2A(b^4-6ab^2c+6a^2c^2)-ab(b^2-6ac)C\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}} \\
&\quad-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}
\end{aligned}
$$

```
[Out] 1/2*(6*A*a*c-2*A*b^2+C*a*b)/a^2/(-4*a*c+b^2)/x^2-1/2*B*(-10*a*c+3*b^2)/a^2/
(-4*a*c+b^2)/x+1/2*B*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1
/2*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2
+a)-1/2*(2*A*(6*a^2*c^2-6*a*b^2*c+b^4)-a*b*(-6*a*c+b^2)*C)*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(2*A*b-C*a)*ln(x)/a^3+1/4*(2
*A*b-C*a)*ln(c*x^4+b*x^2+a)/a^3-1/4*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2
))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*B*arctan
```

```
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-
10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1676, 1265, 836, 814, 648, 632, 212, 642, 12, 1135, 1295, 1180, 211}

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx \\
&=\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{-6aAc-abC+2Ab^2}{2a^2x^2\left(b^2-4ac\right)} \\
&-\frac{B\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&+\frac{B\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} \\
&-\frac{B\left(3b^2-10ac\right)}{2a^2x\left(b^2-4ac\right)}-\frac{\left(2A\left(6a^2c^2-6ab^2c+b^4\right)-abC\left(b^2-6ac\right)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}} \\
&+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(-2ac+b^2+bcx^2\right)}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}
\end{aligned}$$

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(2*A*b^2 - 6*a*A*c - a*b*C)/(a^2*(b^2 - 4*a*c)*x^2) - (B*(3*b^2 - 10*a
*c))/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*
c)*x*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)
/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - (B*Sqrt[c]*(3*b^3 - 16*a*b*c
 + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (B*Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*A*(b^4 - 6*a*b^2*c
+ 6*a^2*c^2) - a*b*(b^2 - 6*a*c)*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C
)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
```

```
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
```

```
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{B}{x^2\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + B\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2Ab^2+6aAc+abC-2c(Ab-2aC)x}{x^2(a+bx+cx^2)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)} - \frac{B\int \frac{-3b^2+10ac-3bcx^2}{x^2(a+bx^2+cx^4)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x} + \frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-2Ab^2+6aAc+abC}{ax^2} + \frac{(-b^2+4ac)(-2Ab+aC)}{a^2x} + \frac{-2A(b^4-5ab^2c+3a^2c^2)+ab(b^2-5ac)C-c(b^2-4ac)(2Ab-aC)x}{a^2(a+bx+cx^2)}\right.\right.}{2a\left(b^2-4ac\right)} + \frac{B\int \frac{-b(3b^2-13ac)-c(3b^2-10ac)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2} - \frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x} + \frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aC)\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{-2A(b^4-5ab^2c+3a^2c^2)+ab(b^2-5ac)C-c(b^2-4ac)(2Ab-aC)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^3\left(b^2-4ac\right)} - \frac{\left(Bc\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a^2\left(b^2-4ac\right)} - \frac{\left(Bc\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a^2\left(b^2-4ac\right)}$$

$$= -\frac{2Ab^2-6aAc-abC}{2a^2(b^2-4ac)x^2} - \frac{B(3b^2-10ac)}{2a^2(b^2-4ac)x} + \frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)x(a+bx^2+cx^4)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \frac{B\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(2Ab-aC)\log(x)}{a^3} + \frac{(2Ab-aC)\text{Subst}\left(\int\frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^3} + \frac{(2A(b^4-6ab^2c+6a^2c^2)-ab(b^2-6ac)C)\,\text{Subst}\left(\int\frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^3(b^2-4ac)}$$

$$= -\frac{2Ab^2-6aAc-abC}{2a^2(b^2-4ac)x^2} - \frac{B(3b^2-10ac)}{2a^2(b^2-4ac)x} + \frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)x(a+bx^2+cx^4)} + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \frac{B\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(2Ab-aC)\log(x)}{a^3} + \frac{(2Ab-aC)\log(a+bx^2+cx^4)}{4a^3} - \frac{(2A(b^4-6ab^2c+6a^2c^2)-ab(b^2-6ac)C)\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a^3(b^2-4ac)}$$

$$
\begin{aligned}
&= -\frac{2Ab^2-6aAc-abC}{2a^2(b^2-4ac)x^2}-\frac{B(3b^2-10ac)}{2a^2(b^2-4ac)x} \\
&+\frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)x(a+bx^2+cx^4)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} \\
&-\frac{B\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\
&-\frac{B\sqrt{c}\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} \\
&-\frac{(2A(b^4-6ab^2c+6a^2c^2)-ab(b^2-6ac)C)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3(b^2-4ac)^{3/2}} \\
&-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(2Ab-aC)\log(a+bx^2+cx^4)}{4a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.23

$$
\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2}\,dx
$$

$$
= \frac{-\frac{2aA}{x^2}-\frac{4aB}{x}-\frac{2a(2a^2cC+b^2Bx(b+cx^2)+A(b^3-3abc+b^2cx^2-2ac^2x^2)-a(b^2C+2Bc^2x^3+bcx(3B+Cx)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}aB\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{\right.}}{(b^2-}}{}
$$

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*a*(2*a^2*c*C + b^2*B*x*(b + c*x^2) + A*(b^3
- 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2) - a*(b^2*C + 2*B*c^2*x^3 + b*c*x*(3*B
+ C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(-3*b^
3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sq
rt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 + 4*(-2*A*b + a*C)*Log[x] + ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(-b^3 + 6*a*b*c - b^2*Sqrt[b^
2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b
^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3 - 6*a*b*c - b^2*Sqrt[b^2
- 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.46

method	result
default	$-\frac{A}{2a^2x^2}-\frac{B}{a^2x}+\frac{(-2Ab+Ca)\ln(x)}{a^3}-\frac{\frac{Bac\left(2ac-b^2\right)x^3}{8ac-2b^2}+\frac{ac\left(2Aac-A\,b^2+abC\right)x^2}{8ac-2b^2}+\frac{Bab\left(3ac-b^2\right)x}{8ac-2b^2}+\frac{a\left(3Aabc-A\,b^3-2a^2cC+Ca\,b^2\right)}{8ac-2b^2}}{c\,x^4+b\,x^2+a}$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/a^2/x^2-1/a^2*B/x+(-2*A*b+C*a)/a^3*ln(x)-1/a^3*((1/2*B*a*c*(2*a*c-b^
2)/(4*a*c-b^2)*x^3+1/2*a*c*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2)*x^2+1/2*B*a*b*
(3*a*c-b^2)/(4*a*c-b^2)*x+1/2*a*(3*A*a*b*c-A*b^3-2*C*a^2*c+C*a*b^2)/(4*a*c-
b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(1/4*(24*A*(-4*a*c+
b^2)^(1/2)*a^2*c^2-24*A*(-4*a*c+b^2)^(1/2)*a*b^2*c+4*A*(-4*a*c+b^2)^(1/2)*b
^4-64*A*a^2*b*c^2+32*A*a*b^3*c-4*A*b^5+12*C*(-4*a*c+b^2)^(1/2)*a^2*b*c-2*C*
(-4*a*c+b^2)^(1/2)*a*b^3+32*C*a^3*c^2-16*C*a^2*b^2*c+2*C*a*b^4)/c*ln(2*c*x^
2+(-4*a*c+b^2)^(1/2)+b)+1/2*(16*B*(-4*a*c+b^2)^(1/2)*a^2*b*c-3*B*(-4*a*c+b^
2)^(1/2)*a*b^3+40*a^3*B*c^2-22*B*a^2*b^2*c+3*B*a*b^4)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1
/(16*a*c-4*b^2)*(-1/4*(24*A*(-4*a*c+b^2)^(1/2)*a^2*c^2-24*A*(-4*a*c+b^2)^(1
/2)*a*b^2*c+4*A*(-4*a*c+b^2)^(1/2)*b^4+64*A*a^2*b*c^2-32*A*a*b^3*c+4*A*b^5+
12*C*(-4*a*c+b^2)^(1/2)*a^2*b*c-2*C*(-4*a*c+b^2)^(1/2)*a*b^3-32*C*a^3*c^2+1
6*C*a^2*b^2*c-2*C*a*b^4)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(16*B*(-4*
a*c+b^2)^(1/2)*a^2*b*c-3*B*(-4*a*c+b^2)^(1/2)*a*b^3-40*a^3*B*c^2+22*B*a^2*b
^2*c-3*B*a*b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{x^3 (a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2}{x^3 (a + bx^2 + cx^4)^2} \, dx = \int \frac{Cx^2 + Bx + A}{(cx^4 + bx^2 + a)^2 x^3} \, dx$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*B*b^2*c - 10*B*a*c^2)*x^5 - (6*A*a*c^2 + (C*a*b - 2*A*b^2)*c)*x^4
+ A*a*b^2 - 4*A*a^2*c + (3*B*b^3 - 11*B*a*b*c)*x^3 - (C*a*b^2 - 2*A*b^3 - (
2*C*a^2 - 7*A*a*b)*c)*x^2 + 2*(B*a*b^2 - 4*B*a^2*c)*x)/((a^2*b^2*c - 4*a^3*
c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c)*x^2) - 1/2*integ
rate((3*B*a*b^3 - 13*B*a^2*b*c - 2*(4*(C*a^2 - 2*A*a*b)*c^2 - (C*a*b^2 - 2*
A*b^3)*c)*x^3 + (3*B*a*b^2*c - 10*B*a^2*c^2)*x^2 + 2*(C*a*b^3 - 2*A*b^4 - 6
*A*a^2*c^2 - 5*(C*a^2*b - 2*A*a*b^2)*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2
 - 4*a^4*c) + (C*a - 2*A*b)*log(x)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6939 vs. $2(470) = 940$.

Time = 1.72 (sec) , antiderivative size = 6939, normalized size of antiderivative = 12.99

$$\int \frac{A + Bx + Cx^2}{x^3 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3
 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b
```

^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2
 + 20*(b^2 - 4*a*c)*a*c^3)*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^6*b^9*c - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^2 - 6*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^2 - 6*a^6*b^9*c^2 + 300*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^3 + 74*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^6*b^7*c^3 + 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^4 - 37*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^4 - 600*a^8*b^5*c^4 + 832*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^5 + 416*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^5 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^8*b^3*c^5 + 1632*a^9*b^3*c^5 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^9*b*c^6 - 1664*a^10*b*c^6 + 6*(b^2 - 4*a*c)*a^6*b^7*c^2 - 74*(b^2
- 4*a*c)*a^7*b^5*c^3 + 304*(b^2 - 4*a*c)*a^8*b^3*c^4 - 416*(b^2 - 4*a*c)*a^
9*b*c^5)*B*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) + (6*a^12*b^12*c^4 -
 128*a^13*b^10*c^5 + 1088*a^14*b^8*c^6 - 4608*a^15*b^6*c^7 + 9728*a^16*b^4*
c^8 - 8192*a^17*b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^12*b^12*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^13*b^10*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^12*b^11*c^3 - 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^14*b^8*c^4 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^13*b^9*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^12*b^10*c^4 + 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^15*b^6*c^5 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^14*b^7*c^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^13*b^8*c^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^15*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^14*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^17*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^3*c^7 + 960*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^15*b^4*c^7 - 1024*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^2*c^8 - 6*(b^2 - 4*a*c)
*a^12*b^10*c^4 + 104*(b^2 - 4*a*c)*a^13*b^8*c^5 - 672*(b^2 - 4*a*c)*a^14*b^
6*c^6 + 1920*(b^2 - 4*a*c)*a^15*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^16*b^2*c^8)*
B)*arctan(2*sqrt(1/2)*x/sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3 + sq
rt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3)^2 - 4*(a^7*b^4*c - 8*a^8*b^2*
c^2 + 16*a^9*c^3)*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/(a^6*b^4*c^2
 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/((a^9*b^8*c - 16*a^10*b^6*c^2 - 2*a^9*b^7*
c^2 + 96*a^11*b^4*c^3 + 24*a^10*b^5*c^3 + a^9*b^6*c^3 - 256*a^12*b^2*c^4 -
96*a^11*b^3*c^4 - 12*a^10*b^4*c^4 + 256*a^13*c^5 + 128*a^12*b*c^5 + 48*a^11
*b^2*c^5 - 64*a^12*c^6)*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*abs(c))
 + 1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c
^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2

```
*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c
^2 + 20*(b^2 - 4*a*c)*a*c^3)*B - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^6*b^9*c - 49*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^2 - 6*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^2 + 6*a^6*b^9*c^2 + 300*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^3 + 74*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^6*b^7*c^3 - 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^4 - 37
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^4 + 600*a^8*b^5*c^4 + 83
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^5 + 416*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^5 + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b^3*c^5 - 1632*a^9*b^3*c^5 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^9*b*c^6 + 1664*a^10*b*c^6 - 6*(b^2 - 4*a*c)*a^6*b^7*c^2 + 74*(b^
2 - 4*a*c)*a^7*b^5*c^3 - 304*(b^2 - 4*a*c)*a^8*b^3*c^4 + 416*(b^2 - 4*a*c)*
a^9*b*c^5)*B*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) + (6*a^12*b^12*c^4
 - 128*a^13*b^10*c^5 + 1088*a^14*b^8*c^6 - 4608*a^15*b^6*c^7 + 9728*a^16*b^
4*c^8 - 8192*a^17*b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^12*b^12*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^13*b^10*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^12*b^11*c^3 - 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^14*b^8*c^4 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^13*b^9*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^12*b^10*c^4 + 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^15*b^6*c^5 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^14*b^7*c^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^13*b^8*c^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^16*b^4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^15*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^14*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^17*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^16*b^3*c^7 + 960*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^15*b^4*c^7 - 1024*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^16*b^2*c^8 - 6*(b^2 - 4*a*
c)*a^12*b^10*c^4 + 104*(b^2 - 4*a*c)*a^13*b^8*c^5 - 672*(b^2 - 4*a*c)*a^14*
b^6*c^6 + 1920*(b^2 - 4*a*c)*a^15*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^16*b^2*c^8
)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3 -
sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3)^2 - 4*(a^7*b^4*c - 8*a^8*b^
2*c^2 + 16*a^9*c^3)*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/(a^6*b^4*c
^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/((a^9*b^8*c - 16*a^10*b^6*c^2 - 2*a^9*b^
7*c^2 + 96*a^11*b^4*c^3 + 24*a^10*b^5*c^3 + a^9*b^6*c^3 - 256*a^12*b^2*c^4
- 96*a^11*b^3*c^4 - 12*a^10*b^4*c^4 + 256*a^13*c^5 + 128*a^12*b*c^5 + 48*a^
```

11*b^2*c^5 - 64*a^12*c^6)*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*abs(c
)) - 1/4*(C*a - 2*A*b)*log(abs(c*x^4 + b*x^2 + a))/a^3 + (C*a - 2*A*b)*log(
abs(x))/a^3 + 1/16*(2*(b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 30*a^2*b^3*c^3 +
12*a*b^4*c^3 + b^5*c^3 - 24*a^3*b*c^4 - 12*a^2*b^2*c^4 - 6*a*b^3*c^4 + 6*a^
2*b*c^5 - (b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 30*a^2*b^2*c^3 + 12*a*b^3*c^3
 + b^4*c^3 - 24*a^3*c^4 - 12*a^2*b*c^4 - 6*a*b^2*c^4 + 6*a^2*c^5)*sqrt(b^2
- 4*a*c))*A*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) - (a*b^6*c - 10*a^2
*b^4*c^2 - 2*a*b^5*c^2 + 24*a^3*b^2*c^3 + 12*a^2*b^3*c^3 + a*b^4*c^3 - 6*a^
2*b^2*c^4 + (a*b^5*c - 10*a^2*b^3*c^2 - 2*a*b^4*c^2 + 24*a^3*b*c^3 + 12*a^2
*b^2*c^3 + a*b^3*c^3 - 6*a^2*b*c^4)*sqrt(b^2 - 4*a*c))*C*abs(a^6*b^4*c - 8*
a^7*b^2*c^2 + 16*a^8*c^3) + 2*(a^6*b^11*c^2 - 18*a^7*b^9*c^3 - 2*a^6*b^10*c
^3 + 126*a^8*b^7*c^4 + 28*a^7*b^8*c^4 + a^6*b^9*c^4 - 424*a^9*b^5*c^5 - 140
*a^8*b^6*c^5 - 14*a^7*b^7*c^5 + 672*a^10*b^3*c^6 + 288*a^9*b^4*c^6 + 70*a^8
*b^5*c^6 - 384*a^11*b*c^7 - 192*a^10*b^2*c^7 - 144*a^9*b^3*c^7 + 96*a^10*b*
c^8 + (a^6*b^10*c^2 - 14*a^7*b^8*c^3 - 2*a^6*b^9*c^3 + 70*a^8*b^6*c^4 + 20*
a^7*b^7*c^4 + a^6*b^8*c^4 - 144*a^9*b^4*c^5 - 60*a^8*b^5*c^5 - 10*a^7*b^6*c
^5 + 96*a^10*b^2*c^6 + 48*a^9*b^3*c^6 + 30*a^8*b^4*c^6 - 24*a^9*b^2*c^7)*sq
rt(b^2 - 4*a*c))*A - (a^7*b^10*c^2 - 18*a^8*b^8*c^3 - 2*a^7*b^9*c^3 + 120*a
^9*b^6*c^4 + 28*a^8*b^7*c^4 + a^7*b^8*c^4 - 352*a^10*b^4*c^5 - 128*a^9*b^5*
c^5 - 14*a^8*b^6*c^5 + 384*a^11*b^2*c^6 + 192*a^10*b^3*c^6 + 64*a^9*b^4*c^6
 - 96*a^10*b^2*c^7 + (a^7*b^9*c^2 - 14*a^8*b^7*c^3 - 2*a^7*b^8*c^3 + 64*a^9
*b^5*c^4 + 20*a^8*b^6*c^4 + a^7*b^7*c^4 - 96*a^10*b^3*c^5 - 48*a^9*b^4*c^5
- 10*a^8*b^5*c^5 + 24*a^9*b^3*c^6)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^6
*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3 + sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 1
6*a^8*b*c^3)^2 - 4*(a^7*b^4*c - 8*a^8*b^2*c^2 + 16*a^9*c^3)*(a^6*b^4*c^2 -
8*a^7*b^2*c^3 + 16*a^8*c^4)))/(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4))/(
(a^4*b^4 - 8*a^5*b^2*c - 2*a^4*b^3*c + 16*a^6*c^2 + 8*a^5*b*c^2 + a^4*b^2*c
^2 - 4*a^5*c^3)*c^2*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)) + 1/16*(2*
(b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 30*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^3
 - 24*a^3*b*c^4 - 12*a^2*b^2*c^4 - 6*a*b^3*c^4 + 6*a^2*b*c^5 + (b^6*c - 10*
a*b^4*c^2 - 2*b^5*c^2 + 30*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 24*a^3*c^
4 - 12*a^2*b*c^4 - 6*a*b^2*c^4 + 6*a^2*c^5)*sqrt(b^2 - 4*a*c))*A*abs(a^6*b^
4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) - (a*b^6*c - 10*a^2*b^4*c^2 - 2*a*b^5*c^2
 + 24*a^3*b^2*c^3 + 12*a^2*b^3*c^3 + a*b^4*c^3 - 6*a^2*b^2*c^4 - (a*b^5*c -
 10*a^2*b^3*c^2 - 2*a*b^4*c^2 + 24*a^3*b*c^3 + 12*a^2*b^2*c^3 + a*b^3*c^3 -
 6*a^2*b*c^4)*sqrt(b^2 - 4*a*c))*C*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c
^3) + 2*(a^6*b^11*c^2 - 18*a^7*b^9*c^3 - 2*a^6*b^10*c^3 + 126*a^8*b^7*c^4 +
 28*a^7*b^8*c^4 + a^6*b^9*c^4 - 424*a^9*b^5*c^5 - 140*a^8*b^6*c^5 - 14*a^7*
b^7*c^5 + 672*a^10*b^3*c^6 + 288*a^9*b^4*c^6 + 70*a^8*b^5*c^6 - 384*a^11*b*
c^7 - 192*a^10*b^2*c^7 - 144*a^9*b^3*c^7 + 96*a^10*b*c^8 + (a^6*b^10*c^2 -
14*a^7*b^8*c^3 - 2*a^6*b^9*c^3 + 70*a^8*b^6*c^4 + 20*a^7*b^7*c^4 + a^6*b^8*
c^4 - 144*a^9*b^4*c^5 - 60*a^8*b^5*c^5 - 10*a^7*b^6*c^5 + 96*a^10*b^2*c^6 +
 48*a^9*b^3*c^6 + 30*a^8*b^4*c^6 - 24*a^9*b^2*c^7)*sqrt(b^2 - 4*a*c))*A - (
a^7*b^10*c^2 - 18*a^8*b^8*c^3 - 2*a^7*b^9*c^3 + 120*a^9*b^6*c^4 + 28*a^8*b^
7*c^4 + a^7*b^8*c^4 - 352*a^10*b^4*c^5 - 128*a^9*b^5*c^5 - 14*a^8*b^6*c^5 +

```
 384*a^11*b^2*c^6 + 192*a^10*b^3*c^6 + 64*a^9*b^4*c^6 - 96*a^10*b^2*c^7 - (
a^7*b^9*c^2 - 14*a^8*b^7*c^3 - 2*a^7*b^8*c^3 + 64*a^9*b^5*c^4 + 20*a^8*b^6*
c^4 + a^7*b^7*c^4 - 96*a^10*b^3*c^5 - 48*a^9*b^4*c^5 - 10*a^8*b^5*c^5 + 24*
a^9*b^3*c^6)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^6*b^5*c - 8*a^7*b^3*c^2
 + 16*a^8*b*c^3 - sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3)^2 - 4*(a^
7*b^4*c - 8*a^8*b^2*c^2 + 16*a^9*c^3)*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8
*c^4)))/(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4))/((a^4*b^4 - 8*a^5*b^2*c
 - 2*a^4*b^3*c + 16*a^6*c^2 + 8*a^5*b*c^2 + a^4*b^2*c^2 - 4*a^5*c^3)*c^2*ab
s(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)) - 1/2*((3*B*a*b^2*c - 10*B*a^2*c
^2)*x^5 + A*a^2*b^2 - 4*A*a^3*c - (C*a^2*b*c - 2*A*a*b^2*c + 6*A*a^2*c^2)*x
^4 + (3*B*a*b^3 - 11*B*a^2*b*c)*x^3 - (C*a^2*b^2 - 2*A*a*b^3 - 2*C*a^3*c +
7*A*a^2*b*c)*x^2 + 2*(B*a^2*b^2 - 4*B*a^3*c)*x)/((c*x^4 + b*x^2 + a)*(b^2 -
 4*a*c)*a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 10595, normalized size of antiderivative = 19.84

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 327680
*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*a^
12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*b^
4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*A*
a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 + 12
2880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3 -
12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + 512*
A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - 1794
048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^7*
c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3*b^1
0*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2*a*b
^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 88576
*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5*z
^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*a^
3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2 -
761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b^1
0*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12*z
^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6*z -
 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3*z
 + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c^5
*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*a^
4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 8294
```

4*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c^3
*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*a^
2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^3*
a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448*A
^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A^2
*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C^3
*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*C*
a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^2*
C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2*C
^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A^3
*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*a*
b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 +
1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^4*
a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b^5
*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^4*
a^2*c^8, z, k)*(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 3
27680*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 10485
76*a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a
^8*b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 31457
28*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3
 + 122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z
^3 - 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 +
 512*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 -
 1794048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4
*b^7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^
3*b^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^
2*a*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 +
88576*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*
c^5*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B
^2*a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z
^2 - 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^
2*b^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b
^12*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^
6*z - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*
c^3*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^
3*c^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C
^2*a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z +
 82944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^
7*c^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B
^2*a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328
*C^3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104
448*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 76
8*A^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 6553
6*C^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B
^2*C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680

*B^2*C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*
A^2*C^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 3225
6*A^3*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C
^3*a*b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c
^7 + 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200
*B^4*a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^
2*b^5*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736
*A^4*a^2*c^8, z, k)*(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^
4 + 327680*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 -
1048576*a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 98304
0*C*a^8*b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 -
3145728*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^
3*z^3 + 122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*
c^6*z^3 - 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*
z^3 + 512*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*
z^2 - 1794048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*
C*a^4*b^7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C
^2*a^3*b^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 61
44*A^2*a*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z
^2 + 88576*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6
*b^3*c^5*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33
232*B^2*a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*
c^4*z^2 - 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A
^2*a^2*b^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*
a^2*b^12*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5
*b*c^6*z - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a
*b^8*c^3*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a
^4*b^3*c^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 23347
2*A*C^2*a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^
6*z + 82944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a
^2*b^7*c^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 931
2*A*B^2*a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z +
3328*C^3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z
+ 104448*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z
- 768*A^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z -
65536*C^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 164
8*A*B^2*C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 -
7680*B^2*C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 +
4608*A^2*C^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 +
32256*A^3*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 57
6*A*C^3*a*b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*
b^2*c^7 + 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 +
4200*B^4*a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A
^2*B^2*b^5*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 -
20736*A^4*a^2*c^8, z, k)*((x*(983040*C*a^11*c^8 - 1867776*A*a^10*b*c^8 - 38

4*A*a^4*b^13*c^2 + 9472*A*a^5*b^11*c^3 - 97408*A*a^6*b^9*c^4 + 534528*A*a^7
*b^7*c^5 - 1650688*A*a^8*b^5*c^6 + 2719744*A*a^9*b^3*c^7 + 192*C*a^5*b^12*c
^2 - 4736*C*a^6*b^10*c^3 + 48896*C*a^7*b^8*c^4 - 270336*C*a^8*b^6*c^5 + 843
776*C*a^9*b^4*c^6 - 1409024*C*a^10*b^2*c^7))/(16*(a^6*b^8 + 256*a^10*c^4 -
16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)) - (10240*B*a^10*c^7 - 48*
B*a^5*b^10*c^2 + 832*B*a^6*b^8*c^3 - 5536*B*a^7*b^6*c^4 + 17280*B*a^8*b^4*c
^5 - 24064*B*a^9*b^2*c^6)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*
b^2*c^2)) + (root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 3276
80*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*
a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*
b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*
A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 +
122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3
- 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + 51
2*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - 17
94048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^
7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3*b
^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2*a
*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 885
76*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5
*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*
a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2
- 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b
^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12
*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6*z
- 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3
*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c
^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*
a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 82
944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c
^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*
a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^
3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448
*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A
^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C
^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*
C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^
2*C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2
*C^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A
^3*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*
a*b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7
+ 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^
4*a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b
^5*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^
4*a^2*c^8, z, k)*x*(1310720*a^13*c^8 + 384*a^7*b^12*c^2 - 8960*a^8*b^10*c^3

+ 87040*a^9*b^8*c^4 - 450560*a^10*b^6*c^5 + 1310720*a^11*b^4*c^6 - 2031616
*a^12*b^2*c^7))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2
- 256*a^9*b^2*c^3))) + (5120*B*C*a^8*c^7 + 96*A*B*a^2*b^11*c^2 - 1664*A*B*
a^3*b^9*c^3 + 11072*A*B*a^4*b^7*c^4 - 34752*A*B*a^5*b^5*c^5 + 49792*A*B*a^6
*b^3*c^6 - 48*B*C*a^3*b^10*c^2 + 832*B*C*a^4*b^8*c^3 - 5392*B*C*a^5*b^6*c^4
+ 15744*B*C*a^6*b^4*c^5 - 18944*B*C*a^7*b^2*c^6 - 24064*A*B*a^7*b*c^7)/(8*
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)) + (x*(331776*A^2*a^
8*c^9 + 245760*C^2*a^9*c^8 - 512*A^2*a^2*b^12*c^3 + 10112*A^2*a^3*b^10*c^4
- 78592*A^2*a^4*b^8*c^5 + 294784*A^2*a^5*b^6*c^6 - 498432*A^2*a^6*b^4*c^7 +
159744*A^2*a^7*b^2*c^8 + 144*B^2*a^2*b^13*c^2 - 3408*B^2*a^3*b^11*c^3 + 33
304*B^2*a^4*b^9*c^4 - 171768*B^2*a^5*b^7*c^5 + 492320*B^2*a^6*b^5*c^6 - 742
016*B^2*a^7*b^3*c^7 - 128*C^2*a^4*b^10*c^3 + 2912*C^2*a^5*b^8*c^4 - 26560*C
^2*a^6*b^6*c^5 + 120832*C^2*a^7*b^4*c^6 - 273408*C^2*a^8*b^2*c^7 + 458240*B
^2*a^8*b*c^8 + 512*A*C*a^3*b^11*c^3 - 10880*A*C*a^4*b^9*c^4 + 92416*A*C*a^5
*b^7*c^5 - 391936*A*C*a^6*b^5*c^6 + 829440*A*C*a^7*b^3*c^7 - 700416*A*C*a^8
*b*c^8))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*
a^9*b^2*c^3))) + (63*B^3*a^4*b^3*c^6 + 1440*A^2*B*a^5*c^8 + 4480*B*C^2*a^6*
c^7 - 300*B^3*a^5*b*c^7 - 384*A^2*B*a^2*b^6*c^5 + 3440*A^2*B*a^3*b^4*c^6 -
8000*A^2*B*a^4*b^2*c^7 - 144*B*C^2*a^3*b^6*c^4 + 1536*B*C^2*a^4*b^4*c^5 - 4
984*B*C^2*a^5*b^2*c^6 - 6112*A*B*C*a^5*b*c^7 + 288*A*B*C*a^2*b^7*c^4 - 2880
*A*B*C*a^3*b^5*c^5 + 8464*A*B*C*a^4*b^3*c^6)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2)) + (x*(256*A^3*b^11*c^4 + 20480*C^3*a^7*c^8 + 3
4048*A^3*a^2*b^7*c^6 - 130816*A^3*a^3*b^5*c^7 + 264320*A^3*a^4*b^3*c^8 - 32
*C^3*a^3*b^8*c^4 + 192*C^3*a^4*b^6*c^5 + 1216*C^3*a^5*b^4*c^6 - 11008*C^3*a
^6*b^2*c^7 - 163200*A*B^2*a^6*c^9 + 119808*A^2*C*a^6*c^9 - 4608*A^3*a*b^9*c
^5 - 225792*A^3*a^5*b*c^9 + 144*A*B^2*a*b^10*c^4 - 46080*A*C^2*a^6*b*c^8 -
384*A^2*C*a*b^10*c^4 + 112320*B^2*C*a^6*b*c^8 - 3120*A*B^2*a^2*b^8*c^5 + 26
272*A*B^2*a^3*b^6*c^6 - 107416*A*B^2*a^4*b^4*c^7 + 212928*A*B^2*a^5*b^2*c^8
+ 192*A*C^2*a^2*b^9*c^4 - 1920*A*C^2*a^3*b^7*c^5 + 3360*A*C^2*a^4*b^5*c^6
+ 16512*A*C^2*a^5*b^3*c^7 + 5376*A^2*C*a^2*b^8*c^5 - 28608*A^2*C*a^3*b^6*c^
6 + 76416*A^2*C*a^4*b^4*c^7 - 123648*A^2*C*a^5*b^2*c^8 + 360*B^2*C*a^2*b^9*
c^4 - 6072*B^2*C*a^3*b^7*c^5 + 38284*B^2*C*a^4*b^5*c^6 - 107104*B^2*C*a^5*b
^3*c^7))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*
a^9*b^2*c^3))) + (224*A^3*B*b^5*c^6 + 640*B*C^3*a^4*c^7 - 1440*A^2*B*C*a^3*
c^8 + 126*A*B^3*a*b^4*c^6 - 1664*A^3*B*a*b^3*c^7 + 2880*A^3*B*a^2*b*c^8 + 3
00*B^3*C*a^3*b*c^7 - 600*A*B^3*a^2*b^2*c^7 - 136*B*C^3*a^3*b^2*c^6 - 63*B^3
*C*a^2*b^3*c^6 - 1824*A*B*C^2*a^3*b*c^7 - 336*A^2*B*C*a*b^4*c^6 + 384*A*B*C
^2*a^2*b^3*c^6 + 1920*A^2*B*C*a^2*b^2*c^7)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2)) + (x*(20736*A^4*a^3*c^10 - 512*A^4*b^6*c^7 + 100
00*B^4*a^4*c^9 + 9216*A^2*C^2*a^4*c^9 - 18432*A^4*a^2*b^2*c^9 + 441*B^4*a^2
*b^4*c^7 - 4200*B^4*a^3*b^2*c^8 - 48*C^4*a^3*b^4*c^6 + 256*C^4*a^4*b^2*c^7
+ 384*A^3*C*b^7*c^6 + 5376*A^4*a*b^4*c^8 - 28800*A*B^2*C*a^4*c^9 + 3072*A*C
^3*a^4*b*c^8 - 3584*A^3*C*a*b^5*c^7 - 9216*A^3*C*a^3*b*c^9 - 288*A^2*B^2*a*
b^5*c^7 - 2880*A^2*B^2*a^3*b*c^9 + 288*A*C^3*a^2*b^5*c^6 - 2048*A*C^3*a^3*b
^3*c^7 - 576*A^2*C^2*a*b^6*c^6 + 10368*A^3*C*a^2*b^3*c^8 + 5440*B^2*C^2*a^4

*b*c^8 + 1936*A^2*B^2*a^2*b^3*c^8 + 4992*A^2*C^2*a^2*b^4*c^7 - 12672*A^2*C^
2*a^3*b^2*c^8 + 216*B^2*C^2*a^2*b^5*c^6 - 2160*B^2*C^2*a^3*b^3*c^7 + 216*A*
B^2*C*a*b^6*c^6 - 3096*A*B^2*C*a^2*b^4*c^7 + 15872*A*B^2*C*a^3*b^2*c^8))/(1
6*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3
)))*root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 327680*a^9*b^
6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*a^12*c^6*
z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*b^4*c^4*z
^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*A*a^8*b^3
*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 + 122880*A*
a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3 - 12288*A
*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + 512*A*a^3*b
^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - 1794048*A*C
*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^7*c^3*z^2
 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3*b^10*c*z^2
 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2*a*b^12*c*z
^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 88576*C^2*a^
5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5*z^2 - 48
3840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*a^3*b^9*c
^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2 - 761856*
A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b^10*c^2*z
^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12*z^2 - 29
4912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6*z - 432*B^
2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3*z + 768*
A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c^5*z - 46
656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*a^4*b^3*c
^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 82944*A*C^2
*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c^3*z - 15
2064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*a^2*b^6*c
^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^3*a^3*b^6
*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448*A^3*a^2*
b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A^2*C*b^10
*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C^3*a^6*c^
6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*C*a*b^4*c
^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^2*C^2*a^3
*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2*C^2*a*b^
4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A^3*C*a^2*
b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*a*b^5*c^4
 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 + 1536*A^
3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^4*a^2*b^2
*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b^5*c^5 -
4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^4*a^2*c^8
, z, k), k, 1, 4) - (A/(2*a) + (B*x)/a - (x^2*(2*A*b^3 - C*a*b^2 + 2*C*a^2*
c - 7*A*a*b*c))/(2*a^2*(4*a*c - b^2)) + (B*x^5*(10*a*c^2 - 3*b^2*c))/(2*a^2
*(4*a*c - b^2)) + (c*x^4*(6*A*a*c - 2*A*b^2 + C*a*b))/(2*a^2*(4*a*c - b^2))
 + (B*b*x^3*(11*a*c - 3*b^2))/(2*a^2*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6

```
) - (log(x)*(2*A*b - C*a))/a^3
```

3.37 $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal result

Integrand size = 30, antiderivative size = 399

$$\begin{aligned}
&\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx \\
&= \frac{a^3A(dx)^{1+m}}{d(1+m)} + \frac{a^3B(dx)^{2+m}}{d^2(2+m)} + \frac{a^2(3Ab+aC)(dx)^{3+m}}{d^3(3+m)} + \frac{3a^2bB(dx)^{4+m}}{d^4(4+m)} \\
&+ \frac{3a(A(b^2+ac)+abC)\,(dx)^{5+m}}{d^5(5+m)} + \frac{3aB(b^2+ac)\,(dx)^{6+m}}{d^6(6+m)} \\
&+ \frac{(A(b^3+6abc)+3a(b^2+ac)\,C)\,(dx)^{7+m}}{d^7(7+m)} + \frac{bB(b^2+6ac)\,(dx)^{8+m}}{d^8(8+m)} \\
&+ \frac{(3Ac(b^2+ac)+b(b^2+6ac)\,C)\,(dx)^{9+m}}{d^9(9+m)} + \frac{3Bc(b^2+ac)\,(dx)^{10+m}}{d^{10}(10+m)} \\
&+ \frac{3c(Abc+(b^2+ac)\,C)\,(dx)^{11+m}}{d^{11}(11+m)} + \frac{3bBc^2(dx)^{12+m}}{d^{12}(12+m)} \\
&+ \frac{c^2(Ac+3bC)(dx)^{13+m}}{d^{13}(13+m)} + \frac{Bc^3(dx)^{14+m}}{d^{14}(14+m)} + \frac{c^3C(dx)^{15+m}}{d^{15}(15+m)}
\end{aligned}$$

```
[Out] a^3*A*(d*x)^(1+m)/d/(1+m)+a^3*B*(d*x)^(2+m)/d^2/(2+m)+a^2*(3*A*b+C*a)*(d*x)
^(3+m)/d^3/(3+m)+3*a^2*b*B*(d*x)^(4+m)/d^4/(4+m)+3*a*(A*(a*c+b^2)+a*b*C)*(d
*x)^(5+m)/d^5/(5+m)+3*a*B*(a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(A*(6*a*b*c+b^3)+
3*a*(a*c+b^2)*C)*(d*x)^(7+m)/d^7/(7+m)+b*B*(6*a*c+b^2)*(d*x)^(8+m)/d^8/(8+m
)+(3*A*c*(a*c+b^2)+b*(6*a*c+b^2)*C)*(d*x)^(9+m)/d^9/(9+m)+3*B*c*(a*c+b^2)*(
d*x)^(10+m)/d^10/(10+m)+3*c*(A*b*c+(a*c+b^2)*C)*(d*x)^(11+m)/d^11/(11+m)+3*
b*B*c^2*(d*x)^(12+m)/d^12/(12+m)+c^2*(A*c+3*C*b)*(d*x)^(13+m)/d^13/(13+m)+B
*c^3*(d*x)^(14+m)/d^14/(14+m)+c^3*C*(d*x)^(15+m)/d^15/(15+m)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1642}

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 dx$$
$$= \frac{a^3A(dx)^{m+1}}{d(m+1)} + \frac{a^3B(dx)^{m+2}}{d^2(m+2)} + \frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)} + \frac{3a^2bB(dx)^{m+4}}{d^4(m+4)}$$
$$+ \frac{3c(dx)^{m+11}\left(C(ac+b^2)+Abc\right)}{d^{11}(m+11)} + \frac{(dx)^{m+9}\left(3Ac(ac+b^2)+bC(6ac+b^2)\right)}{d^9(m+9)}$$
$$+ \frac{3a(dx)^{m+5}\left(A(ac+b^2)+abC\right)}{d^5(m+5)} + \frac{(dx)^{m+7}\left(A(6abc+b^3)+3aC(ac+b^2)\right)}{d^7(m+7)}$$
$$+ \frac{3Bc(ac+b^2)(dx)^{m+10}}{d^{10}(m+10)} + \frac{bB(6ac+b^2)(dx)^{m+8}}{d^8(m+8)} + \frac{3aB(ac+b^2)(dx)^{m+6}}{d^6(m+6)}$$
$$+ \frac{c^2(dx)^{m+13}(Ac+3bC)}{d^{13}(m+13)} + \frac{3bBc^2(dx)^{m+12}}{d^{12}(m+12)} + \frac{Bc^3(dx)^{m+14}}{d^{14}(m+14)} + \frac{c^3C(dx)^{m+15}}{d^{15}(m+15)}$$

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2*b*B*(d*x)^(4 + m))/
(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) +
 (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c) + 3*
a*(b^2 + a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*x)^(8
 + m))/(d^8*(8 + m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)^(9 +
m))/(d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m)) + (3
*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) + (3*b*B*c^2*(d*
x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*(d*x)^(13 + m))/(d^13*(13
 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(14 + m)) + (c^3*C*(d*x)^(15 + m))/(d
^15*(15 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(a^3A(dx)^m + \frac{a^3B(dx)^{1+m}}{d} + \frac{a^2(3Ab+aC)(dx)^{2+m}}{d^2} + \frac{3a^2bB(dx)^{3+m}}{d^3} + \frac{3a(A(b^2+ac)+abC)(dx)^{4+m}}{d^4} + \frac{3aB(b^2+ac)(dx)^{5+m}}{d^5} + \frac{(A(b^3+6abc)+3a(b^2+ac)C)(dx)^{6+m}}{d^6} + \frac{bB(b^2+6ac)(dx)^{7+m}}{d^7} + \frac{(3Ac(b^2+ac)+b(b^2+6ac)C)(dx)^{8+m}}{d^8} + \frac{3Bc(b^2+ac)(dx)^{9+m}}{d^9} + \frac{3c(Abc+(b^2+ac)C)(dx)^{10+m}}{d^{10}} + \frac{3bBc^2(dx)^{11+m}}{d^{11}} + \frac{c^2(Ac+3bC)(dx)^{12+m}}{d^{12}} + \frac{Bc^3(dx)^{13+m}}{d^{13}} + \frac{c^3C(dx)^{14+m}}{d^{14}}\right) dx$$

$$= \frac{a^3A(dx)^{1+m}}{d(1+m)} + \frac{a^3B(dx)^{2+m}}{d^2(2+m)} + \frac{a^2(3Ab+aC)(dx)^{3+m}}{d^3(3+m)} + \frac{3a^2bB(dx)^{4+m}}{d^4(4+m)} + \frac{3a(A(b^2+ac)+abC)(dx)^{5+m}}{d^5(5+m)} + \frac{3aB(b^2+ac)(dx)^{6+m}}{d^6(6+m)} + \frac{(A(b^3+6abc)+3a(b^2+ac)C)(dx)^{7+m}}{d^7(7+m)} + \frac{bB(b^2+6ac)(dx)^{8+m}}{d^8(8+m)} + \frac{(3Ac(b^2+ac)+b(b^2+6ac)C)(dx)^{9+m}}{d^9(9+m)} + \frac{3Bc(b^2+ac)(dx)^{10+m}}{d^{10}(10+m)} + \frac{3c(Abc+(b^2+ac)C)(dx)^{11+m}}{d^{11}(11+m)} + \frac{3bBc^2(dx)^{12+m}}{d^{12}(12+m)} + \frac{c^2(Ac+3bC)(dx)^{13+m}}{d^{13}(13+m)} + \frac{Bc^3(dx)^{14+m}}{d^{14}(14+m)} + \frac{c^3C(dx)^{15+m}}{d^{15}(15+m)}$$

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.74

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 dx = x(dx)^m \left(\frac{a^3A}{1+m} + \frac{a^3Bx}{2+m} + \frac{a^2(3Ab+aC)x^2}{3+m} + \frac{3a^2bBx^3}{4+m} + \frac{3a(A(b^2+ac)+abC)x^4}{5+m} + \frac{3aB(b^2+ac)x^5}{6+m} + \frac{(A(b^3+6abc)+3a(b^2+ac)C)x^6}{7+m} + \frac{bB(b^2+6ac)x^7}{8+m} + \frac{(3Ac(b^2+ac)+b(b^2+6ac)C)x^8}{9+m} + \frac{3Bc(b^2+ac)x^9}{10+m} + \frac{3c(Abc+(b^2+ac)C)x^{10}}{11+m} + \frac{3bBc^2x^{11}}{12+m} + \frac{c^2(Ac+3bC)x^{12}}{13+m} + \frac{Bc^3x^{13}}{14+m} + \frac{c^3Cx^{14}}{15+m}\right)$$

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(d*x)^m*((a^3*A)/(1 + m) + (a^3*B*x)/(2 + m) + (a^2*(3*A*b + a*C)*x^2)/(3
 + m) + (3*a^2*b*B*x^3)/(4 + m) + (3*a*(A*(b^2 + a*c) + a*b*C)*x^4)/(5 + m)
 + (3*a*B*(b^2 + a*c)*x^5)/(6 + m) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*c)*
C)*x^6)/(7 + m) + (b*B*(b^2 + 6*a*c)*x^7)/(8 + m) + ((3*A*c*(b^2 + a*c) + b
*(b^2 + 6*a*c)*C)*x^8)/(9 + m) + (3*B*c*(b^2 + a*c)*x^9)/(10 + m) + (3*c*(A
*b*c + (b^2 + a*c)*C)*x^10)/(11 + m) + (3*b*B*c^2*x^11)/(12 + m) + (c^2*(A*
c + 3*b*C)*x^12)/(13 + m) + (B*c^3*x^13)/(14 + m) + (c^3*C*x^14)/(15 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5519 vs. 2(399) = 798.

Time = 0.41 (sec) , antiderivative size = 5520, normalized size of antiderivative = 13.83

method	result	size
gosper	Expression too large to display	5520
risch	Expression too large to display	5520
parallelrisch	Expression too large to display	7809

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3898 vs. 2(399) = 798.

Time = 0.38 (sec) , antiderivative size = 3898, normalized size of antiderivative = 9.77

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((C*c^3*m^14 + 105*C*c^3*m^13 + 5005*C*c^3*m^12 + 143325*C*c^3*m^11 + 27497
47*C*c^3*m^10 + 37312275*C*c^3*m^9 + 368411615*C*c^3*m^8 + 2681453775*C*c^3
*m^7 + 14409322928*C*c^3*m^6 + 56663366760*C*c^3*m^5 + 159721605680*C*c^3*m
^4 + 310989260400*C*c^3*m^3 + 392156797824*C*c^3*m^2 + 283465647360*C*c^3*m
 + 87178291200*C*c^3)*x^15 + (B*c^3*m^14 + 106*B*c^3*m^13 + 5096*B*c^3*m^12
 + 147056*B*c^3*m^11 + 2840838*B*c^3*m^10 + 38786748*B*c^3*m^9 + 385081268*
B*c^3*m^8 + 2816490248*B*c^3*m^7 + 15200266081*B*c^3*m^6 + 59999485546*B*c^
3*m^5 + 169679309436*B*c^3*m^4 + 331303013496*B*c^3*m^3 + 418753514880*B*c^
3*m^2 + 303268406400*B*c^3*m + 93405312000*B*c^3)*x^14 + ((3*C*b*c^2 + A*c^
3)*m^14 + 107*(3*C*b*c^2 + A*c^3)*m^13 + 5189*(3*C*b*c^2 + A*c^3)*m^12 + 15
0943*(3*C*b*c^2 + A*c^3)*m^11 + 2937363*(3*C*b*c^2 + A*c^3)*m^10 + 40372761
*(3*C*b*c^2 + A*c^3)*m^9 + 403249847*(3*C*b*c^2 + A*c^3)*m^8 + 2965379989*(
```

3*C*b*c^2 + A*c^3)*m^7 + 16081189696*(3*C*b*c^2 + A*c^3)*m^6 + 63747744632*(3*C*b*c^2 + A*c^3)*m^5 + 180951426864*(3*C*b*c^2 + A*c^3)*m^4 + 301771008000*C*b*c^2 + 100590336000*A*c^3 + 354444796368*(3*C*b*c^2 + A*c^3)*m^3 + 449213351040*(3*C*b*c^2 + A*c^3)*m^2 + 326044051200*(3*C*b*c^2 + A*c^3)*m)*x^13 + 3*(B*b*c^2*m^14 + 108*B*b*c^2*m^13 + 5284*B*b*c^2*m^12 + 154992*B*b*c^2*m^11 + 3039718*B*b*c^2*m^10 + 42081864*B*b*c^2*m^9 + 423113372*B*b*c^2*m^8 + 3130267536*B*b*c^2*m^7 + 17067919121*B*b*c^2*m^6 + 67988181228*B*b*c^2*m^5 + 193813932344*B*b*c^2*m^4 + 381046157472*B*b*c^2*m^3 + 484441814160*B*b*c^2*m^2 + 352515844800*B*b*c^2*m + 108972864000*B*b*c^2)*x^12 + 3*((C*b^2*c + (C*a + A*b)*c^2)*m^14 + 109*(C*b^2*c + (C*a + A*b)*c^2)*m^13 + 5381*(C*b^2*c + (C*a + A*b)*c^2)*m^12 + 159209*(C*b^2*c + (C*a + A*b)*c^2)*m^11 + 3148323*(C*b^2*c + (C*a + A*b)*c^2)*m^10 + 43926927*(C*b^2*c + (C*a + A*b)*c^2)*m^9 + 444899543*(C*b^2*c + (C*a + A*b)*c^2)*m^8 + 3313733027*(C*b^2*c + (C*a + A*b)*c^2)*m^7 + 18180066256*(C*b^2*c + (C*a + A*b)*c^2)*m^6 + 72822481864*(C*b^2*c + (C*a + A*b)*c^2)*m^5 + 208624806576*(C*b^2*c + (C*a + A*b)*c^2)*m^4 + 118879488000*C*b^2*c + 411940473264*(C*b^2*c + (C*a + A*b)*c^2)*m^3 + 118879488000*(C*a + A*b)*c^2 + 525650497920*(C*b^2*c + (C*a + A*b)*c^2)*m^2 + 383662137600*(C*b^2*c + (C*a + A*b)*c^2)*m)*x^11 + 3*((B*b^2*c + B*a*c^2)*m^14 + 110*(B*b^2*c + B*a*c^2)*m^13 + 5480*(B*b^2*c + B*a*c^2)*m^12 + 163600*(B*b^2*c + B*a*c^2)*m^11 + 3263622*(B*b^2*c + B*a*c^2)*m^10 + 45922260*(B*b^2*c + B*a*c^2)*m^9 + 468873140*(B*b^2*c + B*a*c^2)*m^8 + 3518896600*(B*b^2*c + B*a*c^2)*m^7 + 19442163553*(B*b^2*c + B*a*c^2)*m^6 + 78381575150*(B*b^2*c + B*a*c^2)*m^5 + 225856355580*(B*b^2*c + B*a*c^2)*m^4 + 130767436800*B*b^2*c + 130767436800*B*a*c^2 + 448249789800*(B*b^2*c + B*a*c^2)*m^3 + 574497805824*(B*b^2*c + B*a*c^2)*m^2 + 420839556480*(B*b^2*c + B*a*c^2)*m)*x^10 + ((C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^14 + 111*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^13 + 5581*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^12 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^11 + 3386083*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^10 + 48083733*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^9 + 495342143*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^8 + 3749548713*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^7 + 20885191136*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^6 + 84836490456*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^5 + 246143692976*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^4 + 145297152000*C*b^3 + 435891456000*A*a*c^2 + 491520108816*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^3 + 633314724480*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^2 + 435891456000*(2*C*a*b + A*b^2)*c + 465985094400*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m)*x^9 + ((B*b^3 + 6*B*a*b*c)*m^14 + 112*(B*b^3 + 6*B*a*b*c)*m^13 + 5684*(B*b^3 + 6*B*a*b*c)*m^12 + 172928*(B*b^3 + 6*B*a*b*c)*m^11 + 3516198*(B*b^3 + 6*B*a*b*c)*m^10 + 50428896*(B*b^3 + 6*B*a*b*c)*m^9 + 524664572*(B*b^3 + 6*B*a*b*c)*m^8 + 4010311424*(B*b^3 + 6*B*a*b*c)*m^7 + 22548638161*(B*b^3 + 6*B*a*b*c)*m^6 + 92414105392*(B*b^3 + 6*B*a*b*c)*m^5 + 270359263944*(B*b^3 + 6*B*a*b*c)*m^4 + 163459296000*B*b^3 + 980755776000*B*a*b*c + 543939234048*(B*b^3 + 6*B*a*b*c)*m^3 + 705481831440*(B*b^3 + 6*B*a*b*c)*m^2 + 521962963200*(B*b^3 + 6*B*a*b*c)*m)*x^8 + ((3*C*a*b^2 + A*b^3

```
+ 3*(C*a^2 + 2*A*a*b)*c)*m^14 + 113*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b
)*c)*m^13 + 5789*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^12 + 177877*
(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^11 + 3654483*(3*C*a*b^2 + A*b
^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^10 + 52977099*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 +
 2*A*a*b)*c)*m^9 + 557256047*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^
8 + 4306835671*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^7 + 2448327985
6*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^6 + 101420251688*(3*C*a*b^2
 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^5 + 299730345264*(3*C*a*b^2 + A*b^3 + 3
*(C*a^2 + 2*A*a*b)*c)*m^4 + 560431872000*C*a*b^2 + 186810624000*A*b^3 + 608
700928752*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^3 + 796089202560*(3
*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^2 + 560431872000*(C*a^2 + 2*A*a
*b)*c + 593193196800*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m)*x^7 + 3
*((B*a*b^2 + B*a^2*c)*m^14 + 114*(B*a*b^2 + B*a^2*c)*m^13 + 5896*(B*a*b^2 +
 B*a^2*c)*m^12 + 183024*(B*a*b^2 + B*a^2*c)*m^11 + 3801478*(B*a*b^2 + B*a^2
*c)*m^10 + 55749612*(B*a*b^2 + B*a^2*c)*m^9 + 593598068*(B*a*b^2 + B*a^2*c)
*m^8 + 4646039592*(B*a*b^2 + B*a^2*c)*m^7 + 26754892001*(B*a*b^2 + B*a^2*c)
*m^6 + 112273858674*(B*a*b^2 + B*a^2*c)*m^5 + 336028955036*(B*a*b^2 + B*a^2
*c)*m^4 + 217945728000*B*a*b^2 + 217945728000*B*a^2*c + 690639615384*(B*a*b
^2 + B*a^2*c)*m^3 + 913158011520*(B*a*b^2 + B*a^2*c)*m^2 + 686869545600*(B*
a*b^2 + B*a^2*c)*m)*x^6 + 3*((C*a^2*b + A*a*b^2 + A*a^2*c)*m^14 + 115*(C*a^
2*b + A*a*b^2 + A*a^2*c)*m^13 + 6005*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^12 + 1
88375*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^11 + 3957747*(C*a^2*b + A*a*b^2 + A*a
^2*c)*m^10 + 58769745*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^9 + 634247015*(C*a^2*
b + A*a*b^2 + A*a^2*c)*m^8 + 5036392925*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^7 +
 29449164928*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^6 + 125557386040*(C*a^2*b + A*
a*b^2 + A*a^2*c)*m^5 + 381885176880*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^4 + 261
534873600*C*a^2*b + 261534873600*A*a*b^2 + 261534873600*A*a^2*c + 797387461
200*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^3 + 1070058397824*(C*a^2*b + A*a*b^2 +
A*a^2*c)*m^2 + 815525625600*(C*a^2*b + A*a*b^2 + A*a^2*c)*m)*x^5 + 3*(B*a^2
*b*m^14 + 116*B*a^2*b*m^13 + 6116*B*a^2*b*m^12 + 193936*B*a^2*b*m^11 + 4123
878*B*a^2*b*m^10 + 62062968*B*a^2*b*m^9 + 679843868*B*a^2*b*m^8 + 548825252
8*B*a^2*b*m^7 + 32678119441*B*a^2*b*m^6 + 142090732916*B*a^2*b*m^5 + 441309
175416*B*a^2*b*m^4 + 941576643936*B*a^2*b*m^3 + 1290689128080*B*a^2*b*m^2 +
 1003061102400*B*a^2*b*m + 326918592000*B*a^2*b)*x^4 + ((C*a^3 + 3*A*a^2*b)
*m^14 + 117*(C*a^3 + 3*A*a^2*b)*m^13 + 6229*(C*a^3 + 3*A*a^2*b)*m^12 + 1997
13*(C*a^3 + 3*A*a^2*b)*m^11 + 4300483*(C*a^3 + 3*A*a^2*b)*m^10 + 65657031*(
C*a^3 + 3*A*a^2*b)*m^9 + 731124647*(C*a^3 + 3*A*a^2*b)*m^8 + 6014254059*(C*
a^3 + 3*A*a^2*b)*m^7 + 36588367376*(C*a^3 + 3*A*a^2*b)*m^6 + 163038108552*(
C*a^3 + 3*A*a^2*b)*m^5 + 520557781424*(C*a^3 + 3*A*a^2*b)*m^4 + 43589145600
0*C*a^3 + 1307674368000*A*a^2*b + 1145140001328*(C*a^3 + 3*A*a^2*b)*m^3 + 1
621575699840*(C*a^3 + 3*A*a^2*b)*m^2 + 1301090515200*(C*a^3 + 3*A*a^2*b)*m)
*x^3 + (B*a^3*m^14 + 118*B*a^3*m^13 + 6344*B*a^3*m^12 + 205712*B*a^3*m^11 +
 4488198*B*a^3*m^10 + 69582084*B*a^3*m^9 + 788931572*B*a^3*m^8 + 6629764856
*B*a^3*m^7 + 41371599841*B*a^3*m^6 + 190060010998*B*a^3*m^5 + 629552085084*
B*a^3*m^4 + 1447709175432*B*a^3*m^3 + 2161577352960*B*a^3*m^2 + 18426629088
```

```
00*B*a^3*m + 653837184000*B*a^3)*x^2 + (A*a^3*m^14 + 119*A*a^3*m^13 + 6461*
A*a^3*m^12 + 211939*A*a^3*m^11 + 4687683*A*a^3*m^10 + 73870797*A*a^3*m^9 +
854224943*A*a^3*m^8 + 7353403057*A*a^3*m^7 + 47277726496*A*a^3*m^6 + 225525
484184*A*a^3*m^5 + 784146622896*A*a^3*m^4 + 1922666722704*A*a^3*m^3 + 31343
28981120*A*a^3*m^2 + 3031488633600*A*a^3*m + 1307674368000*A*a^3)*x)*(d*x)^
m/(m^15 + 120*m^14 + 6580*m^13 + 218400*m^12 + 4899622*m^11 + 78558480*m^10
 + 928095740*m^9 + 8207628000*m^8 + 54631129553*m^7 + 272803210680*m^6 + 10
09672107080*m^5 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614720*m^2
 + 4339163001600*m + 1307674368000)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47658 vs. $2(379) = 758$.

Time = 3.03 (sec) , antiderivative size = 47658, normalized size of antiderivative = 119.44

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-A*a**3/(14*x**14) - A*a**2*b/(4*x**12) - 3*A*a**2*c/(10*x**10)
 - 3*A*a*b**2/(10*x**10) - 3*A*a*b*c/(4*x**8) - A*a*c**2/(2*x**6) - A*b**3/
(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*c**3/(2*x**2) - B*a*
*3/(13*x**13) - 3*B*a**2*b/(11*x**11) - B*a**2*c/(3*x**9) - B*a*b**2/(3*x**
9) - 6*B*a*b*c/(7*x**7) - 3*B*a*c**2/(5*x**5) - B*b**3/(7*x**7) - 3*B*b**2*
c/(5*x**5) - B*b*c**2/x**3 - B*c**3/x - C*a**3/(12*x**12) - 3*C*a**2*b/(10*
x**10) - 3*C*a**2*c/(8*x**8) - 3*C*a*b**2/(8*x**8) - C*a*b*c/x**6 - 3*C*a*c
**2/(4*x**4) - C*b**3/(6*x**6) - 3*C*b**2*c/(4*x**4) - 3*C*b*c**2/(2*x**2)
+ C*c**3*log(x))/d**15, Eq(m, -15)), ((-A*a**3/(13*x**13) - 3*A*a**2*b/(11*
x**11) - A*a**2*c/(3*x**9) - A*a*b**2/(3*x**9) - 6*A*a*b*c/(7*x**7) - 3*A*a
*c**2/(5*x**5) - A*b**3/(7*x**7) - 3*A*b**2*c/(5*x**5) - A*b*c**2/x**3 - A*
c**3/x - B*a**3/(12*x**12) - 3*B*a**2*b/(10*x**10) - 3*B*a**2*c/(8*x**8) -
3*B*a*b**2/(8*x**8) - B*a*b*c/x**6 - 3*B*a*c**2/(4*x**4) - B*b**3/(6*x**6)
- 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2) + B*c**3*log(x) - C*a**3/(11*x*
*11) - C*a**2*b/(3*x**9) - 3*C*a**2*c/(7*x**7) - 3*C*a*b**2/(7*x**7) - 6*C*
a*b*c/(5*x**5) - C*a*c**2/x**3 - C*b**3/(5*x**5) - C*b**2*c/x**3 - 3*C*b*c*
*2/x + C*c**3*x)/d**14, Eq(m, -14)), ((-A*a**3/(12*x**12) - 3*A*a**2*b/(10*
x**10) - 3*A*a**2*c/(8*x**8) - 3*A*a*b**2/(8*x**8) - A*a*b*c/x**6 - 3*A*a*c
**2/(4*x**4) - A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) - 3*A*b*c**2/(2*x**2)
+ A*c**3*log(x) - B*a**3/(11*x**11) - B*a**2*b/(3*x**9) - 3*B*a**2*c/(7*x**
7) - 3*B*a*b**2/(7*x**7) - 6*B*a*b*c/(5*x**5) - B*a*c**2/x**3 - B*b**3/(5*x
**5) - B*b**2*c/x**3 - 3*B*b*c**2/x + B*c**3*x - C*a**3/(10*x**10) - 3*C*a*
*2*b/(8*x**8) - C*a**2*c/(2*x**6) - C*a*b**2/(2*x**6) - 3*C*a*b*c/(2*x**4)
- 3*C*a*c**2/(2*x**2) - C*b**3/(4*x**4) - 3*C*b**2*c/(2*x**2) + 3*C*b*c**2*
log(x) + C*c**3*x**2/2)/d**13, Eq(m, -13)), ((-A*a**3/(11*x**11) - A*a**2*b
```

/(3*x**9) - 3*A*a**2*c/(7*x**7) - 3*A*a*b**2/(7*x**7) - 6*A*a*b*c/(5*x**5)
- A*a*c**2/x**3 - A*b**3/(5*x**5) - A*b**2*c/x**3 - 3*A*b*c**2/x + A*c**3*x
 - B*a**3/(10*x**10) - 3*B*a**2*b/(8*x**8) - B*a**2*c/(2*x**6) - B*a*b**2/(
2*x**6) - 3*B*a*b*c/(2*x**4) - 3*B*a*c**2/(2*x**2) - B*b**3/(4*x**4) - 3*B*
b**2*c/(2*x**2) + 3*B*b*c**2*log(x) + B*c**3*x**2/2 - C*a**3/(9*x**9) - 3*C
*a**2*b/(7*x**7) - 3*C*a**2*c/(5*x**5) - 3*C*a*b**2/(5*x**5) - 2*C*a*b*c/x*
*3 - 3*C*a*c**2/x - C*b**3/(3*x**3) - 3*C*b**2*c/x + 3*C*b*c**2*x + C*c**3*
x**3/3)/d**12, Eq(m, -12)), ((-A*a**3/(10*x**10) - 3*A*a**2*b/(8*x**8) - A*
a**2*c/(2*x**6) - A*a*b**2/(2*x**6) - 3*A*a*b*c/(2*x**4) - 3*A*a*c**2/(2*x*
*2) - A*b**3/(4*x**4) - 3*A*b**2*c/(2*x**2) + 3*A*b*c**2*log(x) + A*c**3*x*
*2/2 - B*a**3/(9*x**9) - 3*B*a**2*b/(7*x**7) - 3*B*a**2*c/(5*x**5) - 3*B*a*
b**2/(5*x**5) - 2*B*a*b*c/x**3 - 3*B*a*c**2/x - B*b**3/(3*x**3) - 3*B*b**2*
c/x + 3*B*b*c**2*x + B*c**3*x**3/3 - C*a**3/(8*x**8) - C*a**2*b/(2*x**6) -
3*C*a**2*c/(4*x**4) - 3*C*a*b**2/(4*x**4) - 3*C*a*b*c/x**2 + 3*C*a*c**2*log
(x) - C*b**3/(2*x**2) + 3*C*b**2*c*log(x) + 3*C*b*c**2*x**2/2 + C*c**3*x**4
/4)/d**11, Eq(m, -11)), ((-A*a**3/(9*x**9) - 3*A*a**2*b/(7*x**7) - 3*A*a**2
*c/(5*x**5) - 3*A*a*b**2/(5*x**5) - 2*A*a*b*c/x**3 - 3*A*a*c**2/x - A*b**3/
(3*x**3) - 3*A*b**2*c/x + 3*A*b*c**2*x + A*c**3*x**3/3 - B*a**3/(8*x**8) -
B*a**2*b/(2*x**6) - 3*B*a**2*c/(4*x**4) - 3*B*a*b**2/(4*x**4) - 3*B*a*b*c/x
2 + 3*B*a*c2*log(x) - B*b**3/(2*x**2) + 3*B*b**2*c*log(x) + 3*B*b*c**2*
x**2/2 + B*c**3*x**4/4 - C*a**3/(7*x**7) - 3*C*a**2*b/(5*x**5) - C*a**2*c/x
3 - C*a*b2/x**3 - 6*C*a*b*c/x + 3*C*a*c**2*x - C*b**3/x + 3*C*b**2*c*x
+ C*b*c**2*x**3 + C*c**3*x**5/5)/d**10, Eq(m, -10)), ((-A*a**3/(8*x**8) - A
*a**2*b/(2*x**6) - 3*A*a**2*c/(4*x**4) - 3*A*a*b**2/(4*x**4) - 3*A*a*b*c/x*
*2 + 3*A*a*c**2*log(x) - A*b**3/(2*x**2) + 3*A*b**2*c*log(x) + 3*A*b*c**2*x
2/2 + A*c3*x**4/4 - B*a**3/(7*x**7) - 3*B*a**2*b/(5*x**5) - B*a**2*c/x*
*3 - B*a*b**2/x**3 - 6*B*a*b*c/x + 3*B*a*c**2*x - B*b**3/x + 3*B*b**2*c*x +
 B*b*c**2*x**3 + B*c**3*x**5/5 - C*a**3/(6*x**6) - 3*C*a**2*b/(4*x**4) - 3*
C*a**2*c/(2*x**2) - 3*C*a*b**2/(2*x**2) + 6*C*a*b*c*log(x) + 3*C*a*c**2*x**
2/2 + C*b**3*log(x) + 3*C*b**2*c*x**2/2 + 3*C*b*c**2*x**4/4 + C*c**3*x**6/6
)/d**9, Eq(m, -9)), ((-A*a**3/(7*x**7) - 3*A*a**2*b/(5*x**5) - A*a**2*c/x**
3 - A*a*b**2/x**3 - 6*A*a*b*c/x + 3*A*a*c**2*x - A*b**3/x + 3*A*b**2*c*x +
A*b*c**2*x**3 + A*c**3*x**5/5 - B*a**3/(6*x**6) - 3*B*a**2*b/(4*x**4) - 3*B
*a**2*c/(2*x**2) - 3*B*a*b**2/(2*x**2) + 6*B*a*b*c*log(x) + 3*B*a*c**2*x**2
/2 + B*b**3*log(x) + 3*B*b**2*c*x**2/2 + 3*B*b*c**2*x**4/4 + B*c**3*x**6/6
- C*a**3/(5*x**5) - C*a**2*b/x**3 - 3*C*a**2*c/x - 3*C*a*b**2/x + 6*C*a*b*c
*x + C*a*c**2*x**3 + C*b**3*x + C*b**2*c*x**3 + 3*C*b*c**2*x**5/5 + C*c**3*
x**7/7)/d**8, Eq(m, -8)), ((-A*a**3/(6*x**6) - 3*A*a**2*b/(4*x**4) - 3*A*a*
*2*c/(2*x**2) - 3*A*a*b**2/(2*x**2) + 6*A*a*b*c*log(x) + 3*A*a*c**2*x**2/2
+ A*b**3*log(x) + 3*A*b**2*c*x**2/2 + 3*A*b*c**2*x**4/4 + A*c**3*x**6/6 - B
*a**3/(5*x**5) - B*a**2*b/x**3 - 3*B*a**2*c/x - 3*B*a*b**2/x + 6*B*a*b*c*x
+ B*a*c**2*x**3 + B*b**3*x + B*b**2*c*x**3 + 3*B*b*c**2*x**5/5 + B*c**3*x**
7/7 - C*a**3/(4*x**4) - 3*C*a**2*b/(2*x**2) + 3*C*a**2*c*log(x) + 3*C*a*b**
2*log(x) + 3*C*a*b*c*x**2 + 3*C*a*c**2*x**4/4 + C*b**3*x**2/2 + 3*C*b**2*c*
x**4/4 + C*b*c**2*x**6/2 + C*c**3*x**8/8)/d**7, Eq(m, -7)), ((-A*a**3/(5*x*

```
*5) - A*a**2*b/x**3 - 3*A*a**2*c/x - 3*A*a*b**2/x + 6*A*a*b*c*x + A*a*c**2*
x**3 + A*b**3*x + A*b**2*c*x**3 + 3*A*b*c**2*x**5/5 + A*c**3*x**7/7 - B*a**
3/(4*x**4) - 3*B*a**2*b/(2*x**2) + 3*B*a**2*c*log(x) + 3*B*a*b**2*log(x) +
3*B*a*b*c*x**2 + 3*B*a*c**2*x**4/4 + B*b**3*x**2/2 + 3*B*b**2*c*x**4/4 + B*
b*c**2*x**6/2 + B*c**3*x**8/8 - C*a**3/(3*x**3) - 3*C*a**2*b/x + 3*C*a**2*c
*x + 3*C*a*b**2*x + 2*C*a*b*c*x**3 + 3*C*a*c**2*x**5/5 + C*b**3*x**3/3 + 3*
C*b**2*c*x**5/5 + 3*C*b*c**2*x**7/7 + C*c**3*x**9/9)/d**6, Eq(m, -6)), ((-A
*a**3/(4*x**4) - 3*A*a**2*b/(2*x**2) + 3*A*a**2*c*log(x) + 3*A*a*b**2*log(x
) + 3*A*a*b*c*x**2 + 3*A*a*c**2*x**4/4 + A*b**3*x**2/2 + 3*A*b**2*c*x**4/4
+ A*b*c**2*x**6/2 + A*c**3*x**8/8 - B*a**3/(3*x**3) - 3*B*a**2*b/x + 3*B*a*
*2*c*x + 3*B*a*b**2*x + 2*B*a*b*c*x**3 + 3*B*a*c**2*x**5/5 + B*b**3*x**3/3
+ 3*B*b**2*c*x**5/5 + 3*B*b*c**2*x**7/7 + B*c**3*x**9/9 - C*a**3/(2*x**2) +
 3*C*a**2*b*log(x) + 3*C*a**2*c*x**2/2 + 3*C*a*b**2*x**2/2 + 3*C*a*b*c*x**4
/2 + C*a*c**2*x**6/2 + C*b**3*x**4/4 + C*b**2*c*x**6/2 + 3*C*b*c**2*x**8/8
+ C*c**3*x**10/10)/d**5, Eq(m, -5)), ((-A*a**3/(3*x**3) - 3*A*a**2*b/x + 3*
A*a**2*c*x + 3*A*a*b**2*x + 2*A*a*b*c*x**3 + 3*A*a*c**2*x**5/5 + A*b**3*x**
3/3 + 3*A*b**2*c*x**5/5 + 3*A*b*c**2*x**7/7 + A*c**3*x**9/9 - B*a**3/(2*x**
2) + 3*B*a**2*b*log(x) + 3*B*a**2*c*x**2/2 + 3*B*a*b**2*x**2/2 + 3*B*a*b*c*
x**4/2 + B*a*c**2*x**6/2 + B*b**3*x**4/4 + B*b**2*c*x**6/2 + 3*B*b*c**2*x**
8/8 + B*c**3*x**10/10 - C*a**3/x + 3*C*a**2*b*x + C*a**2*c*x**3 + C*a*b**2*
x**3 + 6*C*a*b*c*x**5/5 + 3*C*a*c**2*x**7/7 + C*b**3*x**5/5 + 3*C*b**2*c*x*
*7/7 + C*b*c**2*x**9/3 + C*c**3*x**11/11)/d**4, Eq(m, -4)), ((-A*a**3/(2*x*
*2) + 3*A*a**2*b*log(x) + 3*A*a**2*c*x**2/2 + 3*A*a*b**2*x**2/2 + 3*A*a*b*c
*x**4/2 + A*a*c**2*x**6/2 + A*b**3*x**4/4 + A*b**2*c*x**6/2 + 3*A*b*c**2*x*
*8/8 + A*c**3*x**10/10 - B*a**3/x + 3*B*a**2*b*x + B*a**2*c*x**3 + B*a*b**2
*x**3 + 6*B*a*b*c*x**5/5 + 3*B*a*c**2*x**7/7 + B*b**3*x**5/5 + 3*B*b**2*c*x
**7/7 + B*b*c**2*x**9/3 + B*c**3*x**11/11 + C*a**3*log(x) + 3*C*a**2*b*x**2
/2 + 3*C*a**2*c*x**4/4 + 3*C*a*b**2*x**4/4 + C*a*b*c*x**6 + 3*C*a*c**2*x**8
/8 + C*b**3*x**6/6 + 3*C*b**2*c*x**8/8 + 3*C*b*c**2*x**10/10 + C*c**3*x**12
/12)/d**3, Eq(m, -3)), ((-A*a**3/x + 3*A*a**2*b*x + A*a**2*c*x**3 + A*a*b**
2*x**3 + 6*A*a*b*c*x**5/5 + 3*A*a*c**2*x**7/7 + A*b**3*x**5/5 + 3*A*b**2*c*
x**7/7 + A*b*c**2*x**9/3 + A*c**3*x**11/11 + B*a**3*log(x) + 3*B*a**2*b*x**
2/2 + 3*B*a**2*c*x**4/4 + 3*B*a*b**2*x**4/4 + B*a*b*c*x**6 + 3*B*a*c**2*x**
8/8 + B*b**3*x**6/6 + 3*B*b**2*c*x**8/8 + 3*B*b*c**2*x**10/10 + B*c**3*x**1
2/12 + C*a**3*x + C*a**2*b*x**3 + 3*C*a**2*c*x**5/5 + 3*C*a*b**2*x**5/5 + 6
*C*a*b*c*x**7/7 + C*a*c**2*x**9/3 + C*b**3*x**7/7 + C*b**2*c*x**9/3 + 3*C*b
*c**2*x**11/11 + C*c**3*x**13/13)/d**2, Eq(m, -2)), ((A*a**3*log(x) + 3*A*a
**2*b*x**2/2 + 3*A*a**2*c*x**4/4 + 3*A*a*b**2*x**4/4 + A*a*b*c*x**6 + 3*A*a
*c**2*x**8/8 + A*b**3*x**6/6 + 3*A*b**2*c*x**8/8 + 3*A*b*c**2*x**10/10 + A*
c**3*x**12/12 + B*a**3*x + B*a**2*b*x**3 + 3*B*a**2*c*x**5/5 + 3*B*a*b**2*x
**5/5 + 6*B*a*b*c*x**7/7 + B*a*c**2*x**9/3 + B*b**3*x**7/7 + B*b**2*c*x**9/
3 + 3*B*b*c**2*x**11/11 + B*c**3*x**13/13 + C*a**3*x**2/2 + 3*C*a**2*b*x**4
/4 + C*a**2*c*x**6/2 + C*a*b**2*x**6/2 + 3*C*a*b*c*x**8/4 + 3*C*a*c**2*x**1
0/10 + C*b**3*x**8/8 + 3*C*b**2*c*x**10/10 + C*b*c**2*x**12/4 + C*c**3*x**1
4/14)/d, Eq(m, -1)), (A*a**3*m**14*x*(d*x)**m/(m**15 + 120*m**14 + 6580*m**
```

13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076
28000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27
06813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*
m + 1307674368000) + 119*A*a**3*m**13*x*(d*x)**m/(m**15 + 120*m**14 + 6580*
m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82
07628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630016
00*m + 1307674368000) + 6461*A*a**3*m**12*x*(d*x)**m/(m**15 + 120*m**14 + 6
580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9
 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m*
*5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163
001600*m + 1307674368000) + 211939*A*a**3*m**11*x*(d*x)**m/(m**15 + 120*m**
14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740
*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107
080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4
339163001600*m + 1307674368000) + 4687683*A*a**3*m**10*x*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
2 + 4339163001600*m + 1307674368000) + 73870797*A*a3*m**9*x*(d*x)**m/(m
15 + 120*m14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**
10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**
6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658176
14720*m**2 + 4339163001600*m + 1307674368000) + 854224943*A*a**3*m**8*x*(d*
x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855
8480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321
0680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
6165817614720*m**2 + 4339163001600*m + 1307674368000) + 7353403057*A*a**3*m
7*x*(d*x)m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 4727772649
6*A*a**3*m**6*x*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129
553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
225525484184*A*a**3*m**5*x*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21840
0*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334560
0*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767
4368000) + 784146622896*A*a**3*m**4*x*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 1922666722704*A*a**3*m**3*x*(d*x)**m/(m**15 + 120*m**

14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740
*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107
080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4
339163001600*m + 1307674368000) + 3134328981120*A*a**3*m**2*x*(d*x)**m/(m**
15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
+ 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614
720*m**2 + 4339163001600*m + 1307674368000) + 3031488633600*A*a**3*m*x*(d*x
)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558
480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210
680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6
165817614720*m**2 + 4339163001600*m + 1307674368000) + 1307674368000*A*a**3
x(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
+ 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27
2803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m
3 + 6165817614720*m2 + 4339163001600*m + 1307674368000) + 3*A*a**2*b*m*
*14*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*
m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**
7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570
3824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 351*A*a
2*b*m13*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112
9553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5
056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
 18687*A*a**2*b*m**12*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21840
0*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334560
0*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767
4368000) + 599139*A*a**2*b*m**11*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 12901449*A*a**2*b*m**10*x**3*(d*x)**m/(m**15 + 120*m*
*14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809574
0*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967210
7080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
4339163001600*m + 1307674368000) + 196971093*A*a**2*b*m**9*x**3*(d*x)**m/(m
15 + 120*m14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**
10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**
6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658176
14720*m**2 + 4339163001600*m + 1307674368000) + 2193373941*A*a**2*b*m**8*x*
3(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
+ 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27
2803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m
3 + 6165817614720*m2 + 4339163001600*m + 1307674368000) + 18042762177*A

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*a**2*b*m**7*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546311
29553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
+ 109765102128*A*a**2*b*m**6*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
+ 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076280
00*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068
13345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
 1307674368000) + 489114325656*A*a**2*b*m**5*x**3*(d*x)**m/(m**15 + 120*m**
14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740
*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107
080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4
339163001600*m + 1307674368000) + 1561673344272*A*a**2*b*m**4*x**3*(d*x)**m
/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*
m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*
m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658
17614720*m**2 + 4339163001600*m + 1307674368000) + 3435420003984*A*a**2*b*m
**3*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*
m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**
7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570
3824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 4864727
099520*A*a**2*b*m**2*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600
*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 3903271545600*A*a**2*b*m*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*
m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82
07628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630016
00*m + 1307674368000) + 1307674368000*A*a**2*b*x**3*(d*x)**m/(m**15 + 120*m
**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
 4339163001600*m + 1307674368000) + 3*A*a**2*c*m**14*x**5*(d*x)**m/(m**15 +
 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9
28095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10
09672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*
m**2 + 4339163001600*m + 1307674368000) + 345*A*a**2*c*m**13*x**5*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 18015*A*a**2*c*m**12*x**5
*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 +
78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728
03210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**
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3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 565125*A*a**2*c
*m**11*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 48996
22*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*
m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699
5703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1187
3241*A*a**2*c*m**10*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*
m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*
m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743
68000) + 176309235*A*a**2*c*m**9*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 1902741045*A*a**2*c*m**8*x**5*(d*x)**m/(m**15 + 120*m
**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
 4339163001600*m + 1307674368000) + 15109178775*A*a**2*c*m**7*x**5*(d*x)**m
/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*
m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*
m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658
17614720*m**2 + 4339163001600*m + 1307674368000) + 88347494784*A*a**2*c*m**
6*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
+ 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038
24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 376672158
120*A*a**2*c*m**5*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m*
*12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5
4631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m*
*4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368
000) + 1145655530640*A*a**2*c*m**4*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*
m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82
07628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630016
00*m + 1307674368000) + 2392162383600*A*a**2*c*m**3*x**5*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
**2 + 4339163001600*m + 1307674368000) + 3210175193472*A*a**2*c*m**2*x**5*(
d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78
558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803
210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
+ 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 2446576876800*A*a
**2*c*m*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899
622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553
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*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569
95703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 784
604620800*A*a**2*c*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m
**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m
**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767436
8000) + 3*A*a*b**2*m**14*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21
8400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m
**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334
5600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130
7674368000) + 345*A*a*b**2*m**13*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 18015*A*a*b**2*m**12*x**5*(d*x)**m/(m**15 + 120*m**14
 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m
**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967210708
0*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433
9163001600*m + 1307674368000) + 565125*A*a*b**2*m**11*x**5*(d*x)**m/(m**15
+ 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 +
928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1
009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720
*m**2 + 4339163001600*m + 1307674368000) + 11873241*A*a*b**2*m**10*x**5*(d*
x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855
8480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321
0680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
6165817614720*m**2 + 4339163001600*m + 1307674368000) + 176309235*A*a*b**2*
m**9*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622
*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m*
*7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957
03824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 190274
1045*A*a*b**2*m**8*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m
**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m
**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767436
8000) + 15109178775*A*a*b**2*m**7*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m
**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820
7628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916300160
0*m + 1307674368000) + 88347494784*A*a*b**2*m**6*x**5*(d*x)**m/(m**15 + 120
*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809
5740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967
2107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
 + 4339163001600*m + 1307674368000) + 376672158120*A*a*b**2*m**5*x**5*(d*x)
**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785584
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80*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032106
80*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61
65817614720*m**2 + 4339163001600*m + 1307674368000) + 1145655530640*A*a*b**
2*m**4*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 48996
22*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*
m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699
5703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 2392
162383600*A*a*b**2*m**3*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218
400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m*
*8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345
600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307
674368000) + 3210175193472*A*a*b**2*m**2*x**5*(d*x)**m/(m**15 + 120*m**14 +
 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 2446576876800*A*a*b**2*m*x**5*(d*x)**m/(m**15
 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 +
 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 +
1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761472
0*m**2 + 4339163001600*m + 1307674368000) + 784604620800*A*a*b**2*x**5*(d*x
)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558
480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210
680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6
165817614720*m**2 + 4339163001600*m + 1307674368000) + 6*A*a*b*c*m**14*x**7
*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 +
78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728
03210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**
3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 678*A*a*b*c*m**
13*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m
11 + 78558480*m10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703
824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 34734*A*
a*b*c*m**12*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112
9553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5
056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
 1067262*A*a*b*c*m**11*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2184
00*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**
8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133456
00*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076
74368000) + 21926898*A*a*b*c*m**10*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*
m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82
07628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630016
00*m + 1307674368000) + 317862594*A*a*b*c*m**9*x**7*(d*x)**m/(m**15 + 120*m

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**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
 4339163001600*m + 1307674368000) + 3343536282*A*a*b*c*m**8*x**7*(d*x)**m/(
m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m*
*10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m*
*6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817
614720*m**2 + 4339163001600*m + 1307674368000) + 25841014026*A*a*b*c*m**7*x
**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 146899679136
*A*a*b*c*m**6*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12
+ 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631
129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
 + 608521510128*A*a*b*c*m**5*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
+ 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076280
00*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068
13345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
 1307674368000) + 1798382071584*A*a*b*c*m**4*x**7*(d*x)**m/(m**15 + 120*m**
14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740
*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107
080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4
339163001600*m + 1307674368000) + 3652205572512*A*a*b*c*m**3*x**7*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 4776535215360*A*a*b*c*m**
2*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
+ 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038
24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 355915918
0800*A*a*b*c*m*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12
 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463
1129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4
+ 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000
) + 1120863744000*A*a*b*c*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2
18400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*
m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133
45600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13
07674368000) + 3*A*a*c**2*m**14*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**
13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076
28000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27
06813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*
```

m + 1307674368000) + 333*A*a*c**2*m**13*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 16743*A*a*c**2*m**12*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 504513*A*a*c**2*m**11*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 10158249*A*a*c**2*m**10*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 144251199*A*a*c**2*m**9*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1486026429*A*a*c**2*m**8*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 11248646139*A*a*c**2*m**7*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 62655573408*A*a*c**2*m**6*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 254509471368*A*a*c**2*m**5*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 738431078928*A*a*c**2*m**4*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1474560326448*A*a*c**2*m**3*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706

813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m
+ 1307674368000) + 1899944173440*A*a*c**2*m**2*x**9*(d*x)**m/(m**15 + 120*m
14 + 6580*m13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
4339163001600*m + 1307674368000) + 1397955283200*A*a*c**2*m*x**9*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
10 + 928095740*m9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
6 + 1009672107080*m5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 435891456000*A*a*c**2*x**
9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 +
78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272
803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m*
*3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + A*b**3*m**14*x
7*(d*x)m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
+ 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 113*A*b**3*m
13*x7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622
*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m*
*7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957
03824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 5789*A
*b**3*m**12*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112
9553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5
056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
177877*A*b**3*m**11*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600
*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 3654483*A*b**3*m**10*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**1
3 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762
8000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270
6813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m
+ 1307674368000) + 52977099*A*b**3*m**9*x**7*(d*x)**m/(m**15 + 120*m**14 +
6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 557256047*A*b**3*m**8*x**7*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
2 + 4339163001600*m + 1307674368000) + 4306835671*A*b3*m**7*x**7*(d*x)*
*m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855848
0*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321068

0*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616 5817614720*m**2 + 4339163001600*m + 1307674368000) + 24483279856*A*b**3*m** 6*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m* *11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038 24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 101420251 688*A*b**3*m**5*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**1 2 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546 31129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767436800 0) + 299730345264*A*b**3*m**4*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628 000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706 813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 608700928752*A*b**3*m**3*x**7*(d*x)**m/(m**15 + 120*m**1 4 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740* m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721070 80*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43 39163001600*m + 1307674368000) + 796089202560*A*b**3*m**2*x**7*(d*x)**m/(m* *15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**1 0 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761 4720*m**2 + 4339163001600*m + 1307674368000) + 593193196800*A*b**3*m*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78 558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803 210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 186810624000*A*b* *3*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m **11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703 824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3*A*b**2 *c*m**14*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489 9622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112955 3*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056 995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 33 3*A*b**2*c*m**13*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m** 12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54 631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m** 4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743680 00) + 16743*A*b**2*c*m**12*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000 *m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813 345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1 307674368000) + 504513*A*b**2*c*m**11*x**9*(d*x)**m/(m**15 + 120*m**14 + 65 80*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 +

8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**
5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630
01600*m + 1307674368000) + 10158249*A*b**2*c*m**10*x**9*(d*x)**m/(m**15 + 1
20*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928
095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009
672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m*
*2 + 4339163001600*m + 1307674368000) + 144251199*A*b**2*c*m**9*x**9*(d*x)*
*m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855848
0*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321068
0*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616
5817614720*m**2 + 4339163001600*m + 1307674368000) + 1486026429*A*b**2*c*m*
*8*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m
11 + 78558480*m10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703
824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 11248646
139*A*b**2*c*m**7*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m*
*12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5
4631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m*
*4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368
000) + 62655573408*A*b**2*c*m**6*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 254509471368*A*b**2*c*m**5*x**9*(d*x)**m/(m**15 + 120
*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809
5740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967
2107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
 + 4339163001600*m + 1307674368000) + 738431078928*A*b**2*c*m**4*x**9*(d*x)
m/(m15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785584
80*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032106
80*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61
65817614720*m**2 + 4339163001600*m + 1307674368000) + 1474560326448*A*b**2*
c*m**3*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 48996
22*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*
m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699
5703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1899
944173440*A*b**2*c*m**2*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218
400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m*
*8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345
600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307
674368000) + 1397955283200*A*b**2*c*m*x**9*(d*x)**m/(m**15 + 120*m**14 + 65
80*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 +
 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**
5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630
01600*m + 1307674368000) + 435891456000*A*b**2*c*x**9*(d*x)**m/(m**15 + 120

*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809
5740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967
2107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
 + 4339163001600*m + 1307674368000) + 3*A*b*c**2*m**14*x**11*(d*x)**m/(m**1
5 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
+ 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 +
 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658176147
20*m**2 + 4339163001600*m + 1307674368000) + 327*A*b*c**2*m**13*x**11*(d*x)
m/(m15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785584
80*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032106
80*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61
65817614720*m**2 + 4339163001600*m + 1307674368000) + 16143*A*b*c**2*m**12*
x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 477627*A*b
*c**2*m**11*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546311
29553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
+ 9444969*A*b*c**2*m**10*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2
18400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*
m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133
45600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13
07674368000) + 131780781*A*b*c**2*m**9*x**11*(d*x)**m/(m**15 + 120*m**14 +
6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9
 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m
5 + 2706813345600*m4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916
3001600*m + 1307674368000) + 1334698629*A*b*c**2*m**8*x**11*(d*x)**m/(m**15
 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 +
 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 +
1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761472
0*m**2 + 4339163001600*m + 1307674368000) + 9941199081*A*b*c**2*m**7*x**11*
(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7
8558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280
3210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 54540198768*A*b*
c**2*m**6*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129
553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
218467445592*A*b*c**2*m**5*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 +
 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762800
0*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681
3345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +

1307674368000) + 625874419728*A*b*c**2*m**4*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1235821419792*A*b*c**2*m**3*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1576951493760*A*b*c**2*m**2*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1150986412800*A*b*c**2*m*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 356638464000*A*b*c**2*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + A*c**3*m**14*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 107*A*c**3*m**13*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 5189*A*c**3*m**12*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 150943*A*c**3*m**11*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 2937363*A*c**3*m**10*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 40372761*A*c**3*m**9*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569

95703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 403
249847*A*c**3*m**8*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*
m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*
m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743
68000) + 2965379989*A*c**3*m**7*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 16081189696*A*c**3*m**6*x**13*(d*x)**m/(m**15 + 120*m
14 + 6580*m13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
4339163001600*m + 1307674368000) + 63747744632*A*c**3*m**5*x**13*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
10 + 928095740*m9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
6 + 1009672107080*m5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 180951426864*A*c**3*m**4*
x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3544447963
68*A*c**3*m**3*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**1
2 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546
31129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4
+ 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767436800
0) + 449213351040*A*c**3*m**2*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**1
3 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762
8000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270
6813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m
+ 1307674368000) + 326044051200*A*c**3*m*x**13*(d*x)**m/(m**15 + 120*m**14
+ 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m
9 + 8207628000*m8 + 54631129553*m**7 + 272803210680*m**6 + 100967210708
0*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433
9163001600*m + 1307674368000) + 100590336000*A*c**3*x**13*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9
28095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10
09672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*
m**2 + 4339163001600*m + 1307674368000) + B*a**3*m**14*x**2*(d*x)**m/(m**15
+ 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 +
928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 +
1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761472
0*m**2 + 4339163001600*m + 1307674368000) + 118*B*a**3*m**13*x**2*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
10 + 928095740*m9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m

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**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 6344*B*a**3*m**12*x**2*(d
*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785
58480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032
10680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 205712*B*a**3*m**1
1*x**2*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038
24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 4488198*B
*a**3*m**10*x**2*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112
9553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5
056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
 69582084*B*a**3*m**9*x**2*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21840
0*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334560
0*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767
4368000) + 788931572*B*a**3*m**8*x**2*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 6629764856*B*a**3*m**7*x**2*(d*x)**m/(m**15 + 120*m**
14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740
*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107
080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4
339163001600*m + 1307674368000) + 41371599841*B*a**3*m**6*x**2*(d*x)**m/(m*
*15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**1
0 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761
4720*m**2 + 4339163001600*m + 1307674368000) + 190060010998*B*a**3*m**5*x**
2*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 +
 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272
803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m*
*3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 629552085084*B
*a**3*m**4*x**2*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129
553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
1447709175432*B*a**3*m**3*x**2*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2
18400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*
m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133
45600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13
07674368000) + 2161577352960*B*a**3*m**2*x**2*(d*x)**m/(m**15 + 120*m**14 +
 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
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9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 1842662908800*B*a**3*m*x**2*(d*x)**m/(m**15 +
 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9
28095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10
09672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*
m**2 + 4339163001600*m + 1307674368000) + 653837184000*B*a**3*x**2*(d*x)**m
/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*
m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*
m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658
17614720*m**2 + 4339163001600*m + 1307674368000) + 3*B*a**2*b*m**14*x**4*(d
*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785
58480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032
10680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 348*B*a**2*b*m**13
*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 18348*B*a*
*2*b*m**12*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129
553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
581808*B*a**2*b*m**11*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21840
0*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334560
0*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767
4368000) + 12371634*B*a**2*b*m**10*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*
m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82
07628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630016
00*m + 1307674368000) + 186188904*B*a**2*b*m**9*x**4*(d*x)**m/(m**15 + 120*
m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095
740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672
107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
+ 4339163001600*m + 1307674368000) + 2039531604*B*a**2*b*m**8*x**4*(d*x)**m
/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*
m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*
m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658
17614720*m**2 + 4339163001600*m + 1307674368000) + 16464757584*B*a**2*b*m**
7*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
+ 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038
24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 980343583
23*B*a**2*b*m**6*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**
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12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54
631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**
4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743680
00) + 426272198748*B*a**2*b*m**5*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 1323927526248*B*a**2*b*m**4*x**4*(d*x)**m/(m**15 + 12
0*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280
95740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096
72107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**
2 + 4339163001600*m + 1307674368000) + 2824729931808*B*a**2*b*m**3*x**4*(d*
x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855
8480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321
0680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3872067384240*B*a**
2*b*m**2*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489
9622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112955
3*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056
995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 30
09183307200*B*a**2*b*m*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2184
00*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**
8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133456
00*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076
74368000) + 980755776000*B*a**2*b*x**4*(d*x)**m/(m**15 + 120*m**14 + 6580*m
13 + 218400*m12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820
7628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916300160
0*m + 1307674368000) + 3*B*a**2*c*m**14*x**6*(d*x)**m/(m**15 + 120*m**14 +
6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9
 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m
5 + 2706813345600*m4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916
3001600*m + 1307674368000) + 342*B*a**2*c*m**13*x**6*(d*x)**m/(m**15 + 120*
m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095
740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672
107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
+ 4339163001600*m + 1307674368000) + 17688*B*a**2*c*m**12*x**6*(d*x)**m/(m*
*15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**1
0 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761
4720*m**2 + 4339163001600*m + 1307674368000) + 549072*B*a**2*c*m**11*x**6*(
d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78
558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803
210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
+ 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 11404434*B*a**2*c

*m**10*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 48996
22*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*
m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699
5703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1672
48836*B*a**2*c*m**9*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*
m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*
m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743
68000) + 1780794204*B*a**2*c*m**8*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m
13 + 218400*m12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820
7628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916300160
0*m + 1307674368000) + 13938118776*B*a**2*c*m**7*x**6*(d*x)**m/(m**15 + 120
*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809
5740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967
2107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
 + 4339163001600*m + 1307674368000) + 80264676003*B*a**2*c*m**6*x**6*(d*x)*
*m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855848
0*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321068
0*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616
5817614720*m**2 + 4339163001600*m + 1307674368000) + 336821576022*B*a**2*c*
m**5*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622
*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m*
*7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957
03824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 100808
6865108*B*a**2*c*m**4*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21840
0*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334560
0*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767
4368000) + 2071918846152*B*a**2*c*m**3*x**6*(d*x)**m/(m**15 + 120*m**14 + 6
580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9
+ 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m*
*5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163
001600*m + 1307674368000) + 2739474034560*B*a**2*c*m**2*x**6*(d*x)**m/(m**1
5 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
+ 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 +
 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658176147
20*m**2 + 4339163001600*m + 1307674368000) + 2060608636800*B*a**2*c*m*x**6*
(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7
8558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280
3210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 653837184000*B*a
2*c*x6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489962
2*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m
7 + 272803210680*m6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995

703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3*B*a
*b**2*m**14*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112
9553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5
056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
 342*B*a*b**2*m**13*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*
m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*
m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743
68000) + 17688*B*a*b**2*m**12*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628
000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706
813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m
+ 1307674368000) + 549072*B*a*b**2*m**11*x**6*(d*x)**m/(m**15 + 120*m**14 +
 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 11404434*B*a*b**2*m**10*x**6*(d*x)**m/(m**15
+ 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 +
928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1
009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720
*m**2 + 4339163001600*m + 1307674368000) + 167248836*B*a*b**2*m**9*x**6*(d*
x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855
8480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321
0680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1780794204*B*a*b**2
*m**8*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489962
2*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m
7 + 272803210680*m6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995
703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 13938
118776*B*a*b**2*m**7*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600
*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 80264676003*B*a*b**2*m**6*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580
*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8
207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5
+ 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001
600*m + 1307674368000) + 336821576022*B*a*b**2*m**5*x**6*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
2 + 4339163001600*m + 1307674368000) + 1008086865108*B*a*b2*m**4*x**6*(
d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78
558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803

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210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
+ 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 2071918846152*B*a
*b**2*m**3*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129
553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
2739474034560*B*a*b**2*m**2*x**6*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 +
 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762800
0*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681
3345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
1307674368000) + 2060608636800*B*a*b**2*m*x**6*(d*x)**m/(m**15 + 120*m**14
+ 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m*
*9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080
*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339
163001600*m + 1307674368000) + 653837184000*B*a*b**2*x**6*(d*x)**m/(m**15 +
 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9
28095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10
09672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*
m**2 + 4339163001600*m + 1307674368000) + 6*B*a*b*c*m**14*x**8*(d*x)**m/(m*
*15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**1
0 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761
4720*m**2 + 4339163001600*m + 1307674368000) + 672*B*a*b*c*m**13*x**8*(d*x)
**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785584
80*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032106
80*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61
65817614720*m**2 + 4339163001600*m + 1307674368000) + 34104*B*a*b*c*m**12*x
**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1037568*B*a*
b*c*m**11*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 48
99622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546311295
53*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505
6995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 2
1097188*B*a*b*c*m**10*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21840
0*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334560
0*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767
4368000) + 302573376*B*a*b*c*m**9*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m
**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820
7628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916300160
0*m + 1307674368000) + 3147987432*B*a*b*c*m**8*x**8*(d*x)**m/(m**15 + 120*m
**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
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40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
4339163001600*m + 1307674368000) + 24061868544*B*a*b*c*m**7*x**8*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
10 + 928095740*m9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
6 + 1009672107080*m5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 135291828966*B*a*b*c*m**6
*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 5544846323
52*B*a*b*c*m**5*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**1
2 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546
31129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4
+ 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767436800
0) + 1622155583664*B*a*b*c*m**4*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**
13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076
28000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27
06813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*
m + 1307674368000) + 3263635404288*B*a*b*c*m**3*x**8*(d*x)**m/(m**15 + 120*
m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095
740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672
107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
+ 4339163001600*m + 1307674368000) + 4232890988640*B*a*b*c*m**2*x**8*(d*x)*
*m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855848
0*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321068
0*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616
5817614720*m**2 + 4339163001600*m + 1307674368000) + 3131777779200*B*a*b*c*
m*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
+ 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038
24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 980755776
000*B*a*b*c*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112
9553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5
056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
3*B*a*c**2*m**14*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m
12 + 4899622*m11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m
4 + 5056995703824*m3 + 6165817614720*m**2 + 4339163001600*m + 130767436
8000) + 330*B*a*c**2*m**13*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 +
218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762800
0*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681
3345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
1307674368000) + 16440*B*a*c**2*m**12*x**10*(d*x)**m/(m**15 + 120*m**14 + 6

580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 490800*B*a*c**2*m**11*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 9790866*B*a*c**2*m**10*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 137766780*B*a*c**2*m**9*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1406619420*B*a*c**2*m**8*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 10556689800*B*a*c**2*m**7*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 58326490659*B*a*c**2*m**6*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 235144725450*B*a*c**2*m**5*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 677569066740*B*a*c**2*m**4*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1344749369400*B*a*c**2*m**3*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1723493417472*B*a*c**2*m**2*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4

339163001600*m + 1307674368000) + 1262518669440*B*a*c**2*m*x**10*(d*x)**m/(
m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m*
*10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m*
*6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817
614720*m**2 + 4339163001600*m + 1307674368000) + 392302310400*B*a*c**2*x**1
0*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 +
 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272
803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m*
*3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + B*b**3*m**14*x
8*(d*x)m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 112*B*b**3*m
13*x8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622
*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m*
*7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957
03824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 5684*B
*b**3*m**12*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112
9553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5
056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
 172928*B*b**3*m**11*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600
*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 3516198*B*b**3*m**10*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**1
3 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762
8000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270
6813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m
 + 1307674368000) + 50428896*B*b**3*m**9*x**8*(d*x)**m/(m**15 + 120*m**14 +
 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 524664572*B*b**3*m**8*x**8*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
2 + 4339163001600*m + 1307674368000) + 4010311424*B*b3*m**7*x**8*(d*x)*
*m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855848
0*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321068
0*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616
5817614720*m**2 + 4339163001600*m + 1307674368000) + 22548638161*B*b**3*m**
6*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
+ 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038

24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 924141053
92*B*b**3*m**5*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12
 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463
1129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4
+ 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000
) + 270359263944*B*b**3*m**4*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
+ 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076280
00*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068
13345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
 1307674368000) + 543939234048*B*b**3*m**3*x**8*(d*x)**m/(m**15 + 120*m**14
 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m
9 + 8207628000*m8 + 54631129553*m**7 + 272803210680*m**6 + 100967210708
0*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433
9163001600*m + 1307674368000) + 705481831440*B*b**3*m**2*x**8*(d*x)**m/(m**
15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
+ 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614
720*m**2 + 4339163001600*m + 1307674368000) + 521962963200*B*b**3*m*x**8*(d
*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785
58480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032
10680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 163459296000*B*b**
3*x**8*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
+ 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038
24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3*B*b**2*
c*m**14*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489
9622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112955
3*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056
995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 33
0*B*b**2*c*m**13*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m*
*12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5
4631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m*
*4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368
000) + 16440*B*b**2*c*m**12*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
+ 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076280
00*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068
13345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
 1307674368000) + 490800*B*b**2*c*m**11*x**10*(d*x)**m/(m**15 + 120*m**14 +
 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 9790866*B*b**2*c*m**10*x**10*(d*x)**m/(m**15
+ 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 +
928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1

009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720
*m**2 + 4339163001600*m + 1307674368000) + 137766780*B*b**2*c*m**9*x**10*(d
*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785
58480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032
10680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1406619420*B*b**2*
c*m**8*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899
622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553
*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569
95703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 105
56689800*B*b**2*c*m**7*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218
400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m*
*8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345
600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307
674368000) + 58326490659*B*b**2*c*m**6*x**10*(d*x)**m/(m**15 + 120*m**14 +
6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9
 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m
5 + 2706813345600*m4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916
3001600*m + 1307674368000) + 235144725450*B*b**2*c*m**5*x**10*(d*x)**m/(m**
15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
+ 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614
720*m**2 + 4339163001600*m + 1307674368000) + 677569066740*B*b**2*c*m**4*x*
10(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 134474936940
0*B*b**2*c*m**3*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**
12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54
631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**
4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743680
00) + 1723493417472*B*b**2*c*m**2*x**10*(d*x)**m/(m**15 + 120*m**14 + 6580*
m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82
07628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630016
00*m + 1307674368000) + 1262518669440*B*b**2*c*m*x**10*(d*x)**m/(m**15 + 12
0*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280
95740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096
72107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**
2 + 4339163001600*m + 1307674368000) + 392302310400*B*b**2*c*x**10*(d*x)**m
/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*
m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*
m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658
17614720*m**2 + 4339163001600*m + 1307674368000) + 3*B*b*c**2*m**14*x**12*(
d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78

558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803
210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
+ 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 324*B*b*c**2*m**1
3*x**12*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m
11 + 78558480*m10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703
824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 15852*B*
b*c**2*m**12*x**12*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12
+ 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631
129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
 + 464976*B*b*c**2*m**11*x**12*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2
18400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*
m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133
45600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13
07674368000) + 9119154*B*b*c**2*m**10*x**12*(d*x)**m/(m**15 + 120*m**14 + 6
580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9
+ 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m*
*5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163
001600*m + 1307674368000) + 126245592*B*b*c**2*m**9*x**12*(d*x)**m/(m**15 +
 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9
28095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10
09672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*
m**2 + 4339163001600*m + 1307674368000) + 1269340116*B*b*c**2*m**8*x**12*(d
*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785
58480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032
10680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 9390802608*B*b*c**
2*m**7*x**12*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899
622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553
*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569
95703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 512
03757363*B*b*c**2*m**6*x**12*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218
400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m*
*8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345
600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307
674368000) + 203964543684*B*b*c**2*m**5*x**12*(d*x)**m/(m**15 + 120*m**14 +
 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 581441797032*B*b*c**2*m**4*x**12*(d*x)**m/(m*
*15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**1
0 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761
4720*m**2 + 4339163001600*m + 1307674368000) + 1143138472416*B*b*c**2*m**3*

x**12*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1453325442
480*B*b*c**2*m**2*x**12*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m
12 + 4899622*m11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m
4 + 5056995703824*m3 + 6165817614720*m**2 + 4339163001600*m + 130767436
8000) + 1057547534400*B*b*c**2*m*x**12*(d*x)**m/(m**15 + 120*m**14 + 6580*m
13 + 218400*m12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820
7628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916300160
0*m + 1307674368000) + 326918592000*B*b*c**2*x**12*(d*x)**m/(m**15 + 120*m*
*14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809574
0*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967210
7080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
4339163001600*m + 1307674368000) + B*c**3*m**14*x**14*(d*x)**m/(m**15 + 120
*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809
5740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967
2107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
+ 4339163001600*m + 1307674368000) + 106*B*c**3*m**13*x**14*(d*x)**m/(m**1
5 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
+ 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 +
1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658176147
20*m**2 + 4339163001600*m + 1307674368000) + 5096*B*c**3*m**12*x**14*(d*x)*
*m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855848
0*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321068
0*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616
5817614720*m**2 + 4339163001600*m + 1307674368000) + 147056*B*c**3*m**11*x*
14(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
+ 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 2840838*B*c*
*3*m**10*x**14*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 48
99622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546311295
53*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505
6995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3
8786748*B*c**3*m**9*x**14*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600
*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 385081268*B*c**3*m**8*x**14*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600

*m + 1307674368000) + 2816490248*B*c**3*m**7*x**14*(d*x)**m/(m**15 + 120*m*
*14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809574
0*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967210
7080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
4339163001600*m + 1307674368000) + 15200266081*B*c**3*m**6*x**14*(d*x)**m/(
m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m*
*10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m*
*6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817
614720*m**2 + 4339163001600*m + 1307674368000) + 59999485546*B*c**3*m**5*x*
14(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 169679309436
*B*c**3*m**4*x**14*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12
+ 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631
129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
 + 331303013496*B*c**3*m**3*x**14*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
+ 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076280
00*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068
13345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
 1307674368000) + 418753514880*B*c**3*m**2*x**14*(d*x)**m/(m**15 + 120*m**1
4 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*
m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721070
80*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43
39163001600*m + 1307674368000) + 303268406400*B*c**3*m*x**14*(d*x)**m/(m**1
5 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
+ 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 +
 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658176147
20*m**2 + 4339163001600*m + 1307674368000) + 93405312000*B*c**3*x**14*(d*x)
m/(m15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785584
80*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032106
80*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61
65817614720*m**2 + 4339163001600*m + 1307674368000) + C*a**3*m**14*x**3*(d*
x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855
8480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321
0680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
6165817614720*m**2 + 4339163001600*m + 1307674368000) + 117*C*a**3*m**13*x*
3(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
+ 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27
2803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m
3 + 6165817614720*m2 + 4339163001600*m + 1307674368000) + 6229*C*a**3*m
12*x3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622
*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m*
*7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957

03824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 199713
*C*a**3*m**11*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12
+ 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631
129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
+ 4300483*C*a**3*m**10*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218
400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m*
*8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345
600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307
674368000) + 65657031*C*a**3*m**9*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m
13 + 218400*m12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820
7628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916300160
0*m + 1307674368000) + 731124647*C*a**3*m**8*x**3*(d*x)**m/(m**15 + 120*m**
14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740
*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107
080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4
339163001600*m + 1307674368000) + 6014254059*C*a**3*m**7*x**3*(d*x)**m/(m**
15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
+ 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
+ 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614
720*m**2 + 4339163001600*m + 1307674368000) + 36588367376*C*a**3*m**6*x**3*
(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7
8558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280
3210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
+ 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 163038108552*C*a
3*m5*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489
9622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112955
3*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056
995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 52
0557781424*C*a**3*m**4*x**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2184
00*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**
8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133456
00*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076
74368000) + 1145140001328*C*a**3*m**3*x**3*(d*x)**m/(m**15 + 120*m**14 + 65
80*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 +
8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**
5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630
01600*m + 1307674368000) + 1621575699840*C*a**3*m**2*x**3*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9
28095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10
09672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*
m**2 + 4339163001600*m + 1307674368000) + 1301090515200*C*a**3*m*x**3*(d*x)
m/(m15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785584
80*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032106

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80*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61
65817614720*m**2 + 4339163001600*m + 1307674368000) + 435891456000*C*a**3*x
**3*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3*C*a**2*b*m
**14*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622
*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m*
*7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957
03824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 345*C*
a**2*b*m**13*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546311
29553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
+ 18015*C*a**2*b*m**12*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2184
00*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**
8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133456
00*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076
74368000) + 565125*C*a**2*b*m**11*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m
**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820
7628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916300160
0*m + 1307674368000) + 11873241*C*a**2*b*m**10*x**5*(d*x)**m/(m**15 + 120*m
**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
 4339163001600*m + 1307674368000) + 176309235*C*a**2*b*m**9*x**5*(d*x)**m/(
m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m*
*10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m*
*6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817
614720*m**2 + 4339163001600*m + 1307674368000) + 1902741045*C*a**2*b*m**8*x
**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 15109178775*
C*a**2*b*m**7*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12
+ 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631
129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
 + 88347494784*C*a**2*b*m**6*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
+ 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076280
00*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068
13345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
 1307674368000) + 376672158120*C*a**2*b*m**5*x**5*(d*x)**m/(m**15 + 120*m**
14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740
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*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107
080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4
339163001600*m + 1307674368000) + 1145655530640*C*a**2*b*m**4*x**5*(d*x)**m
/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*
m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*
m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658
17614720*m**2 + 4339163001600*m + 1307674368000) + 2392162383600*C*a**2*b*m
**3*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*
m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**
7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570
3824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3210175
193472*C*a**2*b*m**2*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600
*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 2446576876800*C*a**2*b*m*x**5*(d*x)**m/(m**15 + 120*m**14 + 6580*
m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82
07628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630016
00*m + 1307674368000) + 784604620800*C*a**2*b*x**5*(d*x)**m/(m**15 + 120*m*
*14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809574
0*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967210
7080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
4339163001600*m + 1307674368000) + 3*C*a**2*c*m**14*x**7*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
**2 + 4339163001600*m + 1307674368000) + 339*C*a**2*c*m**13*x**7*(d*x)**m/(
m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m*
*10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m*
*6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817
614720*m**2 + 4339163001600*m + 1307674368000) + 17367*C*a**2*c*m**12*x**7*
(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7
8558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280
3210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 533631*C*a**2*c*
m**11*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489962
2*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m
**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995
703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 10963
449*C*a**2*c*m**10*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m
**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m
**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767436
8000) + 158931297*C*a**2*c*m**9*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**
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13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076
28000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27
06813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*
m + 1307674368000) + 1671768141*C*a**2*c*m**8*x**7*(d*x)**m/(m**15 + 120*m*
*14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809574
0*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967210
7080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
4339163001600*m + 1307674368000) + 12920507013*C*a**2*c*m**7*x**7*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
10 + 928095740*m9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
6 + 1009672107080*m5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 73449839568*C*a**2*c*m**6
*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3042607550
64*C*a**2*c*m**5*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**
12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54
631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**
4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743680
00) + 899191035792*C*a**2*c*m**4*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m*
*13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207
628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2
706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600
*m + 1307674368000) + 1826102786256*C*a**2*c*m**3*x**7*(d*x)**m/(m**15 + 12
0*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280
95740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096
72107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**
2 + 4339163001600*m + 1307674368000) + 2388267607680*C*a**2*c*m**2*x**7*(d*
x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855
8480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321
0680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1779579590400*C*a**
2*c*m*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489962
2*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m
7 + 272803210680*m6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995
703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 56043
1872000*C*a**2*c*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**
12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54
631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**
4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743680
00) + 3*C*a*b**2*m**14*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2184
00*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**
8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133456
00*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076

74368000) + 339*C*a*b**2*m**13*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**1
3 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762
8000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270
6813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m
 + 1307674368000) + 17367*C*a*b**2*m**12*x**7*(d*x)**m/(m**15 + 120*m**14 +
 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 533631*C*a*b**2*m**11*x**7*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
2 + 4339163001600*m + 1307674368000) + 10963449*C*a*b2*m**10*x**7*(d*x)
m/(m15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785584
80*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032106
80*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61
65817614720*m**2 + 4339163001600*m + 1307674368000) + 158931297*C*a*b**2*m*
*9*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m
11 + 78558480*m10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703
824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 16717681
41*C*a*b**2*m**8*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**
12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54
631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**
4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743680
00) + 12920507013*C*a*b**2*m**7*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**
13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076
28000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27
06813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*
m + 1307674368000) + 73449839568*C*a*b**2*m**6*x**7*(d*x)**m/(m**15 + 120*m
14 + 6580*m13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
 4339163001600*m + 1307674368000) + 304260755064*C*a*b**2*m**5*x**7*(d*x)**
m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480
*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680
*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165
817614720*m**2 + 4339163001600*m + 1307674368000) + 899191035792*C*a*b**2*m
4*x7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*
m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**
7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570
3824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1826102
786256*C*a*b**2*m**3*x**7*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600

*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 2388267607680*C*a*b**2*m**2*x**7*(d*x)**m/(m**15 + 120*m**14 + 65
80*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 +
 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**
5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630
01600*m + 1307674368000) + 1779579590400*C*a*b**2*m*x**7*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
2 + 4339163001600*m + 1307674368000) + 560431872000*C*a*b2*x**7*(d*x)**
m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480
*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680
*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165
817614720*m**2 + 4339163001600*m + 1307674368000) + 6*C*a*b*c*m**14*x**9*(d
*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785
58480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032
10680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 666*C*a*b*c*m**13*
x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**1
1 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824
*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 33486*C*a*b
*c*m**12*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489
9622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112955
3*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056
995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 10
09026*C*a*b*c*m**11*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*
m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 +
 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*
m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743
68000) + 20316498*C*a*b*c*m**10*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**
13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076
28000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27
06813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*
m + 1307674368000) + 288502398*C*a*b*c*m**9*x**9*(d*x)**m/(m**15 + 120*m**1
4 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*
m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721070
80*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43
39163001600*m + 1307674368000) + 2972052858*C*a*b*c*m**8*x**9*(d*x)**m/(m**
15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
+ 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614
720*m**2 + 4339163001600*m + 1307674368000) + 22497292278*C*a*b*c*m**7*x**9
*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 +
78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728

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03210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**
3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 125311146816*C*
a*b*c*m**6*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129
553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
509018942736*C*a*b*c*m**5*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 2
18400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*
m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068133
45600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13
07674368000) + 1476862157856*C*a*b*c*m**4*x**9*(d*x)**m/(m**15 + 120*m**14
+ 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m*
*9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080
*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339
163001600*m + 1307674368000) + 2949120652896*C*a*b*c*m**3*x**9*(d*x)**m/(m*
*15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**1
0 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761
4720*m**2 + 4339163001600*m + 1307674368000) + 3799888346880*C*a*b*c*m**2*x
**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 279591056640
0*C*a*b*c*m*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112
9553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5
056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
 871782912000*C*a*b*c*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21840
0*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334560
0*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767
4368000) + 3*C*a*c**2*m**14*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
+ 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076280
00*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27068
13345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
 1307674368000) + 327*C*a*c**2*m**13*x**11*(d*x)**m/(m**15 + 120*m**14 + 65
80*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 +
 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**
5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630
01600*m + 1307674368000) + 16143*C*a*c**2*m**12*x**11*(d*x)**m/(m**15 + 120
*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809
5740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967
2107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
 + 4339163001600*m + 1307674368000) + 477627*C*a*c**2*m**11*x**11*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
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**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 9444969*C*a*c**2*m**10*x*
*11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2
72803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*
m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 131780781*C*
a*c**2*m**9*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 +
 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546311
29553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
+ 1334698629*C*a*c**2*m**8*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 +
 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762800
0*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681
3345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
1307674368000) + 9941199081*C*a*c**2*m**7*x**11*(d*x)**m/(m**15 + 120*m**14
 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m
**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967210708
0*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433
9163001600*m + 1307674368000) + 54540198768*C*a*c**2*m**6*x**11*(d*x)**m/(m
**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**
10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**
6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658176
14720*m**2 + 4339163001600*m + 1307674368000) + 218467445592*C*a*c**2*m**5*
x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 6258744197
28*C*a*c**2*m**4*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m*
*12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5
4631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m*
*4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368
000) + 1235821419792*C*a*c**2*m**3*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580
*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8
207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5
+ 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001
600*m + 1307674368000) + 1576951493760*C*a*c**2*m**2*x**11*(d*x)**m/(m**15
+ 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 +
928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1
009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720
*m**2 + 4339163001600*m + 1307674368000) + 1150986412800*C*a*c**2*m*x**11*(
d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78
558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803
210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
+ 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 356638464000*C*a*
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c**2*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489962
2*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m
7 + 272803210680*m6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995
703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + C*b**
3*m**14*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899
622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553
*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569
95703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 111
*C*b**3*m**13*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12
+ 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631
129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 +
5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000)
+ 5581*C*b**3*m**12*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600
*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 168171*C*b**3*m**11*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13
+ 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628
000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706
813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m
+ 1307674368000) + 3386083*C*b**3*m**10*x**9*(d*x)**m/(m**15 + 120*m**14 +
6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9
+ 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m
5 + 2706813345600*m4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916
3001600*m + 1307674368000) + 48083733*C*b**3*m**9*x**9*(d*x)**m/(m**15 + 12
0*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280
95740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096
72107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**
2 + 4339163001600*m + 1307674368000) + 495342143*C*b**3*m**8*x**9*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
10 + 928095740*m9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
6 + 1009672107080*m5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 3749548713*C*b**3*m**7*x*
9(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11
+ 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27
2803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m
3 + 6165817614720*m2 + 4339163001600*m + 1307674368000) + 20885191136*C
*b**3*m**6*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129
553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
84836490456*C*b**3*m**5*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218
400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m*
*8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345
600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307

674368000) + 246143692976*C*b**3*m**4*x**9*(d*x)**m/(m**15 + 120*m**14 + 65
80*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 +
 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**
5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630
01600*m + 1307674368000) + 491520108816*C*b**3*m**3*x**9*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92
8095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100
9672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m
2 + 4339163001600*m + 1307674368000) + 633314724480*C*b3*m**2*x**9*(d*x
)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558
480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210
680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6
165817614720*m**2 + 4339163001600*m + 1307674368000) + 465985094400*C*b**3*
m*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
+ 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038
24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 145297152
000*C*b**3*x**9*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129
553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
3*C*b**2*c*m**14*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m*
*12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5
4631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m*
*4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368
000) + 327*C*b**2*c*m**13*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 +
218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000
*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813
345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1
307674368000) + 16143*C*b**2*c*m**12*x**11*(d*x)**m/(m**15 + 120*m**14 + 65
80*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 +
 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**
5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391630
01600*m + 1307674368000) + 477627*C*b**2*c*m**11*x**11*(d*x)**m/(m**15 + 12
0*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280
95740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096
72107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**
2 + 4339163001600*m + 1307674368000) + 9444969*C*b**2*c*m**10*x**11*(d*x)**
m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480
*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680
*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165
817614720*m**2 + 4339163001600*m + 1307674368000) + 131780781*C*b**2*c*m**9
*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
+ 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038

24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 133469862
9*C*b**2*c*m**8*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**
12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54
631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**
4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743680
00) + 9941199081*C*b**2*c*m**7*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**
13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076
28000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27
06813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*
m + 1307674368000) + 54540198768*C*b**2*c*m**6*x**11*(d*x)**m/(m**15 + 120*
m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095
740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672
107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
 + 4339163001600*m + 1307674368000) + 218467445592*C*b**2*c*m**5*x**11*(d*x)
m/(m15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 785584
80*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728032106
80*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61
65817614720*m**2 + 4339163001600*m + 1307674368000) + 625874419728*C*b**2*c
*m**4*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 48996
22*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*
m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699
5703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 1235
821419792*C*b**2*c*m**3*x**11*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21
8400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m
8 + 54631129553*m7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334
5600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130
7674368000) + 1576951493760*C*b**2*c*m**2*x**11*(d*x)**m/(m**15 + 120*m**14
 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m
9 + 8207628000*m8 + 54631129553*m**7 + 272803210680*m**6 + 100967210708
0*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433
9163001600*m + 1307674368000) + 1150986412800*C*b**2*c*m*x**11*(d*x)**m/(m*
*15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**1
0 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6
 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581761
4720*m**2 + 4339163001600*m + 1307674368000) + 356638464000*C*b**2*c*x**11*
(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7
8558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280
3210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3
 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 3*C*b*c**2*m**14
*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m*
*11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7
 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50569957038
24*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 321*C*b*c
2*m13*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4
899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129

553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 50
56995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) +
15567*C*b*c**2*m**12*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 21840
0*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681334560
0*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767
4368000) + 452829*C*b*c**2*m**11*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m
13 + 218400*m12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820
7628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 +
2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916300160
0*m + 1307674368000) + 8812089*C*b*c**2*m**10*x**13*(d*x)**m/(m**15 + 120*m
14 + 6580*m13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9280957
40*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10096721
07080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 +
 4339163001600*m + 1307674368000) + 121118283*C*b*c**2*m**9*x**13*(d*x)**m/
(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m
10 + 928095740*m9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m
6 + 1009672107080*m5 + 2706813345600*m**4 + 5056995703824*m**3 + 616581
7614720*m**2 + 4339163001600*m + 1307674368000) + 1209749541*C*b*c**2*m**8*
x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**
11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 +
 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 505699570382
4*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 8896139967
*C*b*c**2*m**7*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**1
2 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 546
31129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4
 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 130767436800
0) + 48243569088*C*b*c**2*m**6*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**
13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 82076
28000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 27
06813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*
m + 1307674368000) + 191243233896*C*b*c**2*m**5*x**13*(d*x)**m/(m**15 + 120
*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 92809
5740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 100967
2107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2
 + 4339163001600*m + 1307674368000) + 542854280592*C*b*c**2*m**4*x**13*(d*x
)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558
480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210
680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6
165817614720*m**2 + 4339163001600*m + 1307674368000) + 1063334389104*C*b*c*
*2*m**3*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489
9622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 5463112955
3*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056
995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 13
47640053120*C*b*c**2*m**2*x**13*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 +

218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000
*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813
345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1
307674368000) + 978132153600*C*b*c**2*m*x**13*(d*x)**m/(m**15 + 120*m**14 +
6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**
9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*
m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 43391
63001600*m + 1307674368000) + 301771008000*C*b*c**2*x**13*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9
28095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10
09672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*
m**2 + 4339163001600*m + 1307674368000) + C*c**3*m**14*x**15*(d*x)**m/(m**1
5 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10
+ 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 +
1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 61658176147
20*m**2 + 4339163001600*m + 1307674368000) + 105*C*c**3*m**13*x**15*(d*x)**
m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480
*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680
*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165
817614720*m**2 + 4339163001600*m + 1307674368000) + 5005*C*c**3*m**12*x**15
*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 +
78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 2728
03210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**
3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 143325*C*c**3*m
11*x15*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489962
2*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m
7 + 272803210680*m6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995
703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 27497
47*C*c**3*m**10*x**15*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**
12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54
631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**
4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 13076743680
00) + 37312275*C*c**3*m**9*x**15*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 +
218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 820762800
0*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 270681
3345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m +
1307674368000) + 368411615*C*c**3*m**8*x**15*(d*x)**m/(m**15 + 120*m**14 +
6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9
+ 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m
5 + 2706813345600*m4 + 5056995703824*m**3 + 6165817614720*m**2 + 433916
3001600*m + 1307674368000) + 2681453775*C*c**3*m**7*x**15*(d*x)**m/(m**15 +
120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 9
28095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 10
09672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*
m**2 + 4339163001600*m + 1307674368000) + 14409322928*C*c**3*m**6*x**15*(d*

```
x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 7855
8480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 27280321
0680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 +
6165817614720*m**2 + 4339163001600*m + 1307674368000) + 56663366760*C*c**3*
m**5*x**15*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 489962
2*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m
**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995
703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000) + 15972
1605680*C*c**3*m**4*x**15*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400
*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8207628000*m**8
+ 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5 + 2706813345600
*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001600*m + 1307674
368000) + 310989260400*C*c**3*m**3*x**15*(d*x)**m/(m**15 + 120*m**14 + 6580
*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928095740*m**9 + 8
207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009672107080*m**5
+ 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m**2 + 4339163001
600*m + 1307674368000) + 392156797824*C*c**3*m**2*x**15*(d*x)**m/(m**15 + 1
20*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480*m**10 + 928
095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680*m**6 + 1009
672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165817614720*m*
*2 + 4339163001600*m + 1307674368000) + 283465647360*C*c**3*m*x**15*(d*x)**
m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 + 78558480
*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272803210680
*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m**3 + 6165
817614720*m**2 + 4339163001600*m + 1307674368000) + 87178291200*C*c**3*x**1
5*(d*x)**m/(m**15 + 120*m**14 + 6580*m**13 + 218400*m**12 + 4899622*m**11 +
 78558480*m**10 + 928095740*m**9 + 8207628000*m**8 + 54631129553*m**7 + 272
803210680*m**6 + 1009672107080*m**5 + 2706813345600*m**4 + 5056995703824*m*
*3 + 6165817614720*m**2 + 4339163001600*m + 1307674368000), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.53

$$\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4)^3 \, dx = \frac{Cc^3d^mx^{15}x^m}{m+15} + \frac{Bc^3d^mx^{14}x^m}{m+14} + \frac{3\,Cbc^2d^mx^{13}x^m}{m+13} + \frac{Ac^3d^mx^{13}x^m}{m+13} + \frac{3\,Bbc^2d^mx^{12}x^m}{m+12} + \frac{3\,Cb^2cd^mx^{11}x^m}{m+11} + \frac{3\,Cac^2d^mx^{11}x^m}{m+11} + \frac{3\,Abc^2d^mx^{11}x^m}{m+11} + \frac{3\,Bb^2cd^mx^{10}x^m}{m+10} + \frac{3\,Bac^2d^mx^{10}x^m}{m+10} + \frac{Cb^3d^mx^9x^m}{m+9} + \frac{6\,Cabcd^mx^9x^m}{m+9} + \frac{3\,Ab^2cd^mx^9x^m}{m+9} + \frac{3\,Aac^2d^mx^9x^m}{m+9} + \frac{Bb^3d^mx^8x^m}{m+8} + \frac{6\,Babcd^mx^8x^m}{m+8} + \frac{3\,Cab^2d^mx^7x^m}{m+7} + \frac{Ab^3d^mx^7x^m}{m+7} + \frac{3\,Ca^2cd^mx^7x^m}{m+7} + \frac{6\,Aabcd^mx^7x^m}{m+7} + \frac{3\,Bab^2d^mx^6x^m}{m+6} + \frac{3\,Ba^2cd^mx^6x^m}{m+6} + \frac{3\,Ca^2bd^mx^5x^m}{m+5} + \frac{3\,Aab^2d^mx^5x^m}{m+5} + \frac{3\,Aa^2cd^mx^5x^m}{m+5} + \frac{3\,Ba^2bd^mx^4x^m}{m+4} + \frac{Ca^3d^mx^3x^m}{m+3} + \frac{3\,Aa^2bd^mx^3x^m}{m+3} + \frac{Ba^3d^mx^2x^m}{m+2} + \frac{(dx)^{m+1}Aa^3}{d(m+1)}$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] C*c^3*d^m*x^15*x^m/(m + 15) + B*c^3*d^m*x^14*x^m/(m + 14) + 3*C*b*c^2*d^m*x
^13*x^m/(m + 13) + A*c^3*d^m*x^13*x^m/(m + 13) + 3*B*b*c^2*d^m*x^12*x^m/(m
+ 12) + 3*C*b^2*c*d^m*x^11*x^m/(m + 11) + 3*C*a*c^2*d^m*x^11*x^m/(m + 11) +
 3*A*b*c^2*d^m*x^11*x^m/(m + 11) + 3*B*b^2*c*d^m*x^10*x^m/(m + 10) + 3*B*a*
c^2*d^m*x^10*x^m/(m + 10) + C*b^3*d^m*x^9*x^m/(m + 9) + 6*C*a*b*c*d^m*x^9*x
^m/(m + 9) + 3*A*b^2*c*d^m*x^9*x^m/(m + 9) + 3*A*a*c^2*d^m*x^9*x^m/(m + 9)
+ B*b^3*d^m*x^8*x^m/(m + 8) + 6*B*a*b*c*d^m*x^8*x^m/(m + 8) + 3*C*a*b^2*d^m
*x^7*x^m/(m + 7) + A*b^3*d^m*x^7*x^m/(m + 7) + 3*C*a^2*c*d^m*x^7*x^m/(m + 7
) + 6*A*a*b*c*d^m*x^7*x^m/(m + 7) + 3*B*a*b^2*d^m*x^6*x^m/(m + 6) + 3*B*a^2
*c*d^m*x^6*x^m/(m + 6) + 3*C*a^2*b*d^m*x^5*x^m/(m + 5) + 3*A*a*b^2*d^m*x^5*
x^m/(m + 5) + 3*A*a^2*c*d^m*x^5*x^m/(m + 5) + 3*B*a^2*b*d^m*x^4*x^m/(m + 4)
 + C*a^3*d^m*x^3*x^m/(m + 3) + 3*A*a^2*b*d^m*x^3*x^m/(m + 3) + B*a^3*d^m*x^
2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^3/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7808 vs. $2(399) = 798$.

Time = 0.40 (sec) , antiderivative size = 7808, normalized size of antiderivative = 19.57

$$\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

[Out] ((d*x)^m*C*c^3*m^14*x^15 + (d*x)^m*B*c^3*m^14*x^14 + 105*(d*x)^m*C*c^3*m^13
x^15 + 3(d*x)^m*C*b*c^2*m^14*x^13 + (d*x)^m*A*c^3*m^14*x^13 + 106*(d*x)^m
*B*c^3*m^13*x^14 + 5005*(d*x)^m*C*c^3*m^12*x^15 + 3*(d*x)^m*B*b*c^2*m^14*x^
12 + 321*(d*x)^m*C*b*c^2*m^13*x^13 + 107*(d*x)^m*A*c^3*m^13*x^13 + 5096*(d*
x)^m*B*c^3*m^12*x^14 + 143325*(d*x)^m*C*c^3*m^11*x^15 + 3*(d*x)^m*C*b^2*c*m
^14*x^11 + 3*(d*x)^m*C*a*c^2*m^14*x^11 + 3*(d*x)^m*A*b*c^2*m^14*x^11 + 324*
(d*x)^m*B*b*c^2*m^13*x^12 + 15567*(d*x)^m*C*b*c^2*m^12*x^13 + 5189*(d*x)^m*
A*c^3*m^12*x^13 + 147056*(d*x)^m*B*c^3*m^11*x^14 + 2749747*(d*x)^m*C*c^3*m^
10*x^15 + 3*(d*x)^m*B*b^2*c*m^14*x^10 + 3*(d*x)^m*B*a*c^2*m^14*x^10 + 327*(
d*x)^m*C*b^2*c*m^13*x^11 + 327*(d*x)^m*C*a*c^2*m^13*x^11 + 327*(d*x)^m*A*b*
c^2*m^13*x^11 + 15852*(d*x)^m*B*b*c^2*m^12*x^12 + 452829*(d*x)^m*C*b*c^2*m^
11*x^13 + 150943*(d*x)^m*A*c^3*m^11*x^13 + 2840838*(d*x)^m*B*c^3*m^10*x^14
+ 37312275*(d*x)^m*C*c^3*m^9*x^15 + (d*x)^m*C*b^3*m^14*x^9 + 6*(d*x)^m*C*a*
b*c*m^14*x^9 + 3*(d*x)^m*A*b^2*c*m^14*x^9 + 3*(d*x)^m*A*a*c^2*m^14*x^9 + 33
0*(d*x)^m*B*b^2*c*m^13*x^10 + 330*(d*x)^m*B*a*c^2*m^13*x^10 + 16143*(d*x)^m
*C*b^2*c*m^12*x^11 + 16143*(d*x)^m*C*a*c^2*m^12*x^11 + 16143*(d*x)^m*A*b*c^
2*m^12*x^11 + 464976*(d*x)^m*B*b*c^2*m^11*x^12 + 8812089*(d*x)^m*C*b*c^2*m^
10*x^13 + 2937363*(d*x)^m*A*c^3*m^10*x^13 + 38786748*(d*x)^m*B*c^3*m^9*x^14
 + 368411615*(d*x)^m*C*c^3*m^8*x^15 + (d*x)^m*B*b^3*m^14*x^8 + 6*(d*x)^m*B*
a*b*c*m^14*x^8 + 111*(d*x)^m*C*b^3*m^13*x^9 + 666*(d*x)^m*C*a*b*c*m^13*x^9
+ 333*(d*x)^m*A*b^2*c*m^13*x^9 + 333*(d*x)^m*A*a*c^2*m^13*x^9 + 16440*(d*x)
^m*B*b^2*c*m^12*x^10 + 16440*(d*x)^m*B*a*c^2*m^12*x^10 + 477627*(d*x)^m*C*b
^2*c*m^11*x^11 + 477627*(d*x)^m*C*a*c^2*m^11*x^11 + 477627*(d*x)^m*A*b*c^2*
m^11*x^11 + 9119154*(d*x)^m*B*b*c^2*m^10*x^12 + 121118283*(d*x)^m*C*b*c^2*m
^9*x^13 + 40372761*(d*x)^m*A*c^3*m^9*x^13 + 385081268*(d*x)^m*B*c^3*m^8*x^1
4 + 2681453775*(d*x)^m*C*c^3*m^7*x^15 + 3*(d*x)^m*C*a*b^2*m^14*x^7 + (d*x)^
m*A*b^3*m^14*x^7 + 3*(d*x)^m*C*a^2*c*m^14*x^7 + 6*(d*x)^m*A*a*b*c*m^14*x^7
+ 112*(d*x)^m*B*b^3*m^13*x^8 + 672*(d*x)^m*B*a*b*c*m^13*x^8 + 5581*(d*x)^m*
C*b^3*m^12*x^9 + 33486*(d*x)^m*C*a*b*c*m^12*x^9 + 16743*(d*x)^m*A*b^2*c*m^1
2*x^9 + 16743*(d*x)^m*A*a*c^2*m^12*x^9 + 490800*(d*x)^m*B*b^2*c*m^11*x^10 +
 490800*(d*x)^m*B*a*c^2*m^11*x^10 + 9444969*(d*x)^m*C*b^2*c*m^10*x^11 + 944
4969*(d*x)^m*C*a*c^2*m^10*x^11 + 9444969*(d*x)^m*A*b*c^2*m^10*x^11 + 126245
592*(d*x)^m*B*b*c^2*m^9*x^12 + 1209749541*(d*x)^m*C*b*c^2*m^8*x^13 + 403249
847*(d*x)^m*A*c^3*m^8*x^13 + 2816490248*(d*x)^m*B*c^3*m^7*x^14 + 1440932292
8*(d*x)^m*C*c^3*m^6*x^15 + 3*(d*x)^m*B*a*b^2*m^14*x^6 + 3*(d*x)^m*B*a^2*c*m
^14*x^6 + 339*(d*x)^m*C*a*b^2*m^13*x^7 + 113*(d*x)^m*A*b^3*m^13*x^7 + 339*(
d*x)^m*C*a^2*c*m^13*x^7 + 678*(d*x)^m*A*a*b*c*m^13*x^7 + 5684*(d*x)^m*B*b^3
*m^12*x^8 + 34104*(d*x)^m*B*a*b*c*m^12*x^8 + 168171*(d*x)^m*C*b^3*m^11*x^9
+ 1009026*(d*x)^m*C*a*b*c*m^11*x^9 + 504513*(d*x)^m*A*b^2*c*m^11*x^9 + 5045
13*(d*x)^m*A*a*c^2*m^11*x^9 + 9790866*(d*x)^m*B*b^2*c*m^10*x^10 + 9790866*(
d*x)^m*B*a*c^2*m^10*x^10 + 131780781*(d*x)^m*C*b^2*c*m^9*x^11 + 131780781*(
d*x)^m*C*a*c^2*m^9*x^11 + 131780781*(d*x)^m*A*b*c^2*m^9*x^11 + 1269340116*(
d*x)^m*B*b*c^2*m^8*x^12 + 8896139967*(d*x)^m*C*b*c^2*m^7*x^13 + 2965379989*
(d*x)^m*A*c^3*m^7*x^13 + 15200266081*(d*x)^m*B*c^3*m^6*x^14 + 56663366760*(
d*x)^m*C*c^3*m^5*x^15 + 3*(d*x)^m*C*a^2*b*m^14*x^5 + 3*(d*x)^m*A*a*b^2*m^14

x^5 + 3(d*x)^m*A*a^2*c*m^14*x^5 + 342*(d*x)^m*B*a*b^2*m^13*x^6 + 342*(d*x
)^m*B*a^2*c*m^13*x^6 + 17367*(d*x)^m*C*a*b^2*m^12*x^7 + 5789*(d*x)^m*A*b^3*
m^12*x^7 + 17367*(d*x)^m*C*a^2*c*m^12*x^7 + 34734*(d*x)^m*A*a*b*c*m^12*x^7
+ 172928*(d*x)^m*B*b^3*m^11*x^8 + 1037568*(d*x)^m*B*a*b*c*m^11*x^8 + 338608
3*(d*x)^m*C*b^3*m^10*x^9 + 20316498*(d*x)^m*C*a*b*c*m^10*x^9 + 10158249*(d*
x)^m*A*b^2*c*m^10*x^9 + 10158249*(d*x)^m*A*a*c^2*m^10*x^9 + 137766780*(d*x)
^m*B*b^2*c*m^9*x^10 + 137766780*(d*x)^m*B*a*c^2*m^9*x^10 + 1334698629*(d*x)
^m*C*b^2*c*m^8*x^11 + 1334698629*(d*x)^m*C*a*c^2*m^8*x^11 + 1334698629*(d*x
)^m*A*b*c^2*m^8*x^11 + 9390802608*(d*x)^m*B*b*c^2*m^7*x^12 + 48243569088*(d
*x)^m*C*b*c^2*m^6*x^13 + 16081189696*(d*x)^m*A*c^3*m^6*x^13 + 59999485546*(
d*x)^m*B*c^3*m^5*x^14 + 159721605680*(d*x)^m*C*c^3*m^4*x^15 + 3*(d*x)^m*B*a
^2*b*m^14*x^4 + 345*(d*x)^m*C*a^2*b*m^13*x^5 + 345*(d*x)^m*A*a*b^2*m^13*x^5
 + 345*(d*x)^m*A*a^2*c*m^13*x^5 + 17688*(d*x)^m*B*a*b^2*m^12*x^6 + 17688*(d
*x)^m*B*a^2*c*m^12*x^6 + 533631*(d*x)^m*C*a*b^2*m^11*x^7 + 177877*(d*x)^m*A
*b^3*m^11*x^7 + 533631*(d*x)^m*C*a^2*c*m^11*x^7 + 1067262*(d*x)^m*A*a*b*c*m
^11*x^7 + 3516198*(d*x)^m*B*b^3*m^10*x^8 + 21097188*(d*x)^m*B*a*b*c*m^10*x^
8 + 48083733*(d*x)^m*C*b^3*m^9*x^9 + 288502398*(d*x)^m*C*a*b*c*m^9*x^9 + 14
4251199*(d*x)^m*A*b^2*c*m^9*x^9 + 144251199*(d*x)^m*A*a*c^2*m^9*x^9 + 14066
19420*(d*x)^m*B*b^2*c*m^8*x^10 + 1406619420*(d*x)^m*B*a*c^2*m^8*x^10 + 9941
199081*(d*x)^m*C*b^2*c*m^7*x^11 + 9941199081*(d*x)^m*C*a*c^2*m^7*x^11 + 994
1199081*(d*x)^m*A*b*c^2*m^7*x^11 + 51203757363*(d*x)^m*B*b*c^2*m^6*x^12 + 1
91243233896*(d*x)^m*C*b*c^2*m^5*x^13 + 63747744632*(d*x)^m*A*c^3*m^5*x^13 +
 169679309436*(d*x)^m*B*c^3*m^4*x^14 + 310989260400*(d*x)^m*C*c^3*m^3*x^15
+ (d*x)^m*C*a^3*m^14*x^3 + 3*(d*x)^m*A*a^2*b*m^14*x^3 + 348*(d*x)^m*B*a^2*b
*m^13*x^4 + 18015*(d*x)^m*C*a^2*b*m^12*x^5 + 18015*(d*x)^m*A*a*b^2*m^12*x^5
 + 18015*(d*x)^m*A*a^2*c*m^12*x^5 + 549072*(d*x)^m*B*a*b^2*m^11*x^6 + 54907
2*(d*x)^m*B*a^2*c*m^11*x^6 + 10963449*(d*x)^m*C*a*b^2*m^10*x^7 + 3654483*(d
*x)^m*A*b^3*m^10*x^7 + 10963449*(d*x)^m*C*a^2*c*m^10*x^7 + 21926898*(d*x)^m
*A*a*b*c*m^10*x^7 + 50428896*(d*x)^m*B*b^3*m^9*x^8 + 302573376*(d*x)^m*B*a*
b*c*m^9*x^8 + 495342143*(d*x)^m*C*b^3*m^8*x^9 + 2972052858*(d*x)^m*C*a*b*c*
m^8*x^9 + 1486026429*(d*x)^m*A*b^2*c*m^8*x^9 + 1486026429*(d*x)^m*A*a*c^2*m
^8*x^9 + 10556689800*(d*x)^m*B*b^2*c*m^7*x^10 + 10556689800*(d*x)^m*B*a*c^2
*m^7*x^10 + 54540198768*(d*x)^m*C*b^2*c*m^6*x^11 + 54540198768*(d*x)^m*C*a*
c^2*m^6*x^11 + 54540198768*(d*x)^m*A*b*c^2*m^6*x^11 + 203964543684*(d*x)^m*
B*b*c^2*m^5*x^12 + 542854280592*(d*x)^m*C*b*c^2*m^4*x^13 + 180951426864*(d*
x)^m*A*c^3*m^4*x^13 + 331303013496*(d*x)^m*B*c^3*m^3*x^14 + 392156797824*(d
*x)^m*C*c^3*m^2*x^15 + (d*x)^m*B*a^3*m^14*x^2 + 117*(d*x)^m*C*a^3*m^13*x^3
+ 351*(d*x)^m*A*a^2*b*m^13*x^3 + 18348*(d*x)^m*B*a^2*b*m^12*x^4 + 565125*(d
*x)^m*C*a^2*b*m^11*x^5 + 565125*(d*x)^m*A*a*b^2*m^11*x^5 + 565125*(d*x)^m*A
*a^2*c*m^11*x^5 + 11404434*(d*x)^m*B*a*b^2*m^10*x^6 + 11404434*(d*x)^m*B*a^
2*c*m^10*x^6 + 158931297*(d*x)^m*C*a*b^2*m^9*x^7 + 52977099*(d*x)^m*A*b^3*m
^9*x^7 + 158931297*(d*x)^m*C*a^2*c*m^9*x^7 + 317862594*(d*x)^m*A*a*b*c*m^9*
x^7 + 524664572*(d*x)^m*B*b^3*m^8*x^8 + 3147987432*(d*x)^m*B*a*b*c*m^8*x^8
+ 3749548713*(d*x)^m*C*b^3*m^7*x^9 + 22497292278*(d*x)^m*C*a*b*c*m^7*x^9 +
11248646139*(d*x)^m*A*b^2*c*m^7*x^9 + 11248646139*(d*x)^m*A*a*c^2*m^7*x^9 +

58326490659*(d*x)^m*B*b^2*c*m^6*x^10 + 58326490659*(d*x)^m*B*a*c^2*m^6*x^1
0 + 218467445592*(d*x)^m*C*b^2*c*m^5*x^11 + 218467445592*(d*x)^m*C*a*c^2*m^
5*x^11 + 218467445592*(d*x)^m*A*b*c^2*m^5*x^11 + 581441797032*(d*x)^m*B*b*c
^2*m^4*x^12 + 1063334389104*(d*x)^m*C*b*c^2*m^3*x^13 + 354444796368*(d*x)^m
*A*c^3*m^3*x^13 + 418753514880*(d*x)^m*B*c^3*m^2*x^14 + 283465647360*(d*x)^
m*C*c^3*m*x^15 + (d*x)^m*A*a^3*m^14*x + 118*(d*x)^m*B*a^3*m^13*x^2 + 6229*(
d*x)^m*C*a^3*m^12*x^3 + 18687*(d*x)^m*A*a^2*b*m^12*x^3 + 581808*(d*x)^m*B*a
^2*b*m^11*x^4 + 11873241*(d*x)^m*C*a^2*b*m^10*x^5 + 11873241*(d*x)^m*A*a*b^
2*m^10*x^5 + 11873241*(d*x)^m*A*a^2*c*m^10*x^5 + 167248836*(d*x)^m*B*a*b^2*
m^9*x^6 + 167248836*(d*x)^m*B*a^2*c*m^9*x^6 + 1671768141*(d*x)^m*C*a*b^2*m^
8*x^7 + 557256047*(d*x)^m*A*b^3*m^8*x^7 + 1671768141*(d*x)^m*C*a^2*c*m^8*x^
7 + 3343536282*(d*x)^m*A*a*b*c*m^8*x^7 + 4010311424*(d*x)^m*B*b^3*m^7*x^8 +
24061868544*(d*x)^m*B*a*b*c*m^7*x^8 + 20885191136*(d*x)^m*C*b^3*m^6*x^9 +
125311146816*(d*x)^m*C*a*b*c*m^6*x^9 + 62655573408*(d*x)^m*A*b^2*c*m^6*x^9
+ 62655573408*(d*x)^m*A*a*c^2*m^6*x^9 + 235144725450*(d*x)^m*B*b^2*c*m^5*x^
10 + 235144725450*(d*x)^m*B*a*c^2*m^5*x^10 + 625874419728*(d*x)^m*C*b^2*c*m
^4*x^11 + 625874419728*(d*x)^m*C*a*c^2*m^4*x^11 + 625874419728*(d*x)^m*A*b*
c^2*m^4*x^11 + 1143138472416*(d*x)^m*B*b*c^2*m^3*x^12 + 1347640053120*(d*x)
^m*C*b*c^2*m^2*x^13 + 449213351040*(d*x)^m*A*c^3*m^2*x^13 + 303268406400*(d
*x)^m*B*c^3*m*x^14 + 87178291200*(d*x)^m*C*c^3*x^15 + 119*(d*x)^m*A*a^3*m^1
3*x + 6344*(d*x)^m*B*a^3*m^12*x^2 + 199713*(d*x)^m*C*a^3*m^11*x^3 + 599139*
(d*x)^m*A*a^2*b*m^11*x^3 + 12371634*(d*x)^m*B*a^2*b*m^10*x^4 + 176309235*(d
*x)^m*C*a^2*b*m^9*x^5 + 176309235*(d*x)^m*A*a*b^2*m^9*x^5 + 176309235*(d*x)
^m*A*a^2*c*m^9*x^5 + 1780794204*(d*x)^m*B*a*b^2*m^8*x^6 + 1780794204*(d*x)^
m*B*a^2*c*m^8*x^6 + 12920507013*(d*x)^m*C*a*b^2*m^7*x^7 + 4306835671*(d*x)^
m*A*b^3*m^7*x^7 + 12920507013*(d*x)^m*C*a^2*c*m^7*x^7 + 25841014026*(d*x)^m
*A*a*b*c*m^7*x^7 + 22548638161*(d*x)^m*B*b^3*m^6*x^8 + 135291828966*(d*x)^m
*B*a*b*c*m^6*x^8 + 84836490456*(d*x)^m*C*b^3*m^5*x^9 + 509018942736*(d*x)^m
*C*a*b*c*m^5*x^9 + 254509471368*(d*x)^m*A*b^2*c*m^5*x^9 + 254509471368*(d*x
)^m*A*a*c^2*m^5*x^9 + 677569066740*(d*x)^m*B*b^2*c*m^4*x^10 + 677569066740*
(d*x)^m*B*a*c^2*m^4*x^10 + 1235821419792*(d*x)^m*C*b^2*c*m^3*x^11 + 1235821
419792*(d*x)^m*C*a*c^2*m^3*x^11 + 1235821419792*(d*x)^m*A*b*c^2*m^3*x^11 +
1453325442480*(d*x)^m*B*b*c^2*m^2*x^12 + 978132153600*(d*x)^m*C*b*c^2*m*x^1
3 + 326044051200*(d*x)^m*A*c^3*m*x^13 + 93405312000*(d*x)^m*B*c^3*x^14 + 64
61*(d*x)^m*A*a^3*m^12*x + 205712*(d*x)^m*B*a^3*m^11*x^2 + 4300483*(d*x)^m*C
*a^3*m^10*x^3 + 12901449*(d*x)^m*A*a^2*b*m^10*x^3 + 186188904*(d*x)^m*B*a^2
*b*m^9*x^4 + 1902741045*(d*x)^m*C*a^2*b*m^8*x^5 + 1902741045*(d*x)^m*A*a*b^
2*m^8*x^5 + 1902741045*(d*x)^m*A*a^2*c*m^8*x^5 + 13938118776*(d*x)^m*B*a*b^
2*m^7*x^6 + 13938118776*(d*x)^m*B*a^2*c*m^7*x^6 + 73449839568*(d*x)^m*C*a*b
^2*m^6*x^7 + 24483279856*(d*x)^m*A*b^3*m^6*x^7 + 73449839568*(d*x)^m*C*a^2*
c*m^6*x^7 + 146899679136*(d*x)^m*A*a*b*c*m^6*x^7 + 92414105392*(d*x)^m*B*b^
3*m^5*x^8 + 554484632352*(d*x)^m*B*a*b*c*m^5*x^8 + 246143692976*(d*x)^m*C*b
^3*m^4*x^9 + 1476862157856*(d*x)^m*C*a*b*c*m^4*x^9 + 738431078928*(d*x)^m*A
*b^2*c*m^4*x^9 + 738431078928*(d*x)^m*A*a*c^2*m^4*x^9 + 1344749369400*(d*x)
^m*B*b^2*c*m^3*x^10 + 1344749369400*(d*x)^m*B*a*c^2*m^3*x^10 + 157695149376

0*(d*x)^m*C*b^2*c*m^2*x^11 + 1576951493760*(d*x)^m*C*a*c^2*m^2*x^11 + 15769
51493760*(d*x)^m*A*b*c^2*m^2*x^11 + 1057547534400*(d*x)^m*B*b*c^2*m*x^12 +
301771008000*(d*x)^m*C*b*c^2*x^13 + 100590336000*(d*x)^m*A*c^3*x^13 + 21193
9*(d*x)^m*A*a^3*m^11*x + 4488198*(d*x)^m*B*a^3*m^10*x^2 + 65657031*(d*x)^m*
C*a^3*m^9*x^3 + 196971093*(d*x)^m*A*a^2*b*m^9*x^3 + 2039531604*(d*x)^m*B*a^
2*b*m^8*x^4 + 15109178775*(d*x)^m*C*a^2*b*m^7*x^5 + 15109178775*(d*x)^m*A*a
*b^2*m^7*x^5 + 15109178775*(d*x)^m*A*a^2*c*m^7*x^5 + 80264676003*(d*x)^m*B*
a*b^2*m^6*x^6 + 80264676003*(d*x)^m*B*a^2*c*m^6*x^6 + 304260755064*(d*x)^m*
C*a*b^2*m^5*x^7 + 101420251688*(d*x)^m*A*b^3*m^5*x^7 + 304260755064*(d*x)^m
*C*a^2*c*m^5*x^7 + 608521510128*(d*x)^m*A*a*b*c*m^5*x^7 + 270359263944*(d*x
)^m*B*b^3*m^4*x^8 + 1622155583664*(d*x)^m*B*a*b*c*m^4*x^8 + 491520108816*(d
*x)^m*C*b^3*m^3*x^9 + 2949120652896*(d*x)^m*C*a*b*c*m^3*x^9 + 1474560326448
*(d*x)^m*A*b^2*c*m^3*x^9 + 1474560326448*(d*x)^m*A*a*c^2*m^3*x^9 + 17234934
17472*(d*x)^m*B*b^2*c*m^2*x^10 + 1723493417472*(d*x)^m*B*a*c^2*m^2*x^10 + 1
150986412800*(d*x)^m*C*b^2*c*m*x^11 + 1150986412800*(d*x)^m*C*a*c^2*m*x^11
+ 1150986412800*(d*x)^m*A*b*c^2*m*x^11 + 326918592000*(d*x)^m*B*b*c^2*x^12
+ 4687683*(d*x)^m*A*a^3*m^10*x + 69582084*(d*x)^m*B*a^3*m^9*x^2 + 731124647
*(d*x)^m*C*a^3*m^8*x^3 + 2193373941*(d*x)^m*A*a^2*b*m^8*x^3 + 16464757584*(
d*x)^m*B*a^2*b*m^7*x^4 + 88347494784*(d*x)^m*C*a^2*b*m^6*x^5 + 88347494784*
(d*x)^m*A*a*b^2*m^6*x^5 + 88347494784*(d*x)^m*A*a^2*c*m^6*x^5 + 33682157602
2*(d*x)^m*B*a*b^2*m^5*x^6 + 336821576022*(d*x)^m*B*a^2*c*m^5*x^6 + 89919103
5792*(d*x)^m*C*a*b^2*m^4*x^7 + 299730345264*(d*x)^m*A*b^3*m^4*x^7 + 8991910
35792*(d*x)^m*C*a^2*c*m^4*x^7 + 1798382071584*(d*x)^m*A*a*b*c*m^4*x^7 + 543
939234048*(d*x)^m*B*b^3*m^3*x^8 + 3263635404288*(d*x)^m*B*a*b*c*m^3*x^8 + 6
33314724480*(d*x)^m*C*b^3*m^2*x^9 + 3799888346880*(d*x)^m*C*a*b*c*m^2*x^9 +
 1899944173440*(d*x)^m*A*b^2*c*m^2*x^9 + 1899944173440*(d*x)^m*A*a*c^2*m^2*
x^9 + 1262518669440*(d*x)^m*B*b^2*c*m*x^10 + 1262518669440*(d*x)^m*B*a*c^2*
m*x^10 + 356638464000*(d*x)^m*C*b^2*c*x^11 + 356638464000*(d*x)^m*C*a*c^2*x
^11 + 356638464000*(d*x)^m*A*b*c^2*x^11 + 73870797*(d*x)^m*A*a^3*m^9*x + 78
8931572*(d*x)^m*B*a^3*m^8*x^2 + 6014254059*(d*x)^m*C*a^3*m^7*x^3 + 18042762
177*(d*x)^m*A*a^2*b*m^7*x^3 + 98034358323*(d*x)^m*B*a^2*b*m^6*x^4 + 3766721
58120*(d*x)^m*C*a^2*b*m^5*x^5 + 376672158120*(d*x)^m*A*a*b^2*m^5*x^5 + 3766
72158120*(d*x)^m*A*a^2*c*m^5*x^5 + 1008086865108*(d*x)^m*B*a*b^2*m^4*x^6 +
1008086865108*(d*x)^m*B*a^2*c*m^4*x^6 + 1826102786256*(d*x)^m*C*a*b^2*m^3*x
^7 + 608700928752*(d*x)^m*A*b^3*m^3*x^7 + 1826102786256*(d*x)^m*C*a^2*c*m^3
x^7 + 3652205572512(d*x)^m*A*a*b*c*m^3*x^7 + 705481831440*(d*x)^m*B*b^3*m
^2*x^8 + 4232890988640*(d*x)^m*B*a*b*c*m^2*x^8 + 465985094400*(d*x)^m*C*b^3
*m*x^9 + 2795910566400*(d*x)^m*C*a*b*c*m*x^9 + 1397955283200*(d*x)^m*A*b^2*
c*m*x^9 + 1397955283200*(d*x)^m*A*a*c^2*m*x^9 + 392302310400*(d*x)^m*B*b^2*
c*x^10 + 392302310400*(d*x)^m*B*a*c^2*x^10 + 854224943*(d*x)^m*A*a^3*m^8*x
+ 6629764856*(d*x)^m*B*a^3*m^7*x^2 + 36588367376*(d*x)^m*C*a^3*m^6*x^3 + 10
9765102128*(d*x)^m*A*a^2*b*m^6*x^3 + 426272198748*(d*x)^m*B*a^2*b*m^5*x^4 +
 1145655530640*(d*x)^m*C*a^2*b*m^4*x^5 + 1145655530640*(d*x)^m*A*a*b^2*m^4*
x^5 + 1145655530640*(d*x)^m*A*a^2*c*m^4*x^5 + 2071918846152*(d*x)^m*B*a*b^2
*m^3*x^6 + 2071918846152*(d*x)^m*B*a^2*c*m^3*x^6 + 2388267607680*(d*x)^m*C*

```
a*b^2*m^2*x^7 + 796089202560*(d*x)^m*A*b^3*m^2*x^7 + 2388267607680*(d*x)^m*
C*a^2*c*m^2*x^7 + 4776535215360*(d*x)^m*A*a*b*c*m^2*x^7 + 521962963200*(d*x
)^m*B*b^3*m*x^8 + 3131777779200*(d*x)^m*B*a*b*c*m*x^8 + 145297152000*(d*x)^
m*C*b^3*x^9 + 871782912000*(d*x)^m*C*a*b*c*x^9 + 435891456000*(d*x)^m*A*b^2
*c*x^9 + 435891456000*(d*x)^m*A*a*c^2*x^9 + 7353403057*(d*x)^m*A*a^3*m^7*x
+ 41371599841*(d*x)^m*B*a^3*m^6*x^2 + 163038108552*(d*x)^m*C*a^3*m^5*x^3 +
489114325656*(d*x)^m*A*a^2*b*m^5*x^3 + 1323927526248*(d*x)^m*B*a^2*b*m^4*x^
4 + 2392162383600*(d*x)^m*C*a^2*b*m^3*x^5 + 2392162383600*(d*x)^m*A*a*b^2*m
^3*x^5 + 2392162383600*(d*x)^m*A*a^2*c*m^3*x^5 + 2739474034560*(d*x)^m*B*a*
b^2*m^2*x^6 + 2739474034560*(d*x)^m*B*a^2*c*m^2*x^6 + 1779579590400*(d*x)^m
*C*a*b^2*m*x^7 + 593193196800*(d*x)^m*A*b^3*m*x^7 + 1779579590400*(d*x)^m*C
*a^2*c*m*x^7 + 3559159180800*(d*x)^m*A*a*b*c*m*x^7 + 163459296000*(d*x)^m*B
*b^3*x^8 + 980755776000*(d*x)^m*B*a*b*c*x^8 + 47277726496*(d*x)^m*A*a^3*m^6
*x + 190060010998*(d*x)^m*B*a^3*m^5*x^2 + 520557781424*(d*x)^m*C*a^3*m^4*x^
3 + 1561673344272*(d*x)^m*A*a^2*b*m^4*x^3 + 2824729931808*(d*x)^m*B*a^2*b*m
^3*x^4 + 3210175193472*(d*x)^m*C*a^2*b*m^2*x^5 + 3210175193472*(d*x)^m*A*a*
b^2*m^2*x^5 + 3210175193472*(d*x)^m*A*a^2*c*m^2*x^5 + 2060608636800*(d*x)^m
*B*a*b^2*m*x^6 + 2060608636800*(d*x)^m*B*a^2*c*m*x^6 + 560431872000*(d*x)^m
*C*a*b^2*x^7 + 186810624000*(d*x)^m*A*b^3*x^7 + 560431872000*(d*x)^m*C*a^2*
c*x^7 + 1120863744000*(d*x)^m*A*a*b*c*x^7 + 225525484184*(d*x)^m*A*a^3*m^5*
x + 629552085084*(d*x)^m*B*a^3*m^4*x^2 + 1145140001328*(d*x)^m*C*a^3*m^3*x^
3 + 3435420003984*(d*x)^m*A*a^2*b*m^3*x^3 + 3872067384240*(d*x)^m*B*a^2*b*m
^2*x^4 + 2446576876800*(d*x)^m*C*a^2*b*m*x^5 + 2446576876800*(d*x)^m*A*a*b^
2*m*x^5 + 2446576876800*(d*x)^m*A*a^2*c*m*x^5 + 653837184000*(d*x)^m*B*a*b^
2*x^6 + 653837184000*(d*x)^m*B*a^2*c*x^6 + 784146622896*(d*x)^m*A*a^3*m^4*x
 + 1447709175432*(d*x)^m*B*a^3*m^3*x^2 + 1621575699840*(d*x)^m*C*a^3*m^2*x^
3 + 4864727099520*(d*x)^m*A*a^2*b*m^2*x^3 + 3009183307200*(d*x)^m*B*a^2*b*m
*x^4 + 784604620800*(d*x)^m*C*a^2*b*x^5 + 784604620800*(d*x)^m*A*a*b^2*x^5
+ 784604620800*(d*x)^m*A*a^2*c*x^5 + 1922666722704*(d*x)^m*A*a^3*m^3*x + 21
61577352960*(d*x)^m*B*a^3*m^2*x^2 + 1301090515200*(d*x)^m*C*a^3*m*x^3 + 390
3271545600*(d*x)^m*A*a^2*b*m*x^3 + 980755776000*(d*x)^m*B*a^2*b*x^4 + 31343
28981120*(d*x)^m*A*a^3*m^2*x + 1842662908800*(d*x)^m*B*a^3*m*x^2 + 43589145
6000*(d*x)^m*C*a^3*x^3 + 1307674368000*(d*x)^m*A*a^2*b*x^3 + 3031488633600*
(d*x)^m*A*a^3*m*x + 653837184000*(d*x)^m*B*a^3*x^2 + 1307674368000*(d*x)^m*
A*a^3*x)/(m^15 + 120*m^14 + 6580*m^13 + 218400*m^12 + 4899622*m^11 + 785584
80*m^10 + 928095740*m^9 + 8207628000*m^8 + 54631129553*m^7 + 272803210680*m
^6 + 1009672107080*m^5 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614
720*m^2 + 4339163001600*m + 1307674368000)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 2443, normalized size of antiderivative = 6.12

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 dx = \text{Too large to display}$$

[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x)

[Out] (x^7*(d*x)^m*(A*b^3 + 3*C*a*b^2 + 3*C*a^2*c + 6*A*a*b*c)*(593193196800*m + 796089202560*m^2 + 608700928752*m^3 + 299730345264*m^4 + 101420251688*m^5 + 24483279856*m^6 + 4306835671*m^7 + 557256047*m^8 + 52977099*m^9 + 3654483*m^10 + 177877*m^11 + 5789*m^12 + 113*m^13 + m^14 + 186810624000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (x^9*(d*x)^m*(C*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*C*a*b*c)*(465985094400*m + 633314724480*m^2 + 491520108816*m^3 + 246143692976*m^4 + 84836490456*m^5 + 20885191136*m^6 + 3749548713*m^7 + 495342143*m^8 + 48083733*m^9 + 3386083*m^10 + 168171*m^11 + 5581*m^12 + 111*m^13 + m^14 + 145297152000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (B*c^3*x^14*(d*x)^m*(303268406400*m + 418753514880*m^2 + 331303013496*m^3 + 169679309436*m^4 + 59999485546*m^5 + 15200266081*m^6 + 2816490248*m^7 + 385081268*m^8 + 38786748*m^9 + 2840838*m^10 + 147056*m^11 + 5096*m^12 + 106*m^13 + m^14 + 93405312000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (B*a^3*x^2*(d*x)^m*(1842662908800*m + 2161577352960*m^2 + 1447709175432*m^3 + 629552085084*m^4 + 190060010998*m^5 + 41371599841*m^6 + 6629764856*m^7 + 788931572*m^8 + 69582084*m^9 + 4488198*m^10 + 205712*m^11 + 6344*m^12 + 118*m^13 + m^14 + 653837184000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (3*a*x^5*(d*x)^m*(A*b^2 + A*a*c + C*a*b)*(815525625600*m + 1070058397824*m^2 + 797387461200*m^3 + 381885176880*m^4 + 125557386040*m^5 + 29449164928*m^6 + 5036392925*m^7 + 634247015*m^8 + 58769745*m^9 + 3957747*m^10 + 188375*m^11 + 6005*m^12 + 115*m^13 + m^14 + 261534873600))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (3*c*x^11*(d*x)^m*(C*b^2 + A*b *c + C*a*c)*(383662137600*m + 525650497920*m^2 + 411940473264*m^3 + 2086248 06576*m^4 + 72822481864*m^5 + 18180066256*m^6 + 3313733027*m^7 + 444899543* m^8 + 43926927*m^9 + 3148323*m^10 + 159209*m^11 + 5381*m^12 + 109*m^13 + m^ 14 + 118879488000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^ 3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553* m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 21840 0*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (a^2*x^3*(d*x)^m*(3 *A*b + C*a)*(1301090515200*m + 1621575699840*m^2 + 1145140001328*m^3 + 5205 57781424*m^4 + 163038108552*m^5 + 36588367376*m^6 + 6014254059*m^7 + 731124 647*m^8 + 65657031*m^9 + 4300483*m^10 + 199713*m^11 + 6229*m^12 + 117*m^13 + m^14 + 435891456000))/(4339163001600*m + 6165817614720*m^2 + 505699570382 4*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129 553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 2 18400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (c^2*x^13*(d*x) ^m*(A*c + 3*C*b)*(326044051200*m + 449213351040*m^2 + 354444796368*m^3 + 18 0951426864*m^4 + 63747744632*m^5 + 16081189696*m^6 + 2965379989*m^7 + 40324 9847*m^8 + 40372761*m^9 + 2937363*m^10 + 150943*m^11 + 5189*m^12 + 107*m^13 + m^14 + 100590336000))/(4339163001600*m + 6165817614720*m^2 + 50569957038 24*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 5463112 9553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (A*a^3*x*(d*x) ^m*(3031488633600*m + 3134328981120*m^2 + 1922666722704*m^3 + 784146622896* m^4 + 225525484184*m^5 + 47277726496*m^6 + 7353403057*m^7 + 854224943*m^8 + 73870797*m^9 + 4687683*m^10 + 211939*m^11 + 6461*m^12 + 119*m^13 + m^14 + 1307674368000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^ 12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (C*c^3*x^15*(d*x)^m*(28 3465647360*m + 392156797824*m^2 + 310989260400*m^3 + 159721605680*m^4 + 566 63366760*m^5 + 14409322928*m^6 + 2681453775*m^7 + 368411615*m^8 + 37312275* m^9 + 2749747*m^10 + 143325*m^11 + 5005*m^12 + 105*m^13 + m^14 + 8717829120 0))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 270681334560 0*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000 *m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^ 13 + 120*m^14 + m^15 + 1307674368000) + (3*B*c*x^10*(d*x)^m*(a*c + b^2)*(42 0839556480*m + 574497805824*m^2 + 448249789800*m^3 + 225856355580*m^4 + 783 81575150*m^5 + 19442163553*m^6 + 3518896600*m^7 + 468873140*m^8 + 45922260* m^9 + 3263622*m^10 + 163600*m^11 + 5480*m^12 + 110*m^13 + m^14 + 1307674368 00))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 27068133456 00*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 820762800 0*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m ^13 + 120*m^14 + m^15 + 1307674368000) + (3*B*a*x^6*(d*x)^m*(a*c + b^2)*(68 6869545600*m + 913158011520*m^2 + 690639615384*m^3 + 336028955036*m^4 + 112 273858674*m^5 + 26754892001*m^6 + 4646039592*m^7 + 593598068*m^8 + 55749612 *m^9 + 3801478*m^10 + 183024*m^11 + 5896*m^12 + 114*m^13 + m^14 + 217945728

```
000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345
600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 82076280
00*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*
m^13 + 120*m^14 + m^15 + 1307674368000) + (3*B*b*c^2*x^12*(d*x)^m*(35251584
4800*m + 484441814160*m^2 + 381046157472*m^3 + 193813932344*m^4 + 679881812
28*m^5 + 17067919121*m^6 + 3130267536*m^7 + 423113372*m^8 + 42081864*m^9 +
3039718*m^10 + 154992*m^11 + 5284*m^12 + 108*m^13 + m^14 + 108972864000))/(
4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4
 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8
+ 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 +
120*m^14 + m^15 + 1307674368000) + (B*b*x^8*(d*x)^m*(6*a*c + b^2)*(52196296
3200*m + 705481831440*m^2 + 543939234048*m^3 + 270359263944*m^4 + 924141053
92*m^5 + 22548638161*m^6 + 4010311424*m^7 + 524664572*m^8 + 50428896*m^9 +
3516198*m^10 + 172928*m^11 + 5684*m^12 + 112*m^13 + m^14 + 163459296000))/(
4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4
 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8
+ 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 +
120*m^14 + m^15 + 1307674368000) + (3*B*a^2*b*x^4*(d*x)^m*(1003061102400*m
+ 1290689128080*m^2 + 941576643936*m^3 + 441309175416*m^4 + 142090732916*m^
5 + 32678119441*m^6 + 5488252528*m^7 + 679843868*m^8 + 62062968*m^9 + 41238
78*m^10 + 193936*m^11 + 6116*m^12 + 116*m^13 + m^14 + 326918592000))/(43391
63001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 10
09672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928
095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m
^14 + m^15 + 1307674368000)
```

3.38 $\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 30, antiderivative size = 260

$$\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)^2\,dx = \frac{a^2A(dx)^{1+m}}{d(1+m)}+\frac{a^2B(dx)^{2+m}}{d^2(2+m)}+\frac{a(2Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{2abB(dx)^{4+m}}{d^4(4+m)}+\frac{(A(b^2+2ac)+2abC)(dx)^{5+m}}{d^5(5+m)}+\frac{B(b^2+2ac)(dx)^{6+m}}{d^6(6+m)}+\frac{(2Abc+(b^2+2ac)C)(dx)^{7+m}}{d^7(7+m)}+\frac{2bBc(dx)^{8+m}}{d^8(8+m)}+\frac{c(Ac+2bC)(dx)^{9+m}}{d^9(9+m)}+\frac{Bc^2(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2C(dx)^{11+m}}{d^{11}(11+m)}$$

```
[Out] a^2*A*(d*x)^(1+m)/d/(1+m)+a^2*B*(d*x)^(2+m)/d^2/(2+m)+a*(2*A*b+C*a)*(d*x)^(
3+m)/d^3/(3+m)+2*a*b*B*(d*x)^(4+m)/d^4/(4+m)+(A*(2*a*c+b^2)+2*a*b*C)*(d*x)^
(5+m)/d^5/(5+m)+B*(2*a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(2*A*b*c+(2*a*c+b^2)*C)
*(d*x)^(7+m)/d^7/(7+m)+2*b*B*c*(d*x)^(8+m)/d^8/(8+m)+c*(A*c+2*C*b)*(d*x)^(9
+m)/d^9/(9+m)+B*c^2*(d*x)^(10+m)/d^10/(10+m)+c^2*C*(d*x)^(11+m)/d^11/(11+m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used

$= \{1642\}$

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$$
$$= \frac{a^2A(dx)^{m+1}}{d(m+1)} + \frac{a^2B(dx)^{m+2}}{d^2(m+2)} + \frac{(dx)^{m+7}\left(C(2ac+b^2)+2Abc\right)}{d^7(m+7)} + \frac{(dx)^{m+5}\left(A(2ac+b^2)+2abC\right)}{d^5(m+5)} + \frac{a(dx)^{m+3}(aC+2Ab)}{d^3(m+3)} + \frac{B(2ac+b^2)(dx)^{m+6}}{d^6(m+6)} + \frac{2abB(dx)^{m+4}}{d^4(m+4)} + \frac{c(dx)^{m+9}(Ac+2bC)}{d^9(m+9)} + \frac{2bBc(dx)^{m+8}}{d^8(m+8)} + \frac{Bc^2(dx)^{m+10}}{d^{10}(m+10)} + \frac{c^2C(dx)^{m+11}}{d^{11}(m+11)}$$

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B*(d*x)^(4 + m))/(d^4
*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*
(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*C)*(
d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) + (c*(A
*c + 2*b*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(a^2A(dx)^m + \frac{a^2B(dx)^{1+m}}{d} + \frac{a(2Ab+aC)(dx)^{2+m}}{d^2} + \frac{2abB(dx)^{3+m}}{d^3} + \frac{\left(A(b^2+2ac)+2abC\right)(dx)^{4+m}}{d^4} + \frac{B(b^2+2ac)(dx)^{5+m}}{d^5} + \frac{\left(2Abc+(b^2+2ac)C\right)(dx)^{6+m}}{d^6} + \frac{2bBc(dx)^{7+m}}{d^7} + \frac{c(Ac+2bC)(dx)^{8+m}}{d^8} + \frac{Bc^2(dx)^{9+m}}{d^9} + \frac{c^2C(dx)^{10+m}}{d^{10}}\right) dx$$
$$= \frac{a^2A(dx)^{1+m}}{d(1+m)} + \frac{a^2B(dx)^{2+m}}{d^2(2+m)} + \frac{a(2Ab+aC)(dx)^{3+m}}{d^3(3+m)} + \frac{2abB(dx)^{4+m}}{d^4(4+m)} + \frac{\left(A(b^2+2ac)+2abC\right)(dx)^{5+m}}{d^5(5+m)} + \frac{B(b^2+2ac)(dx)^{6+m}}{d^6(6+m)} + \frac{\left(2Abc+(b^2+2ac)C\right)(dx)^{7+m}}{d^7(7+m)} + \frac{2bBc(dx)^{8+m}}{d^8(8+m)} + \frac{c(Ac+2bC)(dx)^{9+m}}{d^9(9+m)} + \frac{Bc^2(dx)^{10+m}}{d^{10}(10+m)} + \frac{c^2C(dx)^{11+m}}{d^{11}(11+m)}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4)^2 \, dx$$
$$= x(dx)^m \left(\frac{a^2A}{1+m}+\frac{a^2Bx}{2+m}+\frac{a(2Ab+aC)x^2}{3+m}+\frac{2abBx^3}{4+m}+\frac{(A(b^2+2ac)+2abC)x^4}{5+m}+\frac{B(b^2+2ac)x^5}{6+m}+\frac{(2Abc+(b^2+2ac)C)x^6}{7+m}+\frac{2bBcx^7}{8+m}+\frac{c(Ac+2bC)x^8}{9+m}+\frac{Bc^2x^9}{10+m}+\frac{c^2Cx^{10}}{11+m}\right)$$

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(d*x)^m*((a^2*A)/(1 + m) + (a^2*B*x)/(2 + m) + (a*(2*A*b + a*C)*x^2)/(3 +
 m) + (2*a*b*B*x^3)/(4 + m) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^4)/(5 + m) + (
B*(b^2 + 2*a*c)*x^5)/(6 + m) + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6)/(7 + m) +
(2*b*B*c*x^7)/(8 + m) + (c*(A*c + 2*b*C)*x^8)/(9 + m) + (B*c^2*x^9)/(10 + m
) + (c^2*C*x^10)/(11 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2186 vs. 2(260) = 520.

Time = 0.18 (sec) , antiderivative size = 2187, normalized size of antiderivative = 8.41

method	result	size
gosper	Expression too large to display	2187
risch	Expression too large to display	2187
parallelrisch	Expression too large to display	3204

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(C*c^2*m^10*x^10+B*c^2*m^10*x^9+55*C*c^2*m^9*x^10+A*c^2*m^10*x^8+56*B*c^2
*m^9*x^9+2*C*b*c*m^10*x^8+1320*C*c^2*m^8*x^10+57*A*c^2*m^9*x^8+2*B*b*c*m^10
*x^7+1365*B*c^2*m^8*x^9+114*C*b*c*m^9*x^8+18150*C*c^2*m^7*x^10+2*A*b*c*m^10
*x^6+1412*A*c^2*m^8*x^8+116*B*b*c*m^9*x^7+19020*B*c^2*m^7*x^9+2*C*a*c*m^10*
x^6+C*b^2*m^10*x^6+2824*C*b*c*m^8*x^8+157773*C*c^2*m^6*x^10+118*A*b*c*m^9*x
^6+19962*A*c^2*m^7*x^8+2*B*a*c*m^10*x^5+B*b^2*m^10*x^5+2922*B*b*c*m^8*x^7+1
67223*B*c^2*m^6*x^9+118*C*a*c*m^9*x^6+59*C*b^2*m^9*x^6+39924*C*b*c*m^7*x^8+
902055*C*c^2*m^5*x^10+2*A*a*c*m^10*x^4+A*b^2*m^10*x^4+3024*A*b*c*m^8*x^6+17
7765*A*c^2*m^6*x^8+120*B*a*c*m^9*x^5+60*B*b^2*m^9*x^5+41964*B*b*c*m^7*x^7+9
65328*B*c^2*m^5*x^9+2*C*a*b*m^10*x^4+3024*C*a*c*m^8*x^6+1512*C*b^2*m^8*x^6+
355530*C*b*c*m^6*x^8+3416930*C*c^2*m^4*x^10+122*A*a*c*m^9*x^4+61*A*b^2*m^9*
```

x^4+44172*A*b*c*m^7*x^6+1037673*A*c^2*m^5*x^8+2*B*a*b*m^10*x^3+3130*B*a*c*m^8*x^5+1565*B*b^2*m^8*x^5+379134*B*b*c*m^6*x^7+3686255*B*c^2*m^4*x^9+122*C*a*b*m^9*x^4+44172*C*a*c*m^7*x^6+22086*C*b^2*m^7*x^6+2075346*C*b*c*m^5*x^8+8409500*C*c^2*m^3*x^10+2*A*a*b*m^10*x^2+3240*A*a*c*m^8*x^4+1620*A*b^2*m^8*x^4+405642*A*b*c*m^6*x^6+4000478*A*c^2*m^4*x^8+124*B*a*b*m^9*x^3+46560*B*a*c*m^7*x^5+23280*B*b^2*m^7*x^5+2242044*B*b*c*m^5*x^7+9133180*B*c^2*m^3*x^9+C*a^2*m^10*x^2+3240*C*a*b*m^8*x^4+405642*C*a*c*m^6*x^6+202821*C*b^2*m^6*x^6+8000956*C*b*c*m^4*x^8+12753576*C*c^2*m^2*x^10+126*A*a*b*m^9*x^2+49140*A*a*c*m^7*x^4+24570*A*b^2*m^7*x^4+2435622*A*b*c*m^5*x^6+9991428*A*c^2*m^3*x^8+B*a^2*m^10*x+3354*B*a*b*m^8*x^3+435486*B*a*c*m^6*x^5+217743*B*b^2*m^6*x^5+8742718*B*b*c*m^4*x^7+13926276*B*c^2*m^2*x^9+63*C*a^2*m^9*x^2+49140*C*a*b*m^7*x^4+2435622*C*a*c*m^5*x^6+1217811*C*b^2*m^5*x^6+19982856*C*b*c*m^3*x^8+10628640*C*c^2*m*x^10+A*a^2*m^10+3472*A*a*b*m^8*x^2+469146*A*a*c*m^6*x^4+234573*A*b^2*m^6*x^4+9629716*A*b*c*m^4*x^6+15335224*A*c^2*m^2*x^8+64*B*a^2*m^9*x+51924*B*a*b*m^7*x^3+2662200*B*a*c*m^5*x^5+1331100*B*b^2*m^5*x^5+22049716*B*b*c*m^3*x^7+11655216*B*c^2*m*x^9+1736*C*a^2*m^8*x^2+469146*C*a*b*m^6*x^4+9629716*C*a*c*m^4*x^6+4814858*C*b^2*m^4*x^6+30670448*C*b*c*m^2*x^8+3628800*C*c^2*x^10+65*A*a^2*m^9+54924*A*a*b*m^7*x^2+2929386*A*a*c*m^5*x^4+1464693*A*b^2*m^5*x^4+24583448*A*b*c*m^3*x^6+12900960*A*c^2*m*x^8+1797*B*a^2*m^8*x+507150*B*a*b*m^6*x^3+10705870*B*a*c*m^4*x^5+5352935*B*b^2*m^4*x^5+34118424*B*b*c*m^2*x^7+3991680*B*c^2*x^9+27462*C*a^2*m^7*x^2+2929386*C*a*b*m^5*x^4+24583448*C*a*c*m^3*x^6+12291724*C*b^2*m^3*x^6+25801920*C*b*c*m*x^8+1860*A*a^2*m^8+550074*A*a*b*m^6*x^2+12032140*A*a*c*m^4*x^4+6016070*A*b^2*m^4*x^4+38432016*A*b*c*m^2*x^6+4435200*A*c^2*x^8+29076*B*a^2*m^7*x+3246516*B*a*b*m^5*x^3+27756240*B*a*c*m^3*x^5+13878120*B*b^2*m^3*x^5+28888560*B*b*c*m*x^7+275037*C*a^2*m^6*x^2+12032140*C*a*b*m^4*x^4+38432016*C*a*c*m^2*x^6+19216008*C*b^2*m^2*x^6+8870400*C*b*c*x^8+30810*A*a^2*m^7+3624894*A*a*b*m^5*x^2+31830760*A*a*c*m^3*x^4+15915380*A*b^2*m^3*x^4+32811840*A*b*c*m*x^6+299271*B*a^2*m^6*x+13693006*B*a*b*m^4*x^3+43978712*B*a*c*m^2*x^5+21989356*B*b^2*m^2*x^5+9979200*B*b*c*x^7+1812447*C*a^2*m^5*x^2+31830760*C*a*b*m^3*x^4+32811840*C*a*c*m*x^6+16405920*C*b^2*m*x^6+326613*A*a^2*m^6+15804388*A*a*b*m^4*x^2+51362352*A*a*c*m^2*x^4+25681176*A*b^2*m^2*x^4+11404800*A*b*c*x^6+2039016*B*a^2*m^5*x+37219436*B*a*b*m^3*x^3+37963680*B*a*c*m*x^5+18981840*B*b^2*m*x^5+7902194*C*a^2*m^4*x^2+51362352*C*a*b*m^2*x^4+11404800*C*a*c*x^6+5702400*C*b^2*x^6+2310945*A*a^2*m^5+44578296*A*a*b*m^3*x^2+45024192*A*a*c*m*x^4+22512096*A*b^2*m*x^4+9261503*B*a^2*m^4*x+61638408*B*a*b*m^2*x^3+13305600*B*a*c*x^5+6652800*B*b^2*x^5+22289148*C*a^2*m^3*x^2+45024192*C*a*b*m*x^4+11028590*A*a^2*m^4+76781264*A*a*b*m^2*x^2+15966720*A*a*c*x^4+7983360*A*b^2*x^4+27472724*B*a^2*m^3*x+55282320*B*a*b*m*x^3+38390632*C*a^2*m^2*x^2+15966720*C*a*b*x^4+34967140*A*a^2*m^3+71492160*A*a*b*m*x^2+50312628*B*a^2*m^2*x+19958400*B*a*b*x^3+35746080*C*a^2*m*x^2+70290936*A*a^2*m^2+26611200*A*a*b*x^2+50292720*B*a^2*m*x+13305600*C*a^2*x^2+80627040*A*a^2*m+19958400*B*a^2*x+39916800*A*a^2)*(d*x)^m/(11+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(260) = 520$.

Time = 0.33 (sec) , antiderivative size = 1603, normalized size of antiderivative = 6.17

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((C*c^2*m^10 + 55*C*c^2*m^9 + 1320*C*c^2*m^8 + 18150*C*c^2*m^7 + 157773*C*c
^2*m^6 + 902055*C*c^2*m^5 + 3416930*C*c^2*m^4 + 8409500*C*c^2*m^3 + 1275357
6*C*c^2*m^2 + 10628640*C*c^2*m + 3628800*C*c^2)*x^11 + (B*c^2*m^10 + 56*B*c
^2*m^9 + 1365*B*c^2*m^8 + 19020*B*c^2*m^7 + 167223*B*c^2*m^6 + 965328*B*c^2
*m^5 + 3686255*B*c^2*m^4 + 9133180*B*c^2*m^3 + 13926276*B*c^2*m^2 + 1165521
6*B*c^2*m + 3991680*B*c^2)*x^10 + ((2*C*b*c + A*c^2)*m^10 + 57*(2*C*b*c + A
*c^2)*m^9 + 1412*(2*C*b*c + A*c^2)*m^8 + 19962*(2*C*b*c + A*c^2)*m^7 + 1777
65*(2*C*b*c + A*c^2)*m^6 + 1037673*(2*C*b*c + A*c^2)*m^5 + 4000478*(2*C*b*c
 + A*c^2)*m^4 + 9991428*(2*C*b*c + A*c^2)*m^3 + 8870400*C*b*c + 4435200*A*c
^2 + 15335224*(2*C*b*c + A*c^2)*m^2 + 12900960*(2*C*b*c + A*c^2)*m)*x^9 + 2
*(B*b*c*m^10 + 58*B*b*c*m^9 + 1461*B*b*c*m^8 + 20982*B*b*c*m^7 + 189567*B*b
*c*m^6 + 1121022*B*b*c*m^5 + 4371359*B*b*c*m^4 + 11024858*B*b*c*m^3 + 17059
212*B*b*c*m^2 + 14444280*B*b*c*m + 4989600*B*b*c)*x^8 + ((C*b^2 + 2*(C*a +
A*b)*c)*m^10 + 59*(C*b^2 + 2*(C*a + A*b)*c)*m^9 + 1512*(C*b^2 + 2*(C*a + A*
b)*c)*m^8 + 22086*(C*b^2 + 2*(C*a + A*b)*c)*m^7 + 202821*(C*b^2 + 2*(C*a +
A*b)*c)*m^6 + 1217811*(C*b^2 + 2*(C*a + A*b)*c)*m^5 + 4814858*(C*b^2 + 2*(C
*a + A*b)*c)*m^4 + 12291724*(C*b^2 + 2*(C*a + A*b)*c)*m^3 + 5702400*C*b^2 +
 19216008*(C*b^2 + 2*(C*a + A*b)*c)*m^2 + 11404800*(C*a + A*b)*c + 16405920
*(C*b^2 + 2*(C*a + A*b)*c)*m)*x^7 + ((B*b^2 + 2*B*a*c)*m^10 + 60*(B*b^2 + 2
*B*a*c)*m^9 + 1565*(B*b^2 + 2*B*a*c)*m^8 + 23280*(B*b^2 + 2*B*a*c)*m^7 + 21
7743*(B*b^2 + 2*B*a*c)*m^6 + 1331100*(B*b^2 + 2*B*a*c)*m^5 + 5352935*(B*b^2
 + 2*B*a*c)*m^4 + 13878120*(B*b^2 + 2*B*a*c)*m^3 + 6652800*B*b^2 + 13305600
*B*a*c + 21989356*(B*b^2 + 2*B*a*c)*m^2 + 18981840*(B*b^2 + 2*B*a*c)*m)*x^6
 + ((2*C*a*b + A*b^2 + 2*A*a*c)*m^10 + 61*(2*C*a*b + A*b^2 + 2*A*a*c)*m^9 +
 1620*(2*C*a*b + A*b^2 + 2*A*a*c)*m^8 + 24570*(2*C*a*b + A*b^2 + 2*A*a*c)*m
^7 + 234573*(2*C*a*b + A*b^2 + 2*A*a*c)*m^6 + 1464693*(2*C*a*b + A*b^2 + 2*
A*a*c)*m^5 + 6016070*(2*C*a*b + A*b^2 + 2*A*a*c)*m^4 + 15915380*(2*C*a*b +
A*b^2 + 2*A*a*c)*m^3 + 15966720*C*a*b + 7983360*A*b^2 + 15966720*A*a*c + 25
681176*(2*C*a*b + A*b^2 + 2*A*a*c)*m^2 + 22512096*(2*C*a*b + A*b^2 + 2*A*a*
c)*m)*x^5 + 2*(B*a*b*m^10 + 62*B*a*b*m^9 + 1677*B*a*b*m^8 + 25962*B*a*b*m^7
 + 253575*B*a*b*m^6 + 1623258*B*a*b*m^5 + 6846503*B*a*b*m^4 + 18609718*B*a*
b*m^3 + 30819204*B*a*b*m^2 + 27641160*B*a*b*m + 9979200*B*a*b)*x^4 + ((C*a^
2 + 2*A*a*b)*m^10 + 63*(C*a^2 + 2*A*a*b)*m^9 + 1736*(C*a^2 + 2*A*a*b)*m^8 +
 27462*(C*a^2 + 2*A*a*b)*m^7 + 275037*(C*a^2 + 2*A*a*b)*m^6 + 1812447*(C*a^
2 + 2*A*a*b)*m^5 + 7902194*(C*a^2 + 2*A*a*b)*m^4 + 22289148*(C*a^2 + 2*A*a*
```

```
b)*m^3 + 13305600*C*a^2 + 26611200*A*a*b + 38390632*(C*a^2 + 2*A*a*b)*m^2 +
 35746080*(C*a^2 + 2*A*a*b)*m)*x^3 + (B*a^2*m^10 + 64*B*a^2*m^9 + 1797*B*a^
2*m^8 + 29076*B*a^2*m^7 + 299271*B*a^2*m^6 + 2039016*B*a^2*m^5 + 9261503*B*
a^2*m^4 + 27472724*B*a^2*m^3 + 50312628*B*a^2*m^2 + 50292720*B*a^2*m + 1995
8400*B*a^2)*x^2 + (A*a^2*m^10 + 65*A*a^2*m^9 + 1860*A*a^2*m^8 + 30810*A*a^2
*m^7 + 326613*A*a^2*m^6 + 2310945*A*a^2*m^5 + 11028590*A*a^2*m^4 + 34967140
*A*a^2*m^3 + 70290936*A*a^2*m^2 + 80627040*A*a^2*m + 39916800*A*a^2)*x)*(d*
x)^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13
339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 3
9916800)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16323 vs. 2(245) = 490.

Time = 1.42 (sec) , antiderivative size = 16323, normalized size of antiderivative = 62.78

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 dx = \text{Too large to display}$$

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-A*a**2/(10*x**10) - A*a*b/(4*x**8) - A*a*c/(3*x**6) - A*b**2/(
6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*a**2/(9*x**9) - 2*B*a*b/(7*x
**7) - 2*B*a*c/(5*x**5) - B*b**2/(5*x**5) - 2*B*b*c/(3*x**3) - B*c**2/x - C
*a**2/(8*x**8) - C*a*b/(3*x**6) - C*a*c/(2*x**4) - C*b**2/(4*x**4) - C*b*c/
x**2 + C*c**2*log(x))/d**11, Eq(m, -11)), ((-A*a**2/(9*x**9) - 2*A*a*b/(7*x
**7) - 2*A*a*c/(5*x**5) - A*b**2/(5*x**5) - 2*A*b*c/(3*x**3) - A*c**2/x - B
*a**2/(8*x**8) - B*a*b/(3*x**6) - B*a*c/(2*x**4) - B*b**2/(4*x**4) - B*b*c/
x**2 + B*c**2*log(x) - C*a**2/(7*x**7) - 2*C*a*b/(5*x**5) - 2*C*a*c/(3*x**3
) - C*b**2/(3*x**3) - 2*C*b*c/x + C*c**2*x)/d**10, Eq(m, -10)), ((-A*a**2/(
8*x**8) - A*a*b/(3*x**6) - A*a*c/(2*x**4) - A*b**2/(4*x**4) - A*b*c/x**2 +
A*c**2*log(x) - B*a**2/(7*x**7) - 2*B*a*b/(5*x**5) - 2*B*a*c/(3*x**3) - B*b
**2/(3*x**3) - 2*B*b*c/x + B*c**2*x - C*a**2/(6*x**6) - C*a*b/(2*x**4) - C*
a*c/x**2 - C*b**2/(2*x**2) + 2*C*b*c*log(x) + C*c**2*x**2/2)/d**9, Eq(m, -9
)), ((-A*a**2/(7*x**7) - 2*A*a*b/(5*x**5) - 2*A*a*c/(3*x**3) - A*b**2/(3*x*
*3) - 2*A*b*c/x + A*c**2*x - B*a**2/(6*x**6) - B*a*b/(2*x**4) - B*a*c/x**2
- B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2 - C*a**2/(5*x**5) - 2*C*
a*b/(3*x**3) - 2*C*a*c/x - C*b**2/x + 2*C*b*c*x + C*c**2*x**3/3)/d**8, Eq(m
, -8)), ((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*a*c/x**2 - A*b**2/(2*x**2)
+ 2*A*b*c*log(x) + A*c**2*x**2/2 - B*a**2/(5*x**5) - 2*B*a*b/(3*x**3) - 2*B
*a*c/x - B*b**2/x + 2*B*b*c*x + B*c**2*x**3/3 - C*a**2/(4*x**4) - C*a*b/x**
2 + 2*C*a*c*log(x) + C*b**2*log(x) + C*b*c*x**2 + C*c**2*x**4/4)/d**7, Eq(m
, -7)), ((-A*a**2/(5*x**5) - 2*A*a*b/(3*x**3) - 2*A*a*c/x - A*b**2/x + 2*A*
b*c*x + A*c**2*x**3/3 - B*a**2/(4*x**4) - B*a*b/x**2 + 2*B*a*c*log(x) + B*b
**2*log(x) + B*b*c*x**2 + B*c**2*x**4/4 - C*a**2/(3*x**3) - 2*C*a*b/x + 2*C
```

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*a*c*x + C*b**2*x + 2*C*b*c*x**3/3 + C*c**2*x**5/5)/d**6, Eq(m, -6)), ((-A*
a**2/(4*x**4) - A*a*b/x**2 + 2*A*a*c*log(x) + A*b**2*log(x) + A*b*c*x**2 +
A*c**2*x**4/4 - B*a**2/(3*x**3) - 2*B*a*b/x + 2*B*a*c*x + B*b**2*x + 2*B*b*
c*x**3/3 + B*c**2*x**5/5 - C*a**2/(2*x**2) + 2*C*a*b*log(x) + C*a*c*x**2 +
C*b**2*x**2/2 + C*b*c*x**4/2 + C*c**2*x**6/6)/d**5, Eq(m, -5)), ((-A*a**2/(
3*x**3) - 2*A*a*b/x + 2*A*a*c*x + A*b**2*x + 2*A*b*c*x**3/3 + A*c**2*x**5/5
 - B*a**2/(2*x**2) + 2*B*a*b*log(x) + B*a*c*x**2 + B*b**2*x**2/2 + B*b*c*x*
*4/2 + B*c**2*x**6/6 - C*a**2/x + 2*C*a*b*x + 2*C*a*c*x**3/3 + C*b**2*x**3/
3 + 2*C*b*c*x**5/5 + C*c**2*x**7/7)/d**4, Eq(m, -4)), ((-A*a**2/(2*x**2) +
2*A*a*b*log(x) + A*a*c*x**2 + A*b**2*x**2/2 + A*b*c*x**4/2 + A*c**2*x**6/6
- B*a**2/x + 2*B*a*b*x + 2*B*a*c*x**3/3 + B*b**2*x**3/3 + 2*B*b*c*x**5/5 +
B*c**2*x**7/7 + C*a**2*log(x) + C*a*b*x**2 + C*a*c*x**4/2 + C*b**2*x**4/4 +
 C*b*c*x**6/3 + C*c**2*x**8/8)/d**3, Eq(m, -3)), ((-A*a**2/x + 2*A*a*b*x +
2*A*a*c*x**3/3 + A*b**2*x**3/3 + 2*A*b*c*x**5/5 + A*c**2*x**7/7 + B*a**2*lo
g(x) + B*a*b*x**2 + B*a*c*x**4/2 + B*b**2*x**4/4 + B*b*c*x**6/3 + B*c**2*x*
*8/8 + C*a**2*x + 2*C*a*b*x**3/3 + 2*C*a*c*x**5/5 + C*b**2*x**5/5 + 2*C*b*c
*x**7/7 + C*c**2*x**9/9)/d**2, Eq(m, -2)), ((A*a**2*log(x) + A*a*b*x**2 + A
*a*c*x**4/2 + A*b**2*x**4/4 + A*b*c*x**6/3 + A*c**2*x**8/8 + B*a**2*x + 2*B
*a*b*x**3/3 + 2*B*a*c*x**5/5 + B*b**2*x**5/5 + 2*B*b*c*x**7/7 + B*c**2*x**9
/9 + C*a**2*x**2/2 + C*a*b*x**4/2 + C*a*c*x**6/3 + C*b**2*x**6/6 + C*b*c*x*
*8/4 + C*c**2*x**10/10)/d, Eq(m, -1)), (A*a**2*m**10*x*(d*x)**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 65*A*a**2*m**9*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 1860*A*a**2*m**8*x*(d*x)**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 30810*A*a**2*m**7*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 326613*A*a**2*
m**6*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 2310945*A*a**2*m**5*x*(d*x)**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 11028590*A*a**2*m**4*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m
**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525807
6*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 34967140*A*a**2*m**3*x*
(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558
*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1
20543840*m + 39916800) + 70290936*A*a**2*m**2*x*(d*x)**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 80
627040*A*a**2*m*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
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423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 39916800*A*a**2*x*(d*x)**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 2*A*a*b*m**10*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 126*A*a*b*m**9*x**3*
(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558
*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1
20543840*m + 39916800) + 3472*A*a*b*m**8*x**3*(d*x)**m/(m**11 + 66*m**10 +
1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995
730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 5492
4*A*a*b*m**7*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 550074*A*a*b*m**6*x**3*(d*x)**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 3624894*A*a*b*m**5*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 15804388*A
*a*b*m**4*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423
*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150
917976*m**2 + 120543840*m + 39916800) + 44578296*A*a*b*m**3*x**3*(d*x)**m/(
m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13
339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
 + 39916800) + 76781264*A*a*b*m**2*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 71492160*A
*a*b*m*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 26611200*A*a*b*x**3*(d*x)**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m
**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916
800) + 2*A*a*c*m**10*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 122*A*a*c*m**9*x**5*(d*x
)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**
6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12054
3840*m + 39916800) + 3240*A*a*c*m**8*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925
*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*
m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 49140*A*
a*c*m**7*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*
m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509
17976*m**2 + 120543840*m + 39916800) + 469146*A*a*c*m**6*x**5*(d*x)**m/(m**
11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339
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535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
39916800) + 2929386*A*a*c*m**5*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12032140*A*a*c
*m**4*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**
7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509179
76*m**2 + 120543840*m + 39916800) + 31830760*A*a*c*m**3*x**5*(d*x)**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 51362352*A*a*c*m**2*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45024192*A*a*c
*m*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 15966720*A*a*c*x**5*(d*x)**m/(m**11 + 66*m
**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
+ 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
 + A*b**2*m**10*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 61*A*b**2*m**9*x**5*(d*x)**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 1620*A*b**2*m**8*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 24570*A*b**2
*m**7*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**
7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509179
76*m**2 + 120543840*m + 39916800) + 234573*A*b**2*m**6*x**5*(d*x)**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 1464693*A*b**2*m**5*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 6016070*A*b**2*
m**4*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091797
6*m**2 + 120543840*m + 39916800) + 15915380*A*b**2*m**3*x**5*(d*x)**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 25681176*A*b**2*m**2*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9
 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
+ 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 22512096*A*b*
*2*m*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091797
6*m**2 + 120543840*m + 39916800) + 7983360*A*b**2*x**5*(d*x)**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
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5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 2*A*b*c*m**10*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m
**3 + 150917976*m**2 + 120543840*m + 39916800) + 118*A*b*c*m**9*x**7*(d*x)*
*m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
+ 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205438
40*m + 39916800) + 3024*A*b*c*m**8*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 44172*A*b*
c*m**7*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 405642*A*b*c*m**6*x**7*(d*x)**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 2435622*A*b*c*m**5*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1
05258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 9629716*A*b*c*m*
*4*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 24583448*A*b*c*m**3*x**7*(d*x)**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 38432016*A*b*c*m**2*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 32811840*A*b*c*m*
x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26
37558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**
2 + 120543840*m + 39916800) + 11404800*A*b*c*x**7*(d*x)**m/(m**11 + 66*m**1
0 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4
5995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
A*c**2*m**10*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 57*A*c**2*m**9*x**9*(d*x)**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 1412*A*c**2*m**8*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 19962*A*c**2*m*
*7*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 177765*A*c**2*m**6*x**9*(d*x)**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m
**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916
800) + 1037673*A*c**2*m**5*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32
670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105
```

258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 4000478*A*c**2*m**
4*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
2 + 120543840*m + 39916800) + 9991428*A*c2*m**3*x**9*(d*x)**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m
5 + 45995730*m4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916
800) + 15335224*A*c**2*m**2*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12900960*A*c**2*m
*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 4435200*A*c**2*x**9*(d*x)**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
B*a**2*m**10*x**2*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35
7423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 64*B*a**2*m**9*x**2*(d*x)**m/(m
11 + 66*m10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 1797*B*a**2*m**8*x**2*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 29076*B*a**2*m
7*x2*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 299271*B*a**2*m**6*x**2*(d*x)**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 2039016*B*a**2*m**5*x**2*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 9261503*B*a**2*m*
*4*x**2*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 27472724*B*a**2*m**3*x**2*(d*x)**m/(m**11
+ 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535
*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399
16800) + 50312628*B*a**2*m**2*x**2*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 50292720*B*a**2
*m*x**2*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 19958400*B*a**2*x**2*(d*x)**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
+ 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 2*B*a*b*m**10*x**4*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*

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*3 + 150917976*m**2 + 120543840*m + 39916800) + 124*B*a*b*m**9*x**4*(d*x)**
m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12054384
0*m + 39916800) + 3354*B*a*b*m**8*x**4*(d*x)**m/(m**11 + 66*m**10 + 1925*m*
*9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**
4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 51924*B*a*b
*m**7*x**4*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**
7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509179
76*m**2 + 120543840*m + 39916800) + 507150*B*a*b*m**6*x**4*(d*x)**m/(m**11
+ 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535
*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399
16800) + 3246516*B*a*b*m**5*x**4*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 13693006*B*a*b*m*
*4*x**4*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 37219436*B*a*b*m**3*x**4*(d*x)**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 61638408*B*a*b*m**2*x**4*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 55282320*B*a*b*m*
x**4*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26
37558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**
2 + 120543840*m + 39916800) + 19958400*B*a*b*x**4*(d*x)**m/(m**11 + 66*m**1
0 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4
5995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
2*B*a*c*m**10*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35
7423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
 150917976*m**2 + 120543840*m + 39916800) + 120*B*a*c*m**9*x**6*(d*x)**m/(m
**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 3130*B*a*c*m**8*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 46560*B*a*c*m**
7*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 435486*B*a*c*m**6*x**6*(d*x)**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 2662200*B*a*c*m**5*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670
*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258
076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 10705870*B*a*c*m**4*x
**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
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 + 120543840*m + 39916800) + 27756240*B*a*c*m**3*x**6*(d*x)**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 43978712*B*a*c*m**2*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670
*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258
076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 37963680*B*a*c*m*x**6
*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263755
8*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
120543840*m + 39916800) + 13305600*B*a*c*x**6*(d*x)**m/(m**11 + 66*m**10 +
1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995
730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + B*b*
*2*m**10*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*
m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509
17976*m**2 + 120543840*m + 39916800) + 60*B*b**2*m**9*x**6*(d*x)**m/(m**11
+ 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535
*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399
16800) + 1565*B*b**2*m**8*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 326
70*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052
58076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 23280*B*b**2*m**7*x
6*(d*x)m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
 + 120543840*m + 39916800) + 217743*B*b**2*m**6*x**6*(d*x)**m/(m**11 + 66*m
10 + 1925*m9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
+ 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
 + 1331100*B*b**2*m**5*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 5352935*B*b**2*m**4*x*
6(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637
558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
+ 120543840*m + 39916800) + 13878120*B*b**2*m**3*x**6*(d*x)**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 21989356*B*b**2*m**2*x**6*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 18981840*B*b**2*m*x*
6(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637
558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
+ 120543840*m + 39916800) + 6652800*B*b**2*x**6*(d*x)**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 2*
B*b*c*m**10*x**8*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 3574
23*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1
50917976*m**2 + 120543840*m + 39916800) + 116*B*b*c*m**9*x**8*(d*x)**m/(m**
11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339
535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +

39916800) + 2922*B*b*c*m**8*x**8*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 41964*B*b*c*m**7*
x**8*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26
37558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**
2 + 120543840*m + 39916800) + 379134*B*b*c*m**6*x**8*(d*x)**m/(m**11 + 66*m
10 + 1925*m9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
+ 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
+ 2242044*B*b*c*m**5*x**8*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m
8 + 357423*m7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525807
6*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 8742718*B*b*c*m**4*x**8
*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263755
8*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
120543840*m + 39916800) + 22049716*B*b*c*m**3*x**8*(d*x)**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
34118424*B*b*c*m**2*x**8*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 28888560*B*b*c*m*x**8*(d
*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m
6 + 13339535*m5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120
543840*m + 39916800) + 9979200*B*b*c*x**8*(d*x)**m/(m**11 + 66*m**10 + 1925
*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*
m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + B*c**2*m
10*x10*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**
7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509179
76*m**2 + 120543840*m + 39916800) + 56*B*c**2*m**9*x**10*(d*x)**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m
5 + 45995730*m4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916
800) + 1365*B*c**2*m**8*x**10*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 19020*B*c**2*m**7*x*
10(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
+ 120543840*m + 39916800) + 167223*B*c**2*m**6*x**10*(d*x)**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
+ 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 965328*B*c**2*m**5*x**10*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670
*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258
076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3686255*B*c**2*m**4*x
10*(d*x)m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26
37558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**
2 + 120543840*m + 39916800) + 9133180*B*c**2*m**3*x**10*(d*x)**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168

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00) + 13926276*B*c**2*m**2*x**10*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 11655216*B*c**2*m
*x**10*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 3991680*B*c**2*x**10*(d*x)**m/(m**11 + 66*m
**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
+ 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
 + C*a**2*m**10*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 63*C*a**2*m**9*x**3*(d*x)**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 1736*C*a**2*m**8*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 27462*C*a**2
*m**7*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**
7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509179
76*m**2 + 120543840*m + 39916800) + 275037*C*a**2*m**6*x**3*(d*x)**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 1812447*C*a**2*m**5*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 7902194*C*a**2*
m**4*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091797
6*m**2 + 120543840*m + 39916800) + 22289148*C*a**2*m**3*x**3*(d*x)**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 38390632*C*a**2*m**2*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9
 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
+ 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 35746080*C*a*
*2*m*x**3*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091797
6*m**2 + 120543840*m + 39916800) + 13305600*C*a**2*x**3*(d*x)**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 2*C*a*b*m**10*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 122*C*a*b*m**9*x**5*(d*x)
**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543
840*m + 39916800) + 3240*C*a*b*m**8*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*
m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m
**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 49140*C*a
```

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*b*m**7*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 469146*C*a*b*m**6*x**5*(d*x)**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 2929386*C*a*b*m**5*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12032140*C*a*b*
m**4*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091797
6*m**2 + 120543840*m + 39916800) + 31830760*C*a*b*m**3*x**5*(d*x)**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 51362352*C*a*b*m**2*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45024192*C*a*b*
m*x**5*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 15966720*C*a*b*x**5*(d*x)**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
+ 2*C*a*c*m**10*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 118*C*a*c*m**9*x**7*(d*x)**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 3024*C*a*c*m**8*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9
 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
+ 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 44172*C*a*c*m
**7*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 405642*C*a*c*m**6*x**7*(d*x)**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m
**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916
800) + 2435622*C*a*c*m**5*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 326
70*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052
58076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 9629716*C*a*c*m**4*
x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26
37558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**
2 + 120543840*m + 39916800) + 24583448*C*a*c*m**3*x**7*(d*x)**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 38432016*C*a*c*m**2*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 32811840*C*a*c*m*x**
```

7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 11404800*C*a*c*x**7*(d*x)**m/(m**11 + 66*m**10 +
 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4599
5730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + C*b
2*m10*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423
*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150
917976*m**2 + 120543840*m + 39916800) + 59*C*b**2*m**9*x**7*(d*x)**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 1512*C*b**2*m**8*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32
670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105
258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 22086*C*b**2*m**7*
x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26
37558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**
2 + 120543840*m + 39916800) + 202821*C*b**2*m**6*x**7*(d*x)**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 1217811*C*b**2*m**5*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670
*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258
076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 4814858*C*b**2*m**4*x
7*(d*x)m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
 + 120543840*m + 39916800) + 12291724*C*b**2*m**3*x**7*(d*x)**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 19216008*C*b**2*m**2*x**7*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 326
70*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052
58076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 16405920*C*b**2*m*x
7*(d*x)m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
 + 120543840*m + 39916800) + 5702400*C*b**2*x**7*(d*x)**m/(m**11 + 66*m**10
 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45
995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 2
*C*b*c*m**10*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 114*C*b*c*m**9*x**9*(d*x)**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 2824*C*b*c*m**8*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1
05258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 39924*C*b*c*m**7
*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 355530*C*b*c*m**6*x**9*(d*x)**m/(m**11 + 66*

```
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 2075346*C*b*c*m**5*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 8000956*C*b*c*m**4*x**
9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 19982856*C*b*c*m**3*x**9*(d*x)**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
+ 30670448*C*b*c*m**2*x**9*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m
**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525807
6*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 25801920*C*b*c*m*x**9*(
d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 8870400*C*b*c*x**9*(d*x)**m/(m**11 + 66*m**10 + 192
5*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730
*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + C*c**2*
m**10*x**11*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 55*C*c**2*m**9*x**11*(d*x)**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 1320*C*c**2*m**8*x**11*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 326
70*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052
58076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 18150*C*c**2*m**7*x
**11*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26
37558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**
2 + 120543840*m + 39916800) + 157773*C*c**2*m**6*x**11*(d*x)**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 902055*C*c**2*m**5*x**11*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3416930*C*c**2*m**4*
x**11*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 8409500*C*c**2*m**3*x**11*(d*x)**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m
**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916
800) + 12753576*C*c**2*m**2*x**11*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 +
32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1
05258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 10628640*C*c**2*
m*x**11*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 3628800*C*c**2*x**11*(d*x)**m/(m**11 + 66*
```

```
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.32

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx = \frac{Cc^2d^mx^{11}x^m}{m+11} + \frac{Bc^2d^mx^{10}x^m}{m+10} + \frac{2\,Cbcd^mx^9x^m}{m+9} + \frac{Ac^2d^mx^9x^m}{m+9} + \frac{2\,Bbcd^mx^8x^m}{m+8} + \frac{Cb^2d^mx^7x^m}{m+7} + \frac{2\,Cacd^mx^7x^m}{m+7} + \frac{2\,Abcd^mx^7x^m}{m+7} + \frac{Bb^2d^mx^6x^m}{m+6} + \frac{2\,Bacd^mx^6x^m}{m+6} + \frac{2\,Cabd^mx^5x^m}{m+5} + \frac{Ab^2d^mx^5x^m}{m+5} + \frac{2\,Aacd^mx^5x^m}{m+5} + \frac{2\,Babd^mx^4x^m}{m+4} + \frac{Ca^2d^mx^3x^m}{m+3} + \frac{2\,Aabd^mx^3x^m}{m+3} + \frac{Ba^2d^mx^2x^m}{m+2} + \frac{(dx)^{m+1}\,Aa^2}{d(m+1)}$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] C*c^2*d^m*x^11*x^m/(m + 11) + B*c^2*d^m*x^10*x^m/(m + 10) + 2*C*b*c*d^m*x^9
*x^m/(m + 9) + A*c^2*d^m*x^9*x^m/(m + 9) + 2*B*b*c*d^m*x^8*x^m/(m + 8) + C*
b^2*d^m*x^7*x^m/(m + 7) + 2*C*a*c*d^m*x^7*x^m/(m + 7) + 2*A*b*c*d^m*x^7*x^m
/(m + 7) + B*b^2*d^m*x^6*x^m/(m + 6) + 2*B*a*c*d^m*x^6*x^m/(m + 6) + 2*C*a*
b*d^m*x^5*x^m/(m + 5) + A*b^2*d^m*x^5*x^m/(m + 5) + 2*A*a*c*d^m*x^5*x^m/(m
+ 5) + 2*B*a*b*d^m*x^4*x^m/(m + 4) + C*a^2*d^m*x^3*x^m/(m + 3) + 2*A*a*b*d^
m*x^3*x^m/(m + 3) + B*a^2*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^2/(d*(m +
 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3203 vs. 2(260) = 520.

Time = 0.37 (sec) , antiderivative size = 3203, normalized size of antiderivative = 12.32

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c^2*m^10*x^11 + (d*x)^m*B*c^2*m^10*x^10 + 55*(d*x)^m*C*c^2*m^9*x
^11 + 2*(d*x)^m*C*b*c*m^10*x^9 + (d*x)^m*A*c^2*m^10*x^9 + 56*(d*x)^m*B*c^2*
m^9*x^10 + 1320*(d*x)^m*C*c^2*m^8*x^11 + 2*(d*x)^m*B*b*c*m^10*x^8 + 114*(d*
```

x)^m*C*b*c*m^9*x^9 + 57*(d*x)^m*A*c^2*m^9*x^9 + 1365*(d*x)^m*B*c^2*m^8*x^10 + 18150*(d*x)^m*C*c^2*m^7*x^11 + (d*x)^m*C*b^2*m^10*x^7 + 2*(d*x)^m*C*a*c*m^10*x^7 + 2*(d*x)^m*A*b*c*m^10*x^7 + 116*(d*x)^m*B*b*c*m^9*x^8 + 2824*(d*x)^m*C*b*c*m^8*x^9 + 1412*(d*x)^m*A*c^2*m^8*x^9 + 19020*(d*x)^m*B*c^2*m^7*x^10 + 157773*(d*x)^m*C*c^2*m^6*x^11 + (d*x)^m*B*b^2*m^10*x^6 + 2*(d*x)^m*B*a*c*m^10*x^6 + 59*(d*x)^m*C*b^2*m^9*x^7 + 118*(d*x)^m*C*a*c*m^9*x^7 + 118*(d*x)^m*A*b*c*m^9*x^7 + 2922*(d*x)^m*B*b*c*m^8*x^8 + 39924*(d*x)^m*C*b*c*m^7*x^9 + 19962*(d*x)^m*A*c^2*m^7*x^9 + 167223*(d*x)^m*B*c^2*m^6*x^10 + 902055*(d*x)^m*C*c^2*m^5*x^11 + 2*(d*x)^m*C*a*b*m^10*x^5 + (d*x)^m*A*b^2*m^10*x^5 + 2*(d*x)^m*A*a*c*m^10*x^5 + 60*(d*x)^m*B*b^2*m^9*x^6 + 120*(d*x)^m*B*a*c*m^9*x^6 + 1512*(d*x)^m*C*b^2*m^8*x^7 + 3024*(d*x)^m*C*a*c*m^8*x^7 + 3024*(d*x)^m*A*b*c*m^8*x^7 + 41964*(d*x)^m*B*b*c*m^7*x^8 + 355530*(d*x)^m*C*b*c*m^6*x^9 + 177765*(d*x)^m*A*c^2*m^6*x^9 + 965328*(d*x)^m*B*c^2*m^5*x^10 + 3416930*(d*x)^m*C*c^2*m^4*x^11 + 2*(d*x)^m*B*a*b*m^10*x^4 + 122*(d*x)^m*C*a*b*m^9*x^5 + 61*(d*x)^m*A*b^2*m^9*x^5 + 122*(d*x)^m*A*a*c*m^9*x^5 + 1565*(d*x)^m*B*b^2*m^8*x^6 + 3130*(d*x)^m*B*a*c*m^8*x^6 + 22086*(d*x)^m*C*b^2*m^7*x^7 + 44172*(d*x)^m*C*a*c*m^7*x^7 + 44172*(d*x)^m*A*b*c*m^7*x^7 + 379134*(d*x)^m*B*b*c*m^6*x^8 + 2075346*(d*x)^m*C*b*c*m^5*x^9 + 1037673*(d*x)^m*A*c^2*m^5*x^9 + 3686255*(d*x)^m*B*c^2*m^4*x^10 + 8409500*(d*x)^m*C*c^2*m^3*x^11 + (d*x)^m*C*a^2*m^10*x^3 + 2*(d*x)^m*A*a*b*m^10*x^3 + 124*(d*x)^m*B*a*b*m^9*x^4 + 3240*(d*x)^m*C*a*b*m^8*x^5 + 1620*(d*x)^m*A*b^2*m^8*x^5 + 3240*(d*x)^m*A*a*c*m^8*x^5 + 23280*(d*x)^m*B*b^2*m^7*x^6 + 46560*(d*x)^m*B*a*c*m^7*x^6 + 202821*(d*x)^m*C*b^2*m^6*x^7 + 405642*(d*x)^m*C*a*c*m^6*x^7 + 405642*(d*x)^m*A*b*c*m^6*x^7 + 2242044*(d*x)^m*B*b*c*m^5*x^8 + 8000956*(d*x)^m*C*b*c*m^4*x^9 + 4000478*(d*x)^m*A*c^2*m^4*x^9 + 9133180*(d*x)^m*B*c^2*m^3*x^10 + 12753576*(d*x)^m*C*c^2*m^2*x^11 + (d*x)^m*B*a^2*m^10*x^2 + 63*(d*x)^m*C*a^2*m^9*x^3 + 126*(d*x)^m*A*a*b*m^9*x^3 + 3354*(d*x)^m*B*a*b*m^8*x^4 + 49140*(d*x)^m*C*a*b*m^7*x^5 + 24570*(d*x)^m*A*b^2*m^7*x^5 + 49140*(d*x)^m*A*a*c*m^7*x^5 + 217743*(d*x)^m*B*b^2*m^6*x^6 + 435486*(d*x)^m*B*a*c*m^6*x^6 + 1217811*(d*x)^m*C*b^2*m^5*x^7 + 2435622*(d*x)^m*C*a*c*m^5*x^7 + 2435622*(d*x)^m*A*b*c*m^5*x^7 + 8742718*(d*x)^m*B*b*c*m^4*x^8 + 19982856*(d*x)^m*C*b*c*m^3*x^9 + 9991428*(d*x)^m*A*c^2*m^3*x^9 + 13926276*(d*x)^m*B*c^2*m^2*x^10 + 10628640*(d*x)^m*C*c^2*m*x^11 + (d*x)^m*A*a^2*m^10*x + 64*(d*x)^m*B*a^2*m^9*x^2 + 1736*(d*x)^m*C*a^2*m^8*x^3 + 3472*(d*x)^m*A*a*b*m^8*x^3 + 51924*(d*x)^m*B*a*b*m^7*x^4 + 469146*(d*x)^m*C*a*b*m^6*x^5 + 234573*(d*x)^m*A*b^2*m^6*x^5 + 469146*(d*x)^m*A*a*c*m^6*x^5 + 1331100*(d*x)^m*B*b^2*m^5*x^6 + 2662200*(d*x)^m*B*a*c*m^5*x^6 + 4814858*(d*x)^m*C*b^2*m^4*x^7 + 9629716*(d*x)^m*C*a*c*m^4*x^7 + 9629716*(d*x)^m*A*b*c*m^4*x^7 + 22049716*(d*x)^m*B*b*c*m^3*x^8 + 30670448*(d*x)^m*C*b*c*m^2*x^9 + 15335224*(d*x)^m*A*c^2*m^2*x^9 + 11655216*(d*x)^m*B*c^2*m*x^10 + 3628800*(d*x)^m*C*c^2*x^11 + 65*(d*x)^m*A*a^2*m^9*x + 1797*(d*x)^m*B*a^2*m^8*x^2 + 27462*(d*x)^m*C*a^2*m^7*x^3 + 54924*(d*x)^m*A*a*b*m^7*x^3 + 507150*(d*x)^m*B*a*b*m^6*x^4 + 2929386*(d*x)^m*C*a*b*m^5*x^5 + 1464693*(d*x)^m*A*b^2*m^5*x^5 + 2929386*(d*x)^m*A*a*c*m^5*x^5 + 5352935*(d*x)^m*B*b^2*m^4*x^6 + 10705870*(d*x)^m*B*a*c*m^4*x^6 + 12291724*(d*x)^m*C*b^2*m^3*x^7 + 24583448*(d*x)^m*C*a*c*m^3*x^7 + 24583448*(d*x)^m*A*b*c*m^3*

```
x^7 + 34118424*(d*x)^m*B*b*c*m^2*x^8 + 25801920*(d*x)^m*C*b*c*m*x^9 + 12900
960*(d*x)^m*A*c^2*m*x^9 + 3991680*(d*x)^m*B*c^2*x^10 + 1860*(d*x)^m*A*a^2*m
^8*x + 29076*(d*x)^m*B*a^2*m^7*x^2 + 275037*(d*x)^m*C*a^2*m^6*x^3 + 550074*
(d*x)^m*A*a*b*m^6*x^3 + 3246516*(d*x)^m*B*a*b*m^5*x^4 + 12032140*(d*x)^m*C*
a*b*m^4*x^5 + 6016070*(d*x)^m*A*b^2*m^4*x^5 + 12032140*(d*x)^m*A*a*c*m^4*x^
5 + 13878120*(d*x)^m*B*b^2*m^3*x^6 + 27756240*(d*x)^m*B*a*c*m^3*x^6 + 19216
008*(d*x)^m*C*b^2*m^2*x^7 + 38432016*(d*x)^m*C*a*c*m^2*x^7 + 38432016*(d*x)
^m*A*b*c*m^2*x^7 + 28888560*(d*x)^m*B*b*c*m*x^8 + 8870400*(d*x)^m*C*b*c*x^9
 + 4435200*(d*x)^m*A*c^2*x^9 + 30810*(d*x)^m*A*a^2*m^7*x + 299271*(d*x)^m*B
*a^2*m^6*x^2 + 1812447*(d*x)^m*C*a^2*m^5*x^3 + 3624894*(d*x)^m*A*a*b*m^5*x^
3 + 13693006*(d*x)^m*B*a*b*m^4*x^4 + 31830760*(d*x)^m*C*a*b*m^3*x^5 + 15915
380*(d*x)^m*A*b^2*m^3*x^5 + 31830760*(d*x)^m*A*a*c*m^3*x^5 + 21989356*(d*x)
^m*B*b^2*m^2*x^6 + 43978712*(d*x)^m*B*a*c*m^2*x^6 + 16405920*(d*x)^m*C*b^2*
m*x^7 + 32811840*(d*x)^m*C*a*c*m*x^7 + 32811840*(d*x)^m*A*b*c*m*x^7 + 99792
00*(d*x)^m*B*b*c*x^8 + 326613*(d*x)^m*A*a^2*m^6*x + 2039016*(d*x)^m*B*a^2*m
^5*x^2 + 7902194*(d*x)^m*C*a^2*m^4*x^3 + 15804388*(d*x)^m*A*a*b*m^4*x^3 + 3
7219436*(d*x)^m*B*a*b*m^3*x^4 + 51362352*(d*x)^m*C*a*b*m^2*x^5 + 25681176*(
d*x)^m*A*b^2*m^2*x^5 + 51362352*(d*x)^m*A*a*c*m^2*x^5 + 18981840*(d*x)^m*B*
b^2*m*x^6 + 37963680*(d*x)^m*B*a*c*m*x^6 + 5702400*(d*x)^m*C*b^2*x^7 + 1140
4800*(d*x)^m*C*a*c*x^7 + 11404800*(d*x)^m*A*b*c*x^7 + 2310945*(d*x)^m*A*a^2
*m^5*x + 9261503*(d*x)^m*B*a^2*m^4*x^2 + 22289148*(d*x)^m*C*a^2*m^3*x^3 + 4
4578296*(d*x)^m*A*a*b*m^3*x^3 + 61638408*(d*x)^m*B*a*b*m^2*x^4 + 45024192*(
d*x)^m*C*a*b*m*x^5 + 22512096*(d*x)^m*A*b^2*m*x^5 + 45024192*(d*x)^m*A*a*c*
m*x^5 + 6652800*(d*x)^m*B*b^2*x^6 + 13305600*(d*x)^m*B*a*c*x^6 + 11028590*(
d*x)^m*A*a^2*m^4*x + 27472724*(d*x)^m*B*a^2*m^3*x^2 + 38390632*(d*x)^m*C*a^
2*m^2*x^3 + 76781264*(d*x)^m*A*a*b*m^2*x^3 + 55282320*(d*x)^m*B*a*b*m*x^4 +
 15966720*(d*x)^m*C*a*b*x^5 + 7983360*(d*x)^m*A*b^2*x^5 + 15966720*(d*x)^m*
A*a*c*x^5 + 34967140*(d*x)^m*A*a^2*m^3*x + 50312628*(d*x)^m*B*a^2*m^2*x^2 +
 35746080*(d*x)^m*C*a^2*m*x^3 + 71492160*(d*x)^m*A*a*b*m*x^3 + 19958400*(d*
x)^m*B*a*b*x^4 + 70290936*(d*x)^m*A*a^2*m^2*x + 50292720*(d*x)^m*B*a^2*m*x^
2 + 13305600*(d*x)^m*C*a^2*x^3 + 26611200*(d*x)^m*A*a*b*x^3 + 80627040*(d*x
)^m*A*a^2*m*x + 19958400*(d*x)^m*B*a^2*x^2 + 39916800*(d*x)^m*A*a^2*x)/(m^1
1 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13339535*m^
5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916800)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 1314, normalized size of antiderivative = 5.05

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx = \text{Too large to display}$$

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (x^5*(d*x)^m*(A*b^2 + 2*A*a*c + 2*C*a*b)*(22512096*m + 25681176*m^2 + 15915
380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m^6 + 24570*m^7 + 1620*m^8 + 6
```

1*m^9 + m^10 + 7983360))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 459
95730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9
+ 66*m^10 + m^11 + 39916800) + (x^7*(d*x)^m*(C*b^2 + 2*A*b*c + 2*C*a*c)*(16
405920*m + 19216008*m^2 + 12291724*m^3 + 4814858*m^4 + 1217811*m^5 + 202821
*m^6 + 22086*m^7 + 1512*m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m + 1509
17976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357
423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*x^6*(d*x)^
m*(2*a*c + b^2)*(18981840*m + 21989356*m^2 + 13878120*m^3 + 5352935*m^4 + 1
331100*m^5 + 217743*m^6 + 23280*m^7 + 1565*m^8 + 60*m^9 + m^10 + 6652800))/
(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
+ 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 399168
00) + (A*a^2*x*(d*x)^m*(80627040*m + 70290936*m^2 + 34967140*m^3 + 11028590
*m^4 + 2310945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*m^9 + m^10 + 39
916800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 1333
9535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
 + 39916800) + (c*x^9*(d*x)^m*(A*c + 2*C*b)*(12900960*m + 15335224*m^2 + 99
91428*m^3 + 4000478*m^4 + 1037673*m^5 + 177765*m^6 + 19962*m^7 + 1412*m^8 +
 57*m^9 + m^10 + 4435200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 4
5995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^
9 + 66*m^10 + m^11 + 39916800) + (a*x^3*(d*x)^m*(2*A*b + C*a)*(35746080*m +
 38390632*m^2 + 22289148*m^3 + 7902194*m^4 + 1812447*m^5 + 275037*m^6 + 274
62*m^7 + 1736*m^8 + 63*m^9 + m^10 + 13305600))/(120543840*m + 150917976*m^2
 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 +
 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*c^2*x^10*(d*x)^m*(1
1655216*m + 13926276*m^2 + 9133180*m^3 + 3686255*m^4 + 965328*m^5 + 167223*
m^6 + 19020*m^7 + 1365*m^8 + 56*m^9 + m^10 + 3991680))/(120543840*m + 15091
7976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 3574
23*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (C*c^2*x^11*(d
x)^m(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^4 + 902055*m^5 +
 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628800))/(120543840*m
 + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^
6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*a^2
x^2(d*x)^m*(50292720*m + 50312628*m^2 + 27472724*m^3 + 9261503*m^4 + 2039
016*m^5 + 299271*m^6 + 29076*m^7 + 1797*m^8 + 64*m^9 + m^10 + 19958400))/(1
20543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 +
2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800
) + (2*B*b*c*x^8*(d*x)^m*(14444280*m + 17059212*m^2 + 11024858*m^3 + 437135
9*m^4 + 1121022*m^5 + 189567*m^6 + 20982*m^7 + 1461*m^8 + 58*m^9 + m^10 + 4
989600))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 1333
9535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
 + 39916800) + (2*B*a*b*x^4*(d*x)^m*(27641160*m + 30819204*m^2 + 18609718*m
^3 + 6846503*m^4 + 1623258*m^5 + 253575*m^6 + 25962*m^7 + 1677*m^8 + 62*m^9
 + m^10 + 9979200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730
*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*
m^10 + m^11 + 39916800)

3.39 $\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 137

$$\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4) \, dx = \frac{aA(dx)^{1+m}}{d(1+m)} + \frac{aB(dx)^{2+m}}{d^2(2+m)} + \frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)} + \frac{bB(dx)^{4+m}}{d^4(4+m)} + \frac{(Ac+bC)(dx)^{5+m}}{d^5(5+m)} + \frac{Bc(dx)^{6+m}}{d^6(6+m)} + \frac{cC(dx)^{7+m}}{d^7(7+m)}$$

```
[Out] a*A*(d*x)^(1+m)/d/(1+m)+a*B*(d*x)^(2+m)/d^2/(2+m)+(A*b+C*a)*(d*x)^(3+m)/d^3
/(3+m)+b*B*(d*x)^(4+m)/d^4/(4+m)+(A*c+C*b)*(d*x)^(5+m)/d^5/(5+m)+B*c*(d*x)^
(6+m)/d^6/(6+m)+c*C*(d*x)^(7+m)/d^7/(7+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4) \, dx = \frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)} + \frac{aA(dx)^{m+1}}{d(m+1)} + \frac{aB(dx)^{m+2}}{d^2(m+2)} + \frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)} + \frac{bB(dx)^{m+4}}{d^4(m+4)} + \frac{Bc(dx)^{m+6}}{d^6(m+6)} + \frac{cC(dx)^{m+7}}{d^7(m+7)}$$

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m)) + ((A*b
 + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 + m))/(d^4*(4 + m)) +
((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*c*(d*x)^(6 + m))/(d^6*(6 + m
)) + (c*C*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(aA(dx)^m + \frac{aB(dx)^{1+m}}{d} + \frac{(Ab+aC)(dx)^{2+m}}{d^2} + \frac{bB(dx)^{3+m}}{d^3} \right. \\ &\qquad \left. + \frac{(Ac+bC)(dx)^{4+m}}{d^4} + \frac{Bc(dx)^{5+m}}{d^5} + \frac{cC(dx)^{6+m}}{d^6}\right) dx \\ &= \frac{aA(dx)^{1+m}}{d(1+m)} + \frac{aB(dx)^{2+m}}{d^2(2+m)} + \frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)} + \frac{bB(dx)^{4+m}}{d^4(4+m)} \\ &\quad + \frac{(Ac+bC)(dx)^{5+m}}{d^5(5+m)} + \frac{Bc(dx)^{6+m}}{d^6(6+m)} + \frac{cC(dx)^{7+m}}{d^7(7+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\begin{aligned}\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right) dx = x(dx)^m \left(\frac{aA}{1+m} + \frac{aBx}{2+m} + \frac{(Ab+aC)x^2}{3+m} \right. \\ \left. + \frac{bBx^3}{4+m} + \frac{(Ac+bC)x^4}{5+m} + \frac{Bcx^5}{6+m} + \frac{cCx^6}{7+m}\right)\end{aligned}$$

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(d*x)^m*((a*A)/(1 + m) + (a*B*x)/(2 + m) + ((A*b + a*C)*x^2)/(3 + m) + (b
*B*x^3)/(4 + m) + ((A*c + b*C)*x^4)/(5 + m) + (B*c*x^5)/(6 + m) + (c*C*x^6)
/(7 + m))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

method	result
norman	$\frac{(Ab+Ca)x^3\mathrm{e}^{m\ln(dx)}}{3+m}+\frac{(Ac+Cb)x^5\mathrm{e}^{m\ln(dx)}}{5+m}+\frac{Aax\,\mathrm{e}^{m\ln(dx)}}{1+m}+\frac{Ba\,x^2\mathrm{e}^{m\ln(dx)}}{2+m}+\frac{Bb\,x^4\mathrm{e}^{m\ln(dx)}}{4+m}+\frac{Bc\,x^6\mathrm{e}^{m\ln(dx)}}{6+m}+$
gosper	$\frac{x(Cc\,m^6x^6+Bc\,m^6x^5+21Cc\,m^5x^6+Ac\,m^6x^4+22Bc\,m^5x^5+Cb\,m^6x^4+175Cc\,m^4x^6+23Ac\,m^5x^4+Bb\,m^6x^3+190Bc\,m^4x^5+2}{}$
risch	$\frac{x(Cc\,m^6x^6+Bc\,m^6x^5+21Cc\,m^5x^6+Ac\,m^6x^4+22Bc\,m^5x^5+Cb\,m^6x^4+175Cc\,m^4x^6+23Ac\,m^5x^4+Bb\,m^6x^3+190Bc\,m^4x^5+2}{}$
parallelrisch	$\frac{720C\,x^7(dx)^m c+840B\,x^6(dx)^m c+1008A\,x^5(dx)^m c+1008C\,x^5(dx)^m b+1260B\,x^4(dx)^m b+1680A\,x^3(dx)^m b+1680C\,x^3(dx)^m c}{}$

[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] (A*b+C*a)/(3+m)*x^3*exp(m*ln(d*x))+(A*c+C*b)/(5+m)*x^5*exp(m*ln(d*x))+A*a/(1+m)*x*exp(m*ln(d*x))+B*a/(2+m)*x^2*exp(m*ln(d*x))+B*b/(4+m)*x^4*exp(m*ln(d*x))+B*c/(6+m)*x^6*exp(m*ln(d*x))+C*c/(7+m)*x^7*exp(m*ln(d*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(137) = 274.

Time = 0.30 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.24

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx = \frac{((Ccm^6+21\,Ccm^5+175\,Ccm^4+735\,Ccm^3+1624\,Ccm^2+1764\,Ccm+720\,Cc)x^7+(Bcm^6+22\,Bcm}{}$$

[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] ((C*c*m^6 + 21*C*c*m^5 + 175*C*c*m^4 + 735*C*c*m^3 + 1624*C*c*m^2 + 1764*C*c*m + 720*C*c)*x^7 + (B*c*m^6 + 22*B*c*m^5 + 190*B*c*m^4 + 820*B*c*m^3 + 1849*B*c*m^2 + 2038*B*c*m + 840*B*c)*x^6 + ((C*b + A*c)*m^6 + 23*(C*b + A*c)*m^5 + 207*(C*b + A*c)*m^4 + 925*(C*b + A*c)*m^3 + 2144*(C*b + A*c)*m^2 + 1008*C*b + 1008*A*c + 2412*(C*b + A*c)*m)*x^5 + (B*b*m^6 + 24*B*b*m^5 + 226*B*b*m^4 + 1056*B*b*m^3 + 2545*B*b*m^2 + 2952*B*b*m + 1260*B*b)*x^4 + ((C*a + A*b)*m^6 + 25*(C*a + A*b)*m^5 + 247*(C*a + A*b)*m^4 + 1219*(C*a + A*b)*m^3 + 3112*(C*a + A*b)*m^2 + 1680*C*a + 1680*A*b + 3796*(C*a + A*b)*m)*x^3 + (B*a*m^6 + 26*B*a*m^5 + 270*B*a*m^4 + 1420*B*a*m^3 + 3929*B*a*m^2 + 5274*B*a*m + 2520*B*a)*x^2 + (A*a*m^6 + 27*A*a*m^5 + 295*A*a*m^4 + 1665*A*a*m^3 + 5104*A*a*m^2 + 8028*A*a*m + 5040*A*a)*x)*(d*x)^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3628 vs. $2(122) = 244$.

Time = 0.63 (sec) , antiderivative size = 3628, normalized size of antiderivative = 26.48

$$\int (dx)^m \left(A + Bx + Cx^2\right) \left(a + bx^2 + cx^4\right) \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-A*a/(6*x**6) - A*b/(4*x**4) - A*c/(2*x**2) - B*a/(5*x**5) - B*
b/(3*x**3) - B*c/x - C*a/(4*x**4) - C*b/(2*x**2) + C*c*log(x))/d**7, Eq(m,
-7)), ((-A*a/(5*x**5) - A*b/(3*x**3) - A*c/x - B*a/(4*x**4) - B*b/(2*x**2)
+ B*c*log(x) - C*a/(3*x**3) - C*b/x + C*c*x)/d**6, Eq(m, -6)), ((-A*a/(4*x*
*4) - A*b/(2*x**2) + A*c*log(x) - B*a/(3*x**3) - B*b/x + B*c*x - C*a/(2*x**
2) + C*b*log(x) + C*c*x**2/2)/d**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b/x + A
*c*x - B*a/(2*x**2) + B*b*log(x) + B*c*x**2/2 - C*a/x + C*b*x + C*c*x**3/3)
/d**4, Eq(m, -4)), ((-A*a/(2*x**2) + A*b*log(x) + A*c*x**2/2 - B*a/x + B*b*
x + B*c*x**3/3 + C*a*log(x) + C*b*x**2/2 + C*c*x**4/4)/d**3, Eq(m, -3)), ((
-A*a/x + A*b*x + A*c*x**3/3 + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*a*x
+ C*b*x**3/3 + C*c*x**5/5)/d**2, Eq(m, -2)), ((A*a*log(x) + A*b*x**2/2 + A*
c*x**4/4 + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*a*x**2/2 + C*b*x**4/4 + C*c*
x**6/6)/d, Eq(m, -1)), (A*a*m**6*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 27*A*a*m**5*x*(d*x)**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) + 295*A*a*m**4*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1665*A*a*m**3*x*(d*x)**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
+ 5104*A*a*m**2*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m*
*3 + 13132*m**2 + 13068*m + 5040) + 8028*A*a*m*x*(d*x)**m/(m**7 + 28*m**6 +
 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5040*A*a
*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + A*b*m**6*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 25*A*b*m**5*x**3*(d*x)
**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068
*m + 5040) + 247*A*b*m**4*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m
**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1219*A*b*m**3*x**3*(d*x)**
m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
 + 5040) + 3112*A*b*m**2*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3796*A*b*m*x**3*(d*x)**m/(m
**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) + 1680*A*b*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769
*m**3 + 13132*m**2 + 13068*m + 5040) + A*c*m**6*x**5*(d*x)**m/(m**7 + 28*m*
*6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 23*A
*c*m**5*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
```

13132*m**2 + 13068*m + 5040) + 207*A*c*m**4*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 925*A*c*m**3*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2144*A*c*m**2*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2412*A*c*m*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1008*A*c*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*a*m**6*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 26*B*a*m**5*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 270*B*a*m**4*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1420*B*a*m**3*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3929*B*a*m**2*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5274*B*a*m*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2520*B*a*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*b*m**6*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 24*B*b*m**5*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 226*B*b*m**4*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1056*B*b*m**3*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2545*B*b*m**2*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2952*B*b*m*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1260*B*b*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*c*m**6*x**6*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 22*B*c*m**5*x**6*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 190*B*c*m**4*x**6*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 820*B*c*m**3*x**6*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1849*B*c*m**2*x**6*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2038*B*c*m*x**6*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 840*B*c*x**6*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + C*a*m**6*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 25*C*a*m**5*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 247*C*a*m**4*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1219*C*a*m**3*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3112*C*a*m**2*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m*

```
*5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3796*C*a*m*x**3
*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
 13068*m + 5040) + 1680*C*a*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960
*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + C*b*m**6*x**5*(d*x)**m/(
m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 23*C*b*m**5*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 207*C*b*m**4*x**5*(d*x)**m/(m**7
 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040
) + 925*C*b*m**3*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 676
9*m**3 + 13132*m**2 + 13068*m + 5040) + 2144*C*b*m**2*x**5*(d*x)**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
+ 2412*C*b*m*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m*
*3 + 13132*m**2 + 13068*m + 5040) + 1008*C*b*x**5*(d*x)**m/(m**7 + 28*m**6
+ 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + C*c*m**
6*x**7*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*
m**2 + 13068*m + 5040) + 21*C*c*m**5*x**7*(d*x)**m/(m**7 + 28*m**6 + 322*m*
*5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 175*C*c*m**4*x*
*7*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 735*C*c*m**3*x**7*(d*x)**m/(m**7 + 28*m**6 + 322*m**5
+ 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1624*C*c*m**2*x**7
*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
 13068*m + 5040) + 1764*C*c*m*x**7*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 720*C*c*x**7*(d*x)**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13

$$\int (dx)^m\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx=\frac{Ccd^mx^7x^m}{m+7}+\frac{Bcd^mx^6x^m}{m+6}+\frac{Cbd^mx^5x^m}{m+5}+\frac{Acd^mx^5x^m}{m+5}+\frac{Bbd^mx^4x^m}{m+4}+\frac{Cad^mx^3x^m}{m+3}+\frac{Abd^mx^3x^m}{m+3}+\frac{Bad^mx^2x^m}{m+2}+\frac{(dx)^{m+1}Aa}{d(m+1)}$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] C*c*d^m*x^7*x^m/(m + 7) + B*c*d^m*x^6*x^m/(m + 6) + C*b*d^m*x^5*x^m/(m + 5)
 + A*c*d^m*x^5*x^m/(m + 5) + B*b*d^m*x^4*x^m/(m + 4) + C*a*d^m*x^3*x^m/(m +
 3) + A*b*d^m*x^3*x^m/(m + 3) + B*a*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a
/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(137) = 274$.

Time = 0.38 (sec) , antiderivative size = 914, normalized size of antiderivative = 6.67

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4) \, dx = \frac{(dx)^m Ccm^6x^7 + (dx)^m Bcm^6x^6 + 21 (dx)^m Ccm^5x^7 + (dx)^m Cbm^6x^5 + (dx)^m Acm^6x^5 + 22 (dx)^m Bcm^5}{}$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c*m^6*x^7 + (d*x)^m*B*c*m^6*x^6 + 21*(d*x)^m*C*c*m^5*x^7 + (d*x)
^m*C*b*m^6*x^5 + (d*x)^m*A*c*m^6*x^5 + 22*(d*x)^m*B*c*m^5*x^6 + 175*(d*x)^m
*C*c*m^4*x^7 + (d*x)^m*B*b*m^6*x^4 + 23*(d*x)^m*C*b*m^5*x^5 + 23*(d*x)^m*A*
c*m^5*x^5 + 190*(d*x)^m*B*c*m^4*x^6 + 735*(d*x)^m*C*c*m^3*x^7 + (d*x)^m*C*a
*m^6*x^3 + (d*x)^m*A*b*m^6*x^3 + 24*(d*x)^m*B*b*m^5*x^4 + 207*(d*x)^m*C*b*m
^4*x^5 + 207*(d*x)^m*A*c*m^4*x^5 + 820*(d*x)^m*B*c*m^3*x^6 + 1624*(d*x)^m*C
*c*m^2*x^7 + (d*x)^m*B*a*m^6*x^2 + 25*(d*x)^m*C*a*m^5*x^3 + 25*(d*x)^m*A*b*
m^5*x^3 + 226*(d*x)^m*B*b*m^4*x^4 + 925*(d*x)^m*C*b*m^3*x^5 + 925*(d*x)^m*A
*c*m^3*x^5 + 1849*(d*x)^m*B*c*m^2*x^6 + 1764*(d*x)^m*C*c*m*x^7 + (d*x)^m*A*
a*m^6*x + 26*(d*x)^m*B*a*m^5*x^2 + 247*(d*x)^m*C*a*m^4*x^3 + 247*(d*x)^m*A*
b*m^4*x^3 + 1056*(d*x)^m*B*b*m^3*x^4 + 2144*(d*x)^m*C*b*m^2*x^5 + 2144*(d*x
)^m*A*c*m^2*x^5 + 2038*(d*x)^m*B*c*m*x^6 + 720*(d*x)^m*C*c*x^7 + 27*(d*x)^m
*A*a*m^5*x + 270*(d*x)^m*B*a*m^4*x^2 + 1219*(d*x)^m*C*a*m^3*x^3 + 1219*(d*x
)^m*A*b*m^3*x^3 + 2545*(d*x)^m*B*b*m^2*x^4 + 2412*(d*x)^m*C*b*m*x^5 + 2412*
(d*x)^m*A*c*m*x^5 + 840*(d*x)^m*B*c*x^6 + 295*(d*x)^m*A*a*m^4*x + 1420*(d*x
)^m*B*a*m^3*x^2 + 3112*(d*x)^m*C*a*m^2*x^3 + 3112*(d*x)^m*A*b*m^2*x^3 + 295
2*(d*x)^m*B*b*m*x^4 + 1008*(d*x)^m*C*b*x^5 + 1008*(d*x)^m*A*c*x^5 + 1665*(d
*x)^m*A*a*m^3*x + 3929*(d*x)^m*B*a*m^2*x^2 + 3796*(d*x)^m*C*a*m*x^3 + 3796*
(d*x)^m*A*b*m*x^3 + 1260*(d*x)^m*B*b*x^4 + 5104*(d*x)^m*A*a*m^2*x + 5274*(d
*x)^m*B*a*m*x^2 + 1680*(d*x)^m*C*a*x^3 + 1680*(d*x)^m*A*b*x^3 + 8028*(d*x)^
m*A*a*m*x + 2520*(d*x)^m*B*a*x^2 + 5040*(d*x)^m*A*a*x)/(m^7 + 28*m^6 + 322*
m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

Mupad [B] (verification not implemented)

Time = 7.95 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.85

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx$$
$$= \frac{x^3\,(d\,x)^m\,(A\,b+C\,a)\,\left(m^6+25\,m^5+247\,m^4+1219\,m^3+3112\,m^2+3796\,m+1680\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+\frac{x^5\,(d\,x)^m\,(A\,c+C\,b)\,\left(m^6+23\,m^5+207\,m^4+925\,m^3+2144\,m^2+2412\,m+1008\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+\frac{A\,a\,x\,(d\,x)^m\,\left(m^6+27\,m^5+295\,m^4+1665\,m^3+5104\,m^2+8028\,m+5040\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+\frac{B\,a\,x^2\,(d\,x)^m\,\left(m^6+26\,m^5+270\,m^4+1420\,m^3+3929\,m^2+5274\,m+2520\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+\frac{B\,b\,x^4\,(d\,x)^m\,\left(m^6+24\,m^5+226\,m^4+1056\,m^3+2545\,m^2+2952\,m+1260\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+\frac{B\,c\,x^6\,(d\,x)^m\,\left(m^6+22\,m^5+190\,m^4+820\,m^3+1849\,m^2+2038\,m+840\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+\frac{C\,c\,x^7\,(d\,x)^m\,\left(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (x^3*(d*x)^m*(A*b + C*a)*(3796*m + 3112*m^2 + 1219*m^3 + 247*m^4 + 25*m^5 +
 m^6 + 1680))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6
 + m^7 + 5040) + (x^5*(d*x)^m*(A*c + C*b)*(2412*m + 2144*m^2 + 925*m^3 + 20
7*m^4 + 23*m^5 + m^6 + 1008))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 +
322*m^5 + 28*m^6 + m^7 + 5040) + (A*a*x*(d*x)^m*(8028*m + 5104*m^2 + 1665*m
^3 + 295*m^4 + 27*m^5 + m^6 + 5040))/(13068*m + 13132*m^2 + 6769*m^3 + 1960
*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*a*x^2*(d*x)^m*(5274*m + 3929*m^2
 + 1420*m^3 + 270*m^4 + 26*m^5 + m^6 + 2520))/(13068*m + 13132*m^2 + 6769*m
^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*b*x^4*(d*x)^m*(2952*m +
 2545*m^2 + 1056*m^3 + 226*m^4 + 24*m^5 + m^6 + 1260))/(13068*m + 13132*m^2
 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*c*x^6*(d*x)^m*
(2038*m + 1849*m^2 + 820*m^3 + 190*m^4 + 22*m^5 + m^6 + 840))/(13068*m + 13
132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (C*c*x^7*(
d*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068
*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
```

3.40 $\int \frac{(dx)^m(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 368

$$\begin{aligned}
\int \frac{(dx)^m\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx
&= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)}\\
&\quad+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)d(1+m)}\\
&\quad+\frac{2Bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d^2(2+m)}\\
&\quad-\frac{2Bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d^2(2+m)}
\end{aligned}$$

```
[Out] (d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))*(C+(2*A*c-C*b)/(-4*a*c+b^2)^(1/2))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))+2*
B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))/d^2/(2+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+(d*x)^(1+m)*hype
rgeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(C+(-2*A*
c+C*b)/(-4*a*c+b^2)^(1/2))/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))-2*B*c*(d*x)^(2+m)
*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d^2/(2+m
)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1676, 1299, 371, 12, 1145}

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4} \, dx$$
$$= \frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{2Bc(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d^2(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$
$$-\frac{2Bc(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d^2(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) + (2*B*c*(d*x)^(2 + m)*Hypergeometr
ic2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b
^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*(2 + m)) - (2*B*c*(d*x)^(2 + m)*Hyp
ergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1145

```
Int[((d_.)*(x_))^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{B(dx)^{1+m}}{a+bx^2+cx^4}\,dx}{d} + \int \frac{(dx)^m\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\
&\quad + \frac{1}{2}\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{B\int \frac{(dx)^{1+m}}{a+bx^2+cx^4}\,dx}{d} \\
&= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)} \\
&\quad + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)d(1+m)} \\
&\quad + \frac{(Bc)\int \frac{(dx)^{1+m}}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}d} - \frac{(Bc)\int \frac{(dx)^{1+m}}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}d}
\end{aligned}
$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)d(1+m)} + \frac{2Bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d^2(2+m)} - \frac{2Bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d^2(2+m)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 2.26 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.19

$$\int \frac{(dx)^m\,(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{(dx)^m\left(A(2+3m+m^2)\,\text{RootSum}\left[a+b\#1^2+c\#1^4\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1+2c\#1^3}\&\right]\right.}{}$$

```
[In] Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d*x)^m*(A*(2 + 3*m + m^2)*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2
F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ] +
B*(2 + m)*RootSum[a + b*#1^2 + c*#1^4 & , (m*x + (Hypergeometric2F1[-m, -m,
 1 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m + (m*Hypergeometric2F1[-m, -m, 1
 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m)/(b*#1 + 2*c*#1^3) & ] + C*RootSum
[a + b*#1^2 + c*#1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hyperg
eometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hype
rgeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hy
pergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1
^2)/(x/#1)^m)/(b*#1 + 2*c*#1^3) & ]))/(2*m*(1 + m)*(2 + m))
```

Maple [F]

$$\int \frac{(dx)^m\,(C\,x^2+Bx+A)}{c\,x^4+b\,x^2+a}dx$$

```
[In] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(dx)^m\,(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)(dx)^m}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(dx)^m\,(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(dx)^m\,(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
[Out] Integral((d*x)**m*(A + B*x + C*x**2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(dx)^m\,(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)(dx)^m}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)(dx)^m}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(d\,x)^m\,(C\,x^2+B\,x+A)}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
[Out] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x)
```

3.41 $\int \frac{(dx)^m(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 685

$$\int \frac{(dx)^m\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{B(dx)^{2+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d^2\left(a+bx^2+cx^4\right)} + \frac{(dx)^{1+m}\left(A(b^2-2ac)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)}$$
$$+\frac{c\left(2aC\left(2b-\sqrt{b^2-4ac}(1-m)\right)+A\left(b^2(1-m)+b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\right)\right)(dx)^{1+m}\,\text{Hyperg}}{2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}$$
$$-\frac{c\left(2aC\left(2b+\sqrt{b^2-4ac}(1-m)\right)+A\left(b^2(1-m)-b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\right)\right)(dx)^{1+m}\,\text{Hyperg}}{2a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$
$$-\frac{Bc\left(4ac(2-m)+b\left(b+\sqrt{b^2-4ac}\right)m\right)(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d^2(2+m)}$$
$$+\frac{Bc\left(4ac(2-m)+b\left(b-\sqrt{b^2-4ac}\right)m\right)(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d^2(2+m)}$$

```
[Out] 1/2*B*(d*x)^(2+m)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d^2/(c*x^4+b*x^2+a)+1/
2*(d*x)^(1+m)*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/d/(c*
x^4+b*x^2+a)+1/2*B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(2-m)+b*m*(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+
b^2)^(3/2)/d^2/(2+m)/(b+(-4*a*c+b^2)^(1/2))-1/2*B*c*(d*x)^(2+m)*hypergeom([
1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(4*a*c*(2-m)+b*m*(b+
(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d^2/(2+m)/(b-(-4*a*c+b^2)^(1/2))-
1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))*(2*a*C*(2*b+(1-m)*(-4*a*c+b^2)^(1/2))+A*(b^2*(1-m)-4*a*c*(3-m)
```

```
-b*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b+(-4*a*c+b^2)^
(1/2))+1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)))*(2*a*C*(2*b-(1-m)*(-4*a*c+b^2)^(1/2))+A*(b^2*(1-m)-4*a*
c*(3-m)+b*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*
c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1676, 1291, 1299, 371, 12, 1135}

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$$
$$= \frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right)\text{Hypergeon}}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$
$$-\frac{c(dx)^{m+1}\left(-(1-m)\sqrt{b^2-4ac}(Ab-2aC)-4aAc(3-m)+4abC+Ab^2(1-m)\right)\text{Hypergeometric2F1}}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{(dx)^{m+1}\left(A(b^2-2ac)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$-\frac{Bc(dx)^{m+2}\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad^2(m+2)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{Bc(dx)^{m+2}\left(bm\left(b-\sqrt{b^2-4ac}\right)+4ac(2-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad^2(m+2)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{B(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(d*x)^(2 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d^2*(a + b*x^2
 + c*x^4)) + ((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)
)/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) + (c*(2*a*C*(2*b - Sqrt[b^2 - 4
*a*c]*(1 - m)) + A*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 -
m)))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*
(1 + m)) - (c*(4*a*b*C + A*b^2*(1 - m) - Sqrt[b^2 - 4*a*c]*(A*b - 2*a*C)*(1
 - m) - 4*a*A*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)) - (B*c*(4*a*c*(2 - m) + b*(b + Sqrt[b^2 - 4*a*
c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/
```

```
(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*
d^2*(2 + m)) + (B*c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2
+ m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{B(dx)^{1+m}}{(a+bx^2+cx^4)^2}\,dx}{d} + \int \frac{(dx)^m\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{(dx)^{1+m}\left(A(b^2-2ac)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} - \frac{\int \frac{(dx)^m\left(-Ab^2(1-m)+2aAc(3-m)-abC(1+m)-c(Ab-2aC)(1-m)x^2\right)}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)} + \frac{B\int \frac{(dx)^{1+m}}{(a+bx^2+cx^4)^2}\,dx}{d}$$

$$= \frac{B(dx)^{2+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d^2\left(a+bx^2+cx^4\right)} + \frac{(dx)^{1+m}\left(A(b^2-2ac)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} - \frac{B\int \frac{(dx)^{1+m}\left(2ac(2-m)+b^2m+bcmx^2\right)}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)d} - \frac{\left(c\left(4abC+Ab^2(1-m)-\sqrt{b^2-4ac}(Ab-2aC)(1-m)-4aAc(3-m)\right)\right)\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} + \frac{\left(c\left(4abC+Ab^2(1-m)+\sqrt{b^2-4ac}(Ab-2aC)(1-m)-4aAc(3-m)\right)\right)\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{B(dx)^{2+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d^2\left(a+bx^2+cx^4\right)} + \frac{(dx)^{1+m}\left(A(b^2-2ac)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} + \frac{c\left(4abC+Ab^2(1-m)+\sqrt{b^2-4ac}(Ab-2aC)(1-m)-4aAc(3-m)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+}{2}\right.}{2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{c\left(4abC+Ab^2(1-m)-\sqrt{b^2-4ac}(Ab-2aC)(1-m)-4aAc(3-m)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+}{2}\right.}{2a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)} + \frac{\left(Bc\left(4ac(2-m)+b\left(b-\sqrt{b^2-4ac}\right)m\right)\right)\int \frac{(dx)^{1+m}}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}d} - \frac{\left(Bc\left(4ac(2-m)+b\left(b+\sqrt{b^2-4ac}\right)m\right)\right)\int \frac{(dx)^{1+m}}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}d}$$

$$= \frac{B(dx)^{2+m}(b^2-2ac+bcx^2)}{2a(b^2-4ac)d^2(a+bx^2+cx^4)} + \frac{(dx)^{1+m}(A(b^2-2ac)-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)d(a+bx^2+cx^4)}$$
$$+ \frac{c\left(4abC+Ab^2(1-m)+\sqrt{b^2-4ac}(Ab-2aC)(1-m)-4aAc(3-m)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3}{}\right.}{2a(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}$$
$$- \frac{c\left(4abC+Ab^2(1-m)-\sqrt{b^2-4ac}(Ab-2aC)(1-m)-4aAc(3-m)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3}{}\right.}{2a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$
$$- \frac{Bc\left(4ac(2-m)+b\left(b+\sqrt{b^2-4ac}\right)m\right)(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2a(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d^2(2+m)}$$
$$+ \frac{Bc\left(4ac(2-m)+b\left(b-\sqrt{b^2-4ac}\right)m\right)(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d^2(2+m)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.35

$$\int \frac{(dx)^m(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{x(dx)^m\left(A(6+5m+m^2)\,\text{AppellF1}\left(\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+(1+m)x\left(B(3+m)\,\text{App}\right.\right.}{a^2(1+}$$

```
[In] Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*(A*(6 + 5*m + m^2)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x
*(B*(3 + m)*AppellF1[(2 + m)/2, 2, 2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + C*(2 + m)*x*AppellF1[(3 + m
)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])))/(a^2*(1 + m)*(2 + m)*(3 + m))
```

Maple [F]

$$\int \frac{(dx)^m(C\,x^2+Bx+A)}{(c\,x^4+b\,x^2+a)^2}dx$$

```
[In] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx = \int \frac{(Cx^2+Bx+A)(dx)^m}{(cx^4+bx^2+a)^2} dx$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4
 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx = \text{Timed out}$$

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx = \int \frac{(Cx^2+Bx+A)(dx)^m}{(cx^4+bx^2+a)^2} dx$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx = \int \frac{(Cx^2+Bx+A)(dx)^m}{(cx^4+bx^2+a)^2} dx$$

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m\,(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(d\,x)^m\,(C\,x^2+B\,x+A)}{(c\,x^4+b\,x^2+a)^2}\,dx$$

```
[In] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
[Out] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

3.42 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{B(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C + 
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2(A+Cx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)} - \frac{\left(2Ac-bC+\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(bB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(bB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)\,(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.00 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2 Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln\left(x-\_\right.}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2 Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aa\,c^2-2A\,b^2c+4C\right.}{}}{4c\left(4ac\right.}\right.}{}$

[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^2(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} \, dx = \int \frac{(Cx^2 + Bx + A)x^2}{(cx^4 + bx^2 + a)^2} \, dx$$

[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. $2(306) = 612$.

Time = 1.68 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{x^2(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a

^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt

```
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
```

^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153

6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 + 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)

3.43 $\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 356

$$\begin{aligned}\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx &= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\ &\quad - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} \\ &\quad - \frac{bB\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1599, 1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx = -\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{B(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
```

```
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(bB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(bB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aa\,c^2-2A\,b^2c+4C\sqrt{\cdots}\right.}{\cdots}}{4c\left(4ac-b\cdots\right.}\right.}{\cdots}$

[In] int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

[In] integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} \, dx = \int \frac{(Cx^3 + Bx^2 + Ax)x}{(cx^4 + bx^2 + a)^2} \, dx$$

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. 2(306) = 612.

Time = 1.69 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
```

```
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
```

```
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] int((x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x)
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
```

^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
- 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
+ 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153

```
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.44 $\int \frac{Ax^2+Bx^3+Cx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 356

$$\int \frac{Ax^2+Bx^3+Cx^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {1608, 1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx = -\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
```

```
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(bB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(bB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln(x-\_}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aa\,c^2-2A\,b^2c+4C\right.}{}}{4c(4ac}\right.}{}$

[In] int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

[In] integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} \, dx = \int \frac{Cx^4 + Bx^3 + Ax^2}{(cx^4 + bx^2 + a)^2} \, dx$$

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. $2(306) = 612$.

Time = 1.51 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
```

^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt

```
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 8.13 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
```

^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153

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6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.45 $\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 356

$$\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1599, 1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx = -\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
```

```
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(bB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(bB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln\left(x-\_R\right.}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aa\,c^2-2A\,b^2c+4C\sqrt{\cdots}\right.}{\cdots}}{4c\left(4ac-b\cdots\right.}\right.}{\cdots}$

[In] int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

[In] integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x (a + bx^2 + cx^4)^2} \, dx = \int \frac{Cx^5 + Bx^4 + Ax^3}{(cx^4 + bx^2 + a)^2 x} \, dx$$

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. 2(306) = 612.

Time = 1.42 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
```

```
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
```

```
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 8.19 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
```

^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153

```
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
 + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.46 $\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 356

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*C*a+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-C*b+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-C*b+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1599, 1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{bB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
```

```
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(bB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(bB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

```
[In] Integrate[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2Ca}{4ac-b^2}\right)\ln(x-\_ }{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{x^2Bb}{2(4ac-b^2)}+\frac{(Ab-2Ca)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4Abc\sqrt{-4ac+b^2}+8Aa\,c^2-2A\,b^2c+4C\right.}{4c\left(4ac\right.}\right.}{}$

```
[In] int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*B*b+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-1/(4*a*c-b^2)*B*a)/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-
b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(
x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \int \frac{Cx^6 + Bx^5 + Ax^4}{(cx^4 + bx^2 + a)^2 x^2} \, dx$$

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4438 vs. $2(306) = 612$.

Time = 1.50 (sec) , antiderivative size = 4438, normalized size of antiderivative = 12.47

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
```

^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt

```
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
```

^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 - 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153

6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 + 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)

3.47 $\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 273

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4}+\frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3}+\frac{(ce-bf)x^6}{6c^2}+\frac{fx^8}{8c} - \frac{(b^4ce-4ab^2c^2e+2a^2c^3e-b^5f-b^3c(cd-5af)+abc^2(3cd-5af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^5\sqrt{b^2-4ac}} - \frac{(b^3ce-2abc^2e-b^4f-b^2c(cd-3af)+ac^2(cd-af))\log(a+bx^2+cx^4)}{4c^5}$$

```
[Out] 1/2*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*x^2/c^4+1/4*(c^2*d+b^2*f-c*(a*
f+b*e))*x^4/c^3+1/6*(-b*f+c*e)*x^6/c^2+1/8*f*x^8/c-1/4*(b^3*c*e-2*a*b*c^2*e
-b^4*f-b^2*c*(-3*a*f+c*d)+a*c^2*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/c^5-1/2*(b^4*
c*e-4*a*b^2*c^2*e+2*a^2*c^3*e-b^5*f-b^3*c*(-5*a*f+c*d)+a*b*c^2*(-5*a*f+3*c*
d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1677, 1642, 648, 632, 212, 642\}$

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-b^3c(cd-5af)-4ab^2c^2e+abc^2(3cd-5af)+b^5(-f)+b^4ce\right)}{2c^5\sqrt{b^2-4ac}} + \frac{x^4(-c(af+be)+b^2f+c^2d)}{4c^3} + \frac{x^2(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce)}{2c^4} - \frac{\log\left(a+bx^2+cx^4\right)\left(-b^2c(cd-3af)-2abc^2e+ac^2(cd-af)+b^4(-f)+b^3ce\right)}{4c^5} + \frac{x^6(ce-bf)}{6c^2} + \frac{fx^8}{8c}$$

```
[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/(2*c^4) + ((c^2*d + b
^2*f - c*(b*e + a*f))*x^4)/(4*c^3) + ((c*e - b*f)*x^6)/(6*c^2) + (f*x^8)/(8
*c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e - b^5*f - b^3*c*(c*d - 5*a*f)
 + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
5*Sqrt[b^2 - 4*a*c]) - ((b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f
) + a*c^2*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(d+ex+fx^2)}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2ce-ac^2e-b^3f-bc(cd-2af)}{c^4}+\frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^2}{c^2}+\frac{fx^3}{c}+\frac{-a(b^2ce-ac^2e-b^3f-bc(cd-2af))-(b^3ce-2abc^2e-b^4f-b^2c(cd-3af)+ac^2(cd-af)}{c^4\,(a+bx+cx^2)}\right.\right.$$

$$= \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4}+\frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3}+\frac{(ce-bf)x^6}{6c^2}+\frac{fx^8}{8c}+\frac{\text{Subst}\left(\int \frac{-a(b^2ce-ac^2e-b^3f-bc(cd-2af))-(b^3ce-2abc^2e-b^4f-b^2c(cd-3af)+ac^2(cd-af))x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^4}$$

$$= \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4}+\frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3}+\frac{(ce-bf)x^6}{6c^2}+\frac{fx^8}{8c}+\frac{(b^4ce-4ab^2c^2e+2a^2c^3e-b^5f-b^3c(cd-5af)+abc^2(3cd-5af))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx\right.}{4c^5}-\frac{(b^3ce-2abc^2e-b^4f-b^2c(cd-3af)+ac^2(cd-af))\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^5}$$

$$= \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4}+\frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3}+\frac{(ce-bf)x^6}{6c^2}+\frac{fx^8}{8c}-\frac{(b^3ce-2abc^2e-b^4f-b^2c(cd-3af)+ac^2(cd-af))\log\left(a+bx^2+cx^4\right)}{4c^5}-\frac{(b^4ce-4ab^2c^2e+2a^2c^3e-b^5f-b^3c(cd-5af)+abc^2(3cd-5af))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b\right.}{2c^5}$$

$$= \frac{(b^2ce - ac^2e - b^3f - bc(cd - 2af))\,x^2}{2c^4} + \frac{(c^2d + b^2f - c(be + af))\,x^4}{4c^3} + \frac{(ce - bf)x^6}{6c^2} + \frac{fx^8}{8c} - \frac{(b^4ce - 4ab^2c^2e + 2a^2c^3e - b^5f - b^3c(cd - 5af) + abc^2(3cd - 5af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^5\sqrt{b^2 - 4ac}} - \frac{(b^3ce - 2abc^2e - b^4f - b^2c(cd - 3af) + ac^2(cd - af))\log(a + bx^2 + cx^4)}{4c^5}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int \frac{x^7(d + ex^2 + fx^4)}{a + bx^2 + cx^4}\,dx$$

$$= \frac{-12c(-b^2ce + ac^2e + b^3f + bc(cd - 2af))\,x^2 + 6c^2(c^2d + b^2f - c(be + af))\,x^4 + 4c^3(ce - bf)x^6 + 3c^4fx^8}{}$$

```
[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-12*c*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*x^2 + 6*c^2*(c^2*
d + b^2*f - c*(b*e + a*f))*x^4 + 4*c^3*(c*e - b*f)*x^6 + 3*c^4*f*x^8 - (12*
(-(b^4*c*e) + 4*a*b^2*c^2*e - 2*a^2*c^3*e + b^5*f + b^3*c*(c*d - 5*a*f) + a
*b*c^2*(-3*c*d + 5*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^
2 + 4*a*c] + 6*(-(b^3*c*e) + 2*a*b*c^2*e + b^4*f + b^2*c*(c*d - 3*a*f) + a*
c^2*(-(c*d) + a*f))*Log[a + b*x^2 + c*x^4])/(24*c^5)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.21

method	result
default	$\frac{\frac{1}{4}f\,x^8c^3-\frac{1}{3}b\,c^2f\,x^6+\frac{1}{3}c^3e\,x^6-\frac{1}{2}a\,c^2f\,x^4+\frac{1}{2}b^2cf\,x^4-\frac{1}{2}b\,c^2e\,x^4+\frac{1}{2}c^3d\,x^4+2abcf\,x^2-a\,c^2e\,x^2-b^3f\,x^2+b^2ce\,x^2-b\,c^2d\,x^2}{2c^4}+\frac{(a^2c^2f-3a}{}$
risch	Expression too large to display

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^4*(1/4*f*x^8*c^3-1/3*b*c^2*f*x^6+1/3*c^3*e*x^6-1/2*a*c^2*f*x^4+1/2*b^
2*c*f*x^4-1/2*b*c^2*e*x^4+1/2*c^3*d*x^4+2*a*b*c*f*x^2-a*c^2*e*x^2-b^3*f*x^2
+b^2*c*e*x^2-b*c^2*d*x^2)+1/2/c^4*(1/2*(a^2*c^2*f-3*a*b^2*c*f+2*a*b*c^2*e-a
*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(-2*a^2*b*c*f+a^2*c^2
*e+a*b^3*f-a*b^2*c*e+a*b*c^2*d-1/2*(a^2*c^2*f-3*a*b^2*c*f+2*a*b*c^2*e-a*c^3
```

```
*d+b^4*f-b^3*c*e+b^2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.30

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{3\,(b^2c^4-4\,ac^5)fx^8+4\,((b^2c^4-4\,ac^5)e-(b^3c^3-4\,abc^4)f)x^6+6\,((b^2c^4-4\,ac^5)d-(b^3c^3-4\,abc^4)e+(\right.$$

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 -
4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e + (b
^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d - (b
^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*f
)*x^2 + 6*sqrt(b^2 - 4*a*c)*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2
 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*log((2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
+ 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b
*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x
^2 + a))/(b^2*c^5 - 4*a*c^6), 1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c
^4 - 4*a*c^5)*e - (b^3*c^3 - 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d -
 (b^3*c^3 - 4*a*b*c^4)*e + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*
((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c -
 6*a*b^3*c^2 + 8*a^2*b*c^3)*f)*x^2 + 12*sqrt(-b^2 + 4*a*c)*((b^3*c^2 - 3*a*
b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b
*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 6*((b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e +
(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x^2 + a))/(
b^2*c^5 - 4*a*c^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.09

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{3\,c^3fx^8+4\,c^3ex^6-4\,bc^2fx^6+6\,c^3dx^4-6\,bc^2ex^4+6\,b^2cfx^4-6\,ac^2fx^4-12\,bc^2dx^2+12\,b^2cex^2-12\,ac^2e}{24\,c^4} + \frac{(b^2c^2d-ac^3d-b^3ce+2\,abc^2e+b^4f-3\,ab^2cf+a^2c^2f)\log\left(cx^4+bx^2+a\right)}{4\,c^5} - \frac{(b^3c^2d-3\,abc^3d-b^4ce+4\,ab^2c^2e-2\,a^2c^3e+b^5f-5\,ab^3cf+5\,a^2bc^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^5}$$

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/24*(3*c^3*f*x^8 + 4*c^3*e*x^6 - 4*b*c^2*f*x^6 + 6*c^3*d*x^4 - 6*b*c^2*e*x
^4 + 6*b^2*c*f*x^4 - 6*a*c^2*f*x^4 - 12*b*c^2*d*x^2 + 12*b^2*c*e*x^2 - 12*a
*c^2*e*x^2 - 12*b^3*f*x^2 + 24*a*b*c*f*x^2)/c^4 + 1/4*(b^2*c^2*d - a*c^3*d
- b^3*c*e + 2*a*b*c^2*e + b^4*f - 3*a*b^2*c*f + a^2*c^2*f)*log(c*x^4 + b*x^
2 + a)/c^5 - 1/2*(b^3*c^2*d - 3*a*b*c^3*d - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2
*c^3*e + b^5*f - 5*a*b^3*c*f + 5*a^2*b*c^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^
2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 2972, normalized size of antiderivative = 10.89

$$\int \frac{x^7(d + ex^2 + fx^4)}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

```
[In] int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^6*(e/(6*c) - (b*f)/(6*c^2)) - x^4*((b*(e/c - (b*f)/c^2))/(4*c) - d/(4*c)
+ (a*f)/(4*c^2)) - x^2*((a*(e/c - (b*f)/c^2))/(2*c) - (b*((b*(e/c - (b*f)/c
^2))/c - d/c + (a*f)/c^2))/(2*c)) + (f*x^8)/(8*c) - (log(a + b*x^2 + c*x^4)
*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^
2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))
/(2*(16*a*c^6 - 4*b^2*c^5)) + (atan((2*c^8*(4*a*c - b^2)*(x^2*(((((4*a^2*c^
8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d - 16*a*b^2*c^7
*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*f + 8*a^2*c^4*d
 + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f
- 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 - 4*b^2*c^5)
)*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f +
4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) - (b*(b^5*f - 2
*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*
e + 5*a^2*b*c^2*f)*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b
^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e
- 16*a^2*b*c^3*e))/(2*c^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4*b^2*c^5)))/a -
(b*((((4*a^2*c^8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d
 - 16*a*b^2*c^7*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*
f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f
- 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c
^6 - 4*b^2*c^5))*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5
*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e -
16*a^2*b*c^3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) - (b^9*f^2 + b^5*c^4*d^2 + b^7*
c^2*e^2 - 3*a*b^3*c^5*d^2 + 2*a^2*b*c^6*d^2 - 5*a*b^5*c^3*e^2 - 2*a^3*b*c^5
*e^2 + 3*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 7*a^2*b^3*c^4*e^2 + 16*a^2*b^5*c^2*f
^2 - 13*a^3*b^3*c^3*f^2 - 7*a*b^7*c*f^2 + a^3*c^6*d*e - 2*b^6*c^3*d*e - a^4
*c^5*e*f + 2*b^7*c^2*d*f + 8*a*b^4*c^4*d*e - 10*a*b^5*c^3*d*f - 5*a^3*b*c^5
*d*f + 12*a*b^6*c^2*e*f - 8*a^2*b^2*c^5*d*e + 14*a^2*b^3*c^4*d*f - 22*a^2*b
^4*c^3*e*f + 12*a^3*b^2*c^4*e*f)/c^8 + (b*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d
- b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)^2)/(
2*c^8*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a^3*c^7*f - 8*a^2
*c^8*d - 24*a^2*b^2*c^6*f + 8*a*b^2*c^7*d - 8*a*b^3*c^6*e + 16*a^2*b*c^7*e
+ 8*a*b^4*c^5*f)/c^8 + (8*a*c^2*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^
3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12
*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 - 4*b^2*c^5))*(b^5*f - 2*a^2*c^3*
e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2
```

```
*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) + (a*(b^5*f - 2*a^2*c^3*e + b^3*c^2*
d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)*(2
*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c
^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(c
^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4*b^2*c^5)))/a + (b*((a*b^8*f^2 + a^3*c^
6*d^2 + a^5*c^4*f^2 + a*b^4*c^4*d^2 + a*b^6*c^2*e^2 - 6*a^2*b^6*c*f^2 - 2*a
^2*b^2*c^5*d^2 - 4*a^2*b^4*c^3*e^2 + 4*a^3*b^2*c^4*e^2 + 11*a^3*b^4*c^2*f^2
 - 6*a^4*b^2*c^3*f^2 - 2*a^4*c^5*d*f - 2*a*b^5*c^3*d*e - 4*a^3*b*c^5*d*e +
2*a*b^6*c^2*d*f + 4*a^4*b*c^4*e*f + 6*a^2*b^3*c^4*d*e - 8*a^2*b^4*c^3*d*f +
 8*a^3*b^2*c^4*d*f + 10*a^2*b^5*c^2*e*f - 14*a^3*b^3*c^3*e*f - 2*a*b^7*c*e*
f)/c^8 + (((8*a^3*c^7*f - 8*a^2*c^8*d - 24*a^2*b^2*c^6*f + 8*a*b^2*c^7*d -
8*a*b^3*c^6*e + 16*a^2*b*c^7*e + 8*a*b^4*c^5*f)/c^8 + (8*a*c^2*(2*b^6*f + 8
*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*
a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 -
4*b^2*c^5))*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e
+ 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^
2*b*c^3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) - (a*(b^5*f - 2*a^2*c^3*e + b^3*c^2*
d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)^2)
/(c^8*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^10*f^2 + 4*a^4*c^6*e^
2 + b^6*c^4*d^2 + b^8*c^2*e^2 - 6*a*b^4*c^5*d^2 - 8*a*b^6*c^3*e^2 - 2*b^9*c
*e*f + 9*a^2*b^2*c^6*d^2 + 20*a^2*b^4*c^4*e^2 - 16*a^3*b^2*c^5*e^2 + 35*a^2
*b^6*c^2*f^2 - 50*a^3*b^4*c^3*f^2 + 25*a^4*b^2*c^4*f^2 - 10*a*b^8*c*f^2 - 2
*b^7*c^3*d*e + 2*b^8*c^2*d*f + 14*a*b^5*c^4*d*e + 12*a^3*b*c^6*d*e - 16*a*b
^6*c^3*d*f + 18*a*b^7*c^2*e*f - 20*a^4*b*c^5*e*f - 28*a^2*b^3*c^5*d*e + 40*
a^2*b^4*c^4*d*f - 30*a^3*b^2*c^5*d*f - 54*a^2*b^5*c^3*e*f + 60*a^3*b^3*c^4*
e*f))*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*
f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c^5*(4*a*c - b^2)^(1/2))
```

3.48 $\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 203

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\sqrt{b^2-4ac}}+\frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\log\left(a+bx^2+cx^4\right)}{4c^4}$$

```
[Out] 1/2*(c^2*d+b^2*f-c*(a*f+b*e))*x^2/c^3+1/4*(-b*f+c*e)*x^4/c^2+1/6*f*x^6/c+1/
4*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*ln(c*x^4+b*x^2+a)/c^4+1/2*(b^3*c
*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))*arctanh((2*c*x^
2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{1677, 1642, 648, 632, 212, 642\}$

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)}{2c^4\sqrt{b^2-4ac}} + \frac{x^2(-c(af+be)+b^2f+c^2d)}{2c^3} + \frac{\log\left(a+bx^2+cx^4\right)\left(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{4c^4} + \frac{x^4(ce-bf)}{4c^2} + \frac{fx^6}{6c}$$

```
[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/(2*c^3) + ((c*e - b*f)*x^4)/(4*c^2) +
 (f*x^6)/(6*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*
a*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*Sqrt[b^
2 - 4*a*c]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^
2 + c*x^4])/(4*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+ex+fx^2)}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\Bigg(\int \Bigg(\frac{c^2d+b^2f-c(be+af)}{c^3}+\frac{(ce-bf)x}{c^2}+\frac{fx^2}{c} \\
&\quad -\frac{a(c^2d+b^2f-c(be+af))-(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x}{c^3\,(a+bx+cx^2)}\Bigg)\,dx, x, x^2\Bigg) \\
&= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c} \\
&\quad -\frac{\text{Subst}\left(\int \frac{a(c^2d+b^2f-c(be+af))-(b^2ce-ac^2e-b^3f-bc(cd-2af))x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^3} \\
&= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c} \\
&\quad +\frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^4} \\
&\quad -\frac{(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^4} \\
&= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c} \\
&\quad +\frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\log\left(a+bx^2+cx^4\right)}{4c^4} \\
&\quad +\frac{(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^4} \\
&= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c} \\
&\quad +\frac{(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\sqrt{b^2-4ac}} \\
&\quad +\frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\log\left(a+bx^2+cx^4\right)}{4c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$

$$= \frac{6c(c^2d+b^2f-c(be+af))\,x^2+3c^2(ce-bf)x^4+2c^3fx^6+\frac{6(-b^3ce+3abc^2e+b^4f+b^2c(cd-4af)+2ac^2(-cd+af))\arctan\Big(}{\sqrt{-b^2+4ac}}}{12c^4}$$

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*c*(c^2*d + b^2*f - c*(b*e + a*f))*x^2 + 3*c^2*(c*e - b*f)*x^4 + 2*c^3*f*
x^6 + (6*(-(b^3*c*e) + 3*a*b*c^2*e + b^4*f + b^2*c*(c*d - 4*a*f) + 2*a*c^2*
(-(c*d) + a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] - 3*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*
x^4])/(12*c^4)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{3}f\,x^6c^2+\frac{1}{2}bcf\,x^4-\frac{1}{2}c^2e\,x^4+acf\,x^2-b^2f\,x^2+bce\,x^2-c^2d\,x^2}{2c^3}+\frac{\frac{(2abcf-a\,c^2e-b^3f+b^2ce-b\,c^2d)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\big(a^2cf-a\,b^2f+a}{}}{2c^3}$
risch	Expression too large to display

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^3*(-1/3*f*x^6*c^2+1/2*b*c*f*x^4-1/2*c^2*e*x^4+a*c*f*x^2-b^2*f*x^2+b*
c*e*x^2-c^2*d*x^2)+1/2/c^3*(1/2*(2*a*b*c*f-a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/c
*ln(c*x^4+b*x^2+a)+2*(a^2*c*f-a*b^2*f+a*b*c*e-a*c^2*d-1/2*(2*a*b*c*f-a*c^2*
e-b^3*f+b^2*c*e-b*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.33

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{2\,(b^2c^3-4\,ac^4)fx^6+3\,((b^2c^3-4\,ac^4)e-(b^3c^2-4\,abc^3)f)x^4+6\,((b^2c^3-4\,ac^4)d-(b^3c^2-4\,abc^3)e+(\right.$$

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e - (b^3*c^2 -
4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e + (b
^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 + 3*sqrt(b^2 - 4*a*c)*((b^2*c^2 - 2*
a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*log((2*
c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
 + b*x^2 + a)) - 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^
4 - 4*a*c^5), 1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e
- (b^3*c^2 - 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*
b*c^3)*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 - 6*sqrt(-b^2 + 4*a*c)*
((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c
^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^3*c^
2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e + (b^5 - 6*a*b^3*c +
 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^4 - 4*a*c^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx &= \frac{2\,c^2fx^6+3\,c^2ex^4-3\,bcfx^4+6\,c^2dx^2-6\,bcex^2+6\,b^2fx^2-6\,acfx^2}{12\,c^3} \\ &\quad -\frac{(bc^2d-b^2ce+ac^2e+b^3f-2\,abcf)\log\left(cx^4+bx^2+a\right)}{4\,c^4} \\ &\quad +\frac{(b^2c^2d-2\,ac^3d-b^3ce+3\,abc^2e+b^4f-4\,ab^2cf+2\,a^2c^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^4}\end{aligned}$$

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*c^2*f*x^6 + 3*c^2*e*x^4 - 3*b*c*f*x^4 + 6*c^2*d*x^2 - 6*b*c*e*x^2 +
 6*b^2*f*x^2 - 6*a*c*f*x^2)/c^3 - 1/4*(b*c^2*d - b^2*c*e + a*c^2*e + b^3*f
- 2*a*b*c*f)*log(c*x^4 + b*x^2 + a)/c^4 + 1/2*(b^2*c^2*d - 2*a*c^3*d - b^3*
c*e + 3*a*b*c^2*e + b^4*f - 4*a*b^2*c*f + 2*a^2*c^2*f)*arctan((2*c*x^2 + b)
/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 2295, normalized size of antiderivative = 11.31

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^4*(e/(4*c) - (b*f)/(4*c^2)) - x^2*((b*(e/c - (b*f)/c^2))/(2*c) - d/(2*c)
+ (a*f)/(2*c^2)) + (log(a + b*x^2 + c*x^4)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c
^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c
^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)) + (f*x^6)/(6*c) + (atan((2*c^6*(4*a*c - b
^2)*(x^2*(((((6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c
^7*d + 10*a*b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e
 + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e +
16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f -
 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(8*c^4*(4*a*c - b^2)^(1/
2)) + (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2
*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*
c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*c^2*(4*a*c - b^
2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a - (b*((b^7*f^2 + b^3*c^4*d^2 + b^5*c^2*
e^2 - 3*a*b^3*c^3*e^2 + 2*a^2*b*c^4*e^2 - 2*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 7
*a^2*b^3*c^2*f^2 - a*b*c^5*d^2 - 5*a*b^5*c*f^2 - a^2*c^5*d*e - 2*b^4*c^3*d*
e + a^3*c^4*e*f + 2*b^5*c^2*d*f + 4*a*b^2*c^4*d*e - 6*a*b^3*c^3*d*f + 3*a^2
*b*c^4*d*f + 8*a*b^4*c^2*e*f - 8*a^2*b^2*c^3*e*f)/c^6 + (((6*b^2*c^6*d + 4*
a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c^7*d + 10*a*b*c^6*e - 16*a*b^2
*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8
*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4
*b^2*c^4))*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d -
 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)
) - (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e
 - 4*a*b^2*c*f)^2)/(2*c^6*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (((
(8*a^2*c^6*e + 8*a*b*c^6*d - 8*a*b^2*c^5*e + 8*a*b^3*c^4*f - 16*a^2*b*c^5*f
)/c^6 + (8*a*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c
^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c
^4))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e -
 4*a*b^2*c*f))/(8*c^4*(4*a*c - b^2)^(1/2)) + (a*(b^4*f + b^2*c^2*d + 2*a^2*
c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c
^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*
e + 16*a^2*b*c^2*f))/(c^2*(4*a*c - b^2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a -
(b*((a*b^6*f^2 + a^3*c^4*e^2 + a*b^2*c^4*d^2 + a*b^4*c^2*e^2 - 4*a^2*b^4*c*
f^2 - 2*a^2*b^2*c^3*e^2 + 4*a^3*b^2*c^2*f^2 - 2*a*b^3*c^3*d*e + 2*a^2*b*c^4
*d*e + 2*a*b^4*c^2*d*f - 4*a^3*b*c^3*e*f - 4*a^2*b^2*c^3*d*f + 6*a^2*b^3*c^
2*e*f - 2*a*b^5*c*e*f)/c^6 + (((8*a^2*c^6*e + 8*a*b*c^6*d - 8*a*b^2*c^5*e +
```

```
 8*a*b^3*c^4*f - 16*a^2*b*c^5*f)/c^6 + (8*a*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*
b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^
2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d -
2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/
(2*(16*a*c^5 - 4*b^2*c^4)) - (a*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*
d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)^2)/(c^6*(4*a*c - b^2))))/(2*a*(4*a
*c - b^2)^(1/2))))/(b^8*f^2 + 4*a^2*c^6*d^2 + b^4*c^4*d^2 + 4*a^4*c^4*f^2 +
 b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*
c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 8*a^3*c
^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 12*a^2*b*c^5*d*
e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 12*a^3*b*c^4*e*f + 20*a^2*b^2*c^4
*d*f - 28*a^2*b^3*c^3*e*f))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d -
b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*c^4*(4*a*c - b^2)^(1/2))
```

3.49 $\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = \frac{(ce-bf)x^2}{2c^2} + \frac{fx^4}{4c} - \frac{(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(c^2d+b^2f-c(be+af))\log(a+bx^2+cx^4)}{4c^3}$$

```
[Out] 1/2*(-b*f+c*e)*x^2/c^2+1/4*f*x^4/c+1/4*(c^2*d+b^2*f-c*(a*f+b*e))*ln(c*x^4+b
*x^2+a)/c^3-1/2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d))*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1677, 1642, 648, 632, 212, 642}

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = -\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{2c^3\sqrt{b^2-4ac}} + \frac{\log(a+bx^2+cx^4)(-c(af+be)+b^2f+c^2d)}{4c^3} + \frac{x^2(ce-bf)}{2c^2} + \frac{fx^4}{4c}$$

```
[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x^2)/(2*c^2) + (f*x^4)/(4*c) - ((b^2*c*e - 2*a*c^2*e - b^3*f -
 b*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b
^2 - 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*
c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex+fx^2)}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{ce-bf}{c^2}+\frac{fx}{c}-\frac{a(ce-bf)-(c^2d-bce+b^2f-acf)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx,x,x^2\right)$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{\text{Subst}\left(\int \frac{a(ce-bf)-(c^2d-bce+b^2f-acf)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{(-c^2d+bce-b^2f+acf)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3} + \frac{(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}+\frac{(c^2d-bce+b^2f-acf)\log\left(a+bx^2+cx^4\right)}{4c^3} - \frac{(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^3}$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{(b^2ce-2ac^2e-b^3f-bc(cd-3af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(c^2d-bce+b^2f-acf)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$

$$= \frac{2c(ce-bf)x^2+c^2fx^4-\frac{2(-b^2ce+2ac^2e+b^3f+bc(cd-3af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2d+b^2f-c(be+af))\log\left(a+b\right.}{4c^3}$$

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*e - b*f)*x^2 + c^2*f*x^4 - (2*(-(b^2*c*e) + 2*a*c^2*e + b^3*f + b*c
*(c*d - 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] + (c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result	size
default	$-\frac{-\frac{1}{2}cf\,x^4+bf\,x^2-c\,x^2e}{2c^2}+\frac{\frac{\left(-acf+b^2f-ebc+c^2d\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(abf-ace-\frac{\left(-acf+b^2f-ebc+c^2d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c^2}$	146
risch	Expression too large to display	3279

[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/2/c^2*(-1/2*c*f*x^4+b*f*x^2-c*x^2*e)+1/2/c^2*(1/2*(-a*c*f+b^2*f-b*c*e+c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(a*b*f-a*c*e-1/2*(-a*c*f+b^2*f-b*c*e+c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.28

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)fx^4+2\,((b^2c^2-4\,ac^3)e-(b^3c-4\,abc^2)f)x^2-(bc^2d-(b^2c-2\,ac^2)e+(b^3-3\,abc)f)\sqrt{b^2-\cdots}}{\cdots}\right.$$

[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] [1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*c^2)*f)*x^2 - (b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*c^2)*f)*x^2 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{cfx^4+2\,cex^2-2\,bfx^2}{4\,c^2} + \frac{(c^2d-bce+b^2f-acf)\log\left(cx^4+bx^2+a\right)}{4\,c^3} - \frac{(bc^2d-b^2ce+2\,ac^2e+b^3f-3\,abcf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*f*x^4 + 2*c*e*x^2 - 2*b*f*x^2)/c^2 + 1/4*(c^2*d - b*c*e + b^2*f - a*
c*f)*log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b*c^2*d - b^2*c*e + 2*a*c^2*e + b^3*
f - 3*a*b*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*c^3)
```

Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 1689, normalized size of antiderivative = 11.73

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(e/(2*c) - (b*f)/(2*c^2)) + (f*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 1
0*a*b^2*c*f))/(2*(16*a*c^4 - 4*b^2*c^3)) - (atan((2*c^4*(4*a*c - b^2)*(x^2*
(((((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^4*f)/c^4
+ (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8
*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(b^3*f + 2*a*c^2*e + b*
c^2*d - b^2*c*e - 3*a*b*c*f))/(8*c^3*(4*a*c - b^2)^(1/2)) + (b*(b^3*f + 2*a
*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f)*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*
f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(2*c*(4*a*c - b^2)
^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((b^5*f^2 + b*c^4*d^2 + b^3*c^2*e^2
+ 2*a^2*b*c^2*f^2 + a*c^4*d*e - 2*b^4*c*e*f - a*b*c^3*e^2 - 3*a*b^3*c*f^2 -
 2*b^2*c^3*d*e - a^2*c^3*e*f + 2*b^3*c^2*d*f + 4*a*b^2*c^2*e*f - 3*a*b*c^3*
d*f)/c^4 + (((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^
4*f)/c^4 + (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^
3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*f + 2*b
^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f
))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e -
 3*a*b*c*f)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a
^2*c^4*f - 8*a*c^5*d + 8*a*b*c^4*e - 8*a*b^2*c^3*f)/c^4 - (8*a*c^2*(2*b^4*f
 + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b
^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3
*a*b*c*f))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*(b^3*f + 2*a*c^2*e + b*c^2*d -
b^2*c*e - 3*a*b*c*f)*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b
^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*
b^2*c^3)))/a + (b*((((8*a^2*c^4*f - 8*a*c^5*d + 8*a*b*c^4*e - 8*a*b^2*c^3*f
)/c^4 - (8*a*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c
*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*f + 2*b^2*
c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/
(2*(16*a*c^4 - 4*b^2*c^3)) - (a*c^4*d^2 + a*b^4*f^2 + a^3*c^2*f^2 + a*b^2*c
^2*e^2 - 2*a^2*b^2*c*f^2 - 2*a^2*c^3*d*f + 2*a*b^2*c^2*d*f + 2*a^2*b*c^2*e*
f - 2*a*b*c^3*d*e - 2*a*b^3*c*e*f)/c^4 + (a*(b^3*f + 2*a*c^2*e + b*c^2*d -
b^2*c*e - 3*a*b*c*f)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(
b^6*f^2 + 4*a^2*c^4*e^2 + b^2*c^4*d^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b
^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 - 2*b^3*c^3*d*e + 2*b^4*c^2*d*
f - 6*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 12*a^2*b*c^3*e*f + 4*a*b*c^4*d*e))
*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c^3*(4*a*c - b^2)^
(1/2))
```

3.50 $\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{fx^2}{2c} - \frac{(2c^2d-bce+b^2f-2acf)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] 1/2*f*x^2/c+1/4*(-b*f+c*e)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c*f+b^2*f-b*c*e+
2*c^2*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1677, 1671, 648, 632, 212, 642}

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = -\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-2acf+b^2f-bce+2c^2d)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{fx^2}{2c}$$

```
[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x^2)/(2*c) - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*e - b*f)*Log[a + b*x^2
+ c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{f}{c}+\frac{cd-af+(ce-bf)x}{c\,(a+bx+cx^2)}\right)\,dx,x,x^2\right)\\ &= \frac{fx^2}{2c}+\frac{\text{Subst}\left(\int \frac{cd-af+(ce-bf)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\end{aligned}$$

$$= \frac{fx^2}{2c} + \frac{(ce-bf)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2} + \frac{(2c^2d-bce+b^2f-2acf)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}$$

$$= \frac{fx^2}{2c} + \frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2} - \frac{(2c^2d-bce+b^2f-2acf)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^2}$$

$$= \frac{fx^2}{2c} - \frac{(2c^2d-bce+b^2f-2acf)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{2cfx^2 + \frac{2(2c^2d+b^2f-c(be+2af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*f*x^2 + (2*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*ArcTan[(b + 2*c*x^2)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*e - b*f)*Log[a + b*x^2 + c*x^4])
/(4*c^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{f\,x^2}{2c} + \frac{\frac{(-bf+ec)\ln\left(c\,x^4+b\,x^2+a\right)}{2c} + \frac{2\left(-af+cd-\frac{(-bf+ec)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$	101
risch	Expression too large to display	1690

```
[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/c+1/2/c*(1/2*(-b*f+c*e)/c*ln(c*x^4+b*x^2+a)+2*(-a*f+c*d-1/2*(-b*f
+c*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.09

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$$
$$= \left[\frac{2\,(b^2c-4\,ac^2)fx^2-(2\,c^2d-bce+(b^2-2\,ac)f)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+((b}{4\,(b^2c^2-4\,ac^3)} \right.$$

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*f*x^2 - (2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b*c)
*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(b^2*c - 4*a*c^2)*f
*x^2 - 2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*e - (b^3
- 4*a*b*c)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = \text{Timed out}$$

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{fx^2}{2\,c} + \frac{(ce-bf)\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{\left(2\,c^2d-bce+b^2f-2\,acf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/c + 1/4*(c*e - b*f)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*c^2*d - b
*c*e + b^2*f - 2*a*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 1081, normalized size of antiderivative = 10.50

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{f\,x^2}{2\,c} + \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,f\,b^3-2\,e\,b^2\,c-8\,a\,f\,b\,c+8\,a\,e\,c^2\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)} + \frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\,\left(x^2\,\left(\frac{\frac{\left(\frac{6\,f\,b^2\,c^2-6\,e\,b\,c^3+4\,d\,c^4-4\,a\,f\,c^3}{c^2}+\frac{4\,b\,c^2\,\left(2\,f\,b^3-2\,e\,b^2\,c-8\,a\,f\,b\,c+8\,a\,e\,c^2\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\,\left(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{b\,\left(f\,b^2-e\,b\,c\cdots\right.}{\cdots}}{a}\cdots\right.\right.}{\cdots}\right)\cdots}{\cdots}$$

```
[In] int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e -
8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*(x^2*((
(((4*c^4*d + 6*b^2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f +
 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2
*f - 2*a*c*f - b*c*e))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b*(2*c^2*d + b^2*f -
2*a*c*f - b*c*e)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(4*a*c -
```

```
 b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a - (b*((b^3*f^2 + b*c^2*e^2 - c^3*d*e
 - a*b*c*f^2 + a*c^2*e*f + b*c^2*d*f - 2*b^2*c*e*f)/c^2 + (((4*c^4*d + 6*b^
2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^
2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*
e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b*(2*c^2*d + b^2*f - 2*a*c*f
 - b*c*e)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a*c^
3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*
c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(8*c^2*(
4*a*c - b^2)^(1/2)) - (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*(2*b^3*f + 8*a
*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2
)))/a + (b*((((8*a*c^3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e
 - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e - 2
*b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*b^2*f^2 + a*c^2*e^2
 - 2*a*b*c*e*f)/c^2 + (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)^2)/(c^2*(4*a*c
 - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(4*c^4*d^2 + b^4*f^2 + 4*a^2*c^2*f^2
 + b^2*c^2*e^2 - 8*a*c^3*d*f - 4*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 4
*b^2*c^2*d*f + 4*a*b*c^2*e*f))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2*
(4*a*c - b^2)^(1/2))
```

3.51 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx=\frac{(bcd-2ace+abf)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2ac\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}$$

```
[Out] d*ln(x)/a-1/4*(-a*f+c*d)*ln(c*x^4+b*x^2+a)/a/c+1/2*(a*b*f-2*a*c*e+b*c*d)*ar
ctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1677, 1642, 648, 632, 212, 642}

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx=\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{d\log(x)}{a}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*c
*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - ((c*d - a*f)*Log[a + b*x^2 + c*x^4])/(
4*a*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x\,(a+bx+cx^2)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-(cd-af)x}{a\,(a+bx+cx^2)}\right)\,dx, x, x^2\right)\\ &= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-(cd-af)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a}\end{aligned}$$

$$= \frac{d\log(x)}{a} - \frac{(cd-af)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4ac} - \frac{(bcd-2ace+abf)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{d\log(x)}{a} - \frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac} + \frac{(bcd-2ace+abf)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2ac}$$

$$= \frac{(bcd-2ace+abf)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2ac\sqrt{b^2-4ac}} + \frac{d\log(x)}{a} - \frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.84

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx$$

$$= \frac{4c\sqrt{b^2-4ac}d\log(x) - \left(bcd+c\sqrt{b^2-4ac}d-2ace+abf-a\sqrt{b^2-4ac}f\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right) + }{4ac\sqrt{b^2-4ac}}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*c*Sqrt[b^2 - 4*a*c]*d*Log[x] - (b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e
+ a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (b*
c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*Log[
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*c*Sqrt[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

method	result
default	$\frac{d\ln(x)}{a}+\frac{\frac{(af-cd)\ln\left(cx^4+bx^2+a\right)}{2c}+\frac{2\left(ae-bd-\frac{(af-cd)b}{2c}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c^2-ab^2c\right)\_Z^2+\left(-4a^2cf+ab^2f+4ac^2d-b^2cd\right)\_Z+a^2f^2-abef-2acdf+e^2ac+b^2df-bcde+c^2d^2\right)} \_R\ln\left(\left(\left(\right.\right.\right.\right.}{}$

```
[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*ln(x)/a+1/2/a*(1/2*(a*f-c*d)/c*ln(c*x^4+b*x^2+a)+2*(a*e-b*d-1/2*(a*f-c*d)
*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.19

$$\int \frac{d+ex^2+fx^4}{x\,(a+bx^2+cx^4)}\,dx$$
$$= \left[\frac{4\,(b^2c-4\,ac^2)d\log{(x)}+(bcd-2\,ace+abf)\sqrt{b^2-4\,ac}\log{\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}-((b^2c -}{4\,(ab^2c-4\,a^2c^2)} \right.$$

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^2*c - 4*a*c^2)*d*log(x) + (b*c*d - 2*a*c*e + a*b*f)*sqrt(b^2 - 4
*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*c)*f)*l
og(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2), 1/4*(4*(b^2*c - 4*a*c^2)*d*lo
g(x) + 2*(b*c*d - 2*a*c*e + a*b*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*
c)*f)*log(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx = \frac{d\log(x^2)}{2a} - \frac{(cd-af)\log(cx^4+bx^2+a)}{4ac} - \frac{(bcd-2ace+abf)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}ac}$$

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d*log(x^2)/a - 1/4*(c*d - a*f)*log(c*x^4 + b*x^2 + a)/(a*c) - 1/2*(b*c*
d - 2*a*c*e + a*b*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 3927, normalized size of antiderivative = 40.48

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d*log(x))/a - (log((b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^2
 - b*e*f - c*d*f) + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^
2*(4*a*c - b^2)))^(1/2))*(a*b^2*f^2 - x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*
e + 11*a*b*c*f^2 - 9*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) + a*c^2*e^2 - 4
*b*c^2*d*e + 4*b^2*c*d*f + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/
(a^2*c^2*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*c^2*d
- 4*b^2*c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (b*c*
(c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/
2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) - 2*a*b*c*e*f))/(4*a*c) - 2
*b*c*d*e*f)*(b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^2 - b*e*f
 - c*d*f) + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c
 - b^2)))^(1/2))*(x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*e + 11*a*b*c*f^2 - 9
*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) - a*b^2*f^2 - a*c^2*e^2 + 4*b*c^2*d
*e - 4*b^2*c*d*f + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2
*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*c^2*d - 4*b^2*
c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f - (b*c*(a*f - c
*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/2))*(a*b
 + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) + 2*a*b*c*e*f))/(4*a*c) - 2*b*c*d*e
*f))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*
```

b^2*c)) + (atan(((4*a*c - b^2)*(((((a*b*f - 2*a*c*e + b*c*d)*(4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*a^2*c^2 - 4*a*b^2*c)))/(4*a*c*(4*a*c - b^2)^(1/2)) + (b^2*c*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + ((a*b*f - 2*a*c*e + b*c*d)*(a*b^2*f^2 + a*c^2*e^2 + ((4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*a^2*c^2 - 4*a*b^2*c))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 4*b*c^2*d*e + 4*b^2*c*d*f - 2*a*b*c*e*f))/(4*a*c*(4*a*c - b^2)^(1/2)) - (b^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(16*a^2*c*(4*a*c - b^2)^(3/2)))*(6*b^4*d + 20*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 4*a^3*c*f - 28*a*b^2*c*d + 6*a^2*b*c*e))/(c*(a^2*b^2*f^2 + 4*a^2*c^2*e^2 + b^2*c^2*d^2 - 4*a*b*c^2*d*e + 2*a*b^2*c*d*f - 4*a^2*b*c*e*f)*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2*b*e*f - 10*a^2*c*d*f - a*b*c*d*e)) + (16*a^3*c*x^2*(((3*b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 8*a*b*c*d)*(c^2*e^3 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*(3*b^3*f^2 - b*c^2*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*(((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 5*c^3*d*e - 11*a*b*c*f^2 + 9*a*c^2*e*f + 7*b*c^2*d*f - 2*b^2*c*e*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + b^2*e*f^2 - a*b*f^3 + a*c*e*f^2 + b*c*d*f^2 - 2*b*c*e^2*f - c^2*d*e*f - ((((a*b*f - 2*a*c*e + b*c*d)*(((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(4*a*c*(4*a*c - b^2)^(1/2)) + ((12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(8*a*c*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(a*b*f - 2*a*c*e + b*c*d))/(4*a*c*(4*a*c - b^2)^(1/2)) - ((12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(32*a^2*c^2*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)))))/(8*a^3*c^2*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2*b*e*f - 10*a^2*c*d*f - a*b*c*d*e)) + ((((((a*b*f - 2*a*c*e + b*c*d)*(((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(4*a*c*(4*a*c - b^2)^(1/2)) + ((12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(8*a*c*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + ((a*b*f - 2*a*c*e + b*c*d)*(3*b^3*f^2 - b*c^2*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*(((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 5*c^3*d*e - 11*a*b*c*f^2 + 9*a*c^2*e*f + 7*b*c^2*d*f - 2*b^2*c*e*f))/(4*a*c*(4*a*c - b^2)^(1/2)) - ((12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)^3)/(6

4*a^3*c^3*(4*a*c - b^2)^(3/2)))*(6*b^4*d + 20*a^2*c^2*d + 2*a^2*b^2*f - 2*a
*b^3*e - 4*a^3*c*f - 28*a*b^2*c*d + 6*a^2*b*c*e))/(16*a^3*c^2*(4*a*c - b^2)
^(1/2)*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^
2*b*e*f - 10*a^2*c*d*f - a*b*c*d*e)))*(4*a*c - b^2)^(3/2))/(a^2*b^2*f^2 + 4
*a^2*c^2*e^2 + b^2*c^2*d^2 - 4*a*b*c^2*d*e + 2*a*b^2*c*d*f - 4*a^2*b*c*e*f)
 + (2*(4*a*c - b^2)^(3/2)*(3*b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 8*a*b*c*
d)*(b^2*d*f^2 + c^2*d*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f
)*(a*b^2*f^2 + a*c^2*e^2 + ((4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (2*a
*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*a^2*c^2 - 4*a
*b^2*c))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 -
4*a*b^2*c)) - 4*b*c^2*d*e + 4*b^2*c*d*f - 2*a*b*c*e*f))/(2*(16*a^2*c^2 - 4*
a*b^2*c)) - ((((a*b*f - 2*a*c*e + b*c*d)*(4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b
^2*c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*
a^2*c^2 - 4*a*b^2*c)))/(4*a*c*(4*a*c - b^2)^(1/2)) + (b^2*c*(a*b*f - 2*a*c*
e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2
- 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(a*b*f - 2*a*c*e + b*c*d))/(4*a*c*(4*a*c
 - b^2)^(1/2)) - 2*b*c*d*e*f - (b^2*(a*b*f - 2*a*c*e + b*c*d)^2*(8*a*c^2*d
+ 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(8*a*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c
- b^2))))/(c*(a^2*b^2*f^2 + 4*a^2*c^2*e^2 + b^2*c^2*d^2 - 4*a*b*c^2*d*e + 2
*a*b^2*c*d*f - 4*a^2*b*c*e*f)*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c
*d^2 + 3*a*b^2*d*f - a^2*b*e*f - 10*a^2*c*d*f - a*b*c*d*e)))*(a*b*f - 2*a*c
*e + b*c*d))/(2*a*c*(4*a*c - b^2)^(1/2))

3.52 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx=-\frac{d}{2ax^2}-\frac{\left(b^2d-abe-2a(cd-af)\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] -1/2*d/a/x^2-(-a*e+b*d)*ln(x)/a^2+1/4*(-a*e+b*d)*ln(c*x^4+b*x^2+a)/a^2-1/2*
(b^2*d-a*b*e-2*a*(-a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1677, 1642, 648, 632, 212, 642}

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx=-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{2ax^2}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*d/(a*x^2) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/S
qrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + (
(b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^2\,(a+bx+cx^2)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-abe-a(cd-af)+c(bd-ae)x}{a^2\,(a+bx+cx^2)}\right)\,dx, x, x^2\right)
\end{aligned}$$

$$= -\frac{d}{2ax^2} - \frac{(bd-ae)\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2d-abe-a(cd-af)+c(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{d}{2ax^2} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} + \frac{(b^2d-abe-2a(cd-af))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}$$

$$= -\frac{d}{2ax^2} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log(a+bx^2+cx^4)}{4a^2} - \frac{(b^2d-abe-2a(cd-af))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^2}$$

$$= -\frac{d}{2ax^2} - \frac{(b^2d-abe-2a(cd-af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log(a+bx^2+cx^4)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.72

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)}\,dx$$

$$= \frac{-\frac{2ad}{x^2} + 4(-bd+ae)\log(x) + \frac{\left(b^2d+b\left(\sqrt{b^2-4ac}d-ae\right)+a\left(-2cd-\sqrt{b^2-4ac}e+2af\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(-b^2d+b\left(\sqrt{b^2-4ac}\right.\right.}{}}{4a^2}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^2 + 4*(-(b*d) + a*e)*Log[x] + ((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d
- a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d + a*e
) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

method	result
default	$-\frac{d}{2a\,x^{2}}+\frac{(ae-bd)\ln(x)}{a^{2}}+\frac{\frac{(-ace+bcd)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(f\,a^{2}-abe-acd+b^{2}d-\frac{(-ace+bcd)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}$
risch	$-\frac{d}{2a\,x^{2}}+\frac{\ln(x)e}{a}-\frac{\ln(x)bd}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(4a^{2}ce-a\,b^{2}e-4abcd+b^{3}d\right)\_Z+a^{2}f^{2}-abef-2acdf+e^{2}ac+b^{2}df-bcde\right.}\right.}{}$

[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/2*d/a/x^2+(a*e-b*d)/a^2*ln(x)+1/2/a^2*(1/2*(-a*c*e+b*c*d)/c*ln(c*x^4+b*x^2+a)+2*(f*a^2-a*b*e-a*c*d+b^2*d-1/2*(-a*c*e+b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.38

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$=\left[-\frac{(abe-2\,a^2f-(b^2-2\,ac)d)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^3-4\,abc)d-(}{4\,(a^2b^2-}\right.$$

[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] [-1/4*((a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 + b*x^2 + a) + 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) + 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 + b*x^2 + a) - 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2)]

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.11

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{(bd-ae)\log\left(cx^4+bx^2+a\right)}{4\,a^2} - \frac{(bd-ae)\log\left(x^2\right)}{2\,a^2} + \frac{\left(b^2d-2\,acd-abe+2\,a^2f\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2} + \frac{bdx^2-aex^2-ad}{2\,a^2x^2}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/4*(b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 +
 1/2*(b^2*d - 2*a*c*d - a*b*e + 2*a^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4
*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*d*x^2 - a*e*x^2 - a*d)/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 12.76 (sec) , antiderivative size = 4437, normalized size of antiderivative = 37.60

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(a*e - b*d))/a^2 - d/(2*a*x^2) - (log(((c^2*(a*e - b*d)*(a*f - c*d)
^2)/a^3 - ((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4
*a*c - b^2)))^(1/2))*(((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*
d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*(10*a*c^2*d + 4*a*b^2*f + b^2*
c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(b^2*d + a^2*f - a*b*e - a*c*d)
)/a + (b*c^2*(b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*
(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2))/(4*a^2) + (c^2
*(a*f - c*d)*(4*b^2*d + a^2*f - 4*a*b*e - a*c*d))/a^2 - (c^2*x^2*(a*f - c*d
)*(a*b*f + 5*a*c*e - 6*b*c*d))/a^2))/(4*a^2) + (c^2*x^2*(a*f - c*d)^3)/a^3)
*((c^2*(a*e - b*d)*(a*f - c*d)^2)/a^3 - ((a*e - b*d + a^2*(-(b^2*d + 2*a^2*
f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(((a*e - b*d + a^2*(-(b^
2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*
(10*a*c^2*d + 4*a*b^2*f + b^2*c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(
b^2*d + a^2*f - a*b*e - a*c*d))/a - (b*c^2*(a*e - b*d + a^2*(-(b^2*d + 2*a^
2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*
a*c*x^2))/a^2))/(4*a^2) - (c^2*(a*f - c*d)*(4*b^2*d + a^2*f - 4*a*b*e - a*c
*d))/a^2 + (c^2*x^2*(a*f - c*d)*(a*b*f + 5*a*c*e - 6*b*c*d))/a^2))/(4*a^2)
+ (c^2*x^2*(a*f - c*d)^3)/a^3))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*
d))/(2*(16*a^3*c - 4*a^2*b^2)) - (atan((16*a^6*(4*a*c - b^2)^(3/2)*(x^2*(((
(c^5*d^3 - a^3*c^2*f^3 + 3*a^2*c^3*d*f^2 - 3*a*c^4*d^2*f)/a^3 + (((a^3*b*c^
2*f^2 + 6*a*b*c^4*d^2 - 5*a^2*c^4*d*e + 5*a^3*c^3*e*f - 7*a^2*b*c^3*d*f)/a^
3 + (((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*c^2*f - 10
*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2*a*b^2*e +
 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2
*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2
*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (((((20*a^3*c^4*d
 - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*c^2*f - 10*a^3*b*c^3*e)/a^3 +
 ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*
c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/
(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(b^2*d + 2*a
^2*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(8*a
^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e -
2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(b
^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*
b*c*d))/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)))*(3*b^4*d + a^2*c^2*d
 + a^2*b^2*f - 3*a*b^3*e - a^3*c*f - 9*a*b^2*c*d + 8*a^2*b*c*e))/(8*a^3*c^2
```

*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d^2 + 12*a*b
^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f - 49*a^2*b*
c*d*e)) - (((((((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*
c^2*f - 10*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2
*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(b^2*d +
 2*a^2*f - a*b*e - 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 -
 12*a^3*b^3*c^2)*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e +
 8*a^2*c*e - 8*a*b*c*d))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))
)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2))
 + (((a^3*b*c^2*f^2 + 6*a*b*c^4*d^2 - 5*a^2*c^4*d*e + 5*a^3*c^3*e*f - 7*a^2
*b*c^3*d*f)/a^3 + (((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*
b^2*c^2*f - 10*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d
 - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*b
^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(b^2
*d + 2*a^2*f - a*b*e - 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c
^3 - 12*a^3*b^3*c^2)*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^3)/(64*a^9*(4*a*c
- b^2)^(3/2)))*(6*b^5*d + 2*a^2*b^3*f - 20*a^3*c^2*e - 6*a*b^4*e - 30*a*b^3
*c*d - 6*a^3*b*c*f + 26*a^2*b*c^2*d + 28*a^2*b^2*c*e))/(16*a^3*c^2*(4*a*c -
 b^2)^(1/2)*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d
^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f
- 49*a^2*b*c*d*e))) + (((b*c^4*d^3 - a^3*c^2*e*f^2 - a*c^4*d^2*e - 2*a*b*c^
3*d^2*f + 2*a^2*c^3*d*e*f + a^2*b*c^2*d*f^2)/a^3 - (((a^2*c^4*d^2 + a^4*c^2
*f^2 - 4*a*b^2*c^3*d^2 - 2*a^3*c^3*d*f + 4*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f
+ 4*a^2*b^2*c^2*d*f)/a^3 - (((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*e - 4*a^3*b*c
^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e -
 8*a*b*c*d))/(16*a^3*c - 4*a^2*b^2))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a
*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a
*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (((((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*
e - 4*a^3*b*c^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e
+ 8*a^2*c*e - 8*a*b*c*d))/(16*a^3*c - 4*a^2*b^2))*(b^2*d + 2*a^2*f - a*b*e
- 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e
 - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a*(4*a*c - b^
2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/(4*a
^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2*(2
*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(8*a^3*(4*a*c - b^2)*(16*a^3*c
 - 4*a^2*b^2)))*(3*b^4*d + a^2*c^2*d + a^2*b^2*f - 3*a*b^3*e - a^3*c*f - 9*
a*b^2*c*d + 8*a^2*b*c*e))/(8*a^3*c^2*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 -
6*a^2*b^2*e^2 + a^2*c^2*d^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a
*b^2*c*d^2 + a^2*b^2*d*f - 49*a^2*b*c*d*e)) - (((((((4*a^2*b^3*c^2*d - 4*a^
3*b^2*c^2*e - 4*a^3*b*c^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d -
2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(16*a^3*c - 4*a^2*b^2))*(b^2*d + 2*a^2*
f - a*b*e - 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^2*d + 2*a^2
*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a*(
4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*
e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (((a^2*c^4*d^2 + a^4*c^2*f^2 -

```
 4*a*b^2*c^3*d^2 - 2*a^3*c^3*d*f + 4*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f + 4*a^
2*b^2*c^2*d*f)/a^3 - (((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*e - 4*a^3*b*c^3*d +
 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b
*c*d))/(16*a^3*c - 4*a^2*b^2))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d
))/(2*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/(4*a^2*
(4*a*c - b^2)^(1/2)) + (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^3)/(16*
a^5*(4*a*c - b^2)^(3/2)))*(6*b^5*d + 2*a^2*b^3*f - 20*a^3*c^2*e - 6*a*b^4*e
 - 30*a*b^3*c*d - 6*a^3*b*c*f + 26*a^2*b*c^2*d + 28*a^2*b^2*c*e))/(16*a^3*c
^2*(4*a*c - b^2)^(1/2)*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2
+ a^2*c^2*d^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a
^2*b^2*d*f - 49*a^2*b*c*d*e))))/(4*a^2*c^4*d^2 + b^4*c^2*d^2 + 4*a^4*c^2*f^
2 - 4*a*b^2*c^3*d^2 + a^2*b^2*c^2*e^2 - 8*a^3*c^3*d*f - 2*a*b^3*c^2*d*e + 4
*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f + 4*a^2*b^2*c^2*d*f))*(b^2*d + 2*a^2*f - a
*b*e - 2*a*c*d))/(2*a^2*(4*a*c - b^2)^(1/2))
```

3.53 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 174

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx = -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{(b^2d-abe-a(cd-af))\log(x)}{a^3}-\frac{(b^2d-abe-a(cd-af))\log(a+bx^2+cx^4)}{4a^3}$$

```
[Out] -1/4*d/a/x^4+1/2*(-a*e+b*d)/a^2/x^2+(b^2*d-a*b*e-a*(-a*f+c*d))*ln(x)/a^3-1/
4*(b^2*d-a*b*e-a*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/a^3+1/2*(b^3*d-a*b^2*e+2*a^2
*c*e-a*b*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+
b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1677, 1642, 648, 632, 212, 642}

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx = -\frac{\log(a+bx^2+cx^4)\,(-abe-a(cd-af)+b^2d)}{4a^3}+\frac{\log(x)\,(-abe-a(cd-af)+b^2d)}{a^3}+\frac{bd-ae}{2a^2x^2}+\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2a^2ce-ab^2e-ab(3cd-af)+b^3d)}{2a^3\sqrt{b^2-4ac}}-\frac{d}{4ax^4}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*d/(a*x^4) + (b*d - a*e)/(2*a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^2*c*e -
a*b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^
2 - 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*Log[x])/a^3 - ((b^2*d - a*b*
e - a*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^3(a+bx+cx^2)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax^3}+\frac{-bd+ae}{a^2x^2}+\frac{b^2d-abe-a(cd-af)}{a^3x} + \frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c(b^2d-abe-a(cd-af))x}{a^3(a+bx+cx^2)}\right)dx, x, x^2\right)$$
$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{(b^2d-abe-a(cd-af))\log(x)}{a^3} + \frac{\text{Subst}\left(\int \frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c(b^2d-abe-a(cd-af))x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^3}$$
$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{(b^2d-abe-a(cd-af))\log(x)}{a^3} - \frac{(b^2d-abe-a(cd-af))\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^3} - \frac{(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^3}$$
$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{(b^2d-abe-a(cd-af))\log(x)}{a^3} - \frac{(b^2d-abe-a(cd-af))\log(a+bx^2+cx^4)}{4a^3} + \frac{(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^3}$$
$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{(b^3d-ab^2e+2a^2ce-ab(3cd-af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}} + \frac{(b^2d-abe-a(cd-af))\log(x)}{a^3} - \frac{(b^2d-abe-a(cd-af))\log(a+bx^2+cx^4)}{4a^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.80

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx =$$
$$-\frac{\frac{a^2d}{x^4}+\frac{2a(-bd+ae)}{x^2}-4(b^2d-abe+a(-cd+af))\log(x)+\frac{\left(b^3d+b^2\left(\sqrt{b^2-4ac}d-ae\right)+ab\left(-3cd-\sqrt{b^2-4ac}e+af\right)+a\left(-c\sqrt{b\right.\right.}{\sqrt{b^2-4ac}}}{}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*((a^2*d)/x^4 + (2*a*(-(b*d) + a*e))/x^2 - 4*(b^2*d - a*b*e + a*(-(c*d)
 + a*f))*Log[x] + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*b*(-3*c*d -
 Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*Sqr
t[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c]
+ ((-(b^3*d) + b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*b*(-3*c*d + Sqrt[b^2 - 4
*a*c]*e + a*f) + a*(-(c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + a*Sqrt[b^2 - 4*a*c
]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/a^3
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.17

method	result
default	$-\frac{d}{4a\,x^4}-\frac{ae-bd}{2a^2x^2}+\frac{(f\,a^2-abe-acd+b^2d)\ln(x)}{a^3}-\frac{\frac{(a^2cf-abce-a\,c^2d+b^2cd)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(a^2bf+a^2ce-a\,b^2e-2abcd+b^3d-\right.}{}}{2a^3}$
risch	$\frac{-\frac{(ae-bd)x^2}{2a^2}-\frac{d}{4a}}{x^4}+\frac{\ln(x)f}{a}-\frac{\ln(x)be}{a^2}-\frac{\ln(x)cd}{a^2}+\frac{\ln(x)b^2d}{a^3}+\Big(\_R=\mathrm{RootOf}\Big(\big(4c\,a^4-a^3b^2\big)\_Z^2+\big(4a^3cf-a^2b^2f-4a^2bce-4a^2c^2$

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d/a/x^4-1/2*(a*e-b*d)/a^2/x^2+(a^2*f-a*b*e-a*c*d+b^2*d)/a^3*ln(x)-1/2/
a^3*(1/2*(a^2*c*f-a*b*c*e-a*c^2*d+b^2*c*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*b*f+a
^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d-1/2*(a^2*c*f-a*b*c*e-a*c^2*d+b^2*c*d)*b/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.81 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.50

$$\int \frac{d+ex^2+fx^4}{x^5\,(a+bx^2+cx^4)}\,dx$$

$$=\left[\frac{(a^2bf+(b^3-3\,abc)d-(ab^2-2\,a^2c)e)\sqrt{b^2-4\,ac}x^4\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^4-5\right.}{}\right.$$

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(b^2 - 4*a*c)
*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*
a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^4 -
```

```
5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x
^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^2*b^
2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(a^2*b*f + (b^3 - 3*a*b*c
)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^2 + b)*sqr
t(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3
- 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*
f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^
2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx$$
$$= -\frac{\left(b^2d-acd-abe+a^2f\right)\log\left(cx^4+bx^2+a\right)}{4\,a^3}+\frac{\left(b^2d-acd-abe+a^2f\right)\log\left(x^2\right)}{2\,a^3}$$
$$-\frac{\left(b^3d-3\,abcd-ab^2e+2\,a^2ce+a^2bf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^3}$$
$$-\frac{3\,b^2dx^4-3\,acdx^4-3\,abex^4+3\,a^2fx^4-2\,abdx^2+2\,a^2ex^2+a^2d}{4\,a^3x^4}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d - a*c*d - a*b*e + a^2*f)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2*
d - a*c*d - a*b*e + a^2*f)*log(x^2)/a^3 - 1/2*(b^3*d - 3*a*b*c*d - a*b^2*e
+ 2*a^2*c*e + a^2*b*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*a^3) - 1/4*(3*b^2*d*x^4 - 3*a*c*d*x^4 - 3*a*b*e*x^4 + 3*a^2*f*x^4
- 2*a*b*d*x^2 + 2*a^2*e*x^2 + a^2*d)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 6187, normalized size of antiderivative = 35.56

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(b^2*d + a^2*f - a*b*e - a*c*d))/a^3 - (d/(4*a) + (x^2*(a*e - b*d))
/(2*a^2))/x^4 + (log((((((((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2*
c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a
*b*c*d))/a^2 + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^2*d + a^2*f + a^3*(
-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))
^(1/2) - a*b*e - a*c*d))/a^3)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2
*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e - a*c*d)
)/(4*a^3) + (c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*c*e - 5
*a*b*c*d))/a^4 + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5*a*c*
d))/a^4)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*
a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e - a*c*d))/(4*a^3) + (c^4*(a*e
 - b*d)^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a^6 - (c^5*x^2*(a*e - b*d)^3)/a^
6)*((((c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*c*e - 5*a*b*c
*d))/a^4 - (((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2*c*e + 5*a*b*c
*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^2
- (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f +
```

2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2*f - b^2*d + a*b*
e + a*c*d))/a^3)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)
^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2*f - b^2*d + a*b*e + a*c*d))/(4*a^3) + (
c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5*a*c*d))/a^4)*(a^3*(-(b
^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1
/2) - a^2*f - b^2*d + a*b*e + a*c*d))/(4*a^3) - (c^4*(a*e - b*d)^2*(b^2*d +
a^2*f - a*b*e - a*c*d))/a^6 + (c^5*x^2*(a*e - b*d)^3)/a^6))*(2*b^4*d + 8*a
^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e
))/(2*(16*a^4*c - 4*a^3*b^2)) - (atan((16*a^9*(4*a*c - b^2)^(3/2)*(x^2*((((
a^3*c^5*e^3 - b^3*c^5*d^3 + 3*a*b^2*c^5*d^2*e - 3*a^2*b*c^5*d*e^2)/a^6 - ((
(6*a^4*b*c^4*e^2 - 5*a^3*b*c^5*d^2 + 6*a^2*b^3*c^4*d^2 + 5*a^4*c^5*d*e - 5*
a^5*c^4*e*f + 5*a^4*b*c^4*d*f - 12*a^3*b^2*c^4*d*e)/a^6 + (((2*a^4*b^3*c^3*
d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 +
((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*
a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3
b^2)))(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a
*b^2*c*d + 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^2*d
+ 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*(1
6*a^4*c - 4*a^3*b^2)) + (((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3
*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2
)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c
*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b
*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 -
12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)*(2*b^4
*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a
^2*b*c*e))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b
^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40
*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b
*c*d)^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a
*b^2*c*d + 8*a^2*b*c*e))/(32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3
*b^5*d + 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f +
9*a^2*b*c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a
^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^
2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 1
2*a^3*b^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a
^3*b^2*c*d*f - 49*a^4*b*c*e*f)) + (((((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2
*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12
*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f
- 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b
^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40
*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b
*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b
^2*c*d + 8*a^2*b*c*e))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*
(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d
+ 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)) + (((6*a^4*b*c^4*e^2 - 5*a^3*b*

c^5*d^2 + 6*a^2*b^3*c^4*d^2 + 5*a^4*c^5*d*e - 5*a^5*c^4*e*f + 5*a^4*b*c^4*d
*f - 12*a^3*b^2*c^4*d*e)/a^6 + (((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^
2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^
3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*
b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^
2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2
*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*
d))/(4*a^3*(4*a*c - b^2)^(1/2)) - ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d -
a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^3)/(64*a^15*(4*a*c - b^2)^(3/2)
))*(6*b^6*d - 20*a^3*c^3*d + 6*a^2*b^4*f + 20*a^4*c^2*f - 6*a*b^5*e + 54*a^
2*b^2*c^2*d - 36*a*b^4*c*d + 30*a^2*b^3*c*e - 26*a^3*b*c^2*e - 28*a^3*b^2*c
*f))/(16*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*
e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*
a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b
^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*
c*d*f - 49*a^4*b*c*e*f))) - (((b^4*c^4*d^3 - a*b^2*c^5*d^3 - a^3*b*c^4*e^3
- a^3*c^5*d*e^2 + a^4*c^4*e^2*f - 3*a*b^3*c^4*d^2*e + 2*a^2*b*c^5*d^2*e + 3
*a^2*b^2*c^4*d*e^2 + a^2*b^2*c^4*d^2*f - 2*a^3*b*c^4*d*e*f)/a^6 - (((a^5*c^
4*e^2 - 4*a^2*b^4*c^3*d^2 + 5*a^3*b^2*c^4*d^2 - 4*a^4*b^2*c^3*e^2 - 6*a^4*b
*c^4*d*e + 4*a^5*b*c^3*e*f + 8*a^3*b^3*c^3*d*e - 4*a^4*b^2*c^3*d*f)/a^6 - (
((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e + 4*a^6*b^2*c^2*f + 4
*a^6*b*c^3*e)/a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a
*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(16*a^4*c - 4*a^3*b^2))*(
2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d
+ 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*
b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*(16*a^4*c -
4*a^3*b^2)) - (((((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e + 4
*a^6*b^2*c^2*f + 4*a^6*b*c^3*e)/a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d +
2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(16*a^4
*c - 4*a^3*b^2))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^
3*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e -
3*a*b*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 1
0*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^
2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b
^2)^(1/2)) + (b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2
*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*
d + 8*a^2*b*c*e))/(8*a^5*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3*b^5*d +
3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 9*a^2*b*
c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^
2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*
b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3
*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*
d*f - 49*a^4*b*c*e*f)) + (((((((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b
^3*c^2*e + 4*a^6*b^2*c^2*f + 4*a^6*b*c^3*e)/a^6 - (2*a*b^2*c^2*(2*b^4*d + 8
*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c

```
*e))/(16*a^4*c - 4*a^3*b^2))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b
*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f +
2*a^2*c*e - 3*a*b*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8
*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*
c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c
*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)) - (((a^5*c^4*e
^2 - 4*a^2*b^4*c^3*d^2 + 5*a^3*b^2*c^4*d^2 - 4*a^4*b^2*c^3*e^2 - 6*a^4*b*c^
4*d*e + 4*a^5*b*c^3*e*f + 8*a^3*b^3*c^3*d*e - 4*a^4*b^2*c^3*d*f)/a^6 - (((4
*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e + 4*a^6*b^2*c^2*f + 4*a^
6*b*c^3*e)/a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^
3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(16*a^4*c - 4*a^3*b^2))*(2*b
^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8
*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2
*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(b^3*d - a*b^2*e
+ a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^3)/(16*a^8*(4*a*c - b^2)^(3/2)))*(6*b^6*
d - 20*a^3*c^3*d + 6*a^2*b^4*f + 20*a^4*c^2*f - 6*a*b^5*e + 54*a^2*b^2*c^2*
d - 36*a*b^4*c*d + 30*a^2*b^3*c*e - 26*a^3*b*c^2*e - 28*a^3*b^2*c*f))/(16*a
^3*c^2*(4*a*c - b^2)^(1/2)*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a
^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e
- 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 5
0*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49
*a^4*b*c*e*f))))/(4*a^4*c^4*e^2 + b^6*c^2*d^2 - 6*a*b^4*c^3*d^2 + 9*a^2*b^2
*c^4*d^2 + a^2*b^4*c^2*e^2 - 4*a^3*b^2*c^3*e^2 + a^4*b^2*c^2*f^2 - 2*a*b^5*
c^2*d*e - 12*a^3*b*c^4*d*e + 4*a^4*b*c^3*e*f + 10*a^2*b^3*c^3*d*e + 2*a^2*b
^4*c^2*d*f - 6*a^3*b^2*c^3*d*f - 2*a^3*b^3*c^2*e*f))*(b^3*d - a*b^2*e + a^2
*b*f + 2*a^2*c*e - 3*a*b*c*d))/(2*a^3*(4*a*c - b^2)^(1/2))
```

3.54 $\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 244

$$\begin{aligned}
&\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx \\
&= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2} \\
&\quad -\frac{\left(b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\sqrt{b^2-4ac}} \\
&\quad -\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\log(x)}{a^4} \\
&\quad +\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\log\left(a+bx^2+cx^4\right)}{4a^4}
\end{aligned}$$

```
[Out] -1/6*d/a/x^6+1/4*(-a*e+b*d)/a^2/x^4+1/2*(-b^2*d+a*b*e+a*(-a*f+c*d))/a^3/x^2
-(b^3*d-a*b^2*e+a^2*c*e-a*b*(-a*f+2*c*d))*ln(x)/a^4+1/4*(b^3*d-a*b^2*e+a^2*
c*e-a*b*(-a*f+2*c*d))*ln(c*x^4+b*x^2+a)/a^4-1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+
2*a^2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/a^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1677, 1642, 648, 632, 212, 642}

$$\begin{aligned}
&\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx \\
&= -\frac{-abe-a(cd-af)+b^2d}{2a^3x^2}+\frac{bd-ae}{4a^2x^4} \\
&\quad-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d\right)}{2a^4\sqrt{b^2-4ac}} \\
&\quad+\frac{\log\left(a+bx^2+cx^4\right)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{4a^4} \\
&\quad-\frac{\log(x)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{a^4}-\frac{d}{6ax^6}
\end{aligned}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*d/(a*x^6) + (b*d - a*e)/(4*a^2*x^4) - (b^2*d - a*b*e - a*(c*d - a*f))/
(2*a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2
*(4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*Sqrt[b^2 -
 4*a*c]) - ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[x])/a^4 + (
(b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*
a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^4\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax^4}+\frac{-bd+ae}{a^2x^3}+\frac{b^2d-abe-a(cd-af)}{a^3x^2}+\frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)}{a^4x}\right.\right.\\
&\quad\left.\left.+\frac{b^4d-ab^3e+2a^2bce+a^2c(cd-af)-ab^2(3cd-af)+c\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)x}{a^4\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\
&= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\log(x)}{a^4}\\
&\quad+\frac{\text{Subst}\left(\int \frac{b^4d-ab^3e+2a^2bce+a^2c(cd-af)-ab^2(3cd-af)+c\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^4}\\
&= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}\\
&\quad-\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\log(x)}{a^4}\\
&\quad+\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^4}\\
&\quad+\frac{\left(b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^4}
\end{aligned}
$$

$$= -\frac{d}{6ax^6} + \frac{bd-ae}{4a^2x^4} - \frac{b^2d-abe-a(cd-af)}{2a^3x^2} - \frac{(b^3d-ab^2e+a^2ce-ab(2cd-af))\log(x)}{a^4} + \frac{(b^3d-ab^2e+a^2ce-ab(2cd-af))\log(a+bx^2+cx^4)}{4a^4} - \frac{(b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af))\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a^4}$$

$$= -\frac{d}{6ax^6} + \frac{bd-ae}{4a^2x^4} - \frac{b^2d-abe-a(cd-af)}{2a^3x^2} - \frac{(b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\sqrt{b^2-4ac}} - \frac{(b^3d-ab^2e+a^2ce-ab(2cd-af))\log(x)}{a^4} + \frac{(b^3d-ab^2e+a^2ce-ab(2cd-af))\log(a+bx^2+cx^4)}{4a^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.70

$$\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx = \frac{-\frac{2a^3d}{x^6}+\frac{3a^2(bd-ae)}{x^4}+\frac{6a(-b^2d+abe+a(cd-af))}{x^2}-12(b^3d-ab^2e+a^2ce+ab(-2cd+af))\log(x)+\frac{3\left(b^4d+b^3\left(\sqrt{b^2-4\right.\right.}{}}{}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a^3*d)/x^6 + (3*a^2*(b*d - a*e))/x^4 + (6*a*(-(b^2*d) + a*b*e + a*(c*d
 - a*f)))/x^2 - 12*(b^3*d - a*b^2*e + a^2*c*e + a*b*(-2*c*d + a*f))*Log[x]
+ (3*(b^4*d + b^3*(Sqrt[b^2 - 4*a*c]*d - a*e) + a^2*c*(2*c*d + Sqrt[b^2 - 4
*a*c]*e - 2*a*f) + a*b^2*(-4*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-2*c*S
qrt[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (3*(-(b^4*d) + b^3*(Sqrt[b^2 - 4*a*c]
*d + a*e) - a*b^2*(-4*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a^2*c*(-2*c*d + Sq
rt[b^2 - 4*a*c]*e + 2*a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e + a*Sq
rt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c]
)/(12*a^4)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d}{6a\,x^6}-\frac{ae-bd}{4a^2x^4}-\frac{f\,a^2-abe-acd+b^2d}{2a^3x^2}+\frac{\left(-a^2bf-a^2ce+a\,b^2e+2abcd-b^3d\right)\ln(x)}{a^4}-\frac{\left(-a^2bcf-a^2c^2e+a\,b^2ce+2ab\,c^2d-b^3cd\right)\ln}{2c}$
risch	$\frac{-\frac{\left(f\,a^2-abe-acd+b^2d\right)x^4}{2a^3}-\frac{(ae-bd)x^2}{4a^2}-\frac{d}{6a}}{x^6}-\frac{\ln(x)bf}{a^2}-\frac{\ln(x)ce}{a^2}+\frac{\ln(x)b^2e}{a^3}+\frac{2\ln(x)bcd}{a^3}-\frac{\ln(x)b^3d}{a^4}+\left(\_R=\mathrm{RootOf}\left(\left(4c\,a^5\right.\right.\right.$

```
[In] int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*d/a/x^6-1/4*(a*e-b*d)/a^2/x^4-1/2*(a^2*f-a*b*e-a*c*d+b^2*d)/a^3/x^2+1/
a^4*(-a^2*b*f-a^2*c*e+a*b^2*e+2*a*b*c*d-b^3*d)*ln(x)-1/2/a^4*(1/2*(-a^2*b*c
*f-a^2*c^2*e+a*b^2*c*e+2*a*b*c^2*d-b^3*c*d)/c*ln(c*x^4+b*x^2+a)+2*(a^3*c*f-
a^2*b^2*f-2*a^2*b*c*e-a^2*c^2*d+a*b^3*e+3*a*b^2*c*d-d*b^4-1/2*(-a^2*b*c*f-a
^2*c^2*e+a*b^2*c*e+2*a*b*c^2*d-b^3*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.67 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.42

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx$$

$$=\left[-\frac{3\sqrt{b^2-4\,ac}\left((b^4-4\,ab^2c+2\,a^2c^2)d-(ab^3-3\,a^2bc)e+(a^2b^2-2\,a^3c)f\right)x^6\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,c}{cx^4+bx^2+a}\right.}{}\right.$$

$$-\frac{6\sqrt{-b^2+4\,ac}\left((b^4-4\,ab^2c+2\,a^2c^2)d-(ab^3-3\,a^2bc)e+(a^2b^2-2\,a^3c)f\right)x^6\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,a}}{b^2-4\,ac}\right.}{}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(b^2 - 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a
^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^5 - 6*
a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 -
 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b^5 - 6*a*b^3*c + 8*a^2*b*
c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6
*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e
```

```
+ (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^
4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 - 4*a^5*c)*x^6), -1/12*(6*
sqrt(-b^2 + 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e
 + (a^2*b^2 - 2*a^3*c)*f)*x^6*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c +
4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b
^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^
2*b^3 - 4*a^3*b*c)*f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d -
 (a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3
*b*c)*d - (a^3*b^2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 -
 4*a^5*c)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x^7 (a + bx^2 + cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex^2 + fx^4}{x^7 (a + bx^2 + cx^4)} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.24

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx = \frac{\left(b^3d-2\,abcd-ab^2e+a^2ce+a^2bf\right)\log\left(cx^4+bx^2+a\right)}{4\,a^4}$$
$$-\frac{\left(b^3d-2\,abcd-ab^2e+a^2ce+a^2bf\right)\log\left(x^2\right)}{2\,a^4}$$
$$+\frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-ab^3e+3\,a^2bce+a^2b^2f-2\,a^3cf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^4}$$
$$+\frac{11\,b^3dx^6-22\,abcdx^6-11\,ab^2ex^6+11\,a^2cex^6+11\,a^2bfx^6-6\,ab^2dx^4+6\,a^2cdx^4+6\,a^2bex^4-6\,a^3fx^4}{12\,a^4x^6}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + a^2*b*f)*log(c*x^4 + b*x^2 + a
)/a^4 - 1/2*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + a^2*b*f)*log(x^2)/a^4
+ 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e + a^2*b^2*
f - 2*a^3*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^4) + 1/12*(11*b^3*d*x^6 - 22*a*b*c*d*x^6 - 11*a*b^2*e*x^6 + 11*a^2*c*e*x
^6 + 11*a^2*b*f*x^6 - 6*a*b^2*d*x^4 + 6*a^2*c*d*x^4 + 6*a^2*b*e*x^4 - 6*a^3
*f*x^4 + 3*a^2*b*d*x^2 - 3*a^3*e*x^2 - 2*a^3*d)/(a^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 9141, normalized size of antiderivative = 37.46

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((16*a^12*(4*a*c - b^2)^(3/2)*(x^2*((((a^3*c^8*d^3 - b^6*c^5*d^3 - a^6
*c^5*f^3 + 3*a*b^4*c^6*d^3 - 3*a^4*c^7*d^2*f + 3*a^5*c^6*d*f^2 - 3*a^2*b^2*
c^7*d^3 + a^3*b^3*c^5*e^3 + 3*a*b^5*c^5*d^2*e + 3*a^3*b*c^7*d^2*e + 3*a^5*b
*c^5*e*f^2 - 6*a^2*b^3*c^6*d^2*e - 3*a^2*b^4*c^5*d*e^2 + 3*a^3*b^2*c^6*d*e^
2 - 3*a^2*b^4*c^5*d^2*f + 6*a^3*b^2*c^6*d^2*f - 3*a^4*b^2*c^5*d*f^2 - 3*a^4
*b^2*c^5*e^2*f - 6*a^4*b*c^6*d*e*f + 6*a^3*b^3*c^5*d*e*f)/a^9 - (((11*a^5*b
*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 17*a^4*b
^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17*a^6*b*c
^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*d*f - 12*
a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*d + 8
*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a^9 +
```

((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*
a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2
a^9(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^
4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16
*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12
*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c -
 4*a^4*b^2)) + (((((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*d + 8*a^7*b
^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a^9 + ((40*a
^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*
e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*a^9*(
16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*
c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) + ((40*a^10*b
*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c
*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*
b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(8*a
^13*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2
*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c -
b^2)^(1/2)) + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*
b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2*(2*b^5*d + 2*a^2
*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^
2*d + 10*a^2*b^2*c*e))/(32*a^17*(4*a*c - b^2)*(16*a^5*c - 4*a^4*b^2)))*(3*b
^6*d - a^3*c^3*d + 3*a^2*b^4*f + a^4*c^2*f - 3*a*b^5*e + 18*a^2*b^2*c^2*d -
 15*a*b^4*c*d + 12*a^2*b^3*c*e - 9*a^3*b*c^2*e - 9*a^3*b^2*c*f))/(8*a^3*c^2
*(a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2
+ a^6*c^2*f^2 + 36*a^3*b^4*c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^
2*b^4*c^2*d^2 + 96*a^3*b^2*c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 -
12*a^2*b^6*d*f + 12*a^3*b^5*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4
*b*c^3*d*e + 72*a^3*b^4*c*d*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a
^3*b^3*c^2*d*e - 95*a^4*b^2*c^2*d*f)) + (((((((20*a^9*c^4*f - 20*a^8*c^5*d
+ 2*a^6*b^4*c^3*d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 1
0*a^8*b*c^4*e)/a^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3
*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d
+ 10*a^2*b^2*c*e))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a
^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c
- b^2)^(1/2)) + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^
2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2
*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^
2*d + 10*a^2*b^2*c*e))/(8*a^13*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))
*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*
c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)) + (((11*
a^5*b*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 17*
a^4*b^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17*a^
6*b*c^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*d*f
- 12*a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*
d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a

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^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e
 - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e
))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2
*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(
2*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a
^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) - ((40*a^1
0*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^
3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^3)/(64*a^21*(4*a*c - b^2)^(3/2)))*(6*b^7
*d + 6*a^2*b^5*f + 20*a^4*c^3*e - 6*a*b^6*e + 84*a^2*b^3*c^2*d - 54*a^3*b^2
*c^2*e - 42*a*b^5*c*d - 46*a^3*b*c^3*d + 36*a^2*b^4*c*e - 30*a^3*b^3*c*f +
26*a^4*b*c^2*f))/(16*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^4*c^4*d^2 - 6*a^2*b^6*e
^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2 + a^6*c^2*f^2 + 36*a^3*b^4*
c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^2*b^4*c^2*d^2 + 96*a^3*b^2*
c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 - 12*a^2*b^6*d*f + 12*a^3*b^5
*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4*b*c^3*d*e + 72*a^3*b^4*c*d
*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a^3*b^3*c^2*d*e - 95*a^4*b^2
*c^2*d*f))) - (((b^7*c^4*d^3 - 4*a*b^5*c^5*d^3 - 2*a^3*b*c^7*d^3 + a^6*b*c^
4*f^3 + a^4*c^7*d^2*e + a^6*c^5*e*f^2 + 5*a^2*b^3*c^6*d^3 - a^3*b^4*c^4*e^3
 + a^4*b^2*c^5*e^3 - 2*a^5*c^6*d*e*f - 3*a*b^6*c^4*d^2*e + 2*a^4*b*c^6*d*e^
2 + 5*a^4*b*c^6*d^2*f - 4*a^5*b*c^5*d*f^2 - 2*a^5*b*c^5*e^2*f + 9*a^2*b^4*c
^5*d^2*e + 3*a^2*b^5*c^4*d*e^2 - 7*a^3*b^2*c^6*d^2*e - 6*a^3*b^3*c^5*d*e^2
+ 3*a^2*b^5*c^4*d^2*f - 8*a^3*b^3*c^5*d^2*f + 3*a^4*b^3*c^4*d*f^2 + 3*a^4*b
^3*c^4*e^2*f - 3*a^5*b^2*c^4*e*f^2 - 6*a^3*b^4*c^4*d*e*f + 10*a^4*b^2*c^5*d
*e*f)/a^9 - (((a^6*c^6*d^2 + a^8*c^4*f^2 - 4*a^3*b^6*c^3*d^2 + 13*a^4*b^4*c
^4*d^2 - 10*a^5*b^2*c^5*d^2 - 4*a^5*b^4*c^3*e^2 + 5*a^6*b^2*c^4*e^2 - 4*a^7
*b^2*c^3*f^2 - 2*a^7*c^5*d*f + 6*a^6*b*c^5*d*e - 6*a^7*b*c^4*e*f + 8*a^4*b^
5*c^3*d*e - 18*a^5*b^3*c^4*d*e - 8*a^5*b^4*c^3*d*f + 14*a^6*b^2*c^4*d*f + 8
*a^6*b^3*c^3*e*f)/a^9 - (((4*a^6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c
^2*e + 8*a^8*b^2*c^3*e + 4*a^8*b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a
^9 - (2*a*b^2*c^2*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b
^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*
b^2))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a
^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)))*(2
*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f
 + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)) - (((((4*a^
6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c^2*e + 8*a^8*b^2*c^3*e + 4*a^8*
b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a^9 - (2*a*b^2*c^2*(2*b^5*d + 2*
a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b
*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*b^2))*(b^4*d + 2*a^2*c^2*d + a^
2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c -
 b^2)^(1/2)) - (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*
c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a
*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*
a^3*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2
*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c -
```

b^2)^(1/2)) + (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c
*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*
a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(8
a^7(4*a*c - b^2)*(16*a^5*c - 4*a^4*b^2)))*(3*b^6*d - a^3*c^3*d + 3*a^2*b^
4*f + a^4*c^2*f - 3*a*b^5*e + 18*a^2*b^2*c^2*d - 15*a*b^4*c*d + 12*a^2*b^3*
c*e - 9*a^3*b*c^2*e - 9*a^3*b^2*c*f))/(8*a^3*c^2*(a^4*c^4*d^2 - 6*a^2*b^6*e
^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2 + a^6*c^2*f^2 + 36*a^3*b^4*
c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^2*b^4*c^2*d^2 + 96*a^3*b^2*
c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 - 12*a^2*b^6*d*f + 12*a^3*b^5
*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4*b*c^3*d*e + 72*a^3*b^4*c*d
*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a^3*b^3*c^2*d*e - 95*a^4*b^2
*c^2*d*f)) + (((((((4*a^6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c^2*e +
8*a^8*b^2*c^3*e + 4*a^8*b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a^9 - (2
*a*b^2*c^2*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d
- 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*b^2))*(
b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2
*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*
b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2*b
^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*
d + 10*a^2*b^2*c*e))/(2*a^3*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(2
*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f
+ 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)) - (((a^6*c^
6*d^2 + a^8*c^4*f^2 - 4*a^3*b^6*c^3*d^2 + 13*a^4*b^4*c^4*d^2 - 10*a^5*b^2*c
^5*d^2 - 4*a^5*b^4*c^3*e^2 + 5*a^6*b^2*c^4*e^2 - 4*a^7*b^2*c^3*f^2 - 2*a^7*
c^5*d*f + 6*a^6*b*c^5*d*e - 6*a^7*b*c^4*e*f + 8*a^4*b^5*c^3*d*e - 18*a^5*b^
3*c^4*d*e - 8*a^5*b^4*c^3*d*f + 14*a^6*b^2*c^4*d*f + 8*a^6*b^3*c^3*e*f)/a^9
- (((4*a^6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c^2*e + 8*a^8*b^2*c^3*
e + 4*a^8*b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a^9 - (2*a*b^2*c^2*(2*
b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f
+ 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*b^2))*(2*b^5*d + 2*a^
2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c
^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d +
a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c
- b^2)^(1/2)) + (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^
3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^3)/(16*a^11*(4*a*c - b^2)^(3/2)))*(6*b^7
*d + 6*a^2*b^5*f + 20*a^4*c^3*e - 6*a*b^6*e + 84*a^2*b^3*c^2*d - 54*a^3*b^2
*c^2*e - 42*a*b^5*c*d - 46*a^3*b*c^3*d + 36*a^2*b^4*c*e - 30*a^3*b^3*c*f +
26*a^4*b*c^2*f))/(16*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^4*c^4*d^2 - 6*a^2*b^6*e
^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2 + a^6*c^2*f^2 + 36*a^3*b^4*
c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^2*b^4*c^2*d^2 + 96*a^3*b^2*
c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 - 12*a^2*b^6*d*f + 12*a^3*b^5
*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4*b*c^3*d*e + 72*a^3*b^4*c*d
*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a^3*b^3*c^2*d*e - 95*a^4*b^2
*c^2*d*f))))/(4*a^4*c^6*d^2 + b^8*c^2*d^2 + 4*a^6*c^4*f^2 - 8*a*b^6*c^3*d^2
+ 20*a^2*b^4*c^4*d^2 - 16*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 6*a^3*b^4*c^

3*e^2 + 9*a^4*b^2*c^4*e^2 + a^4*b^4*c^2*f^2 - 4*a^5*b^2*c^3*f^2 - 8*a^5*c^5
*d*f - 2*a*b^7*c^2*d*e + 12*a^4*b*c^5*d*e - 12*a^5*b*c^4*e*f + 14*a^2*b^5*c
^3*d*e - 28*a^3*b^3*c^4*d*e + 2*a^2*b^6*c^2*d*f - 12*a^3*b^4*c^3*d*f + 20*a
^4*b^2*c^4*d*f - 2*a^3*b^5*c^2*e*f + 10*a^4*b^3*c^3*e*f))*(b^4*d + 2*a^2*c^
2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(2*a^4*
(4*a*c - b^2)^(1/2)) - (log(((c^4*(b^2*d + a^2*f - a*b*e - a*c*d)^2*(b^3*d
- a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^9 - (((c^3*(4*b^6*d^2 - a^5*c
*f^2 + 4*a^2*b^4*e^2 - a^3*c^3*d^2 + 4*a^4*b^2*f^2 - 5*a^3*b^2*c*e^2 - 8*a*
b^5*d*e + 10*a^2*b^2*c^2*d^2 - 13*a*b^4*c*d^2 + 8*a^2*b^4*d*f - 8*a^3*b^3*e
*f + 2*a^4*c^2*d*f + 18*a^2*b^3*c*d*e - 6*a^3*b*c^2*d*e - 14*a^3*b^2*c*d*f
+ 6*a^4*b*c*e*f))/a^6 - (((4*b*c^2*(b^4*d + a^2*c^2*d + a^2*b^2*f - a*b^3*e
 - a^3*c*f - 3*a*b^2*c*d + 2*a^2*b*c*e))/a^3 + (2*c^3*x^2*(b^4*d - 10*a^2*c
^2*d + a^2*b^2*f - a*b^3*e + 10*a^3*c*f + 4*a*b^2*c*d - 5*a^2*b*c*e))/a^3 +
 (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^3*d + a^4*(-(b^4*d + 2*a^2*c^2*d
+ a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*
c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^4)*(b^3*d +
a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d
+ 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e -
 2*a*b*c*d))/(4*a^4) + (c^4*x^2*(6*b^5*d^2 + 6*a^4*b*f^2 + 6*a^2*b^3*e^2 +
11*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 5*a^4*c*e*f - 17*a*b^3*c*d^2 - 5*a^3*b*c*
e^2 + 12*a^2*b^3*d*f - 5*a^3*c^2*d*e - 12*a^3*b^2*e*f + 22*a^2*b^2*c*d*e -
17*a^3*b*c*d*f))/a^6)*(b^3*d + a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b
^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2)))^(1/2)
- a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/(4*a^4) + (c^5*x^2*(b^2*d + a^2
*f - a*b*e - a*c*d)^3)/a^9)*((c^4*(b^2*d + a^2*f - a*b*e - a*c*d)^2*(b^3*d
- a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^9 - (((c^3*(4*b^6*d^2 - a^5*c
*f^2 + 4*a^2*b^4*e^2 - a^3*c^3*d^2 + 4*a^4*b^2*f^2 - 5*a^3*b^2*c*e^2 - 8*a*
b^5*d*e + 10*a^2*b^2*c^2*d^2 - 13*a*b^4*c*d^2 + 8*a^2*b^4*d*f - 8*a^3*b^3*e
*f + 2*a^4*c^2*d*f + 18*a^2*b^3*c*d*e - 6*a^3*b*c^2*d*e - 14*a^3*b^2*c*d*f
+ 6*a^4*b*c*e*f))/a^6 - (((4*b*c^2*(b^4*d + a^2*c^2*d + a^2*b^2*f - a*b^3*e
 - a^3*c*f - 3*a*b^2*c*d + 2*a^2*b*c*e))/a^3 + (2*c^3*x^2*(b^4*d - 10*a^2*c
^2*d + a^2*b^2*f - a*b^3*e + 10*a^3*c*f + 4*a*b^2*c*d - 5*a^2*b*c*e))/a^3 +
 (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^3*d - a^4*(-(b^4*d + 2*a^2*c^2*d
+ a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*
c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^4)*(b^3*d -
a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d
+ 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e -
 2*a*b*c*d))/(4*a^4) + (c^4*x^2*(6*b^5*d^2 + 6*a^4*b*f^2 + 6*a^2*b^3*e^2 +
11*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 5*a^4*c*e*f - 17*a*b^3*c*d^2 - 5*a^3*b*c*
e^2 + 12*a^2*b^3*d*f - 5*a^3*c^2*d*e - 12*a^3*b^2*e*f + 22*a^2*b^2*c*d*e -
17*a^3*b*c*d*f))/a^6)*(b^3*d - a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b
^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2)))^(1/2)
- a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/(4*a^4) + (c^5*x^2*(b^2*d + a^2
*f - a*b*e - a*c*d)^3)/a^9))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4
*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*

```
a^5*c - 4*a^4*b^2)) - (log(x)*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*
c*d))/a^4 - (d/(6*a) + (x^4*(b^2*d + a^2*f - a*b*e - a*c*d))/(2*a^3) + (x^2
*(a*e - b*d))/(4*a^2))/x^6
```

3.55 $\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 369

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = \frac{(c^2d+b^2f-c(be+af))x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}$$
$$+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)-\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)+\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] (c^2*d+b^2*f-c*(a*f+b*e))*x/c^3+1/3*(-b*f+c*e)*x^3/c^2+1/5*f*x^5/c+1/2*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-a*c^2*e-b^3*f-b
*c*(-2*a*f+c*d)+(-b^3*c*e+3*a*b*c^2*e+b^4*f+b^2*c*(-4*a*f+c*d)-2*a*c^2*(-a*
f+c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-a*c^2*e-b
^3*f-b*c*(-2*a*f+c*d)+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2
*(-a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 3.08 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1678, 1180, 211}

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x(-c(af+be)+b^2f+c^2d)}{c^3}+\frac{x^3(ce-bf)}{3c^2}+\frac{fx^5}{5c}$$

```
[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) - (b^3*c*e - 3*a
*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d
- 2*a*f) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(
c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^2d+b^2f-c(be+af)}{c^3}+\frac{(ce-bf)x^2}{c^2}+\frac{fx^4}{c} - \frac{a(c^2d+b^2f-c(be+af))-(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{c^3\,(a+bx^2+cx^4)}\right) dx$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c} - \frac{\int \frac{a(c^2d+b^2f-c(be+af))+(-b^2ce+ac^2e+b^3f+bc(cd-2af))x^2}{a+bx^2+cx^4}\,dx}{c^3}$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c} + \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)-\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^3} + \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)+\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^3}$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c} + \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)-\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)+\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.24

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c} - \frac{\left(-b^4f-b^2c\left(cd+\sqrt{b^2-4ac}e-4af\right)+ac^2\left(2cd+\sqrt{b^2-4ac}e-2af\right)+b^3\left(ce+\sqrt{b^2-4ac}f\right)+bc\left(c\sqrt{}\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b^4f+b^2c\left(cd-\sqrt{b^2-4ac}e-4af\right)+ac^2\left(-2cd+\sqrt{b^2-4ac}e+2af\right)+b^3\left(-ce+\sqrt{b^2-4ac}f\right)+bc\left(\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) - ((-(b^4*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + a*c^2*(
```

```
2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + b^3*(c*e + Sqrt[b^2 - 4*a*c]*f) + b*
c*(c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^4*f + b^2*c*(c*d - Sqrt[b^2 - 4*a*c
]*e - 4*a*f) + a*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b^3*(-(c*e) +
 Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - 2*a*Sqrt[b^2
 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.44

method	result
risch	$\frac{f\,x^5}{5c}-\frac{bf\,x^3}{3c^2}+\frac{e\,x^3}{3c}-\frac{afx}{c^2}+\frac{b^2fx}{c^3}-\frac{bex}{c^2}+\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\left(2abcf-a\,c^2e-b^3f+b^2ce-b\,c^2d\right)\_R^2+a^2c\ldots\right.}{2c\_R^3+\_\ldots}}{2c^3}$
default	$-\frac{-\frac{1}{5}f\,x^5c^2+\frac{1}{3}bcf\,x^3-\frac{1}{3}c^2e\,x^3+acfx-b^2fx+bcex-c^2dx}{c^3}+\frac{\left(2\sqrt{-4ac+b^2}\,abcf-\sqrt{-4ac+b^2}\,a\,c^2e-b^3f\sqrt{-4ac+b^2}+b^2ce\sqrt{-4ac+b^2}-b\,c^2d\ldots\right.}{2c\sqrt{-\ldots}}$

```
[In] int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*f*x^5/c-1/3/c^2*b*f*x^3+1/3*e*x^3/c-1/c^2*a*f*x+1/c^3*b^2*f*x-1/c^2*b*e
*x+1/c*d*x+1/2/c^3*sum(((2*a*b*c*f-a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)*_R^2+a^2*
c*f-a*b^2*f+a*b*c*e-a*c^2*d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2
*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15467 vs. 2(331) = 662.

Time = 39.65 (sec) , antiderivative size = 15467, normalized size of antiderivative = 41.92

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(fx^4+ex^2+d)x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*f*x^5 + 5*(c^2*e - b*c*f)*x^3 + 15*(c^2*d - b*c*e + (b^2 - a*c)
*f)*x)/c^3 + integrate(-(a*c^2*d - a*b*c*e + (b*c^2*d - (b^2*c - a*c^2)*e +
 (b^3 - 2*a*b*c)*f)*x^2 + (a*b^2 - a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7235 vs. 2(331) = 662.

Time = 1.16 (sec) , antiderivative size = 7235, normalized size of antiderivative = 19.61

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d -
 (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
```

```
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*
(b^2 - 4*a*c)*a^2*c^5)*c^2*e + (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 -
 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2
 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*f + 2*(sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
3*c^5 - 2*a*b^4*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 +
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^6 + 16*a^2*b^2*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*c^7 - 32*a^3*c^7 + 2*(b^2 - 4*a*c)*a*b^2*c^5 - 8*(b^2 -
4*a*c)*a^2*c^6)*d*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 16*a^2
*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 32*a^3*b*c
^6 + 2*(b^2 - 4*a*c)*a*b^3*c^4 - 8*(b^2 - 4*a*c)*a^2*b*c^5)*e*abs(c) + 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 9*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^5*c^3 - 2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2
*c^4 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 18*a^2*b^4*c^4 - 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b*c^5 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 48*a
^3*b^2*c^5 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 32*a^4*c^6
 + 2*(b^2 - 4*a*c)*a*b^4*c^3 - 10*(b^2 - 4*a*c)*a^2*b^2*c^4 + 8*(b^2 - 4*a*
c)*a^3*c^5)*f*abs(c) - (2*b^5*c^6 - 12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)
```

*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)*
a*b*c^7)*d + (2*b^6*c^5 - 14*a*b^4*c^6 + 24*a^2*b^2*c^7 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*(b^2 - 4*a*c)*b^4*c^5 + 6*(b^2 - 4*a*c)*a*b
^2*c^6)*e - (2*b^7*c^4 - 16*a*b^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2 - 4*a
*c)*a*b^3*c^5 - 4*(b^2 - 4*a*c)*a^2*b*c^6)*f)*arctan(2*sqrt(1/2)*x/sqrt((b*
c^5 + sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^
3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) + 1/8*((2*b^
5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d - (2*b^6*c^3
 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
c^5 - 2(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*
c)*a^2*c^5)*c^2*e + (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c

^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2
- 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*f - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*a
*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c^5 + 8*(b^2 - 4*a*c)*a^2*
c^6)*d*abs(c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 -
4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2
- 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^2*b*c^5)*e*abs(c) - 2*(sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 +
2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 10*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^4*c^4 - 18*a^2*b^4*c^4 - 16*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 48*a^3*b^2*c^5
+ 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 - 32*a^4*c^6 - 2*(b^2 -
4*a*c)*a*b^4*c^3 + 10*(b^2 - 4*a*c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)
*f*abs(c) - (2*b^5*c^6 - 12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)*a*b*c^7)*d
+ (2*b^6*c^5 - 14*a*b^4*c^6 + 24*a^2*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^5*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr

```
t(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^4*c^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^6 - 2*(b^2 - 4*a*c)*b^4*c^5 + 6*(b^2 - 4*a*c)*a*b^2*c^6)*e -
 (2*b^7*c^4 - 16*a*b^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2 - 4*a*c)*a*b^3*c
^5 - 4*(b^2 - 4*a*c)*a^2*b*c^6)*f)*arctan(2*sqrt(1/2)*x/sqrt((b*c^5 - sqrt(
b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*
a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) + 1/15*(3*c^4*f*x^5 + 5
*c^4*e*x^3 - 5*b*c^3*f*x^3 + 15*c^4*d*x - 15*b*c^3*e*x + 15*b^2*c^2*f*x - 1
5*a*c^3*f*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 10.50 (sec) , antiderivative size = 23332, normalized size of antiderivative = 63.23

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*(e/(3*c) - (b*f)/(3*c^2)) - x*((b*(e/c - (b*f)/c^2))/c - d/c + (a*f)/c^
2) + atan(((((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*
d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^7 -
 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^
2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*
c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4
*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a
^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*
c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6
*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*
c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5
*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*
c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f
```

+ 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*
e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/
2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*
f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c
^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*
d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2
*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e
 + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*
d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*
b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) -
 (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^
2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b
^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^
3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5
*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*
c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2
+ b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a
*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28
*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 -
 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e -
16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a
^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*
a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c -
 b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*
f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c
^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*
c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5
 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b
^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^

5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^
4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63
*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*
a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*
b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2
*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1
/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^
2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f +
76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^
(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7
- 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*
c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3
*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*
c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^
5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2
*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3
)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*
a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2
*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) + (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^
4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2
*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 -
8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^
4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c
^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^
4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 +
12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 2
0*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2
*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1
/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*
e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*
a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c
- b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^

2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^
2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2)
 - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*
c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*
e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*f - 16*a^2
*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e
 + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d
^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*
a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a
^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^
4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*
a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e -
 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b
^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b
^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(
-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c
^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) -
66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c -
 b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 +
 b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*
b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b
*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^
2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*
b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b
^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b
^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c
^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d
f(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a
^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(
4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*
a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 -
 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 1
6*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c
^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*

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c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)
*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a
^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*
c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(
-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c
*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16
*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2
*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^
(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^
2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4
*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*
d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4
*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(a^4
*b^3*f^3 + a^4*c^3*e^3 + a^2*b*c^4*d^3 + a^2*b^5*d*f^2 + a^3*c^4*d^2*e - a^
3*b^4*e*f^2 + a^5*c^2*e*f^2 - a^3*b^2*c^2*e^3 - 2*a^5*b*c*f^3 - 2*a^4*c^3*d
*e*f - 4*a^3*b*c^3*d^2*f - 4*a^3*b^3*c*d*f^2 + 5*a^4*b*c^2*d*f^2 + 2*a^3*b^
3*c*e^2*f - 3*a^4*b*c^2*e^2*f + a^4*b^2*c*e*f^2 - 2*a^2*b^2*c^3*d^2*e + a^2
*b^3*c^2*d*e^2 + 2*a^2*b^3*c^2*d^2*f - 2*a^2*b^4*c*d*e*f + 4*a^3*b^2*c^2*d*
e*f))/c^5 + (((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6
*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 + (2*x*(4*b^3*c^7
- 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^
2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8
*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^
4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 -
a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7
*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^
6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^
5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3
*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f
 + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3
*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1
/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^
3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e
*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*
```

c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
1*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2
*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^
2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*
e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3
*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2
*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)
+ (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f
^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*
b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a
^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^
5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2
*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2
+ b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 -
a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 2
8*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2
- 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e -
16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*
a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6
*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c
- b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e
*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)
^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*
c^7 - 8*a*b^2*c^8)))^(1/2)))*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*
c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3
*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*
c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^
5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2
*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3
)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*
a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*

(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2
*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2)*2i + atan(((((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c
^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5
- (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^
6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5
d^2(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4
*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)
^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*
a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a
^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b
*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*
b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2
)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f +
76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*
b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 -
8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c
^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^
(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*
b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c
^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5
*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*
c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) -
36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a
^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*
c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2)
+ 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a
*b^2*c^8)))^(1/2) - (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4
*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*
b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8
*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4
*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^
4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4
*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 1
2*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20

*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*
c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 4
2*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/
2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e
 - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a
*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c -
 b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2
*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2)
- 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3
)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e
f(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*f - 16*a^2*
c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e +
 4*a*b^4*c^4*f)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^
2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a
^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^
3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4
e^2(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a
^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2)
- b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e -
2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^
5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^
2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-
(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^
4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6
6*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(
1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*
(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 +
b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b
*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*
c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2
*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b
^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^
4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^
6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^
3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^
2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)

^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a
^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 -
6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16
*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^
2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c
^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*
(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^
2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-
(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*
f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*
a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*
b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(
1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2
*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*
a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d
f(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(
1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((16
*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*
e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(
b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b
^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*
a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2
 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b
^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5
*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2
) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^
3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*
(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*
f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c

^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(a^4*b^3*f^3 + a^4*c^3*e^3 + a^2*b*c^4*d^3 + a^2*b^5*d*f^2 + a^3*c^4*d^2*e - a^3*b^4*e*f^2 + a^5*c^2*e*f^2 - a^3*b^2*c^2*e^3 - 2*a^5*b*c*f^3 - 2*a^4*c^3*d*e*f - 4*a^3*b*c^3*d^2*f - 4*a^3*b^3*c*d*f^2 + 5*a^4*b*c^2*d*f^2 + 2*a^3*b^3*c*e^2*f - 3*a^4*b*c^2*e^2*f + a^4*b^2*c*e*f^2 - 2*a^2*b^2*c^3*d^2*e + a^2*b^3*c^2*d*e^2 + 2*a^2*b^3*c^2*d^2*f - 2*a^2*b^4*c*d*e*f + 4*a^3*b^2*c^2*d*e*f))/c^5 + (((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2

)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63
*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*
a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*
b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2
*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1
/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^
2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f +
76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a
*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^
(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7
- 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*
c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)
^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3
*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*
c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^
5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2
*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3
)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*
a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2
*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2
) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) + (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^
4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2
*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 -
8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^
4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c
^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^
4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 +
12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 2
0*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2
*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1
/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*
e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*
a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c
- b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^
2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2)

- 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^
3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*
c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*
e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)))*(-(b^9*f^2 + b^5*c^4*d^2 +
b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b
*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*
c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2
*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b
^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^
4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^
6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^
3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^
2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)
^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + (f*x^5)/(5*c)

3.56 $\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 282

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4} dx = \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(c^2d-bce+b^2f-acf-\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] (-b*f+c*e)*x/c^2+1/3*f*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*(c^2*d-b*c*e+b^2*f-a*c*f+(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+
c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c^2*d-b*c*e+b^2*f-a
*c*f+(-b^2*c*e+2*a*c^2*e+b^3*f+b*c*(-3*a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2
)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1678, 1180, 211}

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x(ce-bf)}{c^2}+\frac{fx^3}{3c}$$

```
[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a*c*f + (b^
2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*e + b^2*f - a*c*f - (b^2*c*e - 2*a*c^2*e
 - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{ce-bf}{c^2}+\frac{fx^2}{c}-\frac{a(ce-bf)-(c^2d-bce+b^2f-acf)\,x^2}{c^2\,(a+bx^2+cx^4)}\right)dx$$

$$=\frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}-\frac{\int \frac{a(ce-bf)+(-c^2d+bce-b^2f+acf)x^2}{a+bx^2+cx^4}\,dx}{c^2}$$

$$=\frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf-\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}$$
$$+\frac{\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}$$

$$=\frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(c^2d-bce+b^2f-acf-\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.29

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$

$$=\frac{6\sqrt{c}(ce-bf)x+2c^{3/2}fx^3+\frac{3\sqrt{2}\left(-b^3f-bc\left(cd+\sqrt{b^2-4ac}e-3af\right)+b^2\left(ce+\sqrt{b^2-4ac}f\right)+c\left(c\sqrt{b^2-4ac}d-2ace-a\sqrt{b^2-4ac}f\right)\right)\text{arcta}}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{6c}$$

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*(c*e - b*f)*x + 2*c^(3/2)*f*x^3 + (3*Sqrt[2]*(-(b^3*f) - b*c*(c*
d + Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) + c*(c*S
qrt[b^2 - 4*a*c]*d - 2*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (3*Sqrt[2]*(b^3*f + b*c*(c*d - Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b
^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f) + c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*
Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

method	result
risch	$\frac{f x^3}{3c} - \frac{bfx}{c^2} + \frac{xe}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(\left(-acf+b^2f-ebc+c^2d\right)\_R^2+abf-ace\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c^2}$
default	$-\frac{-\frac{1}{3}cf x^3+bfx-xce}{c^2} + \frac{\left(-acf\sqrt{-4ac+b^2}+b^2f\sqrt{-4ac+b^2}-ebc\sqrt{-4ac+b^2}+c^2d\sqrt{-4ac+b^2}-3abcf+2a c^2e+b^3f-b^2ce+b c^2d\right)\sqrt{2}\ \arctan\left(\right.}{2\sqrt{-4ac+b^2}\, c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$

[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/3*f/c*x^3-1/c^2*b*f*x+1/c*x*e+1/2/c^2*sum(((-a*c*f+b^2*f-b*c*e+c^2*d)*_R^2+a*b*f-a*c*e)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9364 vs. 2(246) = 492.

Time = 8.91 (sec) , antiderivative size = 9364, normalized size of antiderivative = 33.21

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(fx^4+ex^2+d)x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*f*x^3 + 3*(c*e - b*f)*x)/c^2 - integrate((a*c*e - a*b*f - (c^2*d - b
*c*e + (b^2 - a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5454 vs. $2(246) = 492$.

Time = 1.09 (sec) , antiderivative size = 5454, normalized size of antiderivative = 19.34

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*((2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d - (2*b^5*c^3
 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2
*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*e + (2*b^6*c^2 - 18*a
*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c
^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
```

+ 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*f - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*e*abs(c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(c) - (2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d + (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*e - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*f)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) - 1/8*((2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 4*sqrt(2)*s

qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^
2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d - (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b
*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8
*(b^2 - 4*a*c)*a*b*c^4)*c^2*e + (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4
- 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^
2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*f + 2*(sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*
b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c
^5)*e*abs(c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c
^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2
- 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(c) - (2*b^4*c^6 - 8*a
*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^
4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^2 - 4*
a*c)*b^2*c^6)*d + (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr

```
t(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c
^6)*e - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^
5)*f)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a
*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c
^6 - 4*a^2*c^7)*c^2) + 1/3*(c^2*f*x^3 + 3*c^2*e*x - 3*b*c*f*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 15674, normalized size of antiderivative = 55.58

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(e/c - (b*f)/c^2) - atan(((((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f
 - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4
*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
+ 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^
3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5
*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f +
12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*
f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*
b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
```

))^(1/2) - (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^
3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2
 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*
c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7
*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^
2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e +
 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*
a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d
f(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*a^2*c^5*e - 4*a*b^2*c
^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*
(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 -
 b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*
b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3
*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f
 + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^
(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2
*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4
*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
+ 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2
+ b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f
 + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^
4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c
^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*
b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c
*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*

e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d
*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14
*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) +
2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*a
^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^
3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(
1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 1
2*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2
*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^
2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2
*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*
e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1
/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f
*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2
*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*
(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 -
b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*
b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3
*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f
+ 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^
(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2
*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6*f^2 - 2*a*c^5*
d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c
^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f -
2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2
*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a
*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b
^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3
*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)
^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*
c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 1
2*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f
+ 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^
(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))
)^(1/2) + (((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)

/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(
4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b
^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b
^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f
+ 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4
*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*
e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^
3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^
6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2
) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a
^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^
6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^
5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e -
14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2)
+ 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-
(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^
2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c
- b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b
^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^
2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 +
4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3
*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2
- c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)
^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c
^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f +
2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*
a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) +
16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-
(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)
^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) - (2*(a*c^4*d^3 - a^4*c*f^3 + a^3*b^2*f^3 - a^2*b*
c^2*e^3 + a^2*c^3*d*e^2 - a^2*b^3*e*f^2 - 3*a^2*c^3*d^2*f + 3*a^3*c^2*d*f^2
- a^3*c^2*e^2*f + a*b^4*d*f^2 - 2*a*b*c^3*d^2*e + a*b^2*c^2*d*e^2 + 2*a*b^
2*c^2*d^2*f - 3*a^2*b^2*c*d*f^2 + 2*a^2*b^2*c*e^2*f - 2*a*b^3*c*d*e*f + 2*a
^2*b*c^2*d*e*f))/c^3))*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3
)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2
+ 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2
- 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*

```
a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4
*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*
e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*
a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i
- atan(((((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c
^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^
3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5
*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f +
12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*
f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*
b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2
*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*
c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*
d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 1
4*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) -
 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4
*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*
c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6
*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*
e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4
*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c
^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 +
 c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*
(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5
*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*
b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*
c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 1
6*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4
*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3
)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*1i - (((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f
 - 16*a^2*b*c^4*f)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4
```

*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
- 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^
3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5
*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f +
12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*
f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*
b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2) + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^
3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2
- 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*
c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7
*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^
2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e +
16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*
a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c
- b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d
f(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*a^2*c^5*e - 4*a*b^2*c
^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*
(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 +
b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*
b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3
*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f
+ 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^

(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2
*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4
*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
- 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2
+ b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f
+ 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^
4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c
^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*
b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c
*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d
*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14
*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) -
2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((16*a^2*
c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 + (2*x*(4*b^3*c
^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2
) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a
^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^
6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^
5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e -
14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2)
- 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-
(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^
2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c
- b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(
b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^
4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3
e^2(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3
*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*

e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(a*c^4*d^3 - a^4*c*f^3 + a^3*b^2*f^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - a^2*b^3*e*f^2 - 3*a^2*c^3*d^2*f + 3*a^3*c^2*d*f^2 - a^3*c^2*e^2*f + a*b^4*d*f^2 - 2*a*b*c^3*d^2*e + a*b^2*c^2*d*e^2 + 2*a*b^2*c^2*d^2*f - 3*a^2*b^2*c*d*f^2 + 2*a^2*b^2*c*e^2*f - 2*a*b^3*c*d*e*f + 2*a^2*b*c^2*d*e*f))/c^3))*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i + (f*x^3)/(3*c)

3.57 $\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx=\frac{fx}{c}+\frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] f*x/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*e-b*f+(
2*c^2*d+b^2*f-c*(2*a*f+b*e))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*(c*e-b*f+(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1690, 1180, 211}

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f - (2*c^2*d - b*c*e + b^2*f -
2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{f}{c}+\frac{cd-af+(ce-bf)x^2}{c\left(a+bx^2+cx^4\right)}\right) dx \\
&= \frac{fx}{c}+\frac{\int \frac{cd-af+(ce-bf)x^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{fx}{c}+\frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&\quad+\frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&= \frac{fx}{c}+\frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.18

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{c}fx + \frac{\sqrt{2}\left(2c^2d+b\left(b-\sqrt{b^2-4ac}\right)f+c\left(-be+\sqrt{b^2-4ac}e-2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(2c^2d+b\left(b+\sqrt{b^2-4ac}\right)f-c\left(be+\sqrt{b^2-4ac}e\right.\right.}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2}}}}{2c^{3/2}}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*f*x + (Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e
) + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*
(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.31

method	result
risch	$\frac{fx}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)} \frac{\left((-bf+ec)\_R^2-af+cd\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{fx}{c} + \frac{\left(-bf\sqrt{-4ac+b^2}+c\sqrt{-4ac+b^2}\,e+2acf-b^2f+ebc-2c^2d\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\left(-bf\sqrt{-4ac+b^2}+c\sqrt{-4ac+b^2}\right.}{2\sqrt{}}$

```
[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] f*x/c+1/2/c*sum(((-b*f+c*e)*_R^2-a*f+c*d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootO
f(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5788 vs. 2(185) = 370.

Time = 4.34 (sec) , antiderivative size = 5788, normalized size of antiderivative = 26.43

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \int \frac{fx^4+ex^2+d}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] f*x/c - integrate(-((c*e - b*f)*x^2 + c*d - a*f)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4082 vs. 2(185) = 370.

Time = 1.00 (sec) , antiderivative size = 4082, normalized size of antiderivative = 18.64

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] f*x/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
```

b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*f + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 32*a^2*c^6 + 2*(b^2 - 4*a*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*d*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*f*abs(c) + 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*e + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*f)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)

```
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4
)*c^2*e - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*f - 2*(
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c^4 + 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*
d*abs(c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b
^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*f*abs(c) + 2*(2*b^3*c^6 - 8*a*b*c^7 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d - (2
*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5
 - 2*(b^2 - 4*a*c)*b^2*c^5)*e + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5)*f)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^
```

```
3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*
c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 10209, normalized size of antiderivative = 46.62

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x)/c - atan(((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)
/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*
c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 +
12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*
a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*
c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b
^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3
 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b
^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^
3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c
^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d
*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*
e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*
a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*
f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2
 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4
*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2
 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 +
16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2
*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*
a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*
c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i - (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*
d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3
*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 +
 a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4
*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*
c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2
```

- c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*(a*c^2*e^3 - a^2*b*f^3 - b^3*d*f^2 + c^3*d^2*e + a*b^2*e*f^2 - b*c^2*d*e^2 - b*c^2*d^2*f + a^2*c*e*f^2 + 2*a*b*c*d*f^2 - 2*a*b*c*e^2*f - 2*a*c^2*d*e*f + 2*b^2*c*d*e*f))/c + (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4* a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)))*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*2i - atan(((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*

e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3
*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e
^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
 - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*
b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i - (((4*b^2*c^3*d + 16*a^
2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(
a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2
- 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2
*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5
*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a
^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2
)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*
d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2
 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*
c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*
(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e
^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*
b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((((4
*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3
 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2
) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) -
a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a
^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3
*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e
f(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^
(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a
*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2
e^2(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*
f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*
e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(

4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*(a*c^2*e^3 - a^2*b*f^3 - b^3*d*f^2 + c^3*d^2*e + a*b^2*e*f^2 - b*c^2*d*e^2 - b*c^2*d^2*f + a^2*c*e*f^2 + 2*a*b*c*d*f^2 - 2*a*b*c*e^2*f - 2*a*c^2*d*e*f + 2*b^2*c*d*e*f))/c + (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)))*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*2i

3.58 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 213

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = -\frac{d}{ax} - \frac{\left(cd-af+\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -d/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-a*f+
(a*b*f-2*a*c*e+b*c*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c
*d-a*f+(-a*b*f+2*a*c*e-b*c*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1678, 1180, 211}

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{d}{ax}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d}{ax^2} + \frac{-bd+ae-(cd-af)x^2}{a\left(a+bx^2+cx^4\right)}\right) dx \\
&= -\frac{d}{ax} + \frac{\int \frac{-bd+ae+(-cd+af)x^2}{a+bx^2+cx^4}\,dx}{a} \\
&= -\frac{d}{ax} - \frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a} \\
&\quad + \frac{\left(-cd+af+\frac{2ace-b(cd+af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a} \\
&= -\frac{d}{ax} - \frac{\left(cd-af-\frac{2ace-b(cd+af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad - \frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.19

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$$
$$=\frac{-\frac{2d}{x}-\frac{\sqrt{2}\left(bcd+c\sqrt{b^2-4ac}d-2ace+abf-a\sqrt{b^2-4ac}f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(bcd-c\sqrt{b^2-4ac}d-2ace+abf+a\sqrt{b^2-4ac}f\right)\arctan\left(\right.}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*d)/x - (Sqrt[2]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f - a*S
qrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b*c*d
 - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03

method	result
default	$\frac{4c\left(\frac{\left(af\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}+abf-2ace+bcd\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(af\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}-abf+2ace-bcd\right)\sqrt{2}\,\mathrm{arcta}}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b\right.}}}\right.}{a}$
risch	Expression too large to display

```
[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*(1/8*(a*f*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1/2)+a*b*f-2*a*c*e+b*c
*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(a*f*(-4*a*c+b^2)^(1/2)-c*d
*(-4*a*c+b^2)^(1/2)-a*b*f+2*a*c*e-b*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)))-d/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5930 vs. $2(177) = 354$.

Time = 1.66 (sec) , antiderivative size = 5930, normalized size of antiderivative = 27.84

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)x^2}\,dx$$

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(-((c*d - a*f)*x^2 + b*d - a*e)/(c*x^4 + b*x^2 + a), x)/a - d/(a*x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3984 vs. $2(177) = 354$.

Time = 1.36 (sec) , antiderivative size = 3984, normalized size of antiderivative = 18.70

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -d/(a*x) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
```

*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*a^2*d - (
2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*a^2*f + 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 16*a^2*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 32*a^3*b*c^4 + 2*(b^2 - 4*a*c)*a*b^3*c^2 -
8*(b^2 - 4*a*c)*a^2*b*c^3)*d*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*a^2*b^4*c^2 + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^3 + 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 -
 32*a^4*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3)*e*abs(
a) + (2*a^2*b^4*c^3 - 8*a^3*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c^3)*d - 2*(2*a^3*b^3*c
^3 - 8*a^4*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c
^3 - 2*(b^2 - 4*a*c)*a^3*b*c^3)*e + (2*a^3*b^4*c^2 - 8*a^4*b^2*c^3 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*(b^2 - 4*a*c)*a^3*b^
2*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))
/((a^3*b^4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a
^3*b^2*c^3 - 4*a^4*c^4)*abs(a)*abs(c)) + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 3
2*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c

```
 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*
(b^2 - 4*a*c)*a*c^4)*a^2*d - (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2
- 4*a*c)*a^2*c^3)*a^2*f - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*
c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 16*a^2*b^
3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 32*a^3*b*c^4
- 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*d*abs(a) + 2*(sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^3*c^2 + 2*a^2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^
3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 32*a^4*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + 8
*(b^2 - 4*a*c)*a^3*c^3)*e*abs(a) + (2*a^2*b^4*c^3 - 8*a^3*b^2*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*(b^2 - 4*a*c)*a
^2*b^2*c^3)*d - 2*(2*a^3*b^3*c^3 - 8*a^4*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a^3*b*c^3)*e + (2*a^3*b^
4*c^2 - 8*a^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((a*b - sq
rt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2
+ 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^3 - 4*a^4*c^4)*abs(a)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 10170, normalized size of antiderivative = 47.75

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x)
[Out] - atan(((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
 (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*b^2*c^2*e + 16*a^
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^
2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/
2)*1i + (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
 (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
```

)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2*d - 4*a^5*b^2*c^2*e - 16*a^
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^
2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/
2)*1i)/((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
- 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*b^2*c^2*e + 16*a^
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^
2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/
2) - (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2 -
2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) + (-
(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d
^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2
- 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*
b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*
b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a^6*

b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2*d - 4*a^5*b^2*c^2*e - 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) - 2*a^6*c*f^3 + 2*a^3*c^4*d^3 + 2*a^4*c^3*d*e^2 - 6*a^4*c^3*d^2*f + 6*a^5*c^2*d*f^2 - 2*a^5*c^2*e^2*f + 2*a^5*b*c*e*f^2 - 2*a^3*b*c^3*d^2*e - 2*a^4*b^2*c*d*f^2 + 2*a^3*b^2*c^2*d^2*f))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*2i - atan(((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*b^2*c^2*e + 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2

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- 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a
^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e -
2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*
b^4*c - 8*a^4*b^2*c^2)))^(1/2)*1i + (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a
^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*
c^3*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^
3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-
(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d
*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a
^4*b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^
3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d
^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 +
 a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b
*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*
b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2
*d - 4*a^5*b^2*c^2*e - 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2
- 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a
^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e -
2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*
b^4*c - 8*a^4*b^2*c^2)))^(1/2)*1i)/((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a
^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*
c^3*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^
3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-
(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d
*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a
^4*b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^
3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d
^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 +
 a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b
*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*
b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2
*d + 4*a^5*b^2*c^2*e + 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^
```

```
2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2
- 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a
^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e -
2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*
b^4*c - 8*a^4*b^2*c^2)))^(1/2) - (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*
c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3
*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
- 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f
^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2
- a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^
2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^
2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2
*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5
*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2*d
- 4*a^5*b^2*c^2*e - 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(
-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a
*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 1
6*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*
c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a
*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4
*c - 8*a^4*b^2*c^2)))^(1/2) - 2*a^6*c*f^3 + 2*a^3*c^4*d^3 + 2*a^4*c^3*d*e^2
 - 6*a^4*c^3*d^2*f + 6*a^5*c^2*d*f^2 - 2*a^5*c^2*e^2*f + 2*a^5*b*c*e*f^2 -
2*a^3*b*c^3*d^2*e - 2*a^4*b^2*c*d*f^2 + 2*a^3*b^2*c^2*d^2*f))*(-(b^5*c*d^2
+ a^3*b^3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2
*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c
^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f -
 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f +
 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*2i - d/(a*x)
```

3.59 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 267

$$\begin{aligned}&\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx\\&=-\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\sqrt{c}\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}\\&\quad-\frac{\sqrt{c}\left(b^2d-b\left(\sqrt{b^2-4ac}d+ae\right)-a\left(2cd-\sqrt{b^2-4ac}e-2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

```
[Out] -1/3*d/a/x^3+(-a*e+b*d)/a^2/x+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*c^(1/2)*(b*d-a*e+(b^2*d-a*b*e-2*a*(-a*f+c*d))/(-4*a*c+b^2)^(1
/2))/a^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-b*(a*e+d*(-4*a*c+b^2)^(1/2))-a
*(2*c*d-2*a*f-e*(-4*a*c+b^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

= {1678, 1180, 211}

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx = \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right)-b\left(d\sqrt{b^2-4ac}+ae\right)+b^2d\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{bd-ae}{a^2x} - \frac{d}{3ax^3}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*d/(a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d - a*b*e
 - 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(
b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e - 2*
a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2
*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d}{ax^4}+\frac{-bd+ae}{a^2x^2}+\frac{b^2d-abe-a(cd-af)+c(bd-ae)x^2}{a^2(a+bx^2+cx^4)}\right)dx \\
&= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\int \frac{b^2d-abe-a(cd-af)+c(bd-ae)x^2}{a+bx^2+cx^4}\,dx}{a^2}
\end{aligned}$$

$$= -\frac{d}{3ax^3} + \frac{bd-ae}{a^2x} + \frac{\left(c\left(bd-ae-\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2} + \frac{\left(c\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2}$$

$$= -\frac{d}{3ax^3} + \frac{bd-ae}{a^2x} + \frac{\sqrt{c}\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(bd-ae-\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx = \frac{-\frac{2ad}{x^3}+\frac{6bd-6ae}{x}+\frac{3\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}d-ae\right)+a\left(-2cd-\sqrt{b^2-4ac}e+2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2d+b\left(\sqrt{b^2-4ac}d+\right.\right.}{}}{6a^2}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^3 + (6*b*d - 6*a*e)/x + (3*Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2
 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d +
 a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/(6*a^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.91

method	result
default	$-\frac{d}{3a\,x^3}-\frac{ae-bd}{a^2x}+\frac{4c\left(\frac{\left(-ae\sqrt{-4ac+b^2}+bd\sqrt{-4ac+b^2}-2f\,a^2+abe+2acd-b^2d\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-ae\sqrt{-4ac+b^2}+bd\sqrt{-\right.}{}\right.}{a^2}$
risch	Expression too large to display

```
[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d/a/x^3-(a*e-b*d)/a^2/x+4/a^2*c*(1/8*(-a*e*(-4*a*c+b^2)^(1/2)+b*d*(-4*
a*c+b^2)^(1/2)-2*f*a^2+a*b*e+2*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))-1/8*(-a*e*(-4*a*c+b^2)^(1/2)+b*d*(-4*a*c+b^2)^(1/2)+2*f*a^2-a*b*e-2*
a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9850 vs. $2(226) = 452$.

Time = 11.66 (sec) , antiderivative size = 9850, normalized size of antiderivative = 36.89

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)x^4}\,dx$$

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*e - a^2*f - (b*c*d - a*c*e)*x^2 - (b^2 - a*c)*d)/(c*x^4 + b
*x^2 + a), x)/a^2 + 1/3*(3*(b*d - a*e)*x^2 - a*d)/(a^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3804 vs. 2(226) = 452.

Time = 0.99 (sec) , antiderivative size = 3804, normalized size of antiderivative = 14.25

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*
c^3 + 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32
*a^3*c^4 - 24*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c
)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4
*a*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b*c^3)*d - (sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 +
16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 32*a^3*b*c^3 + 12*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2
*(b^2 - 4*a*c)*a*b^2*c^2 - 4*(b^2 - 4*a*c)*a^2*c^3)*e + (sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*
c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
```

*a^2*b^2*c^2 + 16*a^3*b^2*c^2 - 2*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*c^3 - 32*a^4*c^3 + 8*a^3*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)
*a^3*c^2 + 2*(b^2 - 4*a*c)*a^2*b*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b +
sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c +
16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/4*((sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c
+ 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^2*c^3 + 14*a*b
^3*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 - 2
4*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5
- 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 12*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 10
*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*c^
3 - 6*(b^2 - 4*a*c)*a*b*c^3)*d - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c
^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32
*a^3*b*c^3 + 12*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2 - 4*a*
c)*a*b^2*c^2 - 4*(b^2 - 4*a*c)*a^2*c^3)*e + (sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2
- 16*a^3*b^2*c^2 - 2*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*

```
c)*a^3*c^3 + 32*a^4*c^3 + 8*a^3*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2 + 2
*(b^2 - 4*a*c)*a^2*b*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b - sqrt(a^4*b^
2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2
+ 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/3*(3*b*d*x^2 - 3*a*e*x
^2 - a*d)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.74 (sec) , antiderivative size = 15505, normalized size of antiderivative = 58.07

$$\int \frac{d + ex^2 + fx^4}{x^4 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^
2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*
e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 +
 a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e
^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2
+ a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3
*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b
^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2
)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*
b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(
4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4
 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-
(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a
^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a
^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^
3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d
*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4
*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*
e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b
^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^11*c^
3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2
```

*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3
)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a
^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c
^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*
a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i
+ (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 -
8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e -
4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2
*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*
c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 -
 a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^
2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4
*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c
*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)
^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7
*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b
*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^
2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e -
 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16
*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c
 - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f
- 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d
f(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16*a^11*c^3*f
+ 4*a^8*b^4*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10*b^2*c^2*f +
 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1
/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7
*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*
a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2
*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b
^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d
*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^

(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*
b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a
c - b^2)^3)^(1/2))/(8(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i)/((x
*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 - 8*a^
7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e - 4*a^
9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2*b^5
*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f
 - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e
 - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/
2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^
7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7*d^2
 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2
*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a
^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2
*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b
^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*
a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(
-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^11*c^3*f - 4*
a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2*f - 16*
a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3
*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^
6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2
d^2(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d
*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f -
 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*
c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - (x*(4*a^8
*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 - 8*a^7*b^2*c
^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e - 4*a^9*b*c^3
*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2*b^5*e^2 +
b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*

(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*
d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a
^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a
^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*
a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 -
 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7*d^2 + a^2*
b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 -
a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2
*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*
e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*
d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^
(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c
d^2(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c
 - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16
*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16*a^11*c^3*f + 4*a^8*b^4
*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10*b^2*c^2*f + 16*a^10*b*
c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b
^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*
e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e +
 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16
*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*
b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e
 + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 2*a^8*c^4*e^3 -
2*a^6*b*c^5*d^3 + 2*a^7*c^5*d^2*e + 2*a^9*c^3*e*f^2 - 4*a^8*c^4*d*e*f - 4*a
^7*b*c^4*d*e^2 + 4*a^7*b*c^4*d^2*f - 2*a^8*b*c^3*d*f^2 - 2*a^8*b*c^3*e^2*f
+ 2*a^6*b^2*c^4*d^2*e - 2*a^6*b^3*c^3*d^2*f + 4*a^7*b^2*c^3*d*e*f))*(-(b^7*
d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*
c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2
*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e -
2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*
a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c
- b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f -
 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^

5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*2i - (d/(3*a) + (x^2*(a*e - b*d))
/a^2)/x^3 + atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^
6*b^4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*
a^8*b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) -
(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 -
a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12
*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*
b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3
*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-
(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*
c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2
*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10
*b^3*c^2)*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4
*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
+ 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f +
16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^
3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d
*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d
+ 16*a^11*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*
a^10*b^2*c^2*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b
*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f +
2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f
+ 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^
2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)
))^(1/2)*1i + (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^
4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*
b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b
^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4
f^2(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4
*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*
c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2

)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e
 - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) +
16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a
*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*
f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2
*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3
c^2)(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3
*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^
2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 2
5*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a
^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3
)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*
e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e -
 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16
*a^11*c^3*f + 4*a^8*b^4*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10
*b^2*c^2*f + 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c
- b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f
^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*
b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24
*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(
1/2)*1i)/((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^
3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^
4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d
^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c
^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*
d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2
*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a
^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c -
 b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f +
3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f
*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5
*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2
)*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2

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 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 +
12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^
2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c
^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*
(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^
2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a
^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^1
1*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2
*c^2*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 +
 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4
*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)
 - (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 -
 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e -
 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^
2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2
+ a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a
^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^
4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*
c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3
)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a
*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 +
16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^
7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*
f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*
b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c
^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e
- 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 1
6*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*
c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f
 + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*
```

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d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(
a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16*a^11*c^3*f
 + 4*a^8*b^4*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10*b^2*c^2*f
+ 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(
1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 -
7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2
*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^
2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*
b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*
d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3
*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 2*a^
8*c^4*e^3 - 2*a^6*b*c^5*d^3 + 2*a^7*c^5*d^2*e + 2*a^9*c^3*e*f^2 - 4*a^8*c^4
*d*e*f - 4*a^7*b*c^4*d*e^2 + 4*a^7*b*c^4*d^2*f - 2*a^8*b*c^3*d*f^2 - 2*a^8*
b*c^3*e^2*f + 2*a^6*b^2*c^4*d^2*e - 2*a^6*b^3*c^3*d^2*f + 4*a^7*b^2*c^3*d*e
*f))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*
f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2
 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25
*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^
4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)
^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e
*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4
*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e -
2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*2i
```

3.60 $\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 329

$$\begin{aligned}
&\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx \\
&= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x} \\
&\quad -\frac{\sqrt{c}\left(b^2d-abe-a(cd-af)+\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad -\frac{\sqrt{c}\left(b^2d-abe-a(cd-af)-\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

```
[Out] -1/5*d/a/x^5+1/3*(-a*e+b*d)/a^2/x^3+(-b^2*d+a*b*e+a*(-a*f+c*d))/a^3/x-1/2*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-a*b*e-
a*(-a*f+c*d)+(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/2))
/a^3*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-a*b*e-a*(-a*f+c*d)+(-b^3*d+a*b^2*e
-2*a^2*c*e+a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/2))/a^3*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1678, 1180, 211}

$$\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$$
$$= -\frac{-abe-a(cd-af)+b^2d}{a^3x}+\frac{bd-ae}{3a^2x^3} - \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{5ax^5}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*d/(a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d - a*f))/
(a^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*d - a*b^2*e + 2*a^
2*c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(b^2*d - a*b*e - a*(c*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(
3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d}{ax^6} + \frac{-bd+ae}{a^2x^4} + \frac{b^2d-abe-a(cd-af)}{a^3x^2} + \frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c(b^2d-abe-a(cd-af))x^2}{a^3(a+bx^2+cx^4)}\right)dx$$

$$= -\frac{d}{5ax^5} + \frac{bd-ae}{3a^2x^3} - \frac{b^2d-abe-a(cd-af)}{a^3x} + \frac{\int \frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c(b^2d-abe-a(cd-af))x^2}{a+bx^2+cx^4}\,dx}{a^3}$$

$$= -\frac{d}{5ax^5} + \frac{bd-ae}{3a^2x^3} - \frac{b^2d-abe-a(cd-af)}{a^3x} - \frac{\left(c\left(b^2d-abe-a(cd-af)-\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^3} - \frac{\left(c\left(b^2d-abe-a(cd-af)+\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^3}$$

$$= -\frac{d}{5ax^5} + \frac{bd-ae}{3a^2x^3} - \frac{b^2d-abe-a(cd-af)}{a^3x} - \frac{\sqrt{c}\left(b^2d-abe-a(cd-af)+\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2d-abe-a(cd-af)-\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$$

$$= \frac{-\frac{6a^2d}{x^5} + \frac{10a(bd-ae)}{x^3} + \frac{30(-b^2d+abe+a(cd-af))}{x} - \frac{15\sqrt{2}\sqrt{c}\left(b^3d+b^2\left(\sqrt{b^2-4ac}d-ae\right)+ab\left(-3cd-\sqrt{b^2-4ac}e+af\right)+a\left(-c\sqrt{b^2-4ac}d+2ac\right.\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-6*a^2*d)/x^5 + (10*a*(b*d - a*e))/x^3 + (30*(-(b^2*d) + a*b*e + a*(c*d -
 a*f)))/x - (15*Sqrt[2]*Sqrt[c]*(b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) +
a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*
a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
```

```
2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (15*Sqrt[2]
*Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-3*c*d + Sqrt[b^2
- 4*a*c]*e + a*f) + a*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*Sqrt[b^2 - 4*a*c
]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(30*a^3)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.09

method	result
default	$-\frac{d}{5a\,x^5}-\frac{ae-bd}{3a^2x^3}-\frac{f\,a^2-abe-acd+b^2d}{a^3x}+\frac{4c\left(\frac{\left(-f\,a^2\sqrt{-4ac+b^2}+abe\sqrt{-4ac+b^2}+acd\sqrt{-4ac+b^2}-b^2d\sqrt{-4ac+b^2}+a^2bf+2a^2ce-a\,b^2}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$
risch	Expression too large to display

```
[In] int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*d/a/x^5-1/3*(a*e-b*d)/a^2/x^3-(a^2*f-a*b*e-a*c*d+b^2*d)/a^3/x+4/a^3*c*
(1/8*(-f*a^2*(-4*a*c+b^2)^(1/2)+a*b*e*(-4*a*c+b^2)^(1/2)+a*c*d*(-4*a*c+b^2)
^(1/2)-b^2*d*(-4*a*c+b^2)^(1/2)+a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-f*a^2*(-4*a*c+b^2)^(1/2)+a*b*e*
(-4*a*c+b^2)^(1/2)+a*c*d*(-4*a*c+b^2)^(1/2)-b^2*d*(-4*a*c+b^2)^(1/2)-a^2*b*
f-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15830 vs. $2(289) = 578$.

Time = 45.94 (sec) , antiderivative size = 15830, normalized size of antiderivative = 48.12

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to display
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^6\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^6\,(a+bx^2+cx^4)}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)x^6}\,dx$$

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*b*f - (a*b*c*e - a^2*c*f - (b^2*c - a*c^2)*d)*x^2 + (b^3 -
2*a*b*c)*d - (a*b^2 - a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/a^3 + 1/15*(15*(a*b
*e - a^2*f - (b^2 - a*c)*d)*x^4 - 3*a^2*d + 5*(a*b*d - a^2*e)*x^2)/(a^3*x^5
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6710 vs. $2(289) = 578$.

Time = 1.35 (sec) , antiderivative size = 6710, normalized size of antiderivative = 20.40

$$\int \frac{d+ex^2+fx^4}{x^6\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(
b^2 - 4*a*c)*a^2*c^4)*a^2*d - (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*a^3*b*c^4
```

- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2
+ 8*(b^2 - 4*a*c)*a^2*b*c^3)*a^2*e + (2*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 32*a
^4*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4
+ 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 2*(b^2 - 4*a*c)*a^2
*b^2*c^2 + 8*(b^2 - 4*a*c)*a^3*c^3)*a^2*f + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^7 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 32*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 12*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c
^2 + 20*a^2*b^5*c^2 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3
- 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 6*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 64*a^3*b^3*c^3 + 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c
- 12*(b^2 - 4*a*c)*a^2*b^3*c^2 + 16*(b^2 - 4*a*c)*a^3*b*c^3)*d*abs(a) - 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^5*c - 2*a^2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^
2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 18*a^3*b^4*c^2 - 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 48*a^
4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 32*a^5*c^4
+ 2*(b^2 - 4*a*c)*a^2*b^4*c - 10*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - 4*a*c
)*a^4*c^3)*e*abs(a) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5 -
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c - 2*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*a^3*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^5*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 16*a^4*b^3*c^
2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 32*a^5*b*c^3 + 2*
(b^2 - 4*a*c)*a^3*b^3*c - 8*(b^2 - 4*a*c)*a^4*b*c^2)*f*abs(a) + (2*a^2*b^6*
c^2 - 14*a^3*b^4*c^3 + 24*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2

- 4*a*c)*c)*a^4*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 2*(b^2 - 4*a*c)*a^2*b^4*c^2 + 6*(b^2 - 4*a*c)*a^3*b^2*c^3)*d - (2*a^3*b^5*c^2 - 12*a^4*b^3*c^3 + 16*a^5*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 2*(b^2 - 4*a*c)*a^3*b^3*c^2 + 4*(b^2 - 4*a*c)*a^4*b*c^3)*e + (2*a^4*b^4*c^2 - 8*a^5*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 2*(b^2 - 4*a*c)*a^4*b^2*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b + sqrt(a^6*b^2 - 4*a^7*c))/(a^3*c)))/((a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 + a^5*b^2*c^2 - 4*a^6*c^3)*abs(a)*abs(c)) + 1/8*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*a^2*d - (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*a^2*e + (2*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 32*a^4*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*

b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c
^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + 8*(b^2 - 4*a*c)*a^3*c^3)*a^2*f - 2*(sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c +
2*a*b^7*c + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 12*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^5*c^2 - 20*a^2*b^5*c^2 - 32*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 64*a^3*b^3*
c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 64*a^4*b*c^4 -
2*(b^2 - 4*a*c)*a*b^5*c + 12*(b^2 - 4*a*c)*a^2*b^3*c^2 - 16*(b^2 - 4*a*c)*a
^3*b*c^3)*d*abs(a) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^5*c + 2*a^2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^2*c^2 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 18*a^3*b^4*
c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b^2*c^3 + 48*a^4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*c^4 - 32*a^5*c^4 - 2*(b^2 - 4*a*c)*a^2*b^4*c + 10*(b^2 - 4*a*c)*a^3*
b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)*e*abs(a) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c
- 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c + 2*a^3*b^5*c + 16*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^3*c^2 - 16*a^4*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b
*c^3 + 32*a^5*b*c^3 - 2*(b^2 - 4*a*c)*a^3*b^3*c + 8*(b^2 - 4*a*c)*a^4*b*c^2
)*f*abs(a) + (2*a^2*b^6*c^2 - 14*a^3*b^4*c^3 + 24*a^4*b^2*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6 + 7*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 2*(b^2 - 4*a*c)*a^2*b^4*c^2 +
6*(b^2 - 4*a*c)*a^3*b^2*c^3)*d - (2*a^3*b^5*c^2 - 12*a^4*b^3*c^3 + 16*a^5*
b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5 +
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^2 - 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 2*(b^2 - 4*a*c)
*a^3*b^3*c^2 + 4*(b^2 - 4*a*c)*a^4*b*c^3)*e + (2*a^4*b^4*c^2 - 8*a^5*b^2*c^
3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4 + 4*s

```
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c + 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 2*(b^2 - 4*a
*c)*a^4*b^2*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b - sqrt(a^6*b^2 - 4*a^7
*c))/(a^3*c)))/((a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b
*c^2 + a^5*b^2*c^2 - 4*a^6*c^3)*abs(a)*abs(c)) - 1/15*(15*b^2*d*x^4 - 15*a*
c*d*x^4 - 15*a*b*e*x^4 + 15*a^2*f*x^4 - 5*a*b*d*x^2 + 5*a^2*e*x^2 + 3*a^2*d
)/(a^3*x^5)
```

Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 23019, normalized size of antiderivative = 69.97

$$\int \frac{d + ex^2 + fx^4}{x^6 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*
d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^
12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e +
12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c
^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e
^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*
a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5
*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2)
 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f -
16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a
^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b
^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a
*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^
2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c
^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f
^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^
2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 4
2*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/
2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^
2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^
```

7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a^15*c^4*e + 4*a^12*b^5*c^2*d - 24*a^13*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^14*b^2*c^3*e + 4*a^14*b^3*c^2*f + 32*a^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i + (x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^

2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) + 16*a^15*c^4*e - 4*a^12*b^5*c^2*d + 24*a^13*b^3*c^3*d + 4*a^13*b^4*c^2*e - 20*a^14*b^2*c^3*e - 4*a^14*b^3*c^2*f - 32*a^14*b*c^4*d + 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i)/((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2)

- 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a^15*c^4*e + 4*a^12*b^5*c^2*d - 24*a^13*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^14*b^2*c^3*e + 4*a^14*b^3*c^2*f + 32*a^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2)

+ 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9
*c^2 - 8*a^8*b^2*c)))^(1/2) - (x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*
c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2
*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*
f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^
3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f)
- (-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2
 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2
+ 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*
a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*
(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*
c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2
*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f -
 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a
^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*
f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^
3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) +
4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a
^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b
^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*
c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^
2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*
b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^
6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*
c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^
4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^
3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a
^8*b^2*c)))^(1/2) + 16*a^15*c^4*e - 4*a^12*b^5*c^2*d + 24*a^13*b^3*c^3*d +
4*a^13*b^4*c^2*e - 20*a^14*b^2*c^3*e - 4*a^14*b^3*c^2*f - 32*a^14*b*c^4*d +
 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1
/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 -
 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) -
2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*

```
c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e
^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 1
6*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 1
8*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b
^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) +
50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(
-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*
b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)
))^(1/2) - 2*a^10*c^7*d^3 + 2*a^13*c^4*f^3 - 2*a^11*b*c^5*e^3 - 2*a^11*c^6*
d*e^2 + 6*a^11*c^6*d^2*f - 6*a^12*c^5*d*f^2 + 2*a^12*c^5*e^2*f + 2*a^9*b^2*
c^6*d^3 - 4*a^12*b*c^4*e*f^2 - 2*a^9*b^3*c^5*d^2*e + 4*a^10*b^2*c^5*d*e^2 +
 2*a^9*b^4*c^4*d^2*f - 6*a^10*b^2*c^5*d^2*f + 4*a^11*b^2*c^4*d*f^2 + 2*a^11
*b^2*c^4*e^2*f + 4*a^11*b*c^5*d*e*f - 4*a^10*b^3*c^4*d*e*f))*(-(b^9*d^2 + a
^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*
d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f
^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2
- 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^
3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^
7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*
f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e +
 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*
f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^
5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-
(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b
*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*2i - (d/(5*a) + (x^4*(b^
2*d + a^2*f - a*b*e - a*c*d))/a^3 + (x^2*(a*e - b*d))/(3*a^2))/x^5 + atan((
(x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 -
12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2
*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^1
3*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f
 - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 - b
^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^
5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c
^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4
*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5
```

*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^
(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4
*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f
*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b
^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f
+ 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^
3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4
 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-
(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12
*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*
b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a
^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4
*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^
2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b
^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*
a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b
^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f +
2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(
1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^
4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a^15*c^
4*e + 4*a^12*b^5*c^2*d - 24*a^13*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^14*b^2
*c^3*e + 4*a^14*b^3*c^2*f + 32*a^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^9*d^2
+ a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c
^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^
2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d
^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^
2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2
*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3
*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*
e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3
*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36
*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f
*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^
3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i + (x*(4*a^13*c^5*e
^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*

d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^1
3*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*
a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c
^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c
- b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5
*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4
e^2(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a
^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3
*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b
^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b
^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)
^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2
*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2
*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1
/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 -
8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b
^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2
- 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 +
a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63
*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*
a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(
1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*
f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c
- b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f +
16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a
^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-
(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^
2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a
*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) + 16*a^15*c^4*e - 4*a^12*b^5
*c^2*d + 24*a^13*b^3*c^3*d + 4*a^13*b^4*c^2*e - 20*a^14*b^2*c^3*e - 4*a^14*
b^3*c^2*f - 32*a^14*b*c^4*d + 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 -
b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b
^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*
c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^
4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^
5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)

^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^
4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c -
b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f
 + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)
^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^
4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i)/((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*
d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^
2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 +
8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e
 + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*
b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a
^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 -
 a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^
6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^
3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c
^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a
^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(
4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*
c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(
1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^
2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*
a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2
 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d
*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f
 - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^
2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*
a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)
^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^
3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*

a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a^15*c^4*e + 4*a^12*b^5*c^2*d - 24*a^13
*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^14*b^2*c^3*e + 4*a^14*b^3*c^2*f + 32*a
^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^
5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2
)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^
4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*
a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^
3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*
b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2
)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) -
2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^
2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 -
 8*a^8*b^2*c)))^(1/2) - (x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^
2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*
b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20
*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*
d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b
^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*
a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a
^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^
5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3
*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a
*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2
+ 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^
5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2
*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*
a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*
b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^
2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*
c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)
^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^
2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4
*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^
2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^

3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) + 16*a^15*c^4*e - 4*a^12*b^5*c^2*d + 24*a^13*b^3*c^3*d + 4*a^13*b^4*c^2*e - 20*a^14*b^2*c^3*e - 4*a^14*b^3*c^2*f - 32*a^14*b*c^4*d + 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 2*a^10*c^7*d^3 + 2*a^13*c^4*f^3 - 2*a^11*b*c^5*e^3 - 2*a^11*c^6*d*e^2 + 6*a^11*c^6*d^2*f - 6*a^12*c^5*d*f^2 + 2*a^12*c^5*e^2*f + 2*a^9*b^2*c^6*d^3 - 4*a^12*b*c^4*e*f^2 - 2*a^9*b^3*c^5*d^2*e + 4*a^10*b^2*c^5*d*e^2 + 2*a^9*b^4*c^4*d^2*f - 6*a^10*b^2*c^5*d^2*f + 4*a^11*b^2*c^4*d*f^2 + 2*a^11*b^2*c^4*e^2*f + 4*a^11*b*c^5*d*e*f - 4*a^10*b^3*c^4*d*e*f))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*

```
c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c
 - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d
*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*2i
```

3.61 $\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 320

$$\begin{aligned}
&\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)} + \frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)} \\
&\quad + \frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad - \frac{(2b^4ce-12ab^2c^2e+12a^2c^3e-3b^5f-b^3c(cd-20af)+6abc^2(cd-5af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\,(b^2-4ac)^{3/2}} \\
&\quad + \frac{(c^2d+3b^2f-2c(be+af))\log(a+bx^2+cx^4)}{4c^4}
\end{aligned}$$

```
[Out] 1/2*(2*b^2*c*e-6*a*c^2*e-3*b^3*f-b*c*(-11*a*f+c*d))*x^2/c^3/(-4*a*c+b^2)+1/
4*(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e))*x^4/c^2/(-4*a*c+b^2)+1/2*x^6*(2*a*c*e-b
*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)-1/2*(2*b^4*c*e-12*a*b^2*c^2*e+12*a^2*c^3*e-3*b^5*f-b^3*c*(-20*a*f+c*d)+6
*a*b*c^2*(-5*a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+
b^2)^(3/2)+1/4*(c^2*d+3*b^2*f-2*c*(a*f+b*e))*ln(c*x^4+b*x^2+a)/c^4
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1677, 1658, 814, 648, 632, 212, 642}

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(12a^2c^3e-b^3c(cd-20af)-12ab^2c^2e+6abc^2(cd-5af)-3b^5f+2b^4ce)}{2c^4(b^2-4ac)^{3/2}} + \frac{x^4(-2c(4af+be)+3b^2f+4c^2d)}{4c^2(b^2-4ac)} + \frac{x^6(-(x^2(-2acf+b^2f-bce+2c^2d))-b(af+cd)+2ace)}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{\log(a+bx^2+cx^4)(-2c(af+be)+3b^2f+c^2d)}{4c^4} + \frac{x^2(-bc(cd-11af)-6ac^2e-3b^3f+2b^2ce)}{2c^3(b^2-4ac)}$$

```
[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*c*e - 6*a*c^2*e - 3*b^3*f - b*c*(c*d - 11*a*f))*x^2)/(2*c^3*(b^2 -
4*a*c)) + ((4*c^2*d + 3*b^2*f - 2*c*(b*e + 4*a*f))*x^4)/(4*c^2*(b^2 - 4*a*c
)) + (x^6*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^
2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*b^4*c*e - 12*a*b^2*c^2*e
+ 12*a^2*c^3*e - 3*b^5*f - b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(3/2)) + ((c
^2*d + 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(d+ex+fx^2)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3\left(2ae-\frac{b(cd+af)}{c}\right)-\frac{(4c^2d-2bce+3b^2f-8acf)x}{c}\right)}{a+bx+cx^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)}$$

$$= \frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\Big(\int \Big(-\frac{2b^2ce-6ac^2e-3b^3f-bc(cd-11af)}{c^3} - \frac{(4c^2d-2bce+3b^2f-8acf)x}{c^2} - \frac{-a(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))+(}{c^3(a+bx+c}}{2\,(b^2-4ac)}$$

$$= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)} + \frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)} + \frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{-a(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))+(b^2-4ac)\,(c^2d+3b^2f-2c(be+af))x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^3\,(b^2-4ac)}$$

$$= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)} + \frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)} + \frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(2b^4ce-12ab^2c^2e+12a^2c^3e-3b^5f-b^3c(cd-20af)+6abc^2(cd-5af))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{4c^4\,(b^2-4ac)} + \frac{(c^2d+3b^2f-2c(be+af))\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^4}$$

$$= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)} + \frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)} + \frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(c^2d+3b^2f-2c(be+af))\log\left(a+bx^2+cx^4\right)}{4c^4} - \frac{(2b^4ce-12ab^2c^2e+12a^2c^3e-3b^5f-b^3c(cd-20af)+6abc^2(cd-5af))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\right.}{2c^4\,(b^2-4ac)}$$

$$
\begin{aligned}
&= \frac{(2b^2ce - 6ac^2e - 3b^3f - bc(cd - 11af))\,x^2}{2c^3\,(b^2 - 4ac)} + \frac{(4c^2d + 3b^2f - 2c(be + 4af))\,x^4}{4c^2\,(b^2 - 4ac)} \\
&\quad + \frac{x^6(2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf)\,x^2)}{2c\,(b^2 - 4ac)\,(a + bx^2 + cx^4)} \\
&\quad - \frac{(2b^4ce - 12ab^2c^2e + 12a^2c^3e - 3b^5f - b^3c(cd - 20af) + 6abc^2(cd - 5af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\,(b^2 - 4ac)^{3/2}} \\
&\quad + \frac{(c^2d + 3b^2f - 2c(be + af))\log(a + bx^2 + cx^4)}{4c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.97

$$
\int \frac{x^7(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2}\,dx
$$

$$
= \frac{2c(ce - 2bf)x^2 + c^2fx^4 + \frac{2(2a^3c^2f + b^3(c^2d - bce + b^2f)x^2 + ab(b^3f - 3c^3dx^2 + bc^2(d + 4ex^2) - b^2c(e + 5fx^2)) + a^2c(-4b^2f - 2c^2(d + ex^2) + b}{(b^2 - 4ac)(a + bx^2 + cx^4)}}{}
$$

```
[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(c*e - 2*b*f)*x^2 + c^2*f*x^4 + (2*(2*a^3*c^2*f + b^3*(c^2*d - b*c*e +
 b^2*f)*x^2 + a*b*(b^3*f - 3*c^3*d*x^2 + b*c^2*(d + 4*e*x^2) - b^2*c*(e + 5
*f*x^2)) + a^2*c*(-4*b^2*f - 2*c^2*(d + e*x^2) + b*c*(3*e + 5*f*x^2))))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(-2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2
*c^3*e + 3*b^5*f + b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(-(c*d) + 5*a*f))*ArcTa
n[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c^2*d + 3*b^2*
f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.35

method	result
default	$\frac{(-cf\,x^2+2bf-ec)^2}{4c^4 f} + \frac{-\frac{(5a^2b\,c^2f-2a^2c^3e-5a\,b^3cf+4a\,b^2c^2e-3ab\,c^3d+b^5f-b^4ec+b^3c^2d)x^2}{c(4ac-b^2)} - \frac{a(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2a\,c^3d+b^4f-b^3c}{c(4ac-b^2)}}{c\,x^4+b\,x^2+a}$
risch	Expression too large to display

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-c*f*x^2+2*b*f-c*e)^2/c^4/f+1/2/c^3*((-(5*a^2*b*c^2*f-2*a^2*c^3*e-5*a*
b^3*c*f+4*a*b^2*c^2*e-3*a*b*c^3*d+b^5*f-b^4*c*e+b^3*c^2*d)/c/(4*a*c-b^2)*x^
```

```
2-a*(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)
/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-8*a^2*c^2*f+14*a*b^2*c
*f-8*a*b*c^2*e+4*a*c^3*d-3*b^4*f+2*b^3*c*e-b^2*c^2*d)/c*ln(c*x^4+b*x^2+a)+2
*(11*a^2*b*c*f-6*a^2*c^2*e-3*a*b^3*f+2*a*b^2*c*e-a*b*c^2*d-1/2*(-8*a^2*c^2*
f+14*a*b^2*c*f-8*a*b*c^2*e+4*a*c^3*d-3*b^4*f+2*b^3*c*e-b^2*c^2*d)*b/c)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(306) = 612$.

Time = 0.53 (sec) , antiderivative size = 2111, normalized size of antiderivative = 6.60

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^
4 + 16*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b
^2*c^3 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d -
 (b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c
+ 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 - (((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^
4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3
)*f)*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^
3)*e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*
c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*
c + 30*a^3*b*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(a*b^4*c^2
 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c - 7*a^2*b^3*c^2 + 12*a^3*b*c^3
)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*f + (((b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e +
 (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*f)*x^4 + ((b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*
e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*f)*x^2 + (a*b^4*c^
2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*f)*log(c*x^4
 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^2), 1/4*(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b^5*c^2
 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3
 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d - (b^6*
c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c + 41*a
```

```
^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 + 2*(((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^4*c^
2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*f)
*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*e
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*c^3)
*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*c +
30*a^3*b*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c
)/(b^2 - 4*a*c)) + 2*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c
 - 7*a^2*b^3*c^2 + 12*a^3*b*c^3)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^
2 - 8*a^4*c^3)*f + (((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 -
32*a^3*c^4)*f)*x^4 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c -
 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 3
2*a^3*b*c^3)*f)*x^2 + (a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5
*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2
*c^2 - 32*a^4*c^3)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 +
16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.30

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(b^3c^2d-6\,abc^3d-2\,b^4ce+12\,ab^2c^2e-12\,a^2c^3e+3\,b^5f-20\,ab^3cf+30\,a^2bc^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^4-4\,ac^5)\sqrt{-b^2+4\,ac}} - \frac{b^2c^3dx^4-4\,ac^4dx^4-2\,b^3c^2ex^4+8\,abc^3ex^4+3\,b^4cfx^4-14\,ab^2c^2fx^4+8\,a^2c^3fx^4-b^3c^2dx^2+2\,abc^3dx}{4\,(b^2c^4-4\,ac^5)(cx} + \frac{(c^2d-2\,bce+3\,b^2f-2\,acf)\log(cx^4+bx^2+a)}{4\,c^4} + \frac{c^2fx^4+2\,c^2ex^2-4\,bcfx^2}{4\,c^4}$$

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*c^2*d - 6*a*b*c^3*d - 2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e +
 3*b^5*f - 20*a*b^3*c*f + 30*a^2*b*c^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c^3*d*x^4 - 4*a
*c^4*d*x^4 - 2*b^3*c^2*e*x^4 + 8*a*b*c^3*e*x^4 + 3*b^4*c*f*x^4 - 14*a*b^2*c
^2*f*x^4 + 8*a^2*c^3*f*x^4 - b^3*c^2*d*x^2 + 2*a*b*c^3*d*x^2 + 4*a^2*c^3*e*
x^2 + b^5*f*x^2 - 4*a*b^3*c*f*x^2 - 2*a^2*b*c^2*f*x^2 - a*b^2*c^2*d + 2*a^2
*b*c^2*e + a*b^4*f - 6*a^2*b^2*c*f + 4*a^3*c^2*f)/((b^2*c^4 - 4*a*c^5)*(c*x
^4 + b*x^2 + a)) + 1/4*(c^2*d - 2*b*c*e + 3*b^2*f - 2*a*c*f)*log(c*x^4 + b*
x^2 + a)/c^4 + 1/4*(c^2*f*x^4 + 2*c^2*e*x^2 - 4*b*c*f*x^2)/c^4
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 3499, normalized size of antiderivative = 10.93

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2*(e/(2*c^2) - (b*f)/c^3) - ((2*a^3*c^2*f - 2*a^2*c^3*d + a*b^4*f - a*b^3
*c*e + a*b^2*c^2*d + 3*a^2*b*c^2*e - 4*a^2*b^2*c*f)/(2*c*(4*a*c - b^2)) + (
x^2*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f
+ 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c*(4*a*c - b^2)))/(a*c^3 + c^4*x^4 + b
*c^3*x^2) - (log(a + b*x^2 + c*x^4)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d
+ 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^
2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*
c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*
```

a^2*b^2*c^6)) + (f*x^4)/(4*c^2) + (atan(((8*a*c^7*(4*a*c - b^2)^3 - 2*b^2*c
^6*(4*a*c - b^2)^3)*(((((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e - 24*a*b^2
*c^4*f)/c^6 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4
*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f -
576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a
^3*b*c^4*e))/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6))*(3
*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f
+ 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3/2)) - (a*(3*b^5
*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12
*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256
*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4
*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e
+ 256*a^3*b*c^4*e))/(c^2*(4*a*c - b^2)^(3/2)*(256*a^3*c^7 - 4*b^6*c^4 + 48
*a*b^4*c^5 - 192*a^2*b^2*c^6)))/(a*(4*a*c - b^2)) - x^2*(((((24*a^2*c^7*e -
6*b^3*c^6*d + 12*b^4*c^5*e - 18*b^5*c^4*f + 28*a*b*c^7*d - 56*a*b^2*c^6*e
+ 96*a*b^3*c^5*f - 92*a^2*b*c^6*f)/(4*a*c^7 - b^2*c^6) - ((8*b^3*c^8 - 32*a
*b*c^9)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e
+ 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^
3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2
*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*
c^6)))*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a
*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3/2)) -
((8*b^3*c^8 - 32*a*b*c^9)*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e -
6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 1
28*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d -
192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f -
24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(16*c^4*(4*a*c - b^2)^
(3/2)*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2
*b^2*c^6)))/(a*(4*a*c - b^2)) + (b*((((24*a^2*c^7*e - 6*b^3*c^6*d + 12*b^4*
c^5*e - 18*b^5*c^4*f + 28*a*b*c^7*d - 56*a*b^2*c^6*e + 96*a*b^3*c^5*f - 92*
a^2*b*c^6*f)/(4*a*c^7 - b^2*c^6) - ((8*b^3*c^8 - 32*a*b*c^9)*(6*b^8*f - 128
*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 1
92*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 2
4*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(4*a*c^7 - b^2*c^6)*(
256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)))*(6*b^8*f - 128*
a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 19
2*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24
*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c
^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) - (9*b^7*f^2 + b^3*c^4*d^2 + 4*b^5*c^
2*e^2 - 20*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - 38*a^3*b*c^3*f^2 - 12*b^6*c*e
*f + 91*a^2*b^3*c^2*f^2 - 5*a*b*c^5*d^2 - 57*a*b^5*c*f^2 - 6*a^2*c^5*d*e -
4*b^4*c^3*d*e + 12*a^3*c^4*e*f + 6*b^5*c^2*d*f + 20*a*b^2*c^4*d*e - 34*a*b^
3*c^3*d*f + 29*a^2*b*c^4*d*f + 68*a*b^4*c^2*e*f - 76*a^2*b^2*c^3*e*f)/(4*a*
c^7 - b^2*c^6) + (((b^3*c^8)/2 - 2*a*b*c^9)*(3*b^5*f - 12*a^2*c^3*e + b^3*c
^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c

```
^2*f)^2)/(c^8*(4*a*c - b^2)^3*(4*a*c^7 - b^2*c^6))))/(2*a*(4*a*c - b^2)^(3/
2))) + (b*((((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e - 24*a*b^2*c^4*f)/c^6
 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*
c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^
2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e)
)/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6))*(6*b^8*f - 12
8*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d -
192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f -
24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6
*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) - (a*c^4*d^2 + 9*a*b^4*f^2 + 4*a^3*
c^2*f^2 + 4*a*b^2*c^2*e^2 - 12*a^2*b^2*c*f^2 - 4*a^2*c^3*d*f + 6*a*b^2*c^2*
d*f + 8*a^2*b*c^2*e*f - 4*a*b*c^3*d*e - 12*a*b^3*c*e*f)/c^6 + (a*(3*b^5*f -
 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b
^2*c^2*e + 30*a^2*b*c^2*f)^2)/(c^6*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3
/2))))/(9*b^10*f^2 + 144*a^4*c^6*e^2 + b^6*c^4*d^2 + 4*b^8*c^2*e^2 - 12*a*b
^4*c^5*d^2 - 48*a*b^6*c^3*e^2 - 12*b^9*c*e*f + 36*a^2*b^2*c^6*d^2 + 192*a^2
*b^4*c^4*e^2 - 288*a^3*b^2*c^5*e^2 + 580*a^2*b^6*c^2*f^2 - 1200*a^3*b^4*c^3
*f^2 + 900*a^4*b^2*c^4*f^2 - 120*a*b^8*c*f^2 - 4*b^7*c^3*d*e + 6*b^8*c^2*d*
f + 48*a*b^5*c^4*d*e + 144*a^3*b*c^6*d*e - 76*a*b^6*c^3*d*f + 152*a*b^7*c^2
*e*f - 720*a^4*b*c^5*e*f - 168*a^2*b^3*c^5*d*e + 300*a^2*b^4*c^4*d*f - 360*
a^3*b^2*c^5*d*f - 672*a^2*b^5*c^3*e*f + 1200*a^3*b^3*c^4*e*f))*(3*b^5*f - 1
2*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2
*c^2*e + 30*a^2*b*c^2*f))/(2*c^4*(4*a*c - b^2)^(3/2))
```

3.62 $\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 236

$$\begin{aligned}\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx &= \frac{(2c^2d+2b^2f-c(be+6af))\,x^2}{2c^2\,(b^2-4ac)}+\frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}\\&\quad-\frac{(12a^2c^2f-b^3(ce-2bf)-2ac(2c^2d-3bce+6b^2f))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{3/2}}\\&\quad+\frac{(ce-2bf)\log\left(a+bx^2+cx^4\right)}{4c^3}\end{aligned}$$

```
[Out] 1/2*(2*c^2*d+2*b^2*f-c*(6*a*f+b*e))*x^2/c^2/(-4*a*c+b^2)+1/2*x^4*(2*a*c*e-b
*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)-1/2*(12*a^2*c^2*f-b^3*(-2*b*f+c*e)-2*a*c*(6*b^2*f-3*b*c*e+2*c^2*d))*arct
anh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/4*(-2*b*f+c*e)
*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used

= {1677, 1658, 787, 648, 632, 212, 642}

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(12a^2c^2f-2ac(6b^2f-3bce+2c^2d)-(b^3(ce-2bf)))}{2c^3(b^2-4ac)^{3/2}}$$
$$+\frac{x^2(-c(6af+be)+2b^2f+2c^2d)}{2c^2(b^2-4ac)}$$
$$+\frac{x^4(-(x^2(-2acf+b^2f-bce+2c^2d))-b(af+cd)+2ace)}{2c(b^2-4ac)(a+bx^2+cx^4)}$$
$$+\frac{(ce-2bf)\log(a+bx^2+cx^4)}{4c^3}$$

```
[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*c^2*d + 2*b^2*f - c*(b*e + 6*a*f))*x^2)/(2*c^2*(b^2 - 4*a*c)) + (x^4*(2
*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((12*a^2*c^2*f - b^3*(c*e - 2*b*f) - 2*a*
c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*(b^2 - 4*a*c)^(3/2)) + ((c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+ex+fx^2)}{(a+bx+cx^2)^2}\,dx, x, x^2\right) \\
&= \frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{x\left(2\left(2ae-\frac{b(cd+af)}{c}\right)-\frac{(2c^2d-bce+2b^2f-6acf)x}{c}\right)}{a+bx+cx^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)}
\end{aligned}
$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)}+\frac{x^4(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a\left(2c^2d-bce+2b^2f-6acf\right)}{c}+\left(\frac{b\left(2c^2d-bce+2b^2f-6acf\right)}{c}+2c\left(2ae-\frac{b(cd+af)}{c}\right)\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c\left(b^2-4ac\right)}$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(ce-2bf)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3} + \frac{\left(12a^2c^2f-b^3(ce-2bf)-2ac(2c^2d-3bce+6b^2f)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3\left(b^2-4ac\right)}$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(ce-2bf)\log\left(a+bx^2+cx^4\right)}{4c^3} - \frac{\left(12a^2c^2f-b^3(ce-2bf)-2ac(2c^2d-3bce+6b^2f)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^3\left(b^2-4ac\right)}$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(12a^2c^2f-b^3(ce-2bf)-2ac(2c^2d-3bce+6b^2f)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{3/2}} + \frac{(ce-2bf)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{2cfx^2 - \frac{2(b^2(c^2d-bce+b^2f)x^2+a^2c(-3bf+2c(e+fx^2))+a(b^3f-2c^3dx^2+bc^2(d+3ex^2)-b^2c(e+4fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{2(12a^2c^2f+b^3(-ce+2bf)-2ac(\cdots}{(-\cdots}}{4c^3}$$

[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]

[Out] (2*c*f*x^2 - (2*(b^2*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(12*a^2*c^2*f + b^3*(-(c*e) + 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3)

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.31

method	result
default	$\frac{f x^2}{2c^2} - \frac{\frac{-\frac{(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2a\,c^3d+b^4f-b^3ce+b^2c^2d)x^2}{c(4ac-b^2)}+\frac{a(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d)}{c(4ac-b^2)}}{c\,x^4+b\,x^2+a} + \frac{\frac{(8abcf-4a\,c^2e-2b^3f+b^2ce)\ln(c\,x^4\cdots}{2c}\cdots}{\cdots}}{2c^2}$
risch	Expression too large to display

[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*f*x^2/c^2-1/2/c^2*((-(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/c/(4*a*c-b^2)*x^2+a*(3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*f-4*a*c^2*e-2*b^3*f+b^2*c*e)/c*ln(c*x^4+b*x^2+a)+2*(6*a^2*c*f-2*a*b^2*f+a*b*c*e-2*a*c^2*d-1/2*(8*a*b*c*f-4*a*c^2*e-2*b^3*f+b^2*c*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(224) = 448$.

Time = 0.35 (sec) , antiderivative size = 1455, normalized size of antiderivative = 6.17

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*c
- 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^
3*c^3)*f)*x^2 + (4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a*b^2*c^2
)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c - 6*a^2*b*c^2)*e
- 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*
c^3)*e - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*e - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*
x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3
*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 +
16*a^2*b*c^5)*x^2), 1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*
a^2*b^2*c^2 - 24*a^3*c^3)*f)*x^2 + 2*(4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 -
 6*a*b*c^3)*e - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d +
 (b^4*c - 6*a*b^2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^
3*c - 6*a^2*b*c^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(-b^2 + 4
*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(a*b^3*c^
2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e - 2*(a*b^5 -
 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e -
 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*x^2 + (a*b^4*c - 8*
a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.15

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{fx^2}{2\,c^2} - \frac{(4\,ac^3d + b^3ce - 6\,abc^2e - 2\,b^4f + 12\,ab^2cf - 12\,a^2c^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^3 - 4\,ac^4)\sqrt{-b^2+4\,ac}}$$
$$- \frac{b^2cex^4 - 4\,ac^2ex^4 - 2\,b^3fx^4 + 8\,abcfx^4 + 2\,b^2cdx^2 - 4\,ac^2dx^2 - b^3ex^2 + 2\,abcex^2 + 4\,a^2cfx^2 + 2\,abcd -}{4\,(cx^4+bx^2+a)(b^2c^2 - 4\,ac^3)}$$
$$+ \frac{(ce - 2\,bf)\log\left(cx^4+bx^2+a\right)}{4\,c^3}$$

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*f*x^2/c^2 - 1/2*(4*a*c^3*d + b^3*c*e - 6*a*b*c^2*e - 2*b^4*f + 12*a*b^2
*c*f - 12*a^2*c^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4
*a*c^4)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*e*x^4 - 4*a*c^2*e*x^4 - 2*b^3*f*x^
4 + 8*a*b*c*f*x^4 + 2*b^2*c*d*x^2 - 4*a*c^2*d*x^2 - b^3*e*x^2 + 2*a*b*c*e*x
^2 + 4*a^2*c*f*x^2 + 2*a*b*c*d - a*b^2*e + 2*a^2*b*f)/((c*x^4 + b*x^2 + a)*
(b^2*c^2 - 4*a*c^3)) + 1/4*(c*e - 2*b*f)*log(c*x^4 + b*x^2 + a)/c^3
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 2450, normalized size of antiderivative = 10.38

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
[Out] ((a*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c*(4*a*c - b^2)
) + (x^2*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2
*e - 4*a*b^2*c*f))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (f*
x^2)/(2*c^2) + (log(a + b*x^2 + c*x^4)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e
 - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 2
56*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c
^5)) - (atan(((8*a*c^5*(4*a*c - b^2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*((
(((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*c^6*d + 28*a*b*c^5*e - 5
6*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*b^7*f + 1
28*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*
c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^
6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(2*b^4*f + 12*a^2*c^2*f -
 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f))/(8*c^3*(4*a*c - b^2)^(3
/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*
c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96
*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^
3*b*c^3*f))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 -
4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((4*b^
5*f^2 + b^3*c^2*e^2 + 12*a^2*b*c^2*f^2 + 2*a*c^4*d*e - 4*b^4*c*e*f - 5*a*b*
c^3*e^2 - 20*a*b^3*c*f^2 - 6*a^2*c^3*e*f + 20*a*b^2*c^2*e*f - 4*a*b*c^3*d*f
)/(4*a*c^5 - b^2*c^4) + (((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*
c^6*d + 28*a*b*c^5*e - 56*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 -
32*a*b*c^7)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a
^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^
5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(
4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f
- 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6
*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (((b^3*c^6)/2 - 2*a*b*c^7)*(2*b^4
*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)^2)/(c
^6*(4*a*c - b^2)^3*(4*a*c^5 - b^2*c^4))))/(2*a*(4*a*c - b^2)^(3/2))) + ((((
8*a*c^4*e - 16*a*b*c^3*f)/c^4 - (8*a*c^2*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c
*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e -
 256*a^3*b*c^3*f))/(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^
5))*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*
c*f))/(8*c^3*(4*a*c - b^2)^(3/2)) - (a*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d
- b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*
```

```
e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e -
256*a^3*b*c^3*f))/(c*(4*a*c - b^2)^(3/2)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^
4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((((8*a*c^4*e - 16*a*b*c^
3*f)/c^4 - (8*a*c^2*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e
 + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2
56*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(4*b^7*f + 128*a^
3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f +
 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*
c^4 - 192*a^2*b^2*c^5)) - (4*a*b^2*f^2 + a*c^2*e^2 - 4*a*b*c*e*f)/c^4 + (a*
(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)
^2)/(c^4*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(4*b^8*f^2 + 16*a^2
*c^6*d^2 + 144*a^4*c^4*f^2 + b^6*c^2*e^2 - 12*a*b^4*c^3*e^2 - 4*b^7*c*e*f +
 36*a^2*b^2*c^4*e^2 + 192*a^2*b^4*c^2*f^2 - 288*a^3*b^2*c^3*f^2 - 48*a*b^6*
c*f^2 - 96*a^3*c^5*d*f + 8*a*b^3*c^4*d*e - 48*a^2*b*c^5*d*e - 16*a*b^4*c^3*
d*f + 48*a*b^5*c^2*e*f + 144*a^3*b*c^4*e*f + 96*a^2*b^2*c^4*d*f - 168*a^2*b
^3*c^3*e*f))*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e -
12*a*b^2*c*f))/(2*c^3*(4*a*c - b^2)^(3/2))
```

3.63 $\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx = \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)x^2)}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{(4ac^2e+b^3f-2bc(cd+3af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}} + \frac{f\log(a+bx^2+cx^4)}{4c^2}$$

```
[Out] 1/2*x^2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+
b^2)/(c*x^4+b*x^2+a)+1/2*(4*a*c^2*e+b^3*f-2*b*c*(3*a*f+c*d))*arctanh((2*c*x
^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*f*ln(c*x^4+b*x^2+a)/c^
2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1677, 1658, 648, 632, 212, 642}

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-2bc(3af+cd)+4ac^2e+b^3f)}{2c^2(b^2-4ac)^{3/2}} + \frac{x^2(-(x^2(-2acf+b^2f-bce+2c^2d))-b(af+cd)+2ace)}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{f\log(a+bx^2+cx^4)}{4c^2}$$

```
[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(
2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a*c^2*e + b^3*f - 2*b*c*(c*d +
 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/
2)) + (f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex+fx^2)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{2ae-\frac{b(cd+af)}{c}-\frac{(b^2-4ac)fx}{c}}{a+bx+cx^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)}$$

$$= \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{f\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} - \frac{(4ac^2e+b^3f-2bc(cd+3af))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2\,(b^2-4ac)}$$

$$= \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{f\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{(4ac^2e+b^3f-2bc(cd+3af))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2\,(b^2-4ac)}$$

$$= \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(4ac^2e+b^3f-2bc(cd+3af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}} + \frac{f\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{\frac{2(-2a^2cf+b(c^2d-bce+b^2f)x^2+a(b^2f+2c^2(d+ex^2)-bc(e+3fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{2(4ac^2e+b^3f-2bc(cd+3af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+f\log(a+bx^2}{4c^2}$$

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 + a*(b^2*f + 2*c^2*(d + e*x
^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(4*a*c^
2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]
)/(-b^2 + 4*a*c)^(3/2) + f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.38

method	result
default	$\frac{\frac{(3abcf-2ac^2e-b^3f+b^2ce-bc^2d)x^2}{(4ac-b^2)c^2}+\frac{a(2acf-b^2f+ebc-2c^2d)}{(4ac-b^2)c^2}}{2cx^4+2bx^2+2a}+\frac{\frac{(4acf-b^2f)\ln(cx^4+bx^2+a)}{2c}+\frac{2\left(-abf+2ace-bcd-\frac{(4acf-b^2f)b}{2c}\right)\arctan}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$
risch	Expression too large to display

```
[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)/c^2*x^2+a*(2*a
*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/2/c/(4*a*c-b^2
)*(1/2*(4*a*c*f-b^2*f)/c*ln(c*x^4+b*x^2+a)+2*(-a*b*f+2*a*c*e-b*c*d-1/2*(4*a
*c*f-b^2*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(155) = 310.

Time = 0.33 (sec) , antiderivative size = 970, normalized size of antiderivative = 5.88

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{2\,((b^3c^2-4\,abc^3)d-(b^4c-6\,ab^2c^2+8\,a^2c^3)e+(b^5-7\,ab^3c+12\,a^2bc^2)f)x^2-(2\,abc^2d-4\,a^2c^2e+(2\,bc^3}{}\right.$$

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e + (b
^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - (2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c
^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e
- (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*sqrt(b^2 - 4*a*c)*log((
2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x
^4 + b*x^2 + a)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*
e + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 +
 16*a^2*b*c^4)*x^2), 1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2
 + 8*a^2*c^3)*e + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - 2*(2*a*b*c^2*d
- 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^
2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e + 2*(a*b^4 - 6*a
^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*f*x^4 + (b^5
- 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*f)*l
og(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{(2\,bc^2d-4\,ac^2e-b^3f+6\,abcf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}}+\frac{f\log\left(cx^4+bx^2+a\right)}{4\,c^2}+\frac{2\,ac^2d-abce+ab^2f-2\,a^2cf+(bc^2d-b^2ce+2\,ac^2e+b^3f-3\,abcf)x^2}{2\,(cx^4+bx^2+a)(b^2-4\,ac)c^2}$$

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c^2*d - 4*a*c^2*e - b^3*f + 6*a*b*c*f)*arctan((2*c*x^2 + b)/sqrt(-
b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*f*log(c*x^4 +
b*x^2 + a)/c^2 + 1/2*(2*a*c^2*d - a*b*c*e + a*b^2*f - 2*a^2*c*f + (b*c^2*d
- b^2*c*e + 2*a*c^2*e + b^3*f - 3*a*b*c*f)*x^2)/((c*x^4 + b*x^2 + a)*(b^2 -
 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 1651, normalized size of antiderivative = 10.01

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2*(4*a*c - b^2)) + (x^2*(b^
3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c^2*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6*f - 128*a^3*c^3*f + 96*a
^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 -
192*a^2*b^2*c^4)) - (atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^
2)^3)*((((8*a*f + (8*a*c^2*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24
*a*b^4*c*f))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(b
^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a
*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f)*(b^3*f + 4*a*c
^2*e - 2*b*c^2*d - 6*a*b*c*f))/((4*a*c - b^2)^(3/2)*(256*a^3*c^5 - 4*b^6*c^
2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - x^2*(((((6*b^3*c^
2*f + 8*a*c^4*e - 4*b*c^4*d - 28*a*b*c^3*f)/(4*a*c^3 - b^2*c^2) + ((8*b^3*c
^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f
))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2
```

```
*b^2*c^4)))*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*c^2*(4*a*c - b^
2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2
*c^2*f - 24*a*b^4*c*f)*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(16*c^2
*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^
4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((b^3*f^2 - 5*a*b*c*f^2 +
 2*a*c^2*e*f - b*c^2*d*f)/(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2*f + 8*a*c^4*e
- 4*b*c^4*d - 28*a*b*c^3*f)/(4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)
*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(4*a*c^3 -
 b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(2*b
^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 -
4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (((b^3*c^4)/2 - 2*a*b*c^5)*(
b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3
 - b^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2))) + (b*(((8*a*f + (8*a*c^2*(2*b^6*f
 - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(256*a^3*c^5 - 4*b^6*c
^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2
*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^
2*b^2*c^4)) + (a*f^2)/c^2 - (a*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f)^
2)/(c^2*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6*f^2 + 16*a^2*c^
4*e^2 + 4*b^2*c^4*d^2 + 36*a^2*b^2*c^2*f^2 - 12*a*b^4*c*f^2 - 4*b^4*c^2*d*f
 + 24*a*b^2*c^3*d*f + 8*a*b^3*c^2*e*f - 48*a^2*b*c^3*e*f - 16*a*b*c^4*d*e))
*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(2*c^2*(4*a*c - b^2)^(3/2))
```

3.64 $\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} \, dx = \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(2cd-be+2af)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1677, 1674, 12, 632, 212}

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2} \, dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{(b^2-4ac)^{3/2}} + \frac{-(x^2(-2acf+b^2f-bce+2c^2d))-b(af+cd)+2ace}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)/(2*c*(b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e + 2*a*f)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right) \\
&= \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{2cd-be+2af}{a+bx+cx^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)} \\
&= \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad - \frac{(2cd-be+2af)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)}
\end{aligned}$$

$$= \frac{2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf)\,x^2}{2c\,(b^2 - 4ac)\,(a + bx^2 + cx^4)} + \frac{(2cd - be + 2af)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx^2\right)}{b^2 - 4ac}$$

$$= \frac{2ace - b(cd + af) - (2c^2d - bce + b^2f - 2acf)\,x^2}{2c\,(b^2 - 4ac)\,(a + bx^2 + cx^4)} + \frac{(2cd - be + 2af)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2}\,dx = \frac{abf + 2c^2dx^2 + b^2fx^2 + bc(d - ex^2) - 2ac(e + fx^2)}{2c\,(-b^2 + 4ac)\,(a + bx^2 + cx^4)} - \frac{(-2cd + be - 2af)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2 + 4ac)^{3/2}}$$

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f*x^2))/(2*
c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*c*d + b*e - 2*a*f)*ArcTan[(b +
 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.13

method	result
default	$\frac{-\frac{\left(2acf-b^2f+ebc-2c^2d\right)x^2}{c\left(4ac-b^2\right)}+\frac{abf-2ace+bcd}{c\left(4ac-b^2\right)}}{2c\,x^4+2b\,x^2+2a}+\frac{(2af-be+2cd)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2acf-b^2f+ebc-2c^2d\right)x^2}{2c\left(4ac-b^2\right)}+\frac{abf-2ace+bcd}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\ln\left(\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)af}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x\right.\right.}{2(-4ac+b^2)^{\frac{3}{2}}}$

```
[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c/(4*a*c-b^2)*x^2+1/c*(a*b*f-2*a*c*e+b*
c*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(117) = 234$.

Time = 0.29 (sec) , antiderivative size = 650, normalized size of antiderivative = 5.28

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[-\frac{(2\,(b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-6\,ab^2c+8\,a^2c^2)f)x^2+((2\,c^3d-bc^2e+2\,ac^2f)x^4+2\,ac^2d}{2\,(ab^4c-8\,a^2b^2c^2-}\right.$$
$$-\frac{(2\,(b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-6\,ab^2c+8\,a^2c^2)f)x^2-2\,((2\,c^3d-bc^2e+2\,ac^2f)x^4+2\,ac^2}{2\,(ab^4c-8\,a^2b^2c^2+16\,a}$$

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
+ 8*a^2*c^2)*f)*x^2 + ((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d - a*
b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(b^2 - 4*a*c
)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c
))/(c*x^4 + b*x^2 + a)) + (b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e
 + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)
, -1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
 + 8*a^2*c^2)*f)*x^2 - 2*((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d -
 a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(-b^2 + 4
*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3*c - 4*
a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(2\,cd-be+2\,af)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,c^2dx^2-bcex^2+b^2fx^2-2\,acfx^2+bcd-2\,ace+abf}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*c*d - b*e + 2*a*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d*x^2 - b*c*e*x^2 + b^2*f*x^2 - 2*a*c*
f*x^2 + b*c*d - 2*a*c*e + a*b*f)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.78

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{a\,b\,f-2\,a\,c\,e+b\,c\,d}{2\,c\,(4\,a\,c-b^2)}+\frac{x^2\,(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c)}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} + \frac{\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\,\left(x^2\,\left(\frac{\left(2\,c^3\,d+2\,a\,c^2\,f-b\,c^2\,e\right)\,(2\,a\,f-b\,e+2\,c\,d)}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)\,(2\,a\,f-b\,e+2\,c\,d)^2}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,\left(b^3-4\,a\,b\,c\right)\,(2\,a\,f-b\,e+2\,c \cdots}{\left(4\,a\,c-b^2\right)^{11/2}}\cdots}{8\,a^2\,c^2\,f^2-8\,a\,b\,c^2\,e\,f+16\,a\,c^3\,d\,f+2\,b^2\,c^2\,e^2-8\,b\,c^3\,d\,e+8\,c^4\,d^2}\right)\cdots}{(4\,a\,c-b^2)^{3/2}}$$

```
[In] int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*b*f - 2*a*c*e + b*c*d)/(2*c*(4*a*c - b^2)) + (x^2*(2*c^2*d + b^2*f - 2*
a*c*f - b*c*e))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (atan(((4*a*c -
b^2)^4*(x^2*(((2*c^3*d + 2*a*c^2*f - b*c^2*e)*(2*a*f - b*e + 2*c*d))/(a*(4*
a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c)*(2*a*f - b*e +
 2*c*d)^2)/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(b^3 - 4*a*b*c)*(2*a*f - b*
e + 2*c*d)^2)/(4*a*c - b^2)^(11/2)))/(8*c^4*d^2 + 8*a^2*c^2*f^2 + 2*b^2*c^2
*e^2 + 16*a*c^3*d*f - 8*b*c^3*d*e - 8*a*b*c^2*e*f))*(2*a*f - b*e + 2*c*d))/
(4*a*c - b^2)^(3/2)
```

3.65 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 166

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx = \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{(b^3d+4a^2ce-2ab(3cd+af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}} + \frac{d\log(x)}{a^2} - \frac{d\log(a+bx^2+cx^4)}{4a^2}$$

```
[Out] 1/2*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)+1/2*(b^3*d+4*a^2*c*e-2*a*b*(a*f+3*c*d))*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+d*ln(x)/a^2-1/4*d*ln(c*x^4+b*x^2
+a)/a^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1677, 1660, 814, 648, 632, 212, 642}

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx = \frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(4a^2ce-2ab(af+3cd)+b^3d)}{2a^2(b^2-4ac)^{3/2}} - \frac{d\log(a+bx^2+cx^4)}{4a^2} + \frac{d\log(x)}{a^2} + \frac{x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)/(2*a*(b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*d + 4*a^2*c*e - 2*a*b*(3*c*d + a*f))
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (d
*Log[x])/a^2 - (d*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
```

```
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\left(\left(\frac{b^2}{a}-4c\right)d\right)-\frac{(bcd-2ace+abf)x}{a}}{x(a+bx+cx^2)}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}\\
&= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{(-b^2+4ac)d}{a^2x}+\frac{b^3d+2a^2ce-ab(5cd+af)+c(b^2-4ac)dx}{a^2(a+bx+cx^2)}\right)dx,x,x^2\right)}{2\left(b^2-4ac\right)}\\
&= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b^3d+2a^2ce-ab(5cd+af)+c(b^2-4ac)dx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2\left(b^2-4ac\right)}\\
&= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{d\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2} - \frac{\left(b^3d+4a^2ce-2ab(3cd+af)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2\left(b^2-4ac\right)}\\
&= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(b^3d+4a^2ce-2ab(3cd+af)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a^2\left(b^2-4ac\right)}
\end{aligned}$$

$$= \frac{b^2 d - abe - 2a(cd - af) + (bcd - 2ace + abf)x^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(b^3 d + 4a^2 ce - 2ab(3cd + af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2 - 4ac)^{3/2}} + \frac{d\log(x)}{a^2} - \frac{d\log(a + bx^2 + cx^4)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.61

$$\int \frac{d + ex^2 + fx^4}{x(a + bx^2 + cx^4)^2} dx = -\frac{-\frac{2a(b^2 d + b(-ae + cdx^2 + afx^2) + 2a(af - c(d + ex^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - 4d\log(x) + \frac{\left(b^3 d + b^2\sqrt{b^2-4ac}d + 4ac\left(-\sqrt{b^2-4ac}d + ae\right) - 2ab(3cd + af)\right)\log\left(b\right.}{(b^2-4ac)^{3/2}}}{4a^2}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((-2*a*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x
^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*d*Log[x] + ((b^3*d + b^2*Sqr
t[b^2 - 4*a*c]*d + 4*a*c*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) - 2*a*b*(3*c*d + a*
f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-(b^3*d)
+ b^2*Sqrt[b^2 - 4*a*c]*d - 4*a*c*(Sqrt[b^2 - 4*a*c]*d + a*e) + 2*a*b*(3*c*
d + a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/a^2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

method	result
default	$\frac{d\ln(x)}{a^2} + \frac{\frac{-\frac{a(abf-2ace+bcd)x^2}{4ac-b^2} - \frac{a(2fa^2-abe-2acd+b^2d)}{4ac-b^2}}{cx^4+bx^2+a} + \frac{\frac{(-4ac^2d+b^2cd)\ln(cx^4+bx^2+a)}{2c} + \frac{2\left(-a^2bf+2a^2ce-5abcd+b^3d-\frac{(-4ac^2d+}{2c}\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}$
risch	$\frac{-\frac{(abf-2ace+bcd)x^2}{2a(4ac-b^2)} - \frac{2fa^2-abe-2acd+b^2d}{2a(4ac-b^2)}}{cx^4+bx^2+a} + \frac{d\ln(x)}{a^2} + \left(\_R = \mathrm{RootOf}\left(\left(64a^5c^3 - 48a^4b^2c^2 + 12cb^4a^3 - b^6a^2\right)\_Z^2 + \left(64c^3a^3d - 48a^2b^2c^2\right.\right.\right.$

```
[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*ln(x)/a^2+1/2/a^2*((-a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^2-a*(2*a^2*f-a
*b*e-2*a*c*d+b^2*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-4*a*c
^2*d+b^2*c*d)/c*ln(c*x^4+b*x^2+a)+2*(-a^2*b*f+2*a^2*c*e-5*a*b*c*d+b^3*d-1/2
```

```
*(-4*a*c^2*d+b^2*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(156) = 312$.

Time = 0.97 (sec) , antiderivative size = 1103, normalized size of antiderivative = 6.64

$$\int \frac{d + ex^2 + fx^4}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3
 - 4*a^3*b*c)*f)*x^2 + (4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f
+ (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*
c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
+ 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e + 4*(a^
3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*
x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(x)
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*((a*b^3*c -
 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^
2 + 2*(4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (b^3*c - 6*a*b*
c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)*d)*x^2 + (a*b^
3 - 6*a^2*b*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*
a^3*b*c)*e + 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)*d)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)*d)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex^2 + fx^4}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{d + ex^2 + fx^4}{x\left(a + bx^2 + cx^4\right)^2}\,dx &= -\frac{\left(b^3 d - 6\,abcd + 4\,a^2 ce - 2\,a^2 bf\right)\arctan\left(\frac{2\,cx^2 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\left(a^2 b^2 - 4\,a^3 c\right)\sqrt{-b^2 + 4\,ac}} - \frac{d\log\left(cx^4 + bx^2 + a\right)}{4\,a^2} + \frac{d\log\left(x^2\right)}{2\,a^2} \\ &\quad + \frac{b^2 cdx^4 - 4\,ac^2 dx^4 + b^3 dx^2 - 2\,abcdx^2 - 4\,a^2 cex^2 + 2\,a^2 bfx^2 + 3\,ab^2 d - 8\,a^2 cd - 2\,a^2 be + 4\,a^3 f}{4\,\left(cx^4 + bx^2 + a\right)\left(a^2 b^2 - 4\,a^3 c\right)}\end{aligned}$$

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(b^3*d - 6*a*b*c*d + 4*a^2*c*e - 2*a^2*b*f)*arctan((2*c*x^2 + b)/sqrt(
-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*d*log(c*x^4 +
 b*x^2 + a)/a^2 + 1/2*d*log(x^2)/a^2 + 1/4*(b^2*c*d*x^4 - 4*a*c^2*d*x^4 + b
^3*d*x^2 - 2*a*b*c*d*x^2 - 4*a^2*c*e*x^2 + 2*a^2*b*f*x^2 + 3*a*b^2*d - 8*a^
2*c*d - 2*a^2*b*e + 4*a^3*f)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

Mupad [B] (verification not implemented)

Time = 16.31 (sec) , antiderivative size = 8706, normalized size of antiderivative = 52.45

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x)
[Out] (d*log(x))/a^2 - ((b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)/(2*a*(4*a*c - b^2)) +
 (x^2*(a*b*f - 2*a*c*e + b*c*d))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) -
 (log((((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*
c - b^2)^3))^(1/2))*(((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d
)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*f - b
^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)) + (b
*c^2*(d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c -
 b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 - (4*b*c^2*(b^3*d - a^
2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2))))/(4*a^2) + (c^2*(a^3*b^2
*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^2*d^2 - 4*a*b^4*d*f - 36*a^
2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e - 4*a^3*b*c*e*f))/(a^2*(4*a*
c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*d^2 + 4*a^2*b*c^2*e^2 - 20*
a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e - 20*a^2*b*c^2*d*f - 4*a^2*
b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) - (c^2*x^2*(a*b
*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) + (c^2*d*(a*b*f - 2*a*c*e +
b*c*d)^2)/(a^3*(4*a*c - b^2)^2))*(((d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*
e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d - a^2*(-(b^3*d - 2*a^2*
b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((2*c^2*x^2*(2
0*a^2*c^2*e + 4*a*b^3*f - b^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c
*f))/(a*(4*a*c - b^2)) + (b*c^2*(d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e -
 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))
/a^2 - (4*b*c^2*(b^3*d - a^2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2)
)))/(4*a^2) + (c^2*(a^3*b^2*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^
2*d^2 - 4*a*b^4*d*f - 36*a^2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e -
 4*a^3*b*c*e*f))/(a^2*(4*a*c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*
d^2 + 4*a^2*b*c^2*e^2 - 20*a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e
- 20*a^2*b*c^2*d*f - 4*a^2*b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2
)))/(4*a^2) - (c^2*x^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) +
 (c^2*d*(a*b*f - 2*a*c*e + b*c*d)^2)/(a^3*(4*a*c - b^2)^2)))*(2*b^6*d - 128
*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*c^3
- 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (atan((x^2*((((b^3*c^5*d^3 - 8*a^3*c^5
*e^3 + a^3*b^3*c^2*f^3 - 6*a*b^2*c^5*d^2*e + 12*a^2*b*c^5*d*e^2 + 3*a*b^3*c
^4*d^2*f + 12*a^3*b*c^4*e^2*f + 3*a^2*b^3*c^3*d*f^2 - 6*a^3*b^2*c^3*e*f^2 -
 12*a^2*b^2*c^4*d*e*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2) - (((6*a*b^5*c^4*d^2 + 80*a^3*b*c^6*d^2 - 16*a^4*b*c^5*e^2 - 44*a^2*b^3*
c^5*d^2 + 4*a^3*b^3*c^4*e^2 + a^3*b^5*c^2*f^2 - 4*a^4*b^3*c^3*f^2 - 160*a^4
```

*c^6*d*e + 80*a^4*b*c^5*d*f - 14*a^2*b^4*c^4*d*e + 96*a^3*b^2*c^5*d*e + 7*a
^2*b^5*c^3*d*f - 48*a^3*b^3*c^4*d*f - 4*a^3*b^4*c^3*e*f + 16*a^4*b^2*c^4*e*
f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((640*a^6*c^6*
e - 2*a^2*b^7*c^3*d + 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3
*e + 168*a^4*b^4*c^4*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c
^3*f + 288*a^5*b^3*c^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f)/(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*d - 128*a^3*c^3*d + 96*
a^2*b^2*c^2*d - 24*a*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^
7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12
*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*
a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))
/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6*d -
128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*
c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((((640*a^6*c^6*e - 2*a^2*b^7*c^3
*d + 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3*e + 168*a^4*b^4*
c^4*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c^3*f + 288*a^5*b^
3*c^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 2
4*a*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*
b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b
^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((
2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b
*f + 4*a^2*c*e - 6*a*b*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*
c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3
*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3
- 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b
*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2
*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2*(2560*
a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*
b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*
(3*b^5*d - a^2*b^3*f - 2*a^3*c^2*e - 21*a*b^3*c*d + a^3*b*c*f + 33*a^2*b*c^
2*d + 2*a^2*b^2*c*e))/(8*a^3*c^2*(4*a*c - b^2)^3*(400*a^3*c^3*d^2 - 6*b^6*d
^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - 291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2 - a
^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^4*b
*c*e*f)) + (((((((640*a^6*c^6*e - 2*a^2*b^7*c^3*d + 36*a^3*b^5*c^4*d - 192*
a^4*b^3*c^5*d - 16*a^3*b^6*c^3*e + 168*a^4*b^4*c^4*e - 576*a^5*b^2*c^5*e +
8*a^3*b^7*c^2*f - 84*a^4*b^5*c^3*f + 288*a^5*b^3*c^4*f + 320*a^5*b*c^6*d -
320*a^6*b*c^5*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (
(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(2560*a^7*b*c^6
+ 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))
/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256
*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e
- 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((2*b^6*d - 128*a^3*c^3*d + 96

*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)*
(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 268
8*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^
2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4
*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((6*a*b^5*c^4*
d^2 + 80*a^3*b*c^6*d^2 - 16*a^4*b*c^5*e^2 - 44*a^2*b^3*c^5*d^2 + 4*a^3*b^3*
c^4*e^2 + a^3*b^5*c^2*f^2 - 4*a^4*b^3*c^3*f^2 - 160*a^4*c^6*d*e + 80*a^4*b*
c^5*d*f - 14*a^2*b^4*c^4*d*e + 96*a^3*b^2*c^5*d*e + 7*a^2*b^5*c^3*d*f - 48*
a^3*b^3*c^4*d*f - 4*a^3*b^4*c^3*e*f + 16*a^4*b^2*c^4*e*f)/(a^3*b^6 - 64*a^6
*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((640*a^6*c^6*e - 2*a^2*b^7*c^3*d
+ 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3*e + 168*a^4*b^4*c^4
*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c^3*f + 288*a^5*b^3*c
^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a
*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5
*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*
b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^
6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256
*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e
 - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) + ((b^3*d - 2*a^2*b*f + 4*a^2*c*
e - 6*a*b*c*d)^3*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*
a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*
a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(96*b^6*d - 1280*a^3*c^3*d - 32*
a^2*b^4*f + 2208*a^2*b^2*c^2*d - 864*a*b^4*c*d + 64*a^2*b^3*c*e - 192*a^3*b
*c^2*e + 96*a^3*b^2*c*f))/(256*a^3*c^2*(4*a*c - b^2)^(7/2)*(400*a^3*c^3*d^2
 - 6*b^6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - 291*a^2*b^2*c^2*d^2 + 72*a*b^4
*c*d^2 - a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a^3*b*c^2*d*e + 6*a^3*b^2*c*d*f
 - 4*a^4*b*c*e*f)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c -
 b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c -
b^2)^(9/2)))/(16*a^4*c^4*e^2 + b^6*c^2*d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*
c^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b*c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b
^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3*b^2*c^3*d*f) + ((16*a^6*b^6*(4*a*c
- b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^
2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*((b^2*c^4*d^3 + 4*a^2*c^4*d
*e^2 - 4*a*b*c^4*d^2*e + 2*a*b^2*c^3*d^2*f + a^2*b^2*c^2*d*f^2 - 4*a^2*b*c^
3*d*e*f)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^4*c^4*e^2 - 4*a*b^4*
c^3*d^2 + 17*a^2*b^2*c^4*d^2 + a^4*b^2*c^2*f^2 - 36*a^3*b*c^4*d*e - 4*a^4*b
*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 18*a^3*b^2*c^3*d*f)/(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d +
80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 3
2*a^5*b*c^4*e)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*
a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d -
 24*a*b^4*c*d))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^
5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2

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*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 19
2*a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d
))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (((((4*
a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d + 80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a
^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 32*a^5*b*c^4*e)/(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d -
 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2))
+ ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3
*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*
b*c*d))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*
a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*
f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 -
 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2
*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2)/(32*a^4*(
4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^
3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(3*b^5*d - a^2*b^3*f - 2*a^3*c^2*e -
21*a*b^3*c*d + a^3*b*c*f + 33*a^2*b*c^2*d + 2*a^2*b^2*c*e))/(8*a^3*c^2*(4*a
*c - b^2)^3*(16*a^4*c^4*e^2 + b^6*c^2*d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*c
^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b*c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b^
3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3*b^2*c^3*d*f)*(400*a^3*c^3*d^2 - 6*b^
6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - 291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2
- a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^
4*b*c*e*f)) - (((((((4*a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d + 80*a^4*b^2*c^4*d
+ 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 32*a^5*b*c^4*e)/(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*
a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^
4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^
2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)
*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2
*b*f + 4*a^2*c*e - 6*a*b*c*d))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c
^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((4*a^4*c^4*e^2 -
 4*a*b^4*c^3*d^2 + 17*a^2*b^2*c^4*d^2 + a^4*b^2*c^2*f^2 - 36*a^3*b*c^4*d*e
- 4*a^4*b*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 18*a^3*b^2*c^3*
d*f)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^2*b^6*c^2*d - 36*a^3*b^4
*c^3*d + 80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*
c^3*f - 32*a^5*b*c^4*e)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*
c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^
2*c^2*d - 24*a*b^4*c*d))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6
 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d
 + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b
```

```
^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a
^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4
)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^3)/(64*a^6*(4*a*c - b^2)^(9/2
)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) -
1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*
a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(96*b^6*d - 1280*a^3*c^3*d - 32*a^2*b^4*f
+ 2208*a^2*b^2*c^2*d - 864*a*b^4*c*d + 64*a^2*b^3*c*e - 192*a^3*b*c^2*e + 9
6*a^3*b^2*c*f))/(256*a^3*c^2*(4*a*c - b^2)^(7/2)*(16*a^4*c^4*e^2 + b^6*c^2*
d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*c^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b*
c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3
*b^2*c^3*d*f)*(400*a^3*c^3*d^2 - 6*b^6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 -
291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2 - a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a
^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^4*b*c*e*f)))*(b^3*d - 2*a^2*b*f + 4*a^
2*c*e - 6*a*b*c*d))/(2*a^2*(4*a*c - b^2)^(3/2))
```

3.66 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 234

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx = -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))x^2}{2a^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(2b^4d-12ab^2cd-ab^3e+6a^2bce+4a^2c(3cd-af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3(b^2-4ac)^{3/2}} - \frac{(2bd-ae)\log(x)}{a^3} + \frac{(2bd-ae)\log(a+bx^2+cx^4)}{4a^3}$$

```
[Out] -1/2*d/a^2/x^2+1/2*(-b^3*d+a*b^2*e-2*a^2*c*e+a*b*(-a*f+3*c*d)-c*(b^2*d-a*b*
e-2*a*(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(2*b^4*d-12*a*b
^2*c*d-a*b^3*e+6*a^2*b*c*e+4*a^2*c*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(-a*e+2*b*d)*ln(x)/a^3+1/4*(-a*e+2*b*d
)*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used

$= \{1677, 1660, 1642, 648, 632, 212, 642\}$

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{(2bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^3} - \frac{\log(x)(2bd-ae)}{a^3}$$
$$- \frac{2a^2ce+cx^2(-abe-2a(cd-af)+b^2d)-ab^2e-ab(3cd-af)+b^3d}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{d}{2a^2x^2}$$
$$- \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+4a^2c(3cd-af)-ab^3e-12ab^2cd+2b^4d\right)}{2a^3\left(b^2-4ac\right)^{3/2}}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*d/(a^2*x^2) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f) + c*(b^
2*d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2*b*c*e + 4*a^2*c*(3*c*d - a*
f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) -
 ((2*b*d - a*e)*Log[x])/a^3 + ((2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^3
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\left(\left(\frac{b^2}{a}-4c\right)d\right)+\frac{\left(b^2-4ac\right)(bd-ae)x}{a^2}+\frac{c\left(b^2d-abe-2a(cd-af)\right)x^2}{a^2}}{x^2(a+bx+cx^2)}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\Big(\int \Big(\frac{(-b^2+4ac)d}{a^2x^2}+\frac{(-b^2+4ac)(-2bd+ae)}{a^3x}+\frac{-2b^4d+10ab^2cd+ab^3e-5a^2bce-2a^2c(3cd-af)-c\left(b^2-4ac\right)(2bd-ae)x}{a^3(a+bx+cx^2)}}{2\left(b^2-4ac\right)}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2bd - ae)\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{-2b^4d + 10ab^2cd + ab^3e - 5a^2bce - 2a^2c(3cd - af) - c(b^2 - 4ac)(2bd - ae)x}{a + bx + cx^2}\,dx, x, x^2\right)}{2a^3(b^2 - 4ac)}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2bd - ae)\log(x)}{a^3} + \frac{(2bd - ae)\text{Subst}\left(\int \frac{b + 2cx}{a + bx + cx^2}\,dx, x, x^2\right)}{4a^3} + \frac{(2b^4d - 12ab^2cd - ab^3e + 6a^2bce + 4a^2c(3cd - af))\,\text{Subst}\left(\int \frac{1}{a + bx + cx^2}\,dx, x, x^2\right)}{4a^3(b^2 - 4ac)}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2bd - ae)\log(x)}{a^3} + \frac{(2bd - ae)\log(a + bx^2 + cx^4)}{4a^3} - \frac{(2b^4d - 12ab^2cd - ab^3e + 6a^2bce + 4a^2c(3cd - af))\,\text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2}\,dx, x, b + 2cx^2\right)}{2a^3(b^2 - 4ac)}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2}{2a^2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2b^4d - 12ab^2cd - ab^3e + 6a^2bce + 4a^2c(3cd - af))\tanh^{-1}\left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}}\right)}{2a^3(b^2 - 4ac)^{3/2}} - \frac{(2bd - ae)\log(x)}{a^3} + \frac{(2bd - ae)\log(a + bx^2 + cx^4)}{4a^3}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.72

$$\int \frac{d + ex^2 + fx^4}{x^3(a + bx^2 + cx^4)^2}\,dx$$

$$= \frac{-\frac{2ad}{x^2} - \frac{2a(b^3d + b^2(-ae + cdx^2) + ab(af - c(3d + ex^2)) + 2ac(-cdx^2 + a(e + fx^2)))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + 4(-2bd + ae)\log(x) + \frac{\left(2b^4d + b^3\left(2\sqrt{b^2 - 4ac}d - a\right.\right.}{}}{}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*d)/x^2 - (2*a*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d +
e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + 4*(-2*b*d + a*e)*Log[x] + ((2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]*d
- a*e) + 2*a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(12*c*d + Sqrt[b^
```

```
2 - 4*a*c]*e) + 4*a^2*c*(3*c*d + Sqrt[b^2 - 4*a*c]*e - a*f))*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-2*b^4*d + b^3*(2*Sqrt[b^2
- 4*a*c]*d + a*e) - 2*a*b*c*(4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(12*c*d
 - Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a*f))*Log
[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.35

method	result
default	$-\frac{d}{2a^2x^2}+\frac{(ae-2bd)\ln(x)}{a^3}+\frac{\frac{ac\left(2f\,a^2-abe-2acd+b^2d\right)x^2}{4ac-b^2}+\frac{a\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-4a^2c^2e+a\,b^2ce+8ab\,c^2d-2b^3cd\right)\ln}{2c}}{}$
risch	$\frac{\frac{c\left(2f\,a^2-abe-6acd+2b^2d\right)x^4}{2a^2\left(4ac-b^2\right)}+\frac{\left(a^2bf+2a^2ce-a\,b^2e-7abcd+2b^3d\right)x^2}{2\left(4ac-b^2\right)a^2}-\frac{d}{2a}}{x^2(c\,x^4+b\,x^2+a)}+\frac{\ln(x)e}{a^2}-\frac{2\ln(x)bd}{a^3}+\frac{\left(\underline{}R=\text{RootOf}\left(\left(64a^6c^3-48b^2a^5c^2+\right.\right.\right.}{}$

```
[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d/a^2/x^2+(a*e-2*b*d)/a^3*ln(x)+1/2/a^3*((a*c*(2*a^2*f-a*b*e-2*a*c*d+b
^2*d)/(4*a*c-b^2)*x^2+a*(a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-
b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-4*a^2*c^2*e+a*b^2*c*e+8*a*b*c^2*
d-2*b^3*c*d)/c*ln(c*x^4+b*x^2+a)+2*(2*a^3*c*f-5*a^2*b*c*e-6*a^2*c^2*d+a*b^3
*e+10*a*b^2*c*d-2*d*b^4-1/2*(-4*a^2*c^2*e+a*b^2*c*e+8*a*b*c^2*d-2*b^3*c*d)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(222) = 444$.

Time = 2.08 (sec) , antiderivative size = 1764, normalized size of antiderivative = 7.54

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b
*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15*a^2*b^3*c + 2
8*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3*b^3 - 4*a^4*b
*c)*f)*x^2 + ((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a*b^3
*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)
*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
```

```
4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x
^2 + a)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*
(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2)*e)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^
6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)
*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2)*e)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6
+ (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*
a^6*c^2)*x^2), -1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b
^3*c - 4*a^3*b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3
*b^3 - 4*a^4*b*c)*f)*x^2 - 2*((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2
*c^3)*d + (a*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*
c + 6*a^2*b*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 -
 6*a^2*b^2*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d -
(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^
2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*
log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*
b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b
^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log
(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x^3 \left(a + bx^2 + cx^4\right)^2}\, dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{\left(2\,b^4d-12\,ab^2cd+12\,a^2c^2d-ab^3e+6\,a^2bce-4\,a^3cf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}$$
$$-\frac{2\,b^2cdx^4-6\,ac^2dx^4-abcex^4+2\,a^2cfx^4+2\,b^3dx^2-7\,abcdx^2-ab^2ex^2+2\,a^2cex^2+a^2bfx^2+ab^2d-}{2\,\left(cx^6+bx^4+ax^2\right)\left(a^2b^2-4\,a^3c\right)}$$
$$+\frac{\left(2\,bd-ae\right)\log\left(cx^4+bx^2+a\right)}{4\,a^3}-\frac{\left(2\,bd-ae\right)\log\left(x^2\right)}{2\,a^3}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - a*b^3*e + 6*a^2*b*c*e - 4*a^3*
c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^
2 + 4*a*c)) - 1/2*(2*b^2*c*d*x^4 - 6*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c*f*
x^4 + 2*b^3*d*x^2 - 7*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*b*f*x
^2 + a*b^2*d - 4*a^2*c*d)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)) + 1
/4*(2*b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^3 - 1/2*(2*b*d - a*e)*log(x^2)/a^
3
```

Mupad [B] (verification not implemented)

Time = 17.16 (sec) , antiderivative size = 11879, normalized size of antiderivative = 50.76

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(2*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 7*a*b*c*d))/(2*a^2*(4*a*c
- b^2)) - d/(2*a) + (c*x^4*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d))/(2*a^2*(4
*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (log(x)*(a*e - 2*b*d))/a^3 + (log((
((((a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*
a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((2*c^3*x^2*(2*b^4
*d - 60*a^2*c^2*d - 8*a^2*b^2*f - a*b^3*e + 20*a^3*c*f + 4*a*b^2*c*d + 10*a
^2*b*c*e))/(a^2*(4*a*c - b^2)) + (4*b*c^2*(2*b^4*d + 6*a^2*c^2*d - a*b^3*e
- 2*a^3*c*f - 10*a*b^2*c*d + 5*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (b*c^2*(a*
e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*
c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*
c*x^2))/a^3))/(4*a^3) + (c^3*(4*a^5*c*f^2 - 16*b^6*d^2 - 4*a^2*b^4*e^2 + 36
*a^3*c^3*d^2 + 17*a^3*b^2*c*e^2 + 16*a*b^5*d*e - 216*a^2*b^2*c^2*d^2 + 116*
a*b^4*c*d^2 - 16*a^2*b^4*d*f + 8*a^3*b^3*e*f - 24*a^4*c^2*d*f - 92*a^2*b^3*
c*d*e + 108*a^3*b*c^2*d*e + 72*a^3*b^2*c*d*f - 36*a^4*b*c*e*f))/(a^4*(4*a*c
 - b^2)^2) - (2*c^4*x^2*(12*b^5*d^2 + 2*a^4*b*f^2 + 3*a^2*b^3*e^2 + 138*a^2
*b*c^2*d^2 - 12*a*b^4*d*e + 20*a^4*c*e*f - 82*a*b^3*c*d^2 - 10*a^3*b*c*e^2
+ 14*a^2*b^3*d*f - 60*a^3*c^2*d*e - 7*a^3*b^2*e*f + 61*a^2*b^2*c*d*e - 52*a
^3*b*c*d*f))/(a^4*(4*a*c - b^2)^2))*(a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*
c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b
^2)^3))^(1/2)))/(4*a^3) + (c^4*(a*e - 2*b*d)*(2*b^2*d + 2*a^2*f - a*b*e - 6
*a*c*d)^2)/(a^6*(4*a*c - b^2)^2) + (c^5*x^2*(2*b^2*d + 2*a^2*f - a*b*e - 6*
a*c*d)^3)/(a^6*(4*a*c - b^2)^3))*(((((2*b*d - a*e + a^3*(-(2*b^4*d + 12*a^2
*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c -
b^2)^3))^(1/2))*((2*c^3*x^2*(2*b^4*d - 60*a^2*c^2*d - 8*a^2*b^2*f - a*b^3*e
 + 20*a^3*c*f + 4*a*b^2*c*d + 10*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (4*b*c^2
*(2*b^4*d + 6*a^2*c^2*d - a*b^3*e - 2*a^3*c*f - 10*a*b^2*c*d + 5*a^2*b*c*e)
)/(a^2*(4*a*c - b^2)) - (b*c^2*(2*b*d - a*e + a^3*(-(2*b^4*d + 12*a^2*c^2*d
 - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3
))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3))/(4*a^3) - (c^3*(4*a^5*c*f^2
 - 16*b^6*d^2 - 4*a^2*b^4*e^2 + 36*a^3*c^3*d^2 + 17*a^3*b^2*c*e^2 + 16*a*b^
5*d*e - 216*a^2*b^2*c^2*d^2 + 116*a*b^4*c*d^2 - 16*a^2*b^4*d*f + 8*a^3*b^3*
e*f - 24*a^4*c^2*d*f - 92*a^2*b^3*c*d*e + 108*a^3*b*c^2*d*e + 72*a^3*b^2*c*
d*f - 36*a^4*b*c*e*f))/(a^4*(4*a*c - b^2)^2) + (2*c^4*x^2*(12*b^5*d^2 + 2*a
^4*b*f^2 + 3*a^2*b^3*e^2 + 138*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 20*a^4*c*e*f
- 82*a*b^3*c*d^2 - 10*a^3*b*c*e^2 + 14*a^2*b^3*d*f - 60*a^3*c^2*d*e - 7*a^3
*b^2*e*f + 61*a^2*b^2*c*d*e - 52*a^3*b*c*d*f))/(a^4*(4*a*c - b^2)^2))*(2*b*
```

d - a*e + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*
d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^4*(a*e - 2*b
d)(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d)^2)/(a^6*(4*a*c - b^2)^2) + (c^5*x
^2*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d)^3)/(a^6*(4*a*c - b^2)^3)))*(4*b^7*
d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a
*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 -
48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (atan((x^2*((((216*a^3*c^8*d^3 - 8*b^6*
c^5*d^3 - 8*a^6*c^5*f^3 + 72*a*b^4*c^6*d^3 - 216*a^4*c^7*d^2*f + 72*a^5*c^6
*d*f^2 - 216*a^2*b^2*c^7*d^3 + a^3*b^3*c^5*e^3 + 12*a*b^5*c^5*d^2*e + 108*a
^3*b*c^7*d^2*e + 12*a^5*b*c^5*e*f^2 - 72*a^2*b^3*c^6*d^2*e - 6*a^2*b^4*c^5*
d*e^2 + 18*a^3*b^2*c^6*d*e^2 - 24*a^2*b^4*c^5*d^2*f + 144*a^3*b^2*c^6*d^2*f
- 24*a^4*b^2*c^5*d*f^2 - 6*a^4*b^2*c^5*e^2*f - 72*a^4*b*c^6*d*e*f + 24*a^3
*b^3*c^5*d*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + ((
(80*a^6*b*c^6*e^2 - 1104*a^5*b*c^7*d^2 - 16*a^7*b*c^5*f^2 + 24*a^2*b^7*c^4*
d^2 - 260*a^3*b^5*c^5*d^2 + 932*a^4*b^3*c^6*d^2 + 6*a^4*b^5*c^4*e^2 - 44*a^
5*b^3*c^5*e^2 + 4*a^6*b^3*c^4*f^2 + 480*a^6*c^7*d*e - 160*a^7*c^6*e*f + 416
*a^6*b*c^6*d*f - 24*a^3*b^6*c^4*d*e + 218*a^4*b^4*c^5*d*e - 608*a^5*b^2*c^6
*d*e + 28*a^4*b^5*c^4*d*f - 216*a^5*b^3*c^5*d*f - 14*a^5*b^4*c^4*e*f + 96*a
^6*b^2*c^5*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + ((
(1920*a^8*c^7*d - 640*a^9*c^6*f - 4*a^4*b^8*c^3*d + 24*a^5*b^6*c^4*d + 120*
a^6*b^4*c^5*d - 1088*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e - 36*a^6*b^5*c^4*e + 1
92*a^7*b^3*c^5*e + 16*a^6*b^6*c^3*f - 168*a^7*b^4*c^4*f + 576*a^8*b^2*c^5*f
- 320*a^8*b*c^6*e)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)
- ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 -
2688*a^9*b^3*c^5)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d
- 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6
*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6
*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d
+ 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2
c^2)))(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b
^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6
- 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) - (((((1920*a^8*c^7*d - 64
0*a^9*c^6*f - 4*a^4*b^8*c^3*d + 24*a^5*b^6*c^4*d + 120*a^6*b^4*c^5*d - 1088
*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e - 36*a^6*b^5*c^4*e + 192*a^7*b^3*c^5*e + 1
6*a^6*b^6*c^3*f - 168*a^7*b^4*c^4*f + 576*a^8*b^2*c^5*f - 320*a^8*b*c^6*e)/
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6
+ 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(
4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e
- 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^6 - 64*a^9*c^
3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c
+ 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b
^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*b*c^6 + 12
*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(2*b^
4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)*(4*b

^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 4
8*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(8*a^3*(4*a*c - b^2)^(3/2)
*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^
6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e
 - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) + (
(2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 26
88*a^9*b^3*c^5)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*
d + 6*a^2*b*c*e)^2*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d
 - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(32
a^6(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)
*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*a^3*c^3*d
 - 6*b^6*d - 2*a^4*c^2*f + 3*a*b^5*e - 72*a^2*b^2*c^2*d + 42*a*b^4*c*d - 21*
a^2*b^3*c*e + 33*a^3*b*c^2*e + 2*a^3*b^2*c*f))/(8*a^3*c^2*(4*a*c - b^2)^3*(
36*a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*a^5*c^3*e^2 + 4*a^6*c^2*f
^2 + 72*a^3*b^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^4*c^2*d^2 + 1528*a^3*b^2*
c^3*d^2 - 291*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 - 24*a^5*c^3*d*f - 288*a^2*
b^5*c*d*e - 1564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f + 2*a^4*b^3*c*e*f - 12*a^5
*b*c^2*e*f + 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2*d*f)) + (((((((1920*a^8*
c^7*d - 640*a^9*c^6*f - 4*a^4*b^8*c^3*d + 24*a^5*b^6*c^4*d + 120*a^6*b^4*c^
5*d - 1088*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e - 36*a^6*b^5*c^4*e + 192*a^7*b^3
*c^5*e + 16*a^6*b^6*c^3*f - 168*a^7*b^4*c^4*f + 576*a^8*b^2*c^5*f - 320*a^8
*b*c^6*e)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a
^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*
b^3*c^5)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*
b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^6 -
 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*
a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*
f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*
b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*
c^5)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b
*c*e)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2
*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(8*a^3*(4*a*c -
b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^
6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*
e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a
^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c +
192*a^5*b^2*c^2)) + (((80*a^6*b*c^6*e^2 - 1104*a^5*b*c^7*d^2 - 16*a^7*b*c^5
*f^2 + 24*a^2*b^7*c^4*d^2 - 260*a^3*b^5*c^5*d^2 + 932*a^4*b^3*c^6*d^2 + 6*a
^4*b^5*c^4*e^2 - 44*a^5*b^3*c^5*e^2 + 4*a^6*b^3*c^4*f^2 + 480*a^6*c^7*d*e -
 160*a^7*c^6*e*f + 416*a^6*b*c^6*d*f - 24*a^3*b^6*c^4*d*e + 218*a^4*b^4*c^5
*d*e - 608*a^5*b^2*c^6*d*e + 28*a^4*b^5*c^4*d*f - 216*a^5*b^3*c^5*d*f - 14*
a^5*b^4*c^4*e*f + 96*a^6*b^2*c^5*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2) + (((1920*a^8*c^7*d - 640*a^9*c^6*f - 4*a^4*b^8*c^3*d + 2
4*a^5*b^6*c^4*d + 120*a^6*b^4*c^5*d - 1088*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e
- 36*a^6*b^5*c^4*e + 192*a^7*b^3*c^5*e + 16*a^6*b^6*c^3*f - 168*a^7*b^4*c^4

*f + 576*a^8*b^2*c^5*f - 320*a^8*b*c^6*e)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(768*b^7*d + 5120*a^4*c^3*e - 384*a*b^6*e + 18432*a^2*b^3*c^2*d - 8832*a^3*b^2*c^2*e - 6912*a*b^5*c*d - 12544*a^3*b*c^3*d + 3456*a^2*b^4*c*e - 256*a^3*b^3*c*f + 768*a^4*b*c^2*f))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(36*a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*a^5*c^3*e^2 + 4*a^6*c^2*f^2 + 72*a^3*b^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^4*c^2*d^2 + 1528*a^3*b^2*c^3*d^2 - 291*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 - 24*a^5*c^3*d*f - 288*a^2*b^5*c*d*e - 1564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f + 2*a^4*b^3*c*e*f - 12*a^5*b*c^2*e*f + 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2*d*f)))*(16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2)))/(144*a^4*c^6*d^2 + 4*b^8*c^2*d^2 + 16*a^6*c^4*f^2 - 48*a*b^6*c^3*d^2 + 192*a^2*b^4*c^4*d^2 - 288*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 12*a^3*b^4*c^3*e^2 + 36*a^4*b^2*c^4*e^2 - 96*a^5*c^5*d*f - 4*a*b^7*c^2*d*e + 144*a^4*b*c^5*d*e - 48*a^5*b*c^4*e*f + 48*a^2*b^5*c^3*d*e - 168*a^3*b^3*c^4*d*e - 16*a^3*b^4*c^3*d*f + 96*a^4*b^2*c^4*d*f + 8*a^4*b^3*c^3*e*f) - ((16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*((8*b^5*c^4*d^3 - 48*a*b^3*c^5*d^3 + 72*a^2*b*c^6*d^3 - 36*a^3*c^6*d^2*e - 4*a^5*c^4*e*f^2 - a^3*b^2*c^4*e^3 + 24*a^4*c^5*d*e*f - 12*a*b^4*c^4*d^2*e - 12*a^3*b*c^5*d*e^2 - 48*a^3*b*c^5*d^2*f + 8*a^4*b*c^4*d*f^2 + 4*a^4*b*c^4*e^2*f + 48*a^2*b^2*c^5*d^2*e + 6*a^2*b^3*c^4*d*e^2 + 16*a^2*b^3*c^4*d^2*f - 16*a^3*b^2*c^4*d*e*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((36*a^5*c^6*d^2 + 4*a^7*c^4*f^2 - 16*a^2*b^6*c^3*d^2 + 116*a^3*b^4*c^4*d^2 - 216*a^4*b^2*c^5*d^2 - 4*a^4*b^4*c^3*e^2 + 17*a^5*b^2*c^4*e^2 - 24*a^6*c^5*d*f + 108*a^5*b*c^5*d*e - 36*a^6*b*c^4*e*f + 16*a^3*b^5*c^3*d*e - 92*a^4*b^3*c^4*d*e - 16*a^4*b^4*c^3*d*f + 72*a^5*b^2*c^4*d*f + 8*a^5*b^3*c^3*e*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((72*a^5*b^5*c^3*d - 8*a^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e + 80*a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^

4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3
*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e
))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d
 + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*
b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 -
48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (((((72*a^5*b^5*c^3*d - 8*a^4*b^7*c^2*d
- 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e + 80*a^7*b^2*c^4*e
 + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)/(a^6*b^4 + 16*a^8*c^2
 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(4*b^7
*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*
a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^4 + 16*a^8*c^2 - 8
*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2
*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/
(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2
c^4)(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*
b*c*e)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^
2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(8*a^3*(4*a*c -
 b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 -
 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^
3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*
b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*
e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2*(4*b^7*d + 128*a^4*c^3*e - 2*
a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c
^3*d + 24*a^2*b^4*c*e))/(32*a^6*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a
^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*a
^3*c^3*d - 6*b^6*d - 2*a^4*c^2*f + 3*a*b^5*e - 72*a^2*b^2*c^2*d + 42*a*b^4*
c*d - 21*a^2*b^3*c*e + 33*a^3*b*c^2*e + 2*a^3*b^2*c*f))/(8*a^3*c^2*(4*a*c -
 b^2)^3*(144*a^4*c^6*d^2 + 4*b^8*c^2*d^2 + 16*a^6*c^4*f^2 - 48*a*b^6*c^3*d^
2 + 192*a^2*b^4*c^4*d^2 - 288*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 12*a^3*b^
4*c^3*e^2 + 36*a^4*b^2*c^4*e^2 - 96*a^5*c^5*d*f - 4*a*b^7*c^2*d*e + 144*a^4
*b*c^5*d*e - 48*a^5*b*c^4*e*f + 48*a^2*b^5*c^3*d*e - 168*a^3*b^3*c^4*d*e -
16*a^3*b^4*c^3*d*f + 96*a^4*b^2*c^4*d*f + 8*a^4*b^3*c^3*e*f)*(36*a^4*c^4*d^
2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*a^5*c^3*e^2 + 4*a^6*c^2*f^2 + 72*a^3*b
^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^4*c^2*d^2 + 1528*a^3*b^2*c^3*d^2 - 291
*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 - 24*a^5*c^3*d*f - 288*a^2*b^5*c*d*e - 1
564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f + 2*a^4*b^3*c*e*f - 12*a^5*b*c^2*e*f +
1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2*d*f)) + (((((((72*a^5*b^5*c^3*d - 8*a
^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e + 80*
a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)/(a^6*b^4
 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^
2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^
2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*
b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6
*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3

+ 64*a^9*b^2*c^4)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2
*c*d + 6*a^2*b*c*e)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*
d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(8
a^3(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 -
256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e -
2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b
*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*
a^5*b^2*c^2)) - (((36*a^5*c^6*d^2 + 4*a^7*c^4*f^2 - 16*a^2*b^6*c^3*d^2 + 11
6*a^3*b^4*c^4*d^2 - 216*a^4*b^2*c^5*d^2 - 4*a^4*b^4*c^3*e^2 + 17*a^5*b^2*c^
4*e^2 - 24*a^6*c^5*d*f + 108*a^5*b*c^5*d*e - 36*a^6*b*c^4*e*f + 16*a^3*b^5*
c^3*d*e - 92*a^4*b^3*c^4*d*e - 16*a^4*b^4*c^3*d*f + 72*a^5*b^2*c^4*d*f + 8*
a^5*b^3*c^3*e*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((72*a^5*b^5*c^3*d
- 8*a^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e
+ 80*a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)/(a
^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*
a^9*b^2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*
a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b
^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 19
2*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d -
96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4
*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^
2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c
- b^2)^(3/2)) + ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*b^4*d
+ 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^3)/(64*
a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))*(16*a^9*b^6*
(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4
*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(768*b^7*d + 5120
*a^4*c^3*e - 384*a*b^6*e + 18432*a^2*b^3*c^2*d - 8832*a^3*b^2*c^2*e - 6912*
a*b^5*c*d - 12544*a^3*b*c^3*d + 3456*a^2*b^4*c*e - 256*a^3*b^3*c*f + 768*a^
4*b*c^2*f))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(144*a^4*c^6*d^2 + 4*b^8*c^2*
d^2 + 16*a^6*c^4*f^2 - 48*a*b^6*c^3*d^2 + 192*a^2*b^4*c^4*d^2 - 288*a^3*b^2
*c^5*d^2 + a^2*b^6*c^2*e^2 - 12*a^3*b^4*c^3*e^2 + 36*a^4*b^2*c^4*e^2 - 96*a
^5*c^5*d*f - 4*a*b^7*c^2*d*e + 144*a^4*b*c^5*d*e - 48*a^5*b*c^4*e*f + 48*a^
2*b^5*c^3*d*e - 168*a^3*b^3*c^4*d*e - 16*a^3*b^4*c^3*d*f + 96*a^4*b^2*c^4*d
*f + 8*a^4*b^3*c^3*e*f)*(36*a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*
a^5*c^3*e^2 + 4*a^6*c^2*f^2 + 72*a^3*b^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^
4*c^2*d^2 + 1528*a^3*b^2*c^3*d^2 - 291*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 -
24*a^5*c^3*d*f - 288*a^2*b^5*c*d*e - 1564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f +
2*a^4*b^3*c*e*f - 12*a^5*b*c^2*e*f + 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2
*d*f)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^
2*b*c*e))/(2*a^3*(4*a*c - b^2)^(3/2))

3.67 $\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 329

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}$$
$$+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,x^2}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\left(3b^5d-2ab^4e+12a^2b^2ce-12a^3c^2e+6a^2bc(5cd-af)-ab^3(20cd-af)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{3/2}}$$
$$+\frac{(3b^2d-2abe-a(2cd-af))\log(x)}{a^4}-\frac{(3b^2d-2abe-a(2cd-af))\log\left(a+bx^2+cx^4\right)}{4a^4}$$

```
[Out] -1/4*d/a^2/x^4+1/2*(-a*e+2*b*d)/a^3/x^2+1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+2*a^
2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*
d))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(3*b^5*d-2*a*b^4*e+12*a^2*b^2
*c*e-12*a^3*c^2*e+6*a^2*b*c*(-a*f+5*c*d)-a*b^3*(-a*f+20*c*d))*arctanh((2*c*
x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(3*b^2*d-2*a*b*e-a*(-a*f+
2*c*d))*ln(x)/a^4-1/4*(3*b^2*d-2*a*b*e-a*(-a*f+2*c*d))*ln(c*x^4+b*x^2+a)/a^
4
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1677, 1660, 1642, 648, 632, 212, 642}

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2}\,dx = -\frac{\log(a+bx^2+cx^4)(-2abe-a(2cd-af)+3b^2d)}{4a^4} + \frac{\log(x)(-2abe-a(2cd-af)+3b^2d)}{a^4} + \frac{2bd-ae}{2a^3x^2} - \frac{d}{4a^2x^4} + \frac{cx^2(2a^2ce-ab^2e-ab(3cd-af)+b^3d)+3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d}{2a^3(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-12a^3c^2e+12a^2b^2ce+6a^2bc(5cd-af)-2ab^4e-ab^3(20cd-af)+3b^5d)}{2a^4(b^2-4ac)^{3/2}}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*d/(a^2*x^4) + (2*b*d - a*e)/(2*a^3*x^2) + (b^4*d - a*b^3*e + 3*a^2*b*c
*e + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f) + c*(b^3*d - a*b^2*e + 2*a^2
*c*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
((3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e + 6*a^2*b*c*(5*c*d -
a*f) - a*b^3*(20*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a
^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x])/a^
4 - ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^3\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,x^2}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\left(\left(\frac{b^2}{a}-4c\right)d\right)+\frac{\left(b^2-4ac\right)(bd-ae)x}{a^2}-\frac{\left(b^2-4ac\right)\left(b^2d-abe-a(cd-af)\right)x^2}{a^3}-\frac{c\left(b^3d-ab^2e+2a^2ce-ab(3cd-af)\right)x^3}{a^3}}{x^3(a+bx+cx^2)}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,x^2}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{(-b^2+4ac)d}{a^2x^3}+\frac{(-b^2+4ac)(-2bd+ae)}{a^3x^2}+\frac{(b^2-4ac)(-3b^2d+2abe+a(2cd-af))}{a^4x}+\frac{3b^5d-2ab^4e+10a^2b^2ce-6a^3c^2e \cdots}{\cdots}\right.\right.}{2\left(b^2-4ac\right)}$$

$$= -\frac{d}{4a^2x^4} + \frac{2bd-ae}{2a^3x^2} + \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd-af) - ab^2(4cd-af) + c(b^3d - ab^2e + 2a^2ce - ab(3cd-af))}{2a^3(b^2-4ac)(a+bx^2+cx^4)} + \frac{(3b^2d - 2abe - a(2cd-af))\log(x)}{a^4} - \frac{\text{Subst}\left(\int \frac{3b^5d - 2ab^4e + 10a^2b^2ce - 6a^3c^2e + a^2bc(19cd-5af) - ab^3(17cd-af) + c(b^2-4ac)(3b^2d-2abe-a(2cd-af))x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^4(b^2-4ac)}$$

$$= -\frac{d}{4a^2x^4} + \frac{2bd-ae}{2a^3x^2} + \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd-af) - ab^2(4cd-af) + c(b^3d - ab^2e + 2a^2ce - ab(3cd-af))}{2a^3(b^2-4ac)(a+bx^2+cx^4)} + \frac{(3b^2d - 2abe - a(2cd-af))\log(x)}{a^4} - \frac{(3b^2d - 2abe - a(2cd-af))\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^4} - \frac{(3b^5d - 2ab^4e + 12a^2b^2ce - 12a^3c^2e + 6a^2bc(5cd-af) - ab^3(20cd-af))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx\right.}{4a^4(b^2-4ac)}$$

$$= -\frac{d}{4a^2x^4} + \frac{2bd-ae}{2a^3x^2} + \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd-af) - ab^2(4cd-af) + c(b^3d - ab^2e + 2a^2ce - ab(3cd-af))}{2a^3(b^2-4ac)(a+bx^2+cx^4)} + \frac{(3b^2d - 2abe - a(2cd-af))\log(x)}{a^4} - \frac{(3b^2d - 2abe - a(2cd-af))\log(a+bx^2+cx^4)}{4a^4} + \frac{(3b^5d - 2ab^4e + 12a^2b^2ce - 12a^3c^2e + 6a^2bc(5cd-af) - ab^3(20cd-af))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\right.}{2a^4(b^2-4ac)}$$

$$= -\frac{d}{4a^2x^4} + \frac{2bd-ae}{2a^3x^2} + \frac{b^4d - ab^3e + 3a^2bce + 2a^2c(cd-af) - ab^2(4cd-af) + c(b^3d - ab^2e + 2a^2ce - ab(3cd-af))}{2a^3(b^2-4ac)(a+bx^2+cx^4)} + \frac{(3b^5d - 2ab^4e + 12a^2b^2ce - 12a^3c^2e + 6a^2bc(5cd-af) - ab^3(20cd-af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{3/2}} + \frac{(3b^2d - 2abe - a(2cd-af))\log(x)}{a^4} - \frac{(3b^2d - 2abe - a(2cd-af))\log(a+bx^2+cx^4)}{4a^4}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.80

$$\int \frac{d+ex^2+fx^4}{x^5 \left(a+bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{\frac{a^2d}{x^4}+\frac{2a(-2bd+ae)}{x^2}+\frac{2a\left(-b^4d+b^3\left(ae-cdx^2\right)+ab^2\left(4cd-af+cex^2\right)-abc\left(3ae-3cdx^2+afx^2\right)+2a^2c\left(af-c\left(d+ex^2\right)\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}-4(3b^2d-2ab\cdots}{\cdots}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((a^2*d)/x^4 + (2*a*(-2*b*d + a*e))/x^2 + (2*a*(-(b^4*d) + b^3*(a*e -
c*d*x^2) + a*b^2*(4*c*d - a*f + c*e*x^2) - a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x
^2) + 2*a^2*c*(a*f - c*(d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
 4*(3*b^2*d - 2*a*b*e + a*(-2*c*d + a*f))*Log[x] + ((3*b^5*d + b^4*(3*Sqrt[
b^2 - 4*a*c]*d - 2*a*e) + 2*a^2*b*c*(15*c*d + 4*Sqrt[b^2 - 4*a*c]*e - 3*a*f
) + a*b^3*(-20*c*d - 2*Sqrt[b^2 - 4*a*c]*e + a*f) - 4*a^2*c*(-2*c*Sqrt[b^2
- 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-14*c*Sqrt[b^2 - 4*a
*c]*d + 12*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*
x^2])/(b^2 - 4*a*c)^(3/2) + ((-3*b^5*d + b^4*(3*Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - a*b^3*(-20*c*d + 2*Sqrt[b^2 - 4*a*c]*e + a*f) + 2*a^2*b*c*(-15*c*d + 4*
Sqrt[b^2 - 4*a*c]*e + 3*a*f) + 4*a^2*c*(2*c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e -
 a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-2*c*(7*Sqrt[b^2 - 4*a*c]*d + 6*a*e) + a*S
qrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3
/2))/a^4
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.42

method	result
default	$-\frac{d}{4a^2x^4}-\frac{ae-2bd}{2a^3x^2}+\frac{\left(f\,a^2-2abe-2acd+3b^2d\right)\ln(x)}{a^4}-\frac{\frac{ac\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)x^2}{4ac-b^2}-\frac{a\left(2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+a\,b\cdots\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}\cdots$
risch	Expression too large to display

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d/a^2/x^4-1/2*(a*e-2*b*d)/a^3/x^2+(a^2*f-2*a*b*e-2*a*c*d+3*b^2*d)/a^4*
ln(x)-1/2/a^4*((a*c*(a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)
*x^2-a*(2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4
*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*a^3*c^2*f-a^2*b^2*c*
f-8*a^2*b*c^2*e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d)/c*ln(c*x^
4+b*x^2+a)+2*(5*a^3*b*c*f+6*a^3*c^2*e-a^2*b^3*f-10*a^2*b^2*c*e-19*a^2*b*c^2
```

```
*d+2*a*b^4*e+17*a*b^3*c*d-3*b^5*d-1/2*(4*a^3*c^2*f-a^2*b^2*c*f-8*a^2*b*c^2*
e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1272 vs. $2(315) = 630$.

Time = 4.47 (sec) , antiderivative size = 2567, normalized size of antiderivative = 7.80

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a
^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 -
 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3
*c + 28*a^4*b*c^2)*e + 2*(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*e)*x^2 + (((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*
a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 2
0*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (
a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d
 - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*
(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d -
 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c - 26*a*b^4*c^2 + 64*a
^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e +
 (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 6
4*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*
e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*
c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(x))/((a^4*b^4*c - 8
*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4), 1/4*(2*((3*a*b^5*c - 23*a^2*b
^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a^3*b^2*c^2 + 12*a^4*c^3)*e + (
a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^
2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*
```

```
b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e)*x^2 + 2*(((3*b^5*c - 2
0*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e +
 (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*
d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^
6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c +
 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d -
2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 +
 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)
*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c
^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c - 26*a*b^4*c^2
 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^
5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^
2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a
^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a
^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(x))/((a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c
^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x^5 (a + bx^2 + cx^4)^2}\, dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex^2 + fx^4}{x^5 (a + bx^2 + cx^4)^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.58

$$
\begin{aligned}
\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = & -\frac{\left(3\,b^5d-20\,ab^3cd+30\,a^2bc^2d-2\,ab^4e+12\,a^2b^2ce-12\,a^3c^2e+a^2b^3f-6\,a^3bcf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4b^2-4\,a^5c\right)\sqrt{-b^2+4\,ac}} \\
& + \frac{3\,b^4cdx^4-14\,ab^2c^2dx^4+8\,a^2c^3dx^4-2\,ab^3cex^4+8\,a^2bc^2ex^4+a^2b^2cfx^4-4\,a^3c^2fx^4+3\,b^5dx^2-12\,ab^3}{} \\
& - \frac{\left(3\,b^2d-2\,acd-2\,abe+a^2f\right)\log\left(cx^4+bx^2+a\right)}{4\,a^4} \\
& + \frac{\left(3\,b^2d-2\,acd-2\,abe+a^2f\right)\log\left(x^2\right)}{2\,a^4} \\
& - \frac{9\,b^2dx^4-6\,acdx^4-6\,abex^4+3\,a^2fx^4-4\,abdx^2+2\,a^2ex^2+a^2d}{4\,a^4x^4}
\end{aligned}
$$

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^5*d - 20*a*b^3*c*d + 30*a^2*b*c^2*d - 2*a*b^4*e + 12*a^2*b^2*c*e
- 12*a^3*c^2*e + a^2*b^3*f - 6*a^3*b*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*d*x^4 - 14*
a*b^2*c^2*d*x^4 + 8*a^2*c^3*d*x^4 - 2*a*b^3*c*e*x^4 + 8*a^2*b*c^2*e*x^4 + a
^2*b^2*c*f*x^4 - 4*a^3*c^2*f*x^4 + 3*b^5*d*x^2 - 12*a*b^3*c*d*x^2 + 2*a^2*b
*c^2*d*x^2 - 2*a*b^4*e*x^2 + 6*a^2*b^2*c*e*x^2 + 4*a^3*c^2*e*x^2 + a^2*b^3*
f*x^2 - 2*a^3*b*c*f*x^2 + 5*a*b^4*d - 22*a^2*b^2*c*d + 12*a^3*c^2*d - 4*a^2
*b^3*e + 14*a^3*b*c*e + 3*a^3*b^2*f - 8*a^4*c*f)/((a^4*b^2 - 4*a^5*c)*(c*x^
4 + b*x^2 + a)) - 1/4*(3*b^2*d - 2*a*c*d - 2*a*b*e + a^2*f)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(3*b^2*d - 2*a*c*d - 2*a*b*e + a^2*f)*log(x^2)/a^4 - 1/4*
(9*b^2*d*x^4 - 6*a*c*d*x^4 - 6*a*b*e*x^4 + 3*a^2*f*x^4 - 4*a*b*d*x^2 + 2*a^
2*e*x^2 + a^2*d)/(a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 24.98 (sec) , antiderivative size = 15905, normalized size of antiderivative = 48.34

$$
\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}
$$

```
[In] int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log(x)*(3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 - (log((((((4*b*c^2*(3*
b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e - 17*a*b^3*c*d - 5*a^3*b*c*f +
```

19*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)) - (b*c^2*(a*b + 3*b^2
*x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e -
 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*
c - b^2)^3))^(1/2) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 + (2*c^3*x^2
*(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a*b^3*c*d - 10*a^3*b*c
*f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)))*(a^4*(-(3*b^5*d
+ a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^
2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) + 3*b^2*d + a^2*
f - 2*a*b*e - 2*a*c*d))/(4*a^4) + (c^3*(36*b^8*d^2 + 16*a^2*b^6*e^2 + 4*a^4
*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c*f^2 - 48*a*b^7
*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b^2*c^2*e^2 - 30
9*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^5*c*d*e + 324*a
^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^5*b*c^2*e*f - 8
32*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^2)^2) + (c^4*x^2
*(54*b^7*d^2 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*c^3*d^2 - 164*a^3
*b^3*c*e^2 + 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*b^3*c^2*d^2 - 441*
a*b^5*c*d^2 - 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*c^3*d*e - 24*a^3*b^
4*e*f - 120*a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^3*b^3*c*d*f + 260*a^4*b
*c^2*d*f + 122*a^4*b^2*c*e*f - 1072*a^3*b^2*c^2*d*e))/(a^6*(4*a*c - b^2)^2)
)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6
*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2
) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/(4*a^4) - (c^4*(3*b^2*d + a^2*f -
 2*a*b*e - 2*a*c*d)*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a*b*c*d
)^2)/(a^9*(4*a*c - b^2)^2) + (c^5*x^2*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^
2*c*e - 11*a*b*c*d)^3)/(a^9*(4*a*c - b^2)^3))*((((c^3*(36*b^8*d^2 + 16*a^2*
b^6*e^2 + 4*a^4*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c
*f^2 - 48*a*b^7*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b
^2*c^2*e^2 - 309*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^
5*c*d*e + 324*a^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^
5*b*c^2*e*f - 832*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^2
)^2) - (((b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f
- 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d +
12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) - 3*b^2*d - a^2*f + 2*a*b*e
+ 2*a*c*d))/a^4 + (4*b*c^2*(3*b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e -
 17*a*b^3*c*d - 5*a^3*b*c*f + 19*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(a^3*(4*a*c
 - b^2)) + (2*c^3*x^2*(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a
*b^3*c*d - 10*a^3*b*c*f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^
2)))*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d
- 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(
1/2) - 3*b^2*d - a^2*f + 2*a*b*e + 2*a*c*d))/(4*a^4) + (c^4*x^2*(54*b^7*d^2
 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*c^3*d^2 - 164*a^3*b^3*c*e^2 +
 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*b^3*c^2*d^2 - 441*a*b^5*c*d^2
- 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*c^3*d*e - 24*a^3*b^4*e*f - 120*
a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^3*b^3*c*d*f + 260*a^4*b*c^2*d*f + 1
22*a^4*b^2*c*e*f - 1072*a^3*b^2*c^2*d*e))/(a^6*(4*a*c - b^2)^2))*(a^4*(-(3*

b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f +
30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) - 3*b^2*d
- a^2*f + 2*a*b*e + 2*a*c*d))/(4*a^4) + (c^4*(3*b^2*d + a^2*f - 2*a*b*e - 2
*a*c*d)*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a*b*c*d)^2)/(a^9*(4
*a*c - b^2)^2) - (c^5*x^2*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a
*b*c*d)^3)/(a^9*(4*a*c - b^2)^3)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f -
128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^
3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*
c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a
^6*b^2*c^2)) - (d/(4*a) + (x^2*(2*a*e - 3*b*d))/(4*a^2) + (x^4*(6*b^4*d + 8
*a^2*c^2*d + 2*a^2*b^2*f - 4*a*b^3*e - 4*a^3*c*f - 25*a*b^2*c*d + 14*a^2*b*
c*e))/(4*a^3*(4*a*c - b^2)) + (c*x^6*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2
*c*e - 11*a*b*c*d))/(2*a^3*(4*a*c - b^2)))/(a*x^4 + b*x^6 + c*x^8) + (atan(
(x^2*((((((1760*a^7*b*c^8*d^2 - 1104*a^8*b*c^7*e^2 + 80*a^9*b*c^6*f^2 + 54*
a^3*b^9*c^4*d^2 - 657*a^4*b^7*c^5*d^2 + 2775*a^5*b^5*c^6*d^2 - 4484*a^6*b^3
*c^7*d^2 + 24*a^5*b^7*c^4*e^2 - 260*a^6*b^5*c^5*e^2 + 932*a^7*b^3*c^6*e^2 +
6*a^7*b^5*c^4*f^2 - 44*a^8*b^3*c^5*f^2 - 960*a^8*c^8*d*e + 480*a^9*c^7*e*f
- 1040*a^8*b*c^7*d*f - 72*a^4*b^8*c^4*d*e + 828*a^5*b^6*c^5*d*e - 3232*a^6
*b^4*c^6*d*e + 4528*a^7*b^2*c^7*d*e + 36*a^5*b^7*c^4*d*f - 351*a^6*b^5*c^5*
d*f + 1088*a^7*b^3*c^6*d*f - 24*a^6*b^6*c^4*e*f + 218*a^7*b^4*c^5*e*f - 608
*a^8*b^2*c^6*e*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)
- (((1920*a^11*c^7*e + 6*a^6*b^9*c^3*d - 40*a^7*b^7*c^4*d - 108*a^8*b^5*c^
5*d + 1248*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*e + 24*a^8*b^6*c^4*e + 120*a^9*b^4
*c^5*e - 1088*a^10*b^2*c^6*e + 2*a^8*b^7*c^3*f - 36*a^9*b^5*c^4*f + 192*a^1
0*b^3*c^5*f - 2240*a^10*b*c^7*d - 320*a^11*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3
- 12*a^10*b^4*c + 48*a^11*b^2*c^2) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 1
84*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8*d + 256*a^4
*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*
a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^
2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^6 - 64*a^12*c^3 -
12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c +
192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f -
4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96
*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b
^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6
*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*
b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a
*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^
6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) - (216*a^6*c^8*e^3 + 27*
b^9*c^5*d^3 - 297*a*b^7*c^6*d^3 + 1089*a^2*b^5*c^7*d^3 - 1331*a^3*b^3*c^8*d
^3 - 8*a^3*b^6*c^5*e^3 + 72*a^4*b^4*c^6*e^3 - 216*a^5*b^2*c^7*e^3 + a^6*b^3
*c^5*f^3 - 54*a*b^8*c^5*d^2*e - 1188*a^5*b*c^8*d*e^2 + 108*a^6*b*c^7*e^2*f
+ 558*a^2*b^6*c^6*d^2*e + 36*a^2*b^7*c^5*d*e^2 - 1914*a^3*b^4*c^7*d^2*e - 3
48*a^3*b^5*c^6*d*e^2 + 2178*a^4*b^2*c^8*d^2*e + 1116*a^4*b^3*c^7*d*e^2 + 27
*a^2*b^7*c^5*d^2*f - 198*a^3*b^5*c^6*d^2*f + 363*a^4*b^3*c^7*d^2*f + 9*a^4*

b^5*c^5*d*f^2 - 33*a^5*b^3*c^6*d*f^2 + 12*a^4*b^5*c^5*e^2*f - 72*a^5*b^3*c^6*e^2*f - 6*a^5*b^4*c^5*e*f^2 + 18*a^6*b^2*c^6*e*f^2 - 36*a^3*b^6*c^5*d*e*f + 240*a^4*b^4*c^6*d*e*f - 396*a^5*b^2*c^7*d*e*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((((1920*a^11*c^7*e + 6*a^6*b^9*c^3*d - 40*a^7*b^7*c^4*d - 108*a^8*b^5*c^5*d + 1248*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*e + 24*a^8*b^6*c^4*e + 120*a^9*b^4*c^5*e - 1088*a^10*b^2*c^6*e + 2*a^8*b^7*c^3*f - 36*a^9*b^5*c^4*f + 192*a^10*b^3*c^5*f - 2240*a^10*b*c^7*d - 320*a^11*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(32*a^8*(4*a*c - b^2)^3*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(9*b^7*d + 3*a^2*b^5*f + 6*a^4*c^3*e - 6*a*b^6*e + 150*a^2*b^3*c^2*d - 72*a^3*b^2*c^2*e - 69*a*b^5*c*d - 75*a^3*b*c^3*d + 42*a^2*b^4*c*e - 21*a^3*b^3*c*f + 33*a^4*b*c^2*f))/(8*a^3*c^2*(4*a*c - b^2)^3*(1600*a^5*c^5*d^2 - 24*a^2*b^8*e^2 - 54*b^10*d^2 - 6*a^4*b^6*f^2 + 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 + 288*a^3*b^6*c*e^2 + 72*a^5*b^4*c*f^2 + 72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2 + 7200*a^3*b^4*c^3*d^2 - 5775*a^4*b^2*c^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528*a^5*b^2*c^3*e^2 - 291*a^6*b^2*c^2*f^2 + 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f + 24*a^3*b^7*e*f - 1600*a^6*c^4*d*f - 912*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e + 456*a^3*b^6*c*d*f - 288*a^4*b^5*c*e*f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^5*c^2*d*e - 6900*a^4*b^3*c^3*d*e - 2025*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*d*f + 1158*a^5*b^3*c^2*e*f)) - (((((1760*a^7*b*c^8*d^2 - 1104*a^8*b*c^7*e^2 + 80*a^9*b*c^6*f^2 + 54*a^3*b^9*c^4*d^2 - 657*a^4*b^7*c^5*d^2 + 2775*a^5*b^5*c^6*d^2 - 4484*a^6*b^

3*c^7*d^2 + 24*a^5*b^7*c^4*e^2 - 260*a^6*b^5*c^5*e^2 + 932*a^7*b^3*c^6*e^2
+ 6*a^7*b^5*c^4*f^2 - 44*a^8*b^3*c^5*f^2 - 960*a^8*c^8*d*e + 480*a^9*c^7*e*
f - 1040*a^8*b*c^7*d*f - 72*a^4*b^8*c^4*d*e + 828*a^5*b^6*c^5*d*e - 3232*a^
6*b^4*c^6*d*e + 4528*a^7*b^2*c^7*d*e + 36*a^5*b^7*c^4*d*f - 351*a^6*b^5*c^5
*d*f + 1088*a^7*b^3*c^6*d*f - 24*a^6*b^6*c^4*e*f + 218*a^7*b^4*c^5*e*f - 60
8*a^8*b^2*c^6*e*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2
) - (((1920*a^11*c^7*e + 6*a^6*b^9*c^3*d - 40*a^7*b^7*c^4*d - 108*a^8*b^5*c
^5*d + 1248*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*e + 24*a^8*b^6*c^4*e + 120*a^9*b^
4*c^5*e - 1088*a^10*b^2*c^6*e + 2*a^8*b^7*c^3*f - 36*a^9*b^5*c^4*f + 192*a^
10*b^3*c^5*f - 2240*a^10*b*c^7*d - 320*a^11*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3
 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 -
184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8*d + 256*a^
4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576
*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a
^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^6 - 64*a^12*c^3 -
 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c +
 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f
- 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 9
6*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*
b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(
3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f
 + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) - (((((192
0*a^11*c^7*e + 6*a^6*b^9*c^3*d - 40*a^7*b^7*c^4*d - 108*a^8*b^5*c^5*d + 124
8*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*e + 24*a^8*b^6*c^4*e + 120*a^9*b^4*c^5*e -
1088*a^10*b^2*c^6*e + 2*a^8*b^7*c^3*f - 36*a^9*b^5*c^4*f + 192*a^10*b^3*c^5
*f - 2240*a^10*b*c^7*d - 320*a^11*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10
*b^4*c + 48*a^11*b^2*c^2) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b
^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8*d + 256*a^4*c^4*d +
2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c
^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e
 + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b
^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b
^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d -
6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2))
 + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^
4 - 2688*a^12*b^3*c^5)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20
*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^8*d + 256*
a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 5
76*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48
*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(8*a^4*(4*a*c - b^2)^(3/2
)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 25
6*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*
a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3
*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e +
 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^

4*c + 192*a^6*b^2*c^2)) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7
*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(3*b^5*d + a^2*b^3*f - 12*a^3
*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b
^2*c*e)^3)/(64*a^12*(4*a*c - b^2)^(9/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^
4*c + 48*a^11*b^2*c^2)))*(4608*b^8*d + 40960*a^4*c^4*d + 1536*a^2*b^6*f - 2
0480*a^5*c^3*f - 3072*a*b^7*e + 138240*a^2*b^4*c^2*d - 145920*a^3*b^2*c^3*d
 - 73728*a^3*b^3*c^2*e + 35328*a^4*b^2*c^2*f - 44544*a*b^6*c*d + 27648*a^2*
b^5*c*e + 50176*a^4*b*c^3*e - 13824*a^3*b^4*c*f))/(4096*a^3*c^2*(4*a*c - b^
2)^(7/2)*(1600*a^5*c^5*d^2 - 24*a^2*b^8*e^2 - 54*b^10*d^2 - 6*a^4*b^6*f^2 +
 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 + 288*a^3*b^6*c*e^2 + 72*a^5*b^4*c*f^2 +
72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2 + 7200*a^3*b^4*c^3*d^2 - 5775*a^4*b^2*c
^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528*a^5*b^2*c^3*e^2 - 291*a^6*b^2*c^2*f^2
+ 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f + 24*a^3*b^7*e*f - 1600*a^6*c^4*d*f - 91
2*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e + 456*a^3*b^6*c*d*f - 288*a^4*b^5*c*e*
f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^5*c^2*d*e - 6900*a^4*b^3*c^3*d*e - 2025
*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*d*f + 1158*a^5*b^3*c^2*e*f)))*(16*a^12*
b^6*(4*a*c - b^2)^(9/2) - 1024*a^15*c^3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*
c*(4*a*c - b^2)^(9/2) + 768*a^14*b^2*c^2*(4*a*c - b^2)^(9/2)))/(144*a^6*c^6
*e^2 + 9*b^10*c^2*d^2 - 120*a*b^8*c^3*d^2 + 580*a^2*b^6*c^4*d^2 - 1200*a^3*
b^4*c^5*d^2 + 900*a^4*b^2*c^6*d^2 + 4*a^2*b^8*c^2*e^2 - 48*a^3*b^6*c^3*e^2
+ 192*a^4*b^4*c^4*e^2 - 288*a^5*b^2*c^5*e^2 + a^4*b^6*c^2*f^2 - 12*a^5*b^4*
c^3*f^2 + 36*a^6*b^2*c^4*f^2 - 12*a*b^9*c^2*d*e - 720*a^5*b*c^6*d*e + 144*a
^6*b*c^5*e*f + 152*a^2*b^7*c^3*d*e - 672*a^3*b^5*c^4*d*e + 1200*a^4*b^3*c^5
*d*e + 6*a^2*b^8*c^2*d*f - 76*a^3*b^6*c^3*d*f + 300*a^4*b^4*c^4*d*f - 360*a
^5*b^2*c^5*d*f - 4*a^3*b^7*c^2*e*f + 48*a^4*b^5*c^3*e*f - 168*a^5*b^3*c^4*e
*f) - ((16*a^12*b^6*(4*a*c - b^2)^(9/2) - 1024*a^15*c^3*(4*a*c - b^2)^(9/2)
 - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^14*b^2*c^2*(4*a*c - b^2)^(9/2
))*((27*b^8*c^4*d^3 - 216*a*b^6*c^5*d^3 - 72*a^5*b*c^6*e^3 - 72*a^5*c^7*d*e
^2 + 36*a^6*c^6*e^2*f + 495*a^2*b^4*c^6*d^3 - 242*a^3*b^2*c^7*d^3 - 8*a^3*b
^5*c^4*e^3 + 48*a^4*b^3*c^5*e^3 + a^6*b^2*c^4*f^3 - 54*a*b^7*c^4*d^2*e + 26
4*a^4*b*c^7*d^2*e + 12*a^6*b*c^5*e*f^2 + 396*a^2*b^5*c^5*d^2*e + 36*a^2*b^6
*c^4*d*e^2 - 798*a^3*b^3*c^6*d^2*e - 240*a^3*b^4*c^5*d*e^2 + 420*a^4*b^2*c^
6*d*e^2 + 27*a^2*b^6*c^4*d^2*f - 144*a^3*b^4*c^5*d^2*f + 165*a^4*b^2*c^6*d^
2*f + 9*a^4*b^4*c^4*d*f^2 - 24*a^5*b^2*c^5*d*f^2 + 12*a^4*b^4*c^4*e^2*f - 4
8*a^5*b^2*c^5*e^2*f - 6*a^5*b^3*c^4*e*f^2 - 156*a^5*b*c^6*d*e*f - 36*a^3*b^
5*c^4*d*e*f + 168*a^4*b^3*c^5*d*e*f)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)
 + (((36*a^8*c^6*e^2 - 36*a^3*b^8*c^3*d^2 + 309*a^4*b^6*c^4*d^2 - 778*a^5*b
^4*c^5*d^2 + 473*a^6*b^2*c^6*d^2 - 16*a^5*b^6*c^3*e^2 + 116*a^6*b^4*c^4*e^2
 - 216*a^7*b^2*c^5*e^2 - 4*a^7*b^4*c^3*f^2 + 17*a^8*b^2*c^4*f^2 - 324*a^7*b
*c^6*d*e + 108*a^8*b*c^5*e*f + 48*a^4*b^7*c^3*d*e - 380*a^5*b^5*c^4*d*e + 8
32*a^6*b^3*c^5*d*e - 24*a^5*b^6*c^3*d*f + 154*a^6*b^4*c^4*d*f - 230*a^7*b^2
*c^5*d*f + 16*a^6*b^5*c^3*e*f - 92*a^7*b^3*c^4*e*f)/(a^9*b^4 + 16*a^11*c^2
 - 8*a^10*b^2*c) + (((12*a^6*b^8*c^2*d - 116*a^7*b^6*c^3*d + 348*a^8*b^4*c^4
*d - 304*a^9*b^2*c^5*d - 8*a^7*b^7*c^2*e + 72*a^8*b^5*c^3*e - 184*a^9*b^3*c
^4*e + 4*a^8*b^6*c^2*f - 36*a^9*b^4*c^3*f + 80*a^10*b^2*c^4*f + 96*a^10*b*c

^5*e)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b
^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*
c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2
*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 2
4*a^3*b^4*c*f))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*
a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^
2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d
 - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 2
56*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*
c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3
*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e
+ 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a
^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2))
 - (((((12*a^6*b^8*c^2*d - 116*a^7*b^6*c^3*d + 348*a^8*b^4*c^4*d - 304*a^9*
b^2*c^5*d - 8*a^7*b^7*c^2*e + 72*a^8*b^5*c^3*e - 184*a^9*b^3*c^4*e + 4*a^8*
b^6*c^2*f - 36*a^9*b^4*c^3*f + 80*a^10*b^2*c^4*f + 96*a^10*b*c^5*e)/(a^9*b^
4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a
^12*b^2*c^4)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b
^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b
^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f
))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*
a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^
4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4
*(4*a*c - b^2)^(3/2)) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^
4)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b
*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^
6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 1
92*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a
^4*b*c^3*e - 24*a^3*b^4*c*f))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^4 + 16*a^11
*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*
c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a
^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) -
((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(3*b^5*d + a^2*b^3*f
- 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d +
12*a^2*b^2*c*e)^2*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f -
4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*
a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^
4*c*f))/(32*a^8*(4*a*c - b^2)^3*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a
^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(9*b^7*d + 3*a^2*b
^5*f + 6*a^4*c^3*e - 6*a*b^6*e + 150*a^2*b^3*c^2*d - 72*a^3*b^2*c^2*e - 69*
a*b^5*c*d - 75*a^3*b*c^3*d + 42*a^2*b^4*c*e - 21*a^3*b^3*c*f + 33*a^4*b*c^2
*f))/(8*a^3*c^2*(4*a*c - b^2)^3*(144*a^6*c^6*e^2 + 9*b^10*c^2*d^2 - 120*a*b
^8*c^3*d^2 + 580*a^2*b^6*c^4*d^2 - 1200*a^3*b^4*c^5*d^2 + 900*a^4*b^2*c^6*d
^2 + 4*a^2*b^8*c^2*e^2 - 48*a^3*b^6*c^3*e^2 + 192*a^4*b^4*c^4*e^2 - 288*a^5
*b^2*c^5*e^2 + a^4*b^6*c^2*f^2 - 12*a^5*b^4*c^3*f^2 + 36*a^6*b^2*c^4*f^2 -

12*a*b^9*c^2*d*e - 720*a^5*b*c^6*d*e + 144*a^6*b*c^5*e*f + 152*a^2*b^7*c^3*
d*e - 672*a^3*b^5*c^4*d*e + 1200*a^4*b^3*c^5*d*e + 6*a^2*b^8*c^2*d*f - 76*a
^3*b^6*c^3*d*f + 300*a^4*b^4*c^4*d*f - 360*a^5*b^2*c^5*d*f - 4*a^3*b^7*c^2*
e*f + 48*a^4*b^5*c^3*e*f - 168*a^5*b^3*c^4*e*f)*(1600*a^5*c^5*d^2 - 24*a^2*
b^8*e^2 - 54*b^10*d^2 - 6*a^4*b^6*f^2 + 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 +
288*a^3*b^6*c*e^2 + 72*a^5*b^4*c*f^2 + 72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2
+ 7200*a^3*b^4*c^3*d^2 - 5775*a^4*b^2*c^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528
*a^5*b^2*c^3*e^2 - 291*a^6*b^2*c^2*f^2 + 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f +
24*a^3*b^7*e*f - 1600*a^6*c^4*d*f - 912*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e
+ 456*a^3*b^6*c*d*f - 288*a^4*b^5*c*e*f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^
5*c^2*d*e - 6900*a^4*b^3*c^3*d*e - 2025*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*
d*f + 1158*a^5*b^3*c^2*e*f)) + (((((((12*a^6*b^8*c^2*d - 116*a^7*b^6*c^3*d
+ 348*a^8*b^4*c^4*d - 304*a^9*b^2*c^5*d - 8*a^7*b^7*c^2*e + 72*a^8*b^5*c^3*
e - 184*a^9*b^3*c^4*e + 4*a^8*b^6*c^2*f - 36*a^9*b^4*c^3*f + 80*a^10*b^2*c^
4*f + 96*a^10*b*c^5*e)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^
6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2
*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d
- 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 25
6*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*
(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2
*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c
^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) + ((4*a^10*b^6*c^2 - 32
*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*
b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^
8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4
*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^
6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(8*a^4*(4*a*c -
b^2)^(3/2)*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3
- 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f
- 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*
a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*
b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192
*a^6*b^2*c^2)) + (((36*a^8*c^6*e^2 - 36*a^3*b^8*c^3*d^2 + 309*a^4*b^6*c^4*d
^2 - 778*a^5*b^4*c^5*d^2 + 473*a^6*b^2*c^6*d^2 - 16*a^5*b^6*c^3*e^2 + 116*a
^6*b^4*c^4*e^2 - 216*a^7*b^2*c^5*e^2 - 4*a^7*b^4*c^3*f^2 + 17*a^8*b^2*c^4*f
^2 - 324*a^7*b*c^6*d*e + 108*a^8*b*c^5*e*f + 48*a^4*b^7*c^3*d*e - 380*a^5*b
^5*c^4*d*e + 832*a^6*b^3*c^5*d*e - 24*a^5*b^6*c^3*d*f + 154*a^6*b^4*c^4*d*f
- 230*a^7*b^2*c^5*d*f + 16*a^6*b^5*c^3*e*f - 92*a^7*b^3*c^4*e*f)/(a^9*b^4
+ 16*a^11*c^2 - 8*a^10*b^2*c) + (((12*a^6*b^8*c^2*d - 116*a^7*b^6*c^3*d + 3
48*a^8*b^4*c^4*d - 304*a^9*b^2*c^5*d - 8*a^7*b^7*c^2*e + 72*a^8*b^5*c^3*e -
184*a^9*b^3*c^4*e + 4*a^8*b^6*c^2*f - 36*a^9*b^4*c^3*f + 80*a^10*b^2*c^4*f
+ 96*a^10*b*c^5*e)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c
^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^
6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 1
92*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a

^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) - ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^3)/(64*a^12*(4*a*c - b^2)^(9/2)*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)))*(16*a^12*b^6*(4*a*c - b^2)^(9/2) - 1024*a^15*c^3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^14*b^2*c^2*(4*a*c - b^2)^(9/2))*(4608*b^8*d + 40960*a^4*c^4*d + 1536*a^2*b^6*f - 20480*a^5*c^3*f - 3072*a*b^7*e + 138240*a^2*b^4*c^2*d - 145920*a^3*b^2*c^3*d - 73728*a^3*b^3*c^2*e + 35328*a^4*b^2*c^2*f - 44544*a*b^6*c*d + 27648*a^2*b^5*c*e + 50176*a^4*b*c^3*e - 13824*a^3*b^4*c*f))/(4096*a^3*c^2*(4*a*c - b^2)^(7/2)*(144*a^6*c^6*e^2 + 9*b^10*c^2*d^2 - 120*a*b^8*c^3*d^2 + 580*a^2*b^6*c^4*d^2 - 1200*a^3*b^4*c^5*d^2 + 900*a^4*b^2*c^6*d^2 + 4*a^2*b^8*c^2*e^2 - 48*a^3*b^6*c^3*e^2 + 192*a^4*b^4*c^4*e^2 - 288*a^5*b^2*c^5*e^2 + a^4*b^6*c^2*f^2 - 12*a^5*b^4*c^3*f^2 + 36*a^6*b^2*c^4*f^2 - 12*a*b^9*c^2*d*e - 720*a^5*b*c^6*d*e + 144*a^6*b*c^5*e*f + 152*a^2*b^7*c^3*d*e - 672*a^3*b^5*c^4*d*e + 1200*a^4*b^3*c^5*d*e + 6*a^2*b^8*c^2*d*f - 76*a^3*b^6*c^3*d*f + 300*a^4*b^4*c^4*d*f - 360*a^5*b^2*c^5*d*f - 4*a^3*b^7*c^2*e*f + 48*a^4*b^5*c^3*e*f - 168*a^5*b^3*c^4*e*f)*(1600*a^5*c^5*d^2 - 24*a^2*b^8*e^2 - 54*b^10*d^2 - 6*a^4*b^6*f^2 + 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 + 288*a^3*b^6*c*e^2 + 72*a^5*b^4*c*f^2 + 72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2 + 7200*a^3*b^4*c^3*d^2 - 5775*a^4*b^2*c^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528*a^5*b^2*c^3*e^2 - 291*a^6*b^2*c^2*f^2 + 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f + 24*a^3*b^7*e*f - 1600*a^6*c^4*d*f - 912*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e + 456*a^3*b^6*c*d*f - 288*a^4*b^5*c*e*f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^5*c^2*d*e - 6900*a^4*b^3*c^3*d*e - 2025*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*d*f + 1158*a^5*b^3*c^2*e*f)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(2*a^4*(4*a*c - b^2)^(3/2))

3.68 $\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 550

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2} + \frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))x^2)}{2c^3(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)-\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd-34af)+4ab}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)+\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd-34af)+4ab}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] (-2*b*f+c*e)*x/c^3+1/3*f*x^3/c^2+1/2*x*(a*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*
a*f+c*d))+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))
*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*e-13*a*b*c^2*e-5*b^4*f-b^2*c*(-24*a*f+c*d)+
2*a*c^2*(-7*a*f+3*c*d)+(-3*b^4*c*e+19*a*b^2*c^2*e-20*a^2*c^3*e+5*b^5*f+b^3*
c*(-34*a*f+c*d)-4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*
a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*e-13*a*b*c^2*e-5*b^4*f-b^2*c*(-24*a*
f+c*d)+2*a*c^2*(-7*a*f+3*c*d)+(3*b^4*c*e-19*a*b^2*c^2*e+20*a^2*c^3*e-5*b^5*
f-b^3*c*(-34*a*f+c*d)+4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2
)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 9.60 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1682, 1690, 1180, 211}

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx =$$
$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x(a(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)+x^2(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$+\frac{x(ce-2bf)}{c^3}+\frac{fx^3}{3c^2}$$

```
[In] Int[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*e - 2*b*f)*x)/c^3 + (f*x^3)/(3*c^2) + (x*(a*(b^2*c*e - 2*a*c^2*e - b^3*
f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*
f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
 ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c
*d - 7*a*f) - (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*
(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b
^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) + (3*b^4*c*e - 19*a*b^2*c^2*e
 + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*
f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

integral

$$= \frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\,x^2)}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{\frac{a^2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)}{c^3}+\frac{a\left(b^3ce-5abc^2e-b^4f-b^2c(cd-6af)+6ac^2(cd-af)\right)x^2}{c^3}-\frac{2a\left(b^2-4ac\right)(ce-bf)x^4}{c^2}+2a\left(4a-\frac{b^2}{c}\right)fx^6}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)}$$

$$= \frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \left(-\frac{2a(b^2-4ac)(ce-2bf)}{c^3}-\frac{2a(b^2-4ac)fx^2}{c^2}-\frac{-a^2(3b^2ce-10ac^2e-5b^3f-bc(cd-19af))-a(3b^3ce-13abc^2e-5b^4f-b^2c(cd-2}{c^3(a+bx^2+cx^4)}\right.}{2a\,(b^2-4ac)}$$

$$= \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2} + \frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\int \frac{-a^2(3b^2ce-10ac^2e-5b^3f-bc(cd-19af))-a(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af))x^2}{a+bx^2+cx^4}\,dx}{2ac^3\,(b^2-4ac)}$$

$$= \frac{(ce-2bf)x}{c^3} + \frac{fx^3}{3c^2} + \frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-a}{2c^3(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)-\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd}{\sqrt{b^2-4ac}}\right.}{4c^3(b^2-4ac)} - \frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)+\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd}{\sqrt{b^2-4ac}}\right.}{4c^3(b^2-4ac)}$$

$$= \frac{(ce-2bf)x}{c^3} + \frac{fx^3}{3c^2} + \frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-a}{2c^3(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)-\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)+\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.18

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{12\sqrt{c}(ce-2bf)x+4c^{3/2}fx^3-\frac{6\sqrt{c}x(b^2(c^2d-bce+b^2f)x^2+a^2c(-3bf+2c(e+fx^2))+a(b^3f-2c^3dx^2+bc^2(d+3ex^2)-b^2c(e+4fx^2))}{(b^2-4ac)(a+bx^2+cx^4)}}{}$$

```
[In] Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*e - 2*b*f)*x + 4*c^(3/2)*f*x^3 - (6*Sqrt[c]*x*(b^2*(c^2*d -
b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*d*
x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*f + a*b*c^2*(8*c*d + 13*Sqrt[b^2 - 4*a*c]*
e - 52*a*f) - b^3*c*(c*d + 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + b^4*(3*c*e + 5
*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt[b^2 - 4*a*c]*d - 19*a*c*e - 24*a*Sqrt
[b^2 - 4*a*c]*f) + 2*a*c^2*(-3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e + 7*a*Sqrt[
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*f + b^
3*c*(c*d - 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + a*b*c^2*(-8*c*d + 13*Sqrt[b^2
- 4*a*c]*e + 52*a*f) + b^4*(-3*c*e + 5*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt
```

```
[b^2 - 4*a*c]*d + 19*a*c*e - 24*a*Sqrt[b^2 - 4*a*c]*f) - 2*a*c^2*(3*c*Sqrt[
b^2 - 4*a*c]*d + 10*a*c*e - 7*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/(12*c^(7/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.59

method	result
risch	$\frac{f\,x^{3}}{3c^{2}}-\frac{2bfx}{c^{3}}+\frac{xe}{c^{2}}+\frac{\frac{\left(2a^{2}c^{2}f-4a\,b^{2}cf+3ab\,c^{2}e-2a\,c^{3}d+b^{4}f-b^{3}ce+b^{2}c^{2}d\right)x^{3}}{8ac-2b^{2}}-\frac{a\left(3abcf-2a\,c^{2}e-b^{3}f+b^{2}ce-b\,c^{2}d\right)x}{2\left(4ac-b^{2}\right)}}{c^{3}(c\,x^{4}+b\,x^{2}+a)}+\frac{\sum}{\_R=\mathrm{RootOf}\left(c\_\right.}$
default	$-\frac{-\frac{1}{3}cf\,x^{3}+2bfx-xce}{c^{3}}+\frac{\frac{\frac{\left(2a^{2}c^{2}f-4a\,b^{2}cf+3ab\,c^{2}e-2a\,c^{3}d+b^{4}f-b^{3}ce+b^{2}c^{2}d\right)x^{3}}{8ac-2b^{2}}-\frac{a\left(3abcf-2a\,c^{2}e-b^{3}f+b^{2}ce-b\,c^{2}d\right)x}{2\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+2c\left(\frac{\left(-14a^{2}c^{2}f\right.}{}\right.}{}$

```
[In] int(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*f*x^3/c^2-2/c^3*b*f*x+1/c^2*x*e+(1/2*(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2
*e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/(4*a*c-b^2)*x^3-1/2*a*(3*a*b*c*f-2*a*
c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)*x)/c^3/(c*x^4+b*x^2+a)+1/4/c^3*sum
((-(14*a^2*c^2*f-24*a*b^2*c*f+13*a*b*c^2*e-6*a*c^3*d+5*b^4*f-3*b^3*c*e+b^2*
c^2*d)/(4*a*c-b^2)*_R^2+a*(19*a*b*c*f-10*a*c^2*e-5*b^3*f+3*b^2*c*e-b*c^2*d)
/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18909 vs. 2(506) = 1012.

Time = 80.29 (sec) , antiderivative size = 18909, normalized size of antiderivative = 34.38

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^6}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2
*a^2*c^2)*f)*x^3 + (a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e + (a*b^3 - 3*a^2*b*
c)*f)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*
b*c^4)*x^2) + 1/2*integrate((a*b*c^2*d + ((b^2*c^2 - 6*a*c^3)*d - (3*b^3*c
- 13*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*f)*x^2 - (3*a*b^2*c - 1
0*a^2*c^2)*e + (5*a*b^3 - 19*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 -
 4*a*c^4) + 1/3*(c*f*x^3 + 3*(c*e - 2*b*f)*x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8946 vs. $2(506) = 1012$.

Time = 2.03 (sec) , antiderivative size = 8946, normalized size of antiderivative = 16.27

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*d*x^3 - 2*a*c^3*d*x^3 - b^3*c*e*x^3 + 3*a*b*c^2*e*x^3 + b^4*f
*x^3 - 4*a*b^2*c*f*x^3 + 2*a^2*c^2*f*x^3 + a*b*c^2*d*x - a*b^2*c*e*x + 2*a^
2*c^2*e*x + a*b^3*f*x - 3*a^2*b*c*f*x)/((b^2*c^3 - 4*a*c^4)*(c*x^4 + b*x^2
+ a)) + 1/16*((2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c
^3 - 4*a*c^4)^2*d - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*
a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*e + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^
2*b^2*c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^5*c - 110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*
c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 +
 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 28*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 24*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4
*a*c)*b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2
*c^3 - 4*a*c^4)^2*f + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 - 2*a*b^5*c^7 + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^8 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 + 16*a^2*b^3*
c^8 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^9 - 32*a^3*b*c^9 +
2*(b^2 - 4*a*c)*a*b^3*c^7 - 8*(b^2 - 4*a*c)*a^2*b*c^8)*d*abs(b^2*c^3 - 4*a*
c^4) - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^5*c^6 - 6*a*b^6*c^6 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^2*c^7 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^
7 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 + 68*a^2*b^4*c^7 -
160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^8 - 80*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^8 - 256*a^3*b^2*c^8 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*c^9 + 320*a^4*c^9 + 6*(b^2 - 4*a*c)*a*b^4*c^6 - 44*(b^2 - 4*a*c)*a^2*
b^2*c^7 + 80*(b^2 - 4*a*c)*a^3*c^8)*e*abs(b^2*c^3 - 4*a*c^4) + 2*(5*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6
*c^5 - 10*a*b^7*c^5 + 232*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c
^6 + 78*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + 5*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 + 118*a^2*b^5*c^6 - 304*sqrt(2)*sqrt
```

(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
^7 - 464*a^3*b^3*c^7 + 76*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8
 + 608*a^4*b*c^8 + 10*(b^2 - 4*a*c)*a*b^5*c^5 - 78*(b^2 - 4*a*c)*a^2*b^3*c^
6 + 152*(b^2 - 4*a*c)*a^3*b*c^7)*f*abs(b^2*c^3 - 4*a*c^4) - (2*b^8*c^10 - 3
2*a*b^6*c^11 + 160*a^2*b^4*c^12 - 256*a^3*b^2*c^13 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c^11 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^12 - 2*(b^2 - 4*a*c)*b^6*c^10 + 24*(b^2 - 4*
a*c)*a*b^4*c^11 - 64*(b^2 - 4*a*c)*a^2*b^2*c^12)*d + (6*b^9*c^9 - 86*a*b^7*
c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 + 640*a^4*b*c^13 - 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^7 + 43*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 + 6*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 - 220*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 - 62*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 31*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - 320*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^11 - 160*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 96*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 80*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^12 - 6*(b^2 - 4*a*c)*b^7*c^9 +
62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^2*b^3*c^11 + 160*(b^2 - 4
*a*c)*a^3*b*c^12)*e - (10*b^10*c^8 - 148*a*b^8*c^9 + 808*a^2*b^6*c^10 - 192
0*a^3*b^4*c^11 + 1664*a^4*b^2*c^12 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^10*c^6 + 74*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^8*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^9*c^7 - 404*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^6*c^8 - 108*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^8*c^8 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^4*c^9 + 376*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^5*c^9 + 54*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^6*c^9 - 832*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^10 - 416*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^3*c^10 - 188*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2

- 4*a*c)*c)*a^2*b^4*c^10 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^11 - 10*(b^2 - 4*a*c)*b^8*c^8 + 108*(b^2 - 4*a*c)*
a*b^6*c^9 - 376*(b^2 - 4*a*c)*a^2*b^4*c^10 + 416*(b^2 - 4*a*c)*a^3*b^2*c^11
)*f)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*a*b*c^4 + sqrt((b^3*c^3 - 4*a*b
c^4)^2 - 4(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4 - 4*a*c^5)))/(b^2*c^4 - 4*a*c^
5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*c^8 + 48*a^3*b^2*c^9 + 16*a^2*b
^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*c^10 - 8*a^2*b^2*c^10 + 16*a^3*
c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) - 1/16*((2*b^4*c^4 - 20*a*b^2*c^5 + 48
*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^
2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 +
 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c^3 - 4*a*c^4)^2*d - (6*b^5*c^3 - 50*a*b^3*c^
4 + 104*a^2*b*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^5*c + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^
3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2
- 4*a*c)*b^3*c^3 + 26*(b^2 - 4*a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*e + (10*
b^6*c^2 - 88*a*b^4*c^3 + 220*a^2*b^2*c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^3 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c^3 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 -
 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2*c^3 - 4*a*c^4)^2*f - 2*(sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 + 2*a*b^5
*c^7 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^3*c^8 - 16*a^2*b^3*c^8 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^9 + 32*a^3*b*c^9 - 2*(b^2 - 4*a*c)*a*b^3*c^7 + 8*(b^2 - 4*a*c)*
a^2*b*c^8)*d*abs(b^2*c^3 - 4*a*c^4) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^6*c^5 - 34*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6
- 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 + 6*a*b^6*c^6 + 128*s

qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 + 44*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^4*c^7 - 68*a^2*b^4*c^7 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*c^8 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + 256*a^3*b^2*c^8 + 40*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^9 - 320*a^4*c^9 - 6*(b^2 - 4*a*c)*a*
b^4*c^6 + 44*(b^2 - 4*a*c)*a^2*b^2*c^7 - 80*(b^2 - 4*a*c)*a^3*c^8)*e*abs(b^
2*c^3 - 4*a*c^4) - 2*(5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 -
 59*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 + 10*a*b^7*c^5 + 232*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^6 + 78*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^4*c^6 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 118
*a^2*b^5*c^6 - 304*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 464*a^3*b^3*c^7 + 76*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 608*a^4*b*c^8 - 10*(b^2 - 4*a*c)*a*b^5*c^
5 + 78*(b^2 - 4*a*c)*a^2*b^3*c^6 - 152*(b^2 - 4*a*c)*a^3*b*c^7)*f*abs(b^2*c
^3 - 4*a*c^4) - (2*b^8*c^10 - 32*a*b^6*c^11 + 160*a^2*b^4*c^12 - 256*a^3*b^
2*c^13 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8
+ 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^9 - 80*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 - 24*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 128*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 + 64*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^11 - 32*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^12 - 2*(b^2 -
 4*a*c)*b^6*c^10 + 24*(b^2 - 4*a*c)*a*b^4*c^11 - 64*(b^2 - 4*a*c)*a^2*b^2*c
^12)*d + (6*b^9*c^9 - 86*a*b^7*c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 +
 640*a^4*b*c^13 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^9*c^7 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^7*c^8 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*
c^8 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5
*c^9 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c
^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^9 +
464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10
+ 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^1
0 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10
 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^11
 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^
11 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
^11 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
12 - 6*(b^2 - 4*a*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a
*c)*a^2*b^3*c^11 + 160*(b^2 - 4*a*c)*a^3*b*c^12)*e - (10*b^10*c^8 - 148*a*b

```
^8*c^9 + 808*a^2*b^6*c^10 - 1920*a^3*b^4*c^11 + 1664*a^4*b^2*c^12 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^10*c^6 + 74*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^7 + 10*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^7 - 404*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^8 - 108*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 960*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^9 + 376*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 + 54*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 832*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^10 - 416*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 - 188*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 208*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 10*(b^2 - 4*a*
c)*b^8*c^8 + 108*(b^2 - 4*a*c)*a*b^6*c^9 - 376*(b^2 - 4*a*c)*a^2*b^4*c^10 +
 416*(b^2 - 4*a*c)*a^3*b^2*c^11)*f)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*
a*b*c^4 - sqrt((b^3*c^3 - 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4
 - 4*a*c^5)))/(b^2*c^4 - 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*
c^8 + 48*a^3*b^2*c^9 + 16*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*
c^10 - 8*a^2*b^2*c^10 + 16*a^3*c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) + 1/3*(
c^4*f*x^3 + 3*c^4*e*x - 6*b*c^3*f*x)/c^6
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 33799, normalized size of antiderivative = 61.45

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x*(e/c^2 - (2*b*f)/c^3) + ((x^3*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*
d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2
*e + a*b^3*f + a*b*c^2*d - a*b^2*c*e - 3*a^2*b*c*f))/(2*(4*a*c - b^2)))/(a*
c^3 + c^4*x^4 + b*c^3*x^2) - atan(((((10240*a^5*c^9*e + 192*a^2*b^5*c^7*d -
 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4*b^2
*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f - 16
*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 19456*a
^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x
*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2
)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c -
 b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7
*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^
4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^
5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
```

+ b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3
*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6
*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*
e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c
^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c -
b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c
f^2(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e
- 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 1
4784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a
^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2
) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f
+ 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c
- b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d
e(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^
9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*
b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^
7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2
+ 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*
d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6
*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 15
04*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^
3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*
a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*
a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9
)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360
*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a
*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e
*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^
5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*
e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f -
69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3
*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e
*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e
*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c -
b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*
(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186
*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24
*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 614
4*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^
2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*

a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 7
18*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4
*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c
^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394*a*b
^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f
 - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*
(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^
13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*
a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 +
 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^
6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2
- 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 2
5*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2
 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*
c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f
^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d
*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*
b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*
(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*
a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*
b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*
c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) -
 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3
*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6
*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^
2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(
1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^1
2*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*
c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i - (((10240*a^5*c^9*e + 192*a^2*b^5*c^7
*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4
*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f
- 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 194
56*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))
+ (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c -
 b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b
*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 384
0*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^
4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(
1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767
*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040
*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^

3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^
11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a
*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b
^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6
*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f
 - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f +
42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^
(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*
e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4
*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c
^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2)
 + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^
8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*
(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2
*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*
c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880
*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2
- 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 1065
6*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*
c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6
366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219
744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^
2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 1
5360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 1
52*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c
^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^
8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^
8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*
f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3
*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c
^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c
^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*
c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) -
 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7
- 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 -
 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^
6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 -
114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2
 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794
*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b
^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394

*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4
*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f))
/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 +
9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3
840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e
^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^
7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*
e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2
 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)
^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4
*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(
4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13
*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c
^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 72
4*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*
f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1
548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*
a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*
b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/
2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132
*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280
*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e
^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*
a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^
9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 +
 b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*
b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i)/((((10240*a^5*c^9*e + 192*a^2*b^5
*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752
*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^
7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f -
 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^
7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-
(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a
^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 +
 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 3024
0*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^
9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 3
5767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 21
5040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^1
2*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*
a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-
(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165
*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6

*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4
*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*
f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)
^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*
c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*
(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a
*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*
a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^
2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1
/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13
*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^
5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 2
6880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*
d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 -
10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*
a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2)
 + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 -
 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c
- b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2
 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f
 + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^
12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-
(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^
2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^
2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^
6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6
*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b
^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b
^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*
c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/
2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*
c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^
11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^
4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^
2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4
*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 +
2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e +
10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f +
 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4
*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e
f))/(2(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^
2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2

- 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^
2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*
c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7
*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2
)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7
*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2
*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*
b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^
13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f
+ 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*
c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
- 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30
720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*
a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)
^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 3
9132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 16
1280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c
^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) -
78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b
^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*
a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (((10240*a^5*c^9*e + 192*a^2*b^
5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 1075
2*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c
^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f
- 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c
^7)) + (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*
a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2
+ 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 302
40*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 -
35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 2
15040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2
e^2(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^
12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258
*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(
-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 16
5*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^
6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^
4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d
*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2
)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6

*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f
*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*
a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^
(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4
*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a
^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(
1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(
16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^1
3*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a
^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 +
26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6
*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 -
 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25
*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2
- 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c
 - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^
2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*
f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b
^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(
-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a
^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b
^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c
^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) -
6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*
b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*
b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2
*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1
/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12
*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c
^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a
^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d
^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^
4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 +
 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e +
 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f
+ 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^
4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*
e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d
^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^
2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*
c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a
^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9
*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^

7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^
2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^
7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^
2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a
*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b
^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f
 + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2
*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 3
0720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352
*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9
)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f +
39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 1
61280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*
c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2)
- 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c -
b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c
^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840
*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (216*a^4*c^6*d^3 + 225*a^4*b^6
*f^3 - 2744*a^7*c^3*f^3 - 1300*a^5*b*c^4*e^3 - 2060*a^5*b^4*c*f^3 + 125*a^2
*b^8*d*f^2 + 600*a^5*c^5*d*e^2 - 175*a^3*b^7*e*f^2 - 1512*a^5*c^5*d^2*f + 3
528*a^6*c^4*d*f^2 - 1400*a^6*c^4*e^2*f + 5*a^2*b^4*c^4*d^3 - 66*a^3*b^2*c^5
*d^3 - 63*a^3*b^5*c^2*e^3 + 573*a^4*b^3*c^3*e^3 + 5334*a^6*b^2*c^2*f^3 - 92
4*a^4*b*c^5*d^2*e - 1350*a^3*b^6*c*d*f^2 + 210*a^3*b^6*c*e^2*f + 1485*a^4*b
^5*c*e*f^2 - 364*a^6*b*c^3*e*f^2 - 30*a^2*b^5*c^3*d^2*e + 45*a^2*b^6*c^2*d*
e^2 + 339*a^3*b^3*c^4*d^2*e - 402*a^3*b^4*c^3*d*e^2 + 762*a^4*b^2*c^4*d*e^2
 + 50*a^2*b^6*c^2*d^2*f - 600*a^3*b^4*c^3*d^2*f + 2002*a^4*b^2*c^4*d^2*f +
4835*a^4*b^4*c^2*d*f^2 - 6598*a^5*b^2*c^3*d*f^2 - 1927*a^4*b^4*c^2*e^2*f +
4722*a^5*b^2*c^3*e^2*f - 3061*a^5*b^3*c^2*e*f^2 - 150*a^2*b^7*c*d*e*f + 231
2*a^5*b*c^4*d*e*f + 1480*a^3*b^5*c^2*d*e*f - 4122*a^4*b^3*c^3*d*e*f)/(4*(64
*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))))*(-(25*b^15*f^2 + b^1
1*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9
*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*
a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f
+ 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*
a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5
*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a
*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928
*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3
*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f
 + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*
c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*
a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*

d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*2i - atan(((((10240*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5 *b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^

5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^
(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c
 - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^
2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^
3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i - (((10240*a
^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224
*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^
6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^
5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b
^4*c^6 - 48*a^2*b^2*c^7)) + (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e
^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^
9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a
^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 -
1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*
a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^
4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 636
6*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 21974
4*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 153
60*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152
*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2
*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c
 - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*
c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*
d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f
- 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c
^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4
*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7
*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c
- b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*
f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 1
86*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 -
24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6
144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 7
68*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 +
 b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a
*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*
e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2
077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800
*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-
(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 11
6928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49
*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1

/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8
*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^
6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) -
246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b
^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*
c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*
d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a
*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10
*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*
c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2)
+ 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2
)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*
(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*
(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*
c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*f^2 - 7
2*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2
*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6
*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3
536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*
d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e
 - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3
*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*
f + 3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*
b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9
)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 -
30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c
^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e
^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*
c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^
3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^
6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c -
b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 358
40*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f
+ 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9
)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-
(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 2240
0*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^
3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 727
8*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 20160
0*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)
^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4
*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^
3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(

1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 128
0*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i)/((((102
40*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e +
4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^
5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^
8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12
*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c
^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*
b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 268
80*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^
2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10
656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^
2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) +
 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 2
19744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 -
 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f +
 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12
*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4
*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*
b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*
c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*
d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b
^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8
*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2
*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^
3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2)
 + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^
7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11
 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10
 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f
^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) -
27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^1
4*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2
 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 4
4800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^
2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2
+ 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2
+ 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7
*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 4352
0*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2
) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c

- b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*
b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*
c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-
(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*
b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*
b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1
/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c -
b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/
(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*
b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2
- 72*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8
*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2
*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2
- 3536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*
c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6
*d*e - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2
*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^
3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-
(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9
)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^
2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b
^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c
^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^
9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^
3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*
c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e +
35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*
d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^
2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^
2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e -
22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 1478
4*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*
c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) -
7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 2
01600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c -
b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*
(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*
a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^
9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 -
1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (((10
240*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e +
4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b

^5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b
^8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 1
2*a*b^4*c^6 - 48*a^2*b^2*c^7)) + (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*
c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5
*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26
880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d
^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 1
0656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a
^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 -
219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2
- 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f
+ 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^1
2*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(
4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2
*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2
*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6
*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*
b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^
8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^
2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c
^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c
^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^1
1 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^1
0 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*
f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) -
27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^
14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^
2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 -
44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d
^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2
+ 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2
+ 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^
9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^
7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 435
20*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/
2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4
*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7
*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(
-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2

*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5
*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(
1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c
- b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2
*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3
*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*f^
2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^
8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^
2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^
2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4
*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^
6*d*e - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^
2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c
^3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(
-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f
^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*
b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*
c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b
^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b
^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a
*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e
+ 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3
*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b
^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f
^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e -
22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 147
84*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2
*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2)
- 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f +
201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c -
b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e
*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184
*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)
^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9
- 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (216
*a^4*c^6*d^3 + 225*a^4*b^6*f^3 - 2744*a^7*c^3*f^3 - 1300*a^5*b*c^4*e^3 - 20
60*a^5*b^4*c*f^3 + 125*a^2*b^8*d*f^2 + 600*a^5*c^5*d*e^2 - 175*a^3*b^7*e*f^
2 - 1512*a^5*c^5*d^2*f + 3528*a^6*c^4*d*f^2 - 1400*a^6*c^4*e^2*f + 5*a^2*b^
4*c^4*d^3 - 66*a^3*b^2*c^5*d^3 - 63*a^3*b^5*c^2*e^3 + 573*a^4*b^3*c^3*e^3 +
5334*a^6*b^2*c^2*f^3 - 924*a^4*b*c^5*d^2*e - 1350*a^3*b^6*c*d*f^2 + 210*a^
3*b^6*c*e^2*f + 1485*a^4*b^5*c*e*f^2 - 364*a^6*b*c^3*e*f^2 - 30*a^2*b^5*c^3

*d^2*e + 45*a^2*b^6*c^2*d*e^2 + 339*a^3*b^3*c^4*d^2*e - 402*a^3*b^4*c^3*d*e
^2 + 762*a^4*b^2*c^4*d*e^2 + 50*a^2*b^6*c^2*d^2*f - 600*a^3*b^4*c^3*d^2*f +
 2002*a^4*b^2*c^4*d^2*f + 4835*a^4*b^4*c^2*d*f^2 - 6598*a^5*b^2*c^3*d*f^2 -
 1927*a^4*b^4*c^2*e^2*f + 4722*a^5*b^2*c^3*e^2*f - 3061*a^5*b^3*c^2*e*f^2 -
 150*a^2*b^7*c*d*e*f + 2312*a^5*b*c^4*d*e*f + 1480*a^3*b^5*c^2*d*e*f - 4122
*a^4*b^3*c^3*d*e*f)/(4*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^
7))))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c
- b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*
a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*
b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 38
40*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a
^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 3576
7*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 21504
0*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c
^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b
^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*
a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*
b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^
6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*
f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f -
 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)
^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5
*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(
4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*
c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2
) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b
^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)
*2i + (f*x^3)/(3*c^2)

3.69 $\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 436

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{fx}{c^2} + \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$
$$+ \frac{\left(b^2ce-6ac^2e-3b^3f+bc(cd+13af)-\frac{b^3ce-8abc^2e-3b^4f+4ac^2(cd-5af)+b^2c(cd+19af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(b^2ce-6ac^2e-3b^3f+bc(cd+13af)+\frac{b^3ce-8abc^2e-3b^4f+4ac^2(cd-5af)+b^2c(cd+19af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] f*x/c^2+1/2*x*(a*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)-(b^2*c*e-2*a*c^2*e-b^3*f-b*
c*(-3*a*f+c*d))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*
c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-6*a*c^2*e-3*b^3*f+b*c*(13*a*
f+c*d)+(-b^3*c*e+8*a*b*c^2*e+3*b^4*f-4*a*c^2*(-5*a*f+c*d)-b^2*c*(19*a*f+c*d
))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-6
*a*c^2*e-3*b^3*f+b*c*(13*a*f+c*d)+(b^3*c*e-8*a*b*c^2*e-3*b^4*f+4*a*c^2*(-5*
a*f+c*d)+b^2*c*(19*a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 3.49 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1682, 1690, 1180, 211}

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x(a(-2acf+b^2f-bce+2c^2d)-x^2(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce))}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{fx}{c^2}$$

```
[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (f*x)/c^2 + (x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e - 2*a*c^2*
e - b^3*f - b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) - (b^3*c*e - 8*a
*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt
[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - 6*a*c^
2*e - 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a
*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
```

```
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\text{integral} = \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{\frac{a^2\left(2c^2d+b^2f-c(be+2af)\right)}{c^2}-\frac{a\left(b^2ce-6ac^2e-b^3f+bc(cd+5af)\right)x^2}{c^2}+2a\left(4a-\frac{b^2}{c}\right)fx^4}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)}$$

$$= \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \left(-\frac{2a(b^2-4ac)f}{c^2}+\frac{a^2(2c^2d-bce+3b^2f-10acf)-a(b^2ce-6ac^2e-3b^3f+bc(cd+13af))x^2}{c^2(a+bx^2+cx^4)}\right)\,dx}{2a\,(b^2-4ac)}$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{a^2(2c^2d-bce+3b^2f-10acf)-a(b^2ce-6ac^2e-3b^3f+bc(cd+13af))x^2}{a+bx^2+cx^4}\,dx}{2ac^2\,(b^2-4ac)}$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\left(b^2ce-6ac^2e-3b^3f+bc(cd+13af)-\frac{b^3ce-8abc^2e-3b^4f+4ac^2(cd-5af)+b^2c(cd+19af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx}}{4c^2\,(b^2-4ac)} + \frac{\left(b^2ce-6ac^2e-3b^3f+bc(cd+13af)+\frac{b^3ce-8abc^2e-3b^4f+4ac^2(cd-5af)+b^2c(cd+19af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx}}{4c^2\,(b^2-4ac)}$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d - bce + b^2f - 2acf) - (b^2ce - 2ac^2e - b^3f - bc(cd - 3af))\,x^2)}{2c^2\,(b^2 - 4ac)\,(a + bx^2 + cx^4)}$$
$$+ \frac{\left(b^2ce - 6ac^2e - 3b^3f + bc(cd + 13af) - \frac{b^3ce - 8abc^2e - 3b^4f + 4ac^2(cd - 5af) + b^2c(cd + 19af)}{\sqrt{b^2 - 4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b - \sqrt{b^2}}}\right.}{2\sqrt{2}c^{5/2}\,(b^2 - 4ac)\,\sqrt{b - \sqrt{b^2 - 4ac}}}$$
$$+ \frac{\left(b^2ce - 6ac^2e - 3b^3f + bc(cd + 13af) + \frac{b^3ce - 8abc^2e - 3b^4f + 4ac^2(cd - 5af) + b^2c(cd + 19af)}{\sqrt{b^2 - 4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b + \sqrt{b^2}}}\right.}{2\sqrt{2}c^{5/2}\,(b^2 - 4ac)\,\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.17

$$\int \frac{x^4(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2}\,dx$$

$$= \frac{4\sqrt{c}fx + \frac{2\sqrt{c}x(-2a^2cf + b(c^2d - bce + b^2f)x^2 + a(b^2f + 2c^2(d + ex^2) - bc(e + 3fx^2)))}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\sqrt{2}\left(-3b^4f + 2ac^2\left(2cd + 3\sqrt{b^2 - 4ac}e - 10af\right) + b^2c\left(cd\right.\right.}{}}{}$$

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*f*x + (2*Sqrt[c]*x*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 +
 a*(b^2*f + 2*c^2*(d + e*x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*f + 2*a*c^2*(2*c*d + 3*Sqrt[b^2 - 4*a*c]*e
 - 10*a*f) + b^2*c*(c*d - Sqrt[b^2 - 4*a*c]*e + 19*a*f) + b^3*(c*e + 3*Sqrt
[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*d + 8*a*c*e + 13*a*Sqrt[b^2 - 4
*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*f + 2*a*c^2*(-2
*c*d + 3*Sqrt[b^2 - 4*a*c]*e + 10*a*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e +
 19*a*f) + b^3*(-(c*e) + 3*Sqrt[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*
d - 8*a*c*e + 13*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
4*c^(5/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.56

method	result
risch	$\frac{fx}{c^2}+\frac{\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^3}{8ac-2b^2}+\frac{a\left(2acf-b^2f+ebc-2c^2d\right)x}{8ac-2b^2}}{c^2(c\,x^4+b\,x^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\left(-\frac{\left(13abcf-6a\,c^2e-3b^3f+b^2ce+b\right.}{4ac-b^2}\right.}{4c^2}$
default	$\frac{fx}{c^2}-\frac{-\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^3}{2\left(4ac-b^2\right)}-\frac{a\left(2acf-b^2f+ebc-2c^2d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(13\sqrt{-4ac+b^2}\,abcf-6\sqrt{-4ac+b^2}\,a\,c^2e-3b^3f\sqrt{-4ac+b^2}+b^2ce\right.}{}\right.}{}$

```
[In] int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/c^2+(1/2*(3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)*x^3+1/
2*a*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)*x)/c^2/(c*x^4+b*x^2+a)+1/4/c^
2*sum((-(13*a*b*c*f-6*a*c^2*e-3*b^3*f+b^2*c*e+b*c^2*d)/(4*a*c-b^2)*_R^2-a*(
10*a*c*f-3*b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=Ro
otOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12597 vs. 2(394) = 788.

Time = 23.67 (sec) , antiderivative size = 12597, normalized size of antiderivative = 28.89

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^4}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*x^3 + (2*a*c^2*d -
 a*b*c*e + (a*b^2 - 2*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*
c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + f*x/c^2 + 1/2*integrate(-(2*a*c^2*d
 - a*b*c*e - (b*c^2*d + (b^2*c - 6*a*c^2)*e - (3*b^3 - 13*a*b*c)*f)*x^2 + (
3*a*b^2 - 10*a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7479 vs. $2(394) = 788$.

Time = 1.78 (sec) , antiderivative size = 7479, normalized size of antiderivative = 17.15

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] f*x/c^2 + 1/2*(b*c^2*d*x^3 - b^2*c*e*x^3 + 2*a*c^2*e*x^3 + b^3*f*x^3 - 3*a*
b*c*f*x^3 + 2*a*c^2*d*x - a*b*c*e*x + a*b^2*f*x - 2*a^2*c*f*x)/((c*x^4 + b*
x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4*a*
c^3)^2*d + (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 -
4*a*c^3)^2*e - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^
3)*(b^2*c^2 - 4*a*c^3)^2*f + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 2*a*b^4*c^6 + 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 + 16*a^2*b
^2*c^7 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^8 - 32*a^3*c^8 + 2
*(b^2 - 4*a*c)*a*b^2*c^6 - 8*(b^2 - 4*a*c)*a^2*c^7)*d*abs(b^2*c^2 - 4*a*c^3
) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^4*c^5 - 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^6 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 16*a^2*b^3*c^6 - 4*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 32*a^3*b*c^7 + 2*(b^2 - 4*a*c)*a*
b^3*c^5 - 8*(b^2 - 4*a*c)*a^2*b*c^6)*e*abs(b^2*c^2 - 4*a*c^3) + 2*(3*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
*c^4 - 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^
5 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 25
6*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 320*a^
4*c^7 + 6*(b^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2*b^2*c^5 + 80*(b^2
- 4*a*c)*a^3*c^6)*f*abs(b^2*c^2 - 4*a*c^3) - (2*b^7*c^8 - 8*a*b^5*c^9 - 32*
a^2*b^3*c^10 + 128*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^7*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^6*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^5*c^8 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b*c^9 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*d - (2*b^8*c^
7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a^3*b^2*c^10 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
```

qrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*
a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*e + (6*b^9*c^6 - 86*a*b^7*c^7 + 4
40*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*
a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9)*f)
*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3
)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))
/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c
^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*ab
s(b^2*c^2 - 4*a*c^3)*abs(c)) - 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4*a*c
^3)^2*d + (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4
*a*c^3)^2*e - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3
)*(b^2*c^2 - 4*a*c^3)^2*f + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*sqrt(2)*s

qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*a*b^4*c^6 + 16*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 16*a^2*b^
2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^8 + 32*a^3*c^8 - 2*
(b^2 - 4*a*c)*a*b^2*c^6 + 8*(b^2 - 4*a*c)*a^2*c^7)*d*abs(b^2*c^2 - 4*a*c^3)
 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^5 + 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b*c^6 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^2*b^3*c^6 - 4*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*c^7 - 2*(b^2 - 4*a*c)*a*b
^3*c^5 + 8*(b^2 - 4*a*c)*a^2*b*c^6)*e*abs(b^2*c^2 - 4*a*c^3) + 2*(3*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*
c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5
 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 256
*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 320*a^4
c^7 - 6(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 - 80*(b^2 -
 4*a*c)*a^3*c^6)*f*abs(b^2*c^2 - 4*a*c^3) - (2*b^7*c^8 - 8*a*b^5*c^9 - 32*a
^2*b^3*c^10 + 128*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^7*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^6*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^5*c^8 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^9 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c
^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*d - (2*b^8*c^7
 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a
*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*e + (6*b^9*c^6 - 86*a*b^7*c^7 + 44
0*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*

```
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a
*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9)*f)*
arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 - sqrt((b^3*c^2 - 4*a*b*c^3)
^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/
((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^
7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs
(b^2*c^2 - 4*a*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 25862, normalized size of antiderivative = 59.32

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (f*x)/c^2 - atan(((((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d +
 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c
^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7
*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^
2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*
d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 38
40*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a
^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^
(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*
c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^
4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2
)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*
b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*
```

a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8
*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6
*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b
^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c
^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b
^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4
*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) -
9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c -
b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 -
512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4
*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5
*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
- b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*
e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e
- 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 19
2*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^
7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e
*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f +
6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1
280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8
*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6
*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^
4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^
3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*
d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c
^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((
768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c
^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2
+ 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*
c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2
*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c
^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*
f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^
6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5
*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
+ 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b
^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*
d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 2
2400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2

)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7

*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 1
14*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*
c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b
*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c -
b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*
f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9
)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3
*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^
6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^
2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b
^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*
b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) -
2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b
^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d
*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 80
64*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*
c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3
*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((63*a^3*b^5*f
^3 - 216*a^4*c^4*e^3 + 3*a*b^3*c^4*d^3 + 4*a^2*b*c^5*d^3 - 573*a^4*b^3*c*f^
3 + 1300*a^5*b*c^2*f^3 - 24*a^3*c^5*d^2*e - 45*a^2*b^6*e*f^2 - 600*a^5*c^3*
e*f^2 - 5*a^2*b^4*c^2*e^3 + 66*a^3*b^2*c^3*e^3 + 27*a*b^7*d*f^2 + 240*a^4*c
^4*d*e*f + 6*a*b^4*c^3*d^2*e + 3*a*b^5*c^2*d*e^2 + 204*a^3*b*c^4*d*e^2 - 18
*a*b^5*c^2*d^2*f - 279*a^2*b^5*c*d*f^2 + 12*a^3*b*c^4*d^2*f - 420*a^4*b*c^3
*d*f^2 + 30*a^2*b^5*c*e^2*f + 402*a^3*b^4*c*e*f^2 + 924*a^4*b*c^3*e^2*f - 4
2*a^2*b^2*c^4*d^2*e - 51*a^2*b^3*c^3*d*e^2 + 81*a^2*b^3*c^3*d^2*f + 801*a^3
*b^3*c^2*d*f^2 - 339*a^3*b^3*c^2*e^2*f - 762*a^4*b^2*c^2*e*f^2 - 18*a*b^6*c
*d*e*f + 246*a^2*b^4*c^2*d*e*f - 804*a^3*b^2*c^3*d*e*f)/(4*(64*a^3*c^6 - b^
6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (((10240*a^5*c^7*f - 2048*a^4*c^8
*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b
^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 3
2*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*
a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^8*d^
2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f
^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^
7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12
*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1
504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a
^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2
f^2(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*
a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^
11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*

a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2

*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4
*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f -
8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^
2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c
^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^
5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 1075
2*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a
*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x
*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^1
1*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*
e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6
*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*
a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^
9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c
^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b
^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360
*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*
a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/
2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^
3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c
^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f
+ 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c -
b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11
+ b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4
*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^
3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768
*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*
e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 +
3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6
*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^
7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*
f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2
- 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)
^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c
^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*
c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^
2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 5
1*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*
c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f
+ 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 2240
0*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9
)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1

2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c ^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^ 5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16* a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 71 8*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^ 6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^ 5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^ 2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3* e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2* b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5* e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3 072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36* a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a *c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2* e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2) ^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d* e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548 *a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5 *b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4 *a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a ^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/ 2)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9* c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880 *a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^ 2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b ^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 1 5360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1 536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9) ^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 12 8*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b ^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4* e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a* c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6* c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840 *a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i - ((x^3*(b^3*f + 2*a*c^2*e + b *c^2*d - b^2*c*e - 3*a*b*c*f))/(2*(4*a*c - b^2)) + (x*(2*a*c^2*d + a*b^2*f - 2*a^2*c*f - a*b*c*e))/(2*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) -

atan(((((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^
2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*
a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 102
4*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 4
8*a^2*b^2*c^5)) - (x*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b
^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8
*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^
9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^
3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c
^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f
^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*
c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*
b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a
*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) +
 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-
(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*
b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*
d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8
064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2
*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^
3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192
*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^
2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 +
 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/
2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*
c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2
 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 4
4800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c
^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c
^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c
^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c
 - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^
5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f -
1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^
3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d
f(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*
(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*
c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8*f^2 + 8*a^2
*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2
*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*
a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f +
2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b

^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374
*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4
*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*
(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*
b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*
b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2
 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 106
56*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2
*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) +
213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f +
6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f -
 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/
2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c
f^2(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e +
 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^
4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4
*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) +
 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 2
4*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144
*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b
^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736
*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d
- 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*
c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/
2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)
^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^
3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^
2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^
5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2
 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 -
 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 3
6*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4
*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^
2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^
2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*
d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 15
48*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a
^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-
(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240
*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(
1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(1
6*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b
^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2)
 + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*

(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*
c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2
- 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 302
40*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)
^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*
a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^
8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c -
b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f
- 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^
(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e +
576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2
*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2
*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b
^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) +
(x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4
*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*
a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f
^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*
a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4
*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
c^4)))((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^1
1*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b
^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26
880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2
- 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077
*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^
5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e
+ 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f
+ 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)
^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e +
128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^
3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c
^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4
*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a
^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3
840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((63*a^3*b^5*f^3 - 216*a^4
*c^4*e^3 + 3*a*b^3*c^4*d^3 + 4*a^2*b*c^5*d^3 - 573*a^4*b^3*c*f^3 + 1300*a^5
*b*c^2*f^3 - 24*a^3*c^5*d^2*e - 45*a^2*b^6*e*f^2 - 600*a^5*c^3*e*f^2 - 5*a^
2*b^4*c^2*e^3 + 66*a^3*b^2*c^3*e^3 + 27*a*b^7*d*f^2 + 240*a^4*c^4*d*e*f + 6
*a*b^4*c^3*d^2*e + 3*a*b^5*c^2*d*e^2 + 204*a^3*b*c^4*d*e^2 - 18*a*b^5*c^2*d
^2*f - 279*a^2*b^5*c*d*f^2 + 12*a^3*b*c^4*d^2*f - 420*a^4*b*c^3*d*f^2 + 30*
a^2*b^5*c*e^2*f + 402*a^3*b^4*c*e*f^2 + 924*a^4*b*c^3*e^2*f - 42*a^2*b^2*c^

4*d^2*e - 51*a^2*b^3*c^3*d*e^2 + 81*a^2*b^3*c^3*d^2*f + 801*a^3*b^3*c^2*d*f
^2 - 339*a^3*b^3*c^2*e^2*f - 762*a^4*b^2*c^2*e*f^2 - 18*a*b^6*c*d*e*f + 246
*a^2*b^4*c^2*d*e*f - 804*a^3*b^2*c^3*d*e*f)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)) + (((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2
*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 7
36*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*
d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^
6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((c^4*d^2*(-(4*a*c - b^2)^9)^(
1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*
c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*
a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*
c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f
^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*
c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2
 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f +
 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-
(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*
c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^
7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f +
1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720
*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*
(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) -
 b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/
2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^
2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^
5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^
2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 3
0240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^
2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 307
2*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*
b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c
 - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*
f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9
)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e
+ 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a
^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b
^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a
c - b^2)^9)^(1/2))/(32(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2
*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)
 - (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c

^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 7
4*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c
*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e -
8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f +
 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b
^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a
*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) -
26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d
^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 20
77*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*
a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*
e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*
f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*
e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^
2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e
 + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*
a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6
*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-
(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 +
 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7*f - 2048*
a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 76
8*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^
6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/
(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^4*d^2*(
-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f
^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a
^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 +
 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*
e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 +
10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*
a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f
 + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*
f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^
(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^
2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*
e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512
*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*
b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5
 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6
144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 76

8*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f

```
^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*
c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^
2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 +
 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(
1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7
*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^
7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)
^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*
a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^
6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f +
 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*
a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^
(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*
c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9
 - 6144*a^5*b^2*c^10)))^(1/2)*2i
```

3.70 $\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 362

$$\begin{aligned}\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx &= -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\ &\quad-\frac{\left(2cd-be+6af-\frac{b^2f}{c}+\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad-\frac{\left(2cd-be+6af-\frac{b^2f}{c}-\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

```
[Out] -1/2*x*(b*c*d-2*a*c*e+a*b*f+(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b
^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*(2*c*d-b*e+6*a*f-b^2*f/c+(b^2*c*e+4*a*c^2*e+b^3*f-4*b*c*(2*a*f+c*d))/c/
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-b*e+6*
a*f-b^2*f/c+(-b^2*c*e-4*a*c^2*e-b^3*f+4*b*c*(2*a*f+c*d))/c/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1682, 1180, 211}

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-be+2cd\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-be+2cd\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{x(x^2(-2acf+b^2f-bce+2c^2d)+abf-2ace+bcd)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)
)/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c
 + (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4*a*c]
))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c*d - b*e + 6*a*f - (b
^2*f)/c - (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 -
 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
```

```
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\text{integral} = -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{-\frac{a(bcd-2ace+abf)}{c}+a\left(2cd-be+6af-\frac{b^2f}{c}\right)x^2}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)}$$

$$= -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(2cd-be+6af-\frac{b^2f}{c}-\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\,(b^2-4ac)} - \frac{\left(2cd-be+6af-\frac{b^2f}{c}+\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\,(b^2-4ac)}$$

$$= -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(2cd-be+6af-\frac{b^2f}{c}+\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2cd-be+6af-\frac{b^2f}{c}-\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.14

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{2\sqrt{c}x(abf+2c^2dx^2+b^2fx^2+bc(d-ex^2)-2ac(e+fx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3f+bc\left(4cd+\sqrt{b^2-4ac}e+8af\right)+b^2\left(-ce+\sqrt{b^2-4ac}f\right)-2c\left(c\sqrt{b^2-4ac}d+2a\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(
e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*f) + b*c
*(4*c*d + Sqrt[b^2 - 4*a*c]*e + 8*a*f) + b^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f)
 - 2*c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*f + b*c*(-4*c*d + Sqrt[b^2 - 4*a*c
]*e - 8*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) - 2*c*(c*Sqrt[b^2 - 4*a*c]*d
 - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.55

method	result
risch	$\frac{-\frac{\left(2acf-b^2f+ebc-2c^2d\right)x^3}{2c\left(4ac-b^2\right)}+\frac{(abf-2ace+bcd)x}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{\left(6acf-b^2f-ebc+2c^2d\right)\_R^2}{4ac-b^2}-\frac{abf-2ace+bcd}{4ac-b^2}\right)\ln\left(x-\_\right.}{2c\_R^3+\_Rb}}{4c}$
default	$\frac{-\frac{\left(2acf-b^2f+ebc-2c^2d\right)x^3}{2c\left(4ac-b^2\right)}+\frac{(abf-2ace+bcd)x}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{\left(6acf\sqrt{-4ac+b^2}-b^2f\sqrt{-4ac+b^2}-ebc\sqrt{-4ac+b^2}+2c^2d\sqrt{-4ac+b^2}+8abcf-4ac^2e-b^3f-b^2\right.}{4c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$

```
[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c/(4*a*c-b^2)*x^3+1/2/c*(a*b*f-2*a*c*e+
b*c*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/c*sum(((6*a*c*f-b^2*f-b*c*e+2*c^2
*d)/(4*a*c-b^2)*_R^2-(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(
x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8951 vs. $2(320) = 640$.

Time = 13.28 (sec) , antiderivative size = 8951, normalized size of antiderivative = 24.73

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^2}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + (b*c*d - 2*a*c*e + a*b*f)*x
)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
 - 1/2*integrate(-(b*c*d - 2*a*c*e + a*b*f - (2*c^2*d - b*c*e - (b^2 - 6*a*
c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6200 vs. $2(320) = 640$.

Time = 1.52 (sec) , antiderivative size = 6200, normalized size of antiderivative = 17.13

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*c^2*d*x^3 - b*c*e*x^3 + b^2*f*x^3 - 2*a*c*f*x^3 + b*c*d*x - 2*a*c*e
*x + a*b*f*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) - 1/16*(2*(2*b^2*c^4
- 8*a*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^4 - 2*(b^2 - 4*a*c)*c^4)
*(b^2*c - 4*a*c^2)^2*d - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*e - (2*b^4
```

*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 -
4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*f - 2*(sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 2
*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c^5 + 16*a*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^6 - 32*a^2*b*c^6 + 2*(b^2 - 4*a*c)*b^3*c^4 - 8*(b^2 - 4*a*c)*a*b*c^
5)*d*abs(b^2*c - 4*a*c^2) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^
2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*
(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*e*abs(b^2*c - 4*a*c^2) -
 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3
c^3 - 8(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(b^2*c - 4*a*c^2) - 4*(2*b^6*c^6 - 1
6*a*b^4*c^7 + 32*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7
 - 2*(b^2 - 4*a*c)*b^4*c^6 + 8*(b^2 - 4*a*c)*a*b^2*c^7)*d + (2*b^7*c^5 - 8*
a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*e

+ (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2
 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*f)*arctan(2*sqrt(1/2)*x
/sqrt((b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*
c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c
^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6
 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c))
 + 1/16*(2*(2*b^2*c^4 - 8*a*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^4 -
 2*(b^2 - 4*a*c)*c^4)*(b^2*c - 4*a*c^2)^2*d - (2*b^3*c^3 - 8*a*b*c^4 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c -
4*a*c^2)^2*e - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*
c^2)^2*f + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^4*c^4 + 2*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*a*b^3*c^5 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + 32*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8
*(b^2 - 4*a*c)*a*b*c^5)*d*abs(b^2*c - 4*a*c^2) - 4*(sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4
 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*

```
c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*e*a
bs(b^2*c - 4*a*c^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*
c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 -
2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(b^2*c - 4*a*c^
2) - 4*(2*b^6*c^6 - 16*a*b^4*c^7 + 32*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 8*(b^2 - 4*a*c)*a*b^2*c^7
)*d + (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 -
 4*a*c)*a^2*b*c^7)*e + (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^
3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c
^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a
*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*f)
*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2
- 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b
^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a
*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*
c - 4*a*c^2)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 19494, normalized size of antiderivative = 53.85

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

[In] int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)

[Out] ((x^3*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c*(4*a*c - b^2)) + (x*(a*b*f - 2*a*c*e + b*c*d))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f

+ 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*
d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c
^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c
f^2(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 +
96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b
^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 1
2*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a
*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^
7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f
 - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b
^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^1
2*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4
*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*1i - (((2048*a^4*c^6*e + 16*b^7*c^3*d + 76
8*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*
f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e
 + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3)) + (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a
*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2
f^2(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2
*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96
*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c
^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2
- 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f
+ 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c
^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f -
3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b
^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*
b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(
16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-
(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a
*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27
*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b
^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*
f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*
d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b
^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*
f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a
^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a
c - b^2)^9)^(1/2))/(32(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*
a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/
2) - (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^

3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*1i)/((((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a

^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*

b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e
*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*
e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*
c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*
c^8)))^(1/2) - (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 +
72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*
c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f -
52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2
*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c
^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^
6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(
1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 -
512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5
*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a
*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b
^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*
f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e
f(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c
^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c
^8)))^(1/2) + (8*a*c^5*d^3 + b^6*d*f^2 + 5*a^2*b^4*f^3 + 6*b^2*c^4*d^3 + 21
6*a^4*c^2*f^3 - 3*a*b^3*c^2*e^3 - 4*a^2*b*c^3*e^3 - 66*a^3*b^2*c*f^3 + 8*a^
2*c^4*d*e^2 + 72*a^2*c^4*d^2*f + 216*a^3*c^3*d*f^2 - 5*b^3*c^3*d^2*e + b^4*
c^2*d*e^2 + 24*a^3*c^3*e^2*f - 5*b^4*c^2*d^2*f - 3*a*b^5*e*f^2 - 28*a*b*c^4
*d^2*e - 12*a*b^4*c*d*f^2 - 6*a*b^4*c*e^2*f + 18*a*b^2*c^3*d*e^2 + 26*a*b^2
*c^3*d^2*f + 51*a^2*b^3*c*e*f^2 - 204*a^3*b*c^2*e*f^2 + 2*b^5*c*d*e*f + 2*a
^2*b^2*c^2*d*f^2 + 42*a^2*b^2*c^2*e^2*f + 6*a*b^3*c^2*d*e*f - 152*a^2*b*c^3
*d*e*f)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*((768*a^
4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 -
a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a
*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 -
9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^
6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 15
04*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6
*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*
f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 12
8*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8
*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f
- 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9
+ a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840
*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*2i - atan(((((2048*a^4*c^6*e + 16*
b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 19
2*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d -
32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4

- 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) -
b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c
^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)
^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 -
512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^
5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*
a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*
b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e
*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*
e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*
c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*
c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5
))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/
2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5
*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c -
b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e
^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 384
0*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e
+ 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/
2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*
a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c
^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*
b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b
^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*
b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d
^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*
b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d
*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e)
)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2
) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*
b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^
9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b
^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^
2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840
*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e +
6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a
^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^
3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b
*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^
10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b
^2*c^8)))^(1/2)*1i - (((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d +

384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c
^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f
 - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
)) + (x*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768
*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b
^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840
*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2
- 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*
b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*
e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*
d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*
b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*
d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536
*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(
32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^
4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*
a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a
*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 +
 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c
 - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 +
3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*
d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*
a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^
7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*
c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*
a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*
c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f +
1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 128
0*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) - (x*(8*a*c^5*
d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e
^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 +
48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^
3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*
b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 +
768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3
840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d
^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a
^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7
*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c
^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a
^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c
^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1
536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)

)/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280
*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*1i)/((((2048*a^
4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b
^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a
^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c
- 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((c^3*d^2*(-(4*a*c - b^
2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2
+ 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a
*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-
(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*
b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*
f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8
*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d
*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*
a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e
*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 -
24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 -
6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 76
8*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c
- b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*
e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^
2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*
a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*
c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a
*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c
- b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c
^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f -
192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10
*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c
^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^
7 - 6144*a^6*b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 -
10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*
e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e
+ 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8
*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c -
b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e
^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2
*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a
^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c
^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*
b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c -
b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^

3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 1
92*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*
c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^
3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7
 - 6144*a^6*b^2*c^8)))^(1/2) + (((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b
^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768
*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a
*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3)) + (x*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11
*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-
(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*
f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^
5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2
- 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*
a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*
a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e
+ 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^
4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*
e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5
 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*
c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2
*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*
b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^
2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^
9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^
2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*
e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1
024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3
584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*
d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 307
2*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*
c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8
*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) - (x
*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 -
 b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^
4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f
 + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*
c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b
^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2
*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9
*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2
*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e
^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 10

24*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 35
84*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d
*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072
*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c
^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*
c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (8*
a*c^5*d^3 + b^6*d*f^2 + 5*a^2*b^4*f^3 + 6*b^2*c^4*d^3 + 216*a^4*c^2*f^3 - 3
*a*b^3*c^2*e^3 - 4*a^2*b*c^3*e^3 - 66*a^3*b^2*c*f^3 + 8*a^2*c^4*d*e^2 + 72*
a^2*c^4*d^2*f + 216*a^3*c^3*d*f^2 - 5*b^3*c^3*d^2*e + b^4*c^2*d*e^2 + 24*a^
3*c^3*e^2*f - 5*b^4*c^2*d^2*f - 3*a*b^5*e*f^2 - 28*a*b*c^4*d^2*e - 12*a*b^4
*c*d*f^2 - 6*a*b^4*c*e^2*f + 18*a*b^2*c^3*d*e^2 + 26*a*b^2*c^3*d^2*f + 51*a
^2*b^3*c*e*f^2 - 204*a^3*b*c^2*e*f^2 + 2*b^5*c*d*e*f + 2*a^2*b^2*c^2*d*f^2
+ 42*a^2*b^2*c^2*e^2*f + 6*a*b^3*c^2*d*e*f - 152*a^2*b*c^3*d*e*f)/(4*(b^6*c
 - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*((c^3*d^2*(-(4*a*c - b^2)
^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 +
768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4
*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^
5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^
2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c
^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)
^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f
 + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^
3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f
 - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 2
4*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 61
44*a^6*b^2*c^8)))^(1/2)*2i

3.71 $\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 346

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} dx = \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(bcd-2ace+abf+\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bcd-2ace+abf-\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*(b*c*d-2*a*c*e+a*b*f+(4*a*b*c*e+b^2*(-a*f+c*d)-4*a*c*(a*f+3*c*d))/(-4*a*c+
b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-2*a*c*e+a*b*
f+(-4*a*b*c*e-b^2*(-a*f+c*d)+4*a*c*(a*f+3*c*d))/(-4*a*c+b^2)^(1/2))/a/(-4*a
*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1692, 1180, 211}

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*e + a*b*f + (4*a*b*c*e
 + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*
c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e +
b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
```

```
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d-abe+2a(3cd+af)+(-bcd+2ace-abf)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}\\
&= \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd-2ace+abf-\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)} + \frac{\left(bcd-2ace+abf+\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)}\\
&= \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd-2ace+abf+\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bcd-2ace+abf-\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.10

$$\int \frac{d+ex^2+fx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{\frac{2x\left(b^2d+b\left(-ae+cdx^2+afx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(b^2(cd-af)-2ac\left(6cd+\sqrt{b^2-4ac}e+2af\right)+b\left(c\sqrt{b^2-4ac}d+4ace+a\sqrt{b^2-4ac}f\right)\right)\text{arcta}}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4a}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2))))
/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*f) - 2*a*c*(6
```

```
*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e +
a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*
(b^2*(-(c*d) + a*f) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sq
rt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.58

method	result
risch	$\frac{-\frac{(abf-2ace+bcd)x^3}{2a(4ac-b^2)}-\frac{(2fa^2-abe-2acd+b^2d)x}{2a(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\text{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{(abf-2ace+bcd)\_R^2}{4ac-b^2}+\frac{2fa^2-abe+6acd-b^2d}{4ac-b^2}\right)\ln(x\cdots}{2c\_R^3+\_Rb}}{4a}$
default	$\frac{-\frac{(abf-2ace+bcd)x^3}{2a(4ac-b^2)}-\frac{(2fa^2-abe-2acd+b^2d)x}{2a(4ac-b^2)}}{cx^4+bx^2+a}+\frac{2c\left(\frac{\left(-\sqrt{-4ac+b^2}\,abf+2ace\sqrt{-4ac+b^2}-bcd\sqrt{-4ac+b^2}-4a^2cf-ab^2f+4abce-12ac^2d+b^2\cdots\right.}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\cdots\right.}{\cdots}$

```
[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a^2*f-a*b*e-2*a*c*d+b^
2*d)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/a*sum((-(a*b*f-2*a*c*e+b*c*d)/(4*
a*c-b^2)*_R^2+(2*a^2*f-a*b*e+6*a*c*d-b^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln
(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8991 vs. 2(304) = 608.

Time = 11.61 (sec) , antiderivative size = 8991, normalized size of antiderivative = 25.99

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*e + a*b*f)*x^3 - (a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*x)
/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
+ 1/2*integrate((a*b*e - 2*a^2*f + (b*c*d - 2*a*c*e + a*b*f)*x^2 + (b^2 - 6
*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6348 vs. $2(304) = 608$.

Time = 1.29 (sec) , antiderivative size = 6348, normalized size of antiderivative = 18.35

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*e*x^3 + a*b*f*x^3 + b^2*d*x - 2*a*c*d*x - a*b*e*x +
2*a^2*f*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) - 1/16*((2*b^3*c^3 - 8*a
*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c
^3)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*e
```

+ (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*f - 2*(sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 2*a*b^
6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^3 + 28*a^2*b^4*c^3 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 128*a^3*b^2*c^4 + 2
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 192*a^4*c^5 + 2*(b^2 -
4*a*c)*a*b^4*c^2 - 20*(b^2 - 4*a*c)*a^2*b^2*c^3 + 48*(b^2 - 4*a*c)*a^3*c^4)
*d*abs(a*b^2 - 4*a^2*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*a^2*b^5*c^2 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 1
6*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 32*a^
4*b*c^4 + 2*(b^2 - 4*a*c)*a^2*b^3*c^2 - 8*(b^2 - 4*a*c)*a^3*b*c^3)*e*abs(a*
b^2 - 4*a^2*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*a^3*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 16*a^4*b^2*c^3
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 32*a^5*c^4 + 2*(b^2
- 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)*f*abs(a*b^2 - 4*a^2*c) + (2
*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c + 20*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d + 4*
(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr

t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2
 - 4*a*c)*a^4*b^2*c^4)*e - (2*a^3*b^7*c^2 - 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4
+ 128*a^6*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6
*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2
- 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 -
32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 +
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 2*
(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a*c)*a^5*b*c^4)*f)*arctan(2*sqrt(1/
2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*
a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6*c - 12*a^4
*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3 + 16*a^4*b^3*c^3 + a^3*b^4*c^3 -
64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4 + 16*a^5*c^5)*abs(a*b^2 - 4*a^2*c
)*abs(c)) + 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^
3 - 8*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4
*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*e + (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4
*a^2*c)^2*f + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^2 + 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 28*a^2*b^4*c^3 -
96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^2*c^4 + 128*a^3*b^2*c^4 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*c^5 - 192*a^4*c^5 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 20*(b^2 - 4*a*c)*a^2*b
^2*c^3 - 48*(b^2 - 4*a*c)*a^3*c^4)*d*abs(a*b^2 - 4*a^2*c) + 2*(sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2
+ 2*a^2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + sqrt(2)*sqrt(b*c - sq

rt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 16*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^4 + 32*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^3*c^2 + 8
*(b^2 - 4*a*c)*a^3*b*c^3)*e*abs(a*b^2 - 4*a^2*c) - 4*(sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 2*a^3*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^3 - 16*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*c^4 + 32*a^5*c^4 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - 4*a*c)*a^
4*c^3)*f*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b
^3*c^5 - 384*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 -
96*(b^2 - 4*a*c)*a^4*b*c^5)*d + 4*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*
b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6
*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^
2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3
 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*
(b^2 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*e - (2*a^3*b^7*c^2
 - 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a
*c)*a^5*b*c^4)*f)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^
3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c -
 4*a^2*c^2)))/((a^3*b^6*c - 12*a^4*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3
 + 16*a^4*b^3*c^3 + a^3*b^4*c^3 - 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4
 + 16*a^5*c^5)*abs(a*b^2 - 4*a^2*c)*abs(c))

Mupad [B] (verification not implemented)

Time = 12.16 (sec) , antiderivative size = 19589, normalized size of antiderivative = 56.62

$$\int \frac{d + ex^2 + fx^4}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4
*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^
4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 1
6*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2)) - (x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c
- b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c -
b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^
2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*
a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*
c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 -
3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f
- 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*
d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 12
8*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^
6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b
^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^
8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f
^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2
*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(
-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^
4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 +
 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^
3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^
6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*
a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*
c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f
- 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2
+ 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)
))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2
 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^
2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f +
 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2)
```

- b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) -
a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b
^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2
+ 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a
^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*
e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(
-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b
^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d
*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384
*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(
32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^
6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i - (((6144*a^5*c
^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b
^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^
3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 102
4*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) +
(x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b
^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2
*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c
d^2(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1
504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b
^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e -
1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*
a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c
^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f +
960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5
*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(
4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^
6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a
^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^
9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(
1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7
*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2
- 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^
6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2
*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 1
92*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b
^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e
*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 -
1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a
^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2
+ a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f

+ 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f
- 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*
c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 384
0*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 76
8*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4
*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96
*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*
f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(
1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^
4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^
3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f -
2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 +
a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^
7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i)/((8*a^3*c^4*e^3 + 5*b^3*c^4*d^3 -
3*a^3*b^3*c*f^3 - 4*a^4*b*c^2*f^3 + 72*a^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 8
*a^4*c^3*e*f^2 + b^5*c^2*d^2*f + 6*a^2*b^2*c^3*e^3 - 36*a*b*c^5*d^3 + a*b^5
*c*d*f^2 + 48*a^3*c^4*d*e*f + 18*a*b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a
^2*b*c^4*d*e^2 - a*b^3*c^3*d^2*f - 60*a^2*b*c^4*d^2*f - 28*a^3*b*c^3*d*f^2
+ a^2*b^4*c*e*f^2 - 28*a^3*b*c^3*e^2*f - 9*a^2*b^3*c^2*d*f^2 - 5*a^2*b^3*c^
2*e^2*f + 18*a^3*b^2*c^2*e*f^2 - 4*a*b^4*c^2*d*e*f + 52*a^2*b^2*c^3*d*e*f)/
(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c^
6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^
2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3
*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024
*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) -
(x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^
11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*
b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 15
04*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^
3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1
024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a
*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^
3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f +
960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*
b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4
096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6
*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^
2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9
)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^
(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1
/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*
c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2

- 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2

+ 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*2i - ((x*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/(2*a*(4*a*c - b^2)) + (x^3*(a*b*f - 2*a*c*e + b*c*d))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d -

1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)
) - (x*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a
*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) -
a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^
2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2
+ 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^
5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e
 - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-
(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^
6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*
f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*
a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6
*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 1
6*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^
2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*
c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)
^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*
b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*
e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072
*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*
a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e
- 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^
3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^
2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^
9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^
3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(7
2*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*
d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d
*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d
*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^
2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 +
3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 +
 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*
c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*
c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 +
 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5
*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9
)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128
*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5
*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f
 - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7
 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840
*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i - (((6144*a^5*c^6*d + 2048*a^6*

c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^
2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 38
4*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(
8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^3*f^2*(-
(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840
*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768
*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*
d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*
a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f
 + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1
/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4
*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3
*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2
*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a
^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7
*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192
*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*
((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^
2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9
*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2
*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*
a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c
^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024
*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d
*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960
*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4
*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^
4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a
^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2
 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*
e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*
e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9
)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2
+ 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2
- a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) +
 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*
b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2
 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*
d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^
8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e +
1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5
*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e -

2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i)/((8*a^3*c^4*e^3 + 5*b^3*c^4*d^3 - 3*a^3*b^3*c*f^3 - 4*a^4*b*c^2*f^3 + 72*a^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 8*a^4*c^3*e*f^2 + b^5*c^2*d^2*f + 6*a^2*b^2*c^3*e^3 - 36*a*b*c^5*d^3 + a*b^5*c*d*f^2 + 48*a^3*c^4*d*e*f + 18*a*b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a^2*b*c^4*d*e^2 - a*b^3*c^3*d^2*f - 60*a^2*b*c^4*d^2*f - 28*a^3*b*c^3*d*f^2 + a^2*b^4*c*e*f^2 - 28*a^3*b*c^3*e^2*f - 9*a^2*b^3*c^2*d*f^2 - 5*a^2*b^3*c^2*e^2*f + 18*a^3*b^2*c^2*e*f^2 - 4*a*b^4*c^2*d*e*f + 52*a^2*b^2*c^3*d*e*f)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)

^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 +

2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*2i

3.72 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 399

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx &= -\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c(b^2d-abe-2a(cd-af))\,x^2\right)}{2a^2\,(b^2-4ac)\,(a+bx^2+cx^4)}\\ &\quad-\frac{\sqrt{c}\left(3b^2d-abe-2a(5cd-af)+\frac{3b^3d-ab^2e+12a^2ce-4ab(4cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad-\frac{\sqrt{c}\left(3b^2d-abe-2a(5cd-af)-\frac{3b^3d-ab^2e+12a^2ce-4ab(4cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

```
[Out] -d/a^2/x-1/2*x*(a*(b^3*d/a-b*(b*e+3*c*d)+a*(b*f+2*c*e))+c*(b^2*d-a*b*e-2*a*
(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2*d-a*b*e-2*a*(-a*f+5*c*d)+(
3*b^3*d-a*b^2*e+12*a^2*c*e-4*a*b*(a*f+4*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a
*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2*d-a*b*e-2*a*(-a*f+5*c*d)+(-3*b^
3*d+a*b^2*e-12*a^2*c*e+4*a*b*(a*f+4*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b
^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1683, 1678, 1180, 211}

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$$
$$= -\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2(-abe-2a(cd-af)+b^2d)\right)}{2a^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{d}{a^2x}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f)) + c*(b
^2*d - a*b*e - 2*a*(c*d - a*f))*x^2))/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) + (3*b^3*d - a*b^2*e +
 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) - (3*b^3
*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a
*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
```

```
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c(b^2d-abe-2a(cd-af))\,x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\
&\quad -\frac{\int \frac{-2(b^2-4ac)d+\frac{\left(b^3d-ab^2e+6a^2ce-ab(5cd+af)\right)x^2}{a}+\frac{c\left(b^2d-abe-2a(cd-af)\right)x^4}{a}}{x^2(a+bx^2+cx^4)}\,dx}{2a\left(b^2-4ac\right)} \\
&= -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c(b^2d-abe-2a(cd-af))\,x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\
&\quad -\frac{\int \left(\frac{2(-b^2+4ac)d}{ax^2}+\frac{3b^3d-ab^2e+6a^2ce-ab(13cd+af)+c(3b^2d-abe-2a(5cd-af))x^2}{a(a+bx^2+cx^4)}\right)dx}{2a\left(b^2-4ac\right)} \\
&= -\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c(b^2d-abe-2a(cd-af))\,x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\
&\quad -\frac{\int \frac{3b^3d-ab^2e+6a^2ce-ab(13cd+af)+c(3b^2d-abe-2a(5cd-af))x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)} \\
&= -\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c(b^2d-abe-2a(cd-af))\,x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\
&\quad -\frac{\left(c\left(3b^2d-abe-2a(5cd-af)-\frac{3b^3d-ab^2e+12a^2ce-4ab(4cd+af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a^2\left(b^2-4ac\right)} \\
&\quad -\frac{\left(c\left(3b^2d-abe-2a(5cd-af)+\frac{3b^3d-ab^2e+12a^2ce-4ab(4cd+af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a^2\left(b^2-4ac\right)}
\end{aligned}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a} - b(3cd + be) + a(2ce + bf)\right) + c(b^2d - abe - 2a(cd - af))\,x^2\right)}{2a^2\,(b^2 - 4ac)\,(a + bx^2 + cx^4)}$$
$$- \frac{\sqrt{c}\left(3b^2d - abe - 2a(5cd - af) + \frac{3b^3d - ab^2e + 12a^2ce - 4ab(4cd + af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\,(b^2 - 4ac)\,\sqrt{b - \sqrt{b^2 - 4ac}}}$$
$$- \frac{\sqrt{c}\left(3b^2d - abe - 2a(5cd - af) - \frac{3b^3d - ab^2e + 12a^2ce - 4ab(4cd + af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\,(b^2 - 4ac)\,\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.11

$$\int \frac{d + ex^2 + fx^4}{x^2\,(a + bx^2 + cx^4)^2}\,dx$$

$$= \frac{-\frac{4d}{x} - \frac{2x(b^3d + b^2(-ae + cdx^2) + ab(af - c(3d + ex^2)) + 2ac(-cdx^2 + a(e + fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^3d + b^2\left(-3\sqrt{b^2-4ac}d + ae\right) + ab\left(16cd + \sqrt{b^2-4ac}\right.\right.}{(b^2-}}{}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*d)/x - (2*x*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d + e*x^
2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3*d + b^2*(-3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b
*(16*c*d + Sqrt[b^2 - 4*a*c]*e + 4*a*f) - 2*a*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6
*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*Sqrt[c]*(3*b^3*d - b^2*(3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-16*c*d + Sqrt
[b^2 - 4*a*c]*e - 4*a*f) + 2*a*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.10

method	result
default	$-\frac{d}{a^2 x}+\frac{\frac{c\left(2f\,a^2-abe-2acd+b^2d\right)x^3}{8ac-2b^2}+\frac{\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(2f\,a^2\sqrt{-4ac+b^2}-abe\sqrt{-4ac+b^2}-10acd\sqrt{-4ac+b^2}+3b^2d\right.}{8\sqrt{-}}\right.}{}$
risch	Expression too large to display

```
[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d/a^2/x+1/a^2*((1/2*c*(2*a^2*f-a*b*e-2*a*c*d+b^2*d)/(4*a*c-b^2)*x^3+1/2*(a
^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/
(4*a*c-b^2)*c*(1/8*(2*f*a^2*(-4*a*c+b^2)^(1/2)-a*b*e*(-4*a*c+b^2)^(1/2)-10*
a*c*d*(-4*a*c+b^2)^(1/2)+3*b^2*d*(-4*a*c+b^2)^(1/2)+4*a^2*b*f-12*a^2*c*e+a*
b^2*e+16*a*b*c*d-3*b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(2*f*a^
2*(-4*a*c+b^2)^(1/2)-a*b*e*(-4*a*c+b^2)^(1/2)-10*a*c*d*(-4*a*c+b^2)^(1/2)+3
*b^2*d*(-4*a*c+b^2)^(1/2)-4*a^2*b*f+12*a^2*c*e-a*b^2*e-16*a*b*c*d+3*b^3*d)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13111 vs. $2(357) = 714$.

Time = 28.00 (sec) , antiderivative size = 13111, normalized size of antiderivative = 32.86

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{fx^4+ex^2+d}{\left(cx^4+bx^2+a\right)^2x^2}\,dx$$

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*f + (3*b^3
 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^
2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) -
 1/2*integrate(-(a^2*b*f + (a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x
^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a
^2*b^2 - 4*a^3*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7173 vs. $2(357) = 714$.

Time = 1.44 (sec) , antiderivative size = 7173, normalized size of antiderivative = 17.98

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c*f*x^4 + 3*b^3*
d*x^2 - 11*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*b*f*x^2 + 2*a*b^
2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((6*b^4
*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
+ 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b
^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*d - (2*
a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 -
2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(2*a^2*b^2*c^2 - 8*a^3
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c - sqrt(2)*sqrt
```

(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2
c^2)(a^2*b^2 - 4*a^3*c)^2*f + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*
c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^
3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a
^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*d*ab
s(a^2*b^2 - 4*a^3*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 -
 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^5*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 28*a^4*b
^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^2*c^3 - 128*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*c^4 + 192*a^6*c^4 + 2*(b^2 - 4*a*c)*a^3*b^4*c - 20*(b^2 - 4*a
*c)*a^4*b^2*c^2 + 48*(b^2 - 4*a*c)*a^5*c^3)*e*abs(a^2*b^2 - 4*a^3*c) - 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^5*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^4*c - 2*a^4*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^2 +
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 16*a^5*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 32*a^6*b*c^3 + 2*(b^2 - 4*a*c)*a^4*b^3*c -
8*(b^2 - 4*a*c)*a^5*b*c^2)*f*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 - 80*a
^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(
b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*d - (2*a^5*b^7*c^
2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sq

rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2
 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*e - 4*(2*a^6*b^6*c^2 -
16*a^7*b^4*c^3 + 32*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^6*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^7*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^6*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^8*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^7*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^6*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^7*b^2*c^3 - 2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 8*(b^2 - 4*a*c)*a^7*b^2*c^3)*
f)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b
c)^2 - 4(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*
c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3
*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4
)*abs(a^2*b^2 - 4*a^3*c)*abs(c)) + 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2
*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^
2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*d - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^
2*b^2 - 4*a^3*c)^2*e + 2*(2*a^2*b^2*c^2 - 8*a^3*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*c^2)*(a^2*b^2 - 4*a^3*c)^2*
f - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*b^5*c^2 - 208*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c

)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50*(b^2 - 4*a*c)*
a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*d*abs(a^2*b^2 - 4*a^3*c) + 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^5*c + 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2
 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 28*a^4*b^4*c^2 - 96*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 128*
a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 - 192*a^6*
c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c + 20*(b^2 - 4*a*c)*a^4*b^2*c^2 - 48*(b^2 -
4*a*c)*a^5*c^3)*e*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^4*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*a^4*b^5*c + 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^5*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
3*c^2 - 16*a^5*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^
3 + 32*a^6*b*c^3 - 2*(b^2 - 4*a*c)*a^4*b^3*c + 8*(b^2 - 4*a*c)*a^5*b*c^2)*f
*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4
 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 1
28*(b^2 - 4*a*c)*a^6*b^2*c^4)*d - (2*a^5*b^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7
*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(
b^2 - 4*a*c)*a^7*b*c^4)*e - 4*(2*a^6*b^6*c^2 - 16*a^7*b^4*c^3 + 32*a^8*b^2*
c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6 + 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c + 2*sq

```
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c - 16*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^2 - 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 - 2*(b^2 - 4*a
*c)*a^6*b^4*c^2 + 8*(b^2 - 4*a*c)*a^7*b^2*c^3)*f)*arctan(2*sqrt(1/2)*x/sqrt
((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c
)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^
4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*
c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs
(c))
```

Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 28164, normalized size of antiderivative = 70.59

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(3*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 11*a*b*c*d))/(2*a^2*(4*a*c
 - b^2)) - d/a + (c*x^4*(3*b^2*d + 2*a^2*f - a*b*e - 10*a*c*d))/(2*a^2*(4*a
*c - b^2)))/(a*x + b*x^3 + c*x^5) - atan(((x*(204800*a^12*c^9*d^2 - 73728*a
^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*
d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*
d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 +
 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 +
160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 1638
4*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^1
3*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^
5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3
*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f
 + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 92
16*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2 - a^2*b^
11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 +
 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e -
2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 4480
0*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840
*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*
d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*
f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*
c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*
```

c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f))*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 2240

0*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9
)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f
- 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2
*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b
^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b
^2*c^5)))^(1/2)*1i + (x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^
14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c
^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2
*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^
2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2
- 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 -
81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*
b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b
^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c
^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7
*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f -
32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-
(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2
6880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2
+ 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a
^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96
*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f
+ 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^
5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)
^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 15
48*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a
^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d
*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128
*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(393216*a^1
5*c^8*e + x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9
)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*
d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7
*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^
7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^
2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6
*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(
1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b
*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*

a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*
d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e
+ 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b
^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*
f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*
a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c
^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 467
2*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^
12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*
c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*
e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 1024
0*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 851968*a^1
4*b*c^8*d - 65536*a^15*b*c^7*f))*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*
d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/
2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^
2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d
^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^
9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^
2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^1
1*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e
*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^
7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 3
0720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7
*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f
 + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c
^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(
(x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*
b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b
^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^1
0*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^
6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f
^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f +
237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a
^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a
^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^
6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3
*e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e
*f) + ((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2
) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 -

9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 76
8*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d
^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c -
b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*
e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 51
2*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e -
2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) -
152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(
-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^
8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e -
8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^
2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3
*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 15
36*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12
+ 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 384
0*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((27*a^3*b^9*c*e^2 - a^2*b^11
*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c
- b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9
*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 20
77*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*
a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(
4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a
^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^
2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f
+ 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*
d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^
4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2)
+ 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^
5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))
)^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 6144
0*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b
^3*c^7) - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47
360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 126156
8*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^
8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c
^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40
960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^1
5*b*c^7*f))*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9
)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*
d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7

*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) - (x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(393216*a^15*c^8*e + x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3

*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a
^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*
b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*
c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b
^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 +
6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*
a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f
+ 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*
(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2
)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^
4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 57
6*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^
6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*
a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1
/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^
12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c
^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256
000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^
9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^
6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*
f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 819
20*a^14*b^3*c^6*f - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f))*((27*a^3*b^9
*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 -
a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*
a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 +
6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5
*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 2
5*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5
*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2
+ 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f -
3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*
e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^
(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2
*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d
*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3
*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f
- 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 -
24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 61
44*a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9*d^3 - 1024*a^13*c^6*f^3 + 4608*a
^11*b*c^7*e^3 + 46080*a^11*c^8*d*e^2 - 76800*a^11*c^8*d^2*f + 15360*a^12*c^
7*d*f^2 - 9216*a^12*c^7*e^2*f + 504*a^6*b^8*c^5*d^3 - 8112*a^7*b^6*c^6*d^3
+ 48704*a^8*b^4*c^7*d^3 - 129280*a^9*b^2*c^8*d^3 - 40*a^8*b^7*c^4*e^3 + 608
*a^9*b^5*c^5*e^3 - 2944*a^10*b^3*c^6*e^3 - 48*a^10*b^6*c^3*f^3 + 320*a^11*b

^4*c^4*f^3 - 256*a^12*b^2*c^5*f^3 - 84480*a^10*b*c^8*d^2*e + 7680*a^12*b*c^
6*e*f^2 - 360*a^6*b^9*c^4*d^2*e + 5736*a^7*b^7*c^5*d^2*e + 240*a^7*b^8*c^4*
d*e^2 - 33888*a^8*b^5*c^6*d^2*e - 3792*a^8*b^6*c^5*d*e^2 + 87936*a^9*b^3*c^
7*d^2*e + 21696*a^9*b^4*c^6*d*e^2 - 52992*a^10*b^2*c^7*d*e^2 + 216*a^6*b^10
*c^3*d^2*f - 3744*a^7*b^8*c^4*d^2*f + 25200*a^8*b^6*c^5*d^2*f + 72*a^8*b^8*
c^3*d*f^2 - 81984*a^9*b^4*c^6*d^2*f - 1296*a^9*b^6*c^4*d*f^2 + 128256*a^10*
b^2*c^7*d^2*f + 7872*a^10*b^4*c^5*d*f^2 - 19200*a^11*b^2*c^6*d*f^2 + 24*a^8
*b^8*c^3*e^2*f - 336*a^9*b^6*c^4*e^2*f - 24*a^9*b^7*c^3*e*f^2 + 960*a^10*b^
4*c^5*e^2*f + 672*a^10*b^5*c^4*e*f^2 + 2304*a^11*b^2*c^6*e^2*f - 4224*a^11*
b^3*c^5*e*f^2 - 21504*a^11*b*c^7*d*e*f - 144*a^7*b^9*c^3*d*e*f + 2256*a^8*b
^7*c^4*d*e*f - 12480*a^9*b^5*c^5*d*e*f + 28416*a^10*b^3*c^6*d*e*f))*((27*a^
3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f
^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 +
3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f
^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^
4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2
) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^
5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3
*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e
*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10
*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2
)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*
a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*
c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(
4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^
6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^
4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*
c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
 - 6144*a^10*b^2*c^5)))^(1/2)*2i - atan(((x*(204800*a^12*c^9*d^2 - 73728*a^
13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d
^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d
^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 +
4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 1
60*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384
*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13
*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5
*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*
d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f
+ 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 921
6*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((9*b^4*d^2*(-(4*a*c - b^2)^
9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2
)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 -
9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2
077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800
*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-

(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*
a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d
^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f
 - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c
*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c
*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c
- b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b
^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2)
 + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a
^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*
e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)
))^(1/2)*(x*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^
2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 +
27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7
*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^
7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^
2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6
*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(
1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b
*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*
a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*
d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e
- 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b
^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*
f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*
a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c
^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*a^15*c^8*e + 192*
a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b
^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2
*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e -
327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a
^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^
3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f))*((9*b^4*d^2*(-(4*a*c -
 b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c
 - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*
e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d
*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 +
 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*
d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 -
 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^

11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c
^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3
*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10
*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(
4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400
*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f -
192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*
b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^
10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^
2*c^5)))^(1/2)*1i + (x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^1
4*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^
5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*
c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2
- 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 -
2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 8
1920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*b
^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^
5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^
4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*
d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f -
32768*a^12*b^3*c^6*e*f) + ((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e
^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^
6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 +
10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^
2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2
) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*
a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f +
15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a
*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5
*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^
9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 154
8*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^
6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*
f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*
a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 12
80*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(393216*a^15
*c^8*e + x*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2
- a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 2
7*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*
c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7

*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2
 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*
d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1
/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*
e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a
^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d
*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e -
 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^
5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f
 + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a
^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^
5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672
*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^1
2*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c
^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e
 - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240
*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 851968*a^14
*b*c^8*d - 65536*a^15*b*c^7*f))*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*
b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26
880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2
*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9
)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2
 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11
*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7
*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c -
 b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2)
 + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30
720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*
c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f
+ 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/((
x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b
^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^
6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10
*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6
*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^
2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 2
37568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^
8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^

11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 409

60*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^15
*b*c^7*f))*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2
 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 2
7*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*
c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7
*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2
 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*
d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1
/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*
e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a
^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d
*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e -
 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^
5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f
 + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) - (x*(204800*a^12*c^9*d^2 -
 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b
^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*
b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c
^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^
7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^
2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 4
0960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^
9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*
b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4
*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*
e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((9*b^4*d^2*(-(4*a*
c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c
^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^1
2*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^
2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c
^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^
2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a
*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^
8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*
a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) -
 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*
(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22
400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)
^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*
f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a

^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(393216*a^15*c^8*e + x*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f))*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 614

4*a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9*d^3 - 1024*a^13*c^6*f^3 + 4608*a^11*b*c^7*e^3 + 46080*a^11*c^8*d*e^2 - 76800*a^11*c^8*d^2*f + 15360*a^12*c^7*d*f^2 - 9216*a^12*c^7*e^2*f + 504*a^6*b^8*c^5*d^3 - 8112*a^7*b^6*c^6*d^3 + 48704*a^8*b^4*c^7*d^3 - 129280*a^9*b^2*c^8*d^3 - 40*a^8*b^7*c^4*e^3 + 608*a^9*b^5*c^5*e^3 - 2944*a^10*b^3*c^6*e^3 - 48*a^10*b^6*c^3*f^3 + 320*a^11*b^4*c^4*f^3 - 256*a^12*b^2*c^5*f^3 - 84480*a^10*b*c^8*d^2*e + 7680*a^12*b*c^6*e*f^2 - 360*a^6*b^9*c^4*d^2*e + 5736*a^7*b^7*c^5*d^2*e + 240*a^7*b^8*c^4*d*e^2 - 33888*a^8*b^5*c^6*d^2*e - 3792*a^8*b^6*c^5*d*e^2 + 87936*a^9*b^3*c^7*d^2*e + 21696*a^9*b^4*c^6*d*e^2 - 52992*a^10*b^2*c^7*d*e^2 + 216*a^6*b^10*c^3*d^2*f - 3744*a^7*b^8*c^4*d^2*f + 25200*a^8*b^6*c^5*d^2*f + 72*a^8*b^8*c^3*d*f^2 - 81984*a^9*b^4*c^6*d^2*f - 1296*a^9*b^6*c^4*d*f^2 + 128256*a^10*b^2*c^7*d^2*f + 7872*a^10*b^4*c^5*d*f^2 - 19200*a^11*b^2*c^6*d*f^2 + 24*a^8*b^8*c^3*e^2*f - 336*a^9*b^6*c^4*e^2*f - 24*a^9*b^7*c^3*e*f^2 + 960*a^10*b^4*c^5*e^2*f + 672*a^10*b^5*c^4*e*f^2 + 2304*a^11*b^2*c^6*e^2*f - 4224*a^11*b^3*c^5*e*f^2 - 21504*a^11*b*c^7*d*e*f - 144*a^7*b^9*c^3*d*e*f + 2256*a^8*b^7*c^4*d*e*f - 12480*a^9*b^5*c^5*d*e*f + 28416*a^10*b^3*c^6*d*e*f))*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i

3.73 $\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 575

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}$$
$$+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))x^2\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(5b^4d+b^3\left(5\sqrt{b^2-4ac}d-3ae\right)+2a^2c\left(14cd+5\sqrt{b^2-4ac}e-6af\right)-ab^2\left(29cd+3\sqrt{b^2-4ac}e-a\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4}}}$$
$$-\frac{\sqrt{c}\left(5b^4d-b^3\left(5\sqrt{b^2-4ac}d+3ae\right)+2a^2c\left(14cd-5\sqrt{b^2-4ac}e-6af\right)-ab^2\left(29cd-3\sqrt{b^2-4ac}e-a\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4}}}$$

```
[Out] -1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*(a^2*(b^4*d/a^2+2*c^2*d+3*b*c*e-b^2
*(b*e+4*c*d)/a+b^2*f-2*a*c*f)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))*
x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4*d+b^3*(-3*a*e+5*d*(-4*a*c+b^2)^(1/2))-
a*b^2*(29*c*d-a*f+3*e*(-4*a*c+b^2)^(1/2))+2*a^2*c*(14*c*d-6*a*f+5*e*(-4*a*c
+b^2)^(1/2))-a*b*(-16*a*c*e+19*c*d*(-4*a*c+b^2)^(1/2)-a*f*(-4*a*c+b^2)^(1/2
)))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4*d-b^3*(3*a*e
+5*d*(-4*a*c+b^2)^(1/2))+2*a^2*c*(14*c*d-6*a*f-5*e*(-4*a*c+b^2)^(1/2))-a*b^
2*(29*c*d-a*f-3*e*(-4*a*c+b^2)^(1/2))+a*b*(16*a*c*e+19*c*d*(-4*a*c+b^2)^(1/
2)-a*f*(-4*a*c+b^2)^(1/2)))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 6.84 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1683, 1678, 1180, 211}

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}dx = \frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(3e\sqrt{b^2-4ac}-af+29cd\right)-ab(19cd\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4a}}} - \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(2a^2c\left(-5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(-3e\sqrt{b^2-4ac}-af+29cd\right)+ab(1\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}}} + \frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/3*d/(a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d +
 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - 2*a*c*f) + c*(b^3*d - a*b^2*e +
2*a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14
*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]
*e - a*f) - a*b*(19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*
f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3
*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4*d - b^3
*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*a^2*c*(14*c*d - 5*Sqrt[b^2 - 4*a*c]*e
- 6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^2 - 4*a*c]*e - a*f) + a*b*(19*c*Sqrt[b^
2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\text{integral} = \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,x^2\right)}{2a^3\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{-2(b^2-4ac)d+\frac{2(b^2-4ac)(bd-ae)x^2}{a}-\left(\frac{b^4d}{a^2}+6c^2d+5bce-\frac{b^2(6cd+be)}{a}+b^2f-6acf\right)x^4-\frac{c\left(b^3d-ab^2e+2a^2ce-ab(3cd-af)\right)x^6}{a^2}}{x^4(a+bx^2+cx^4)}\,dx}{2a\,(b^2-4ac)}$$

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,x^2\right)}{2a^3\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \left(\frac{2(-b^2+4ac)d}{ax^4}+\frac{2(-b^2+4ac)(-2bd+ae)}{a^2x^2}+\frac{-5b^4d+3ab^3e-13a^2bce-2a^2c(7cd-3af)+ab^2(24cd-af)-c(5b^3d-3ab^2e+10a}{a^2(a+bx^2+cx^4)}\right.}{2a\,(b^2-4ac)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x} + \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,x^2\right.}{2a^3\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{-5b^4d+3ab^3e-13a^2bce-2a^2c(7cd-3af)+ab^2(24cd-af)-c(5b^3d-3ab^2e+10a^2ce-ab(19cd-af))x^2}{a+bx^2+cx^4}\,dx}{2a^3\,(b^2-4ac)}$$

$$= -\frac{d}{3a^2x^3} + \frac{2bd-ae}{a^3x} + \frac{x\left(a^2\left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - 2acf\right) + c(b^3d - ab^2e + 2a^2ce - ab(3cd-af))\,x^2\right)}{2a^3\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(c(5b^4d + b^3(5\sqrt{b^2-4ac}d - 3ae) + 2a^2c(14cd + 5\sqrt{b^2-4ac}e - 6af) - ab^2(29cd + 3\sqrt{b^2-4ac}}{4a^3\,(b^2-4ac)^{3/2}} - \frac{(c(5b^4d - b^3(5\sqrt{b^2-4ac}d + 3ae) + 2a^2c(14cd - 5\sqrt{b^2-4ac}e - 6af) - ab^2(29cd - 3\sqrt{b^2-4ac}}{4a^3\,(b^2-4ac)^{3/2}}$$

$$= -\frac{d}{3a^2x^3} + \frac{2bd-ae}{a^3x} + \frac{x\left(a^2\left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - 2acf\right) + c(b^3d - ab^2e + 2a^2ce - ab(3cd-af))\,x^2\right)}{2a^3\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{c}(5b^4d + b^3(5\sqrt{b^2-4ac}d - 3ae) + 2a^2c(14cd + 5\sqrt{b^2-4ac}e - 6af) - ab^2(29cd + 3\sqrt{b^2-4a}{2\sqrt{2}a^3\,(b^2-4ac)^{3/2}\,\sqrt{b-\sqrt{}}} - \frac{\sqrt{c}(5b^4d - b^3(5\sqrt{b^2-4ac}d + 3ae) + 2a^2c(14cd - 5\sqrt{b^2-4ac}e - 6af) - ab^2(29cd - 3\sqrt{b^2-4a}{2\sqrt{2}a^3\,(b^2-4ac)^{3/2}\,\sqrt{b+\sqrt{}}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.95

$$\int \frac{d+ex^2+fx^4}{x^4\,(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{4ad}{x^3} + \frac{24bd-12ae}{x} + \frac{6x(b^4d+b^3(-ae+cdx^2)+abc(3ae-3cdx^2+afx^2)+2a^2c(-af+c(d+ex^2))+ab^2(af-c(4d+ex^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{3\sqrt{2}\sqrt{c}(5b^4d+b^3}{}}{}$$

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
+ a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2) + 2*a^2*c*(-(a*f) + c*(d + e*x^2)) +
a*b^2*(a*f - c*(4*d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sq
rt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*
c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]
*e + a*f) + a*b*(-19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4*d + b^3*(
5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a
*f) + 2*a^2*c*(-14*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 6*a*f) + a*b*(-19*c*Sqrt[b
```

```
^2 - 4*a*c]*d - 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4
*a*c]]))/(12*a^3)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.99

method	result
default	$-\frac{d}{3a^2x^3}-\frac{ae-2bd}{a^3x}+\frac{-\frac{c\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)x^3}{2\left(4ac-b^2\right)}+\frac{\left(2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+a\,b^3e+4a\,b^2cd-d\,b^4\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(-a^2bf\sqrt{-}\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d/a^2/x^3-(a*e-2*b*d)/a^3/x+1/a^3*((-1/2*c*(a^2*b*f+2*a^2*c*e-a*b^2*e-
3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x^3+1/2*(2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a^2
*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b
^2)*c*(1/8*(-a^2*b*f*(-4*a*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b
^2*e*(-4*a*c+b^2)^(1/2)+19*a*b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^
(1/2)-12*a^3*c*f+a^2*b^2*f+16*a^2*b*c*e+28*a^2*c^2*d-3*a*b^3*e-29*a*b^2*c*d
+5*d*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-a^2*b*f*(-4*a*c+b^2)^
(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*a*b*c*d
*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)+12*a^3*c*f-a^2*b^2*f-16*a^2*
b*c*e-28*a^2*c^2*d+3*a*b^3*e+29*a*b^2*c*d-5*d*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19333 vs. $2(510) = 1020$.

Time = 88.78 (sec) , antiderivative size = 19333, normalized size of antiderivative = 33.62

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{fx^4+ex^2+d}{\left(cx^4+bx^2+a\right)^2x^4}\,dx$$

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(a^2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*
x^6 + ((15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d - 3*(3*a*b^3 - 11*a^2*b*c)*e +
3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*b*c)*d - 3*(a^2*b^2 - 4*
a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^
3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) + 1/2*integrate(((a^2*b*c
*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 -
24*a*b^2*c + 14*a^2*c^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)
*f)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8649 vs. $2(510) = 1020$.

Time = 1.65 (sec) , antiderivative size = 8649, normalized size of antiderivative = 15.04

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 - a*b^2*c*e*x^3 + 2*a^2*c^2*e*x^3 + a^2*
b*c*f*x^3 + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x - a*b^3*e*x + 3*a^2*b*c
*e*x + a^2*b^2*f*x - 2*a^3*c*f*x)/((a^3*b^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a))
 + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
```

sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*
(a^3*b^2 - 4*a^4*c)^2*d - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*
(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + (2*a^2*b^3*c^2 - 8*a^3*b*c
^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)
*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 22
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4
*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*d*abs
(a^3*b^2 - 4*a^4*c) - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7
- 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a
^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 416*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c -
 50*(b^2 - 4*a*c)*a^5*b^3*c^2 + 104*(b^2 - 4*a*c)*a^6*b*c^3)*e*abs(a^3*b^2
- 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6 - 14*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^5*c - 2*a^5*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2
 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 28*a^6*b^4*c^2 - 9

```
6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^6*b^2*c^3 - 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^7*c^4 + 192*a^8*c^4 + 2*(b^2 - 4*a*c)*a^5*b^4*c - 20*(b^2 - 4*a*c)*a^6*b^
2*c^2 + 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^
2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 -
 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b
^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*
a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d - (6*a^7*b^8*c^2 - 80*a^8*b^6*
c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 -
 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e + (2*a^8*b^7*c^2 - 4
0*a^9*b^5*c^3 + 224*a^10*b^3*c^4 - 384*a^11*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^11*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(b^
2 - 4*a*c)*a^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f)*arctan(2*sqrt(1/2)
```

*x/sqrt((a^3*b^3 - 4*a^4*b*c + sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - 2*(5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*a^3*b^8*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 572*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 88*(b^2 - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*a*c)*a^6*c^4)*d*abs(a^3*b^2 - 4*a^4*c) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c + 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 - 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*

sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2 - 4*a*c)
*a^4*b^5*c + 50*(b^2 - 4*a*c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*b*c^3)*e*
abs(a^3*b^2 - 4*a^4*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6
 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c + 2*a^5*b^6*c + 64*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^6*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 - 28*a^6
*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^6*b^2*c^3 + 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^7*c^4 - 192*a^8*c^4 - 2*(b^2 - 4*a*c)*a^5*b^4*c + 20*(b^2 - 4
*a*c)*a^6*b^2*c^2 - 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^4*c) + (1
0*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*
a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^
7*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*
c - 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c
^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c
^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^
2 + 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c
^3 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*
c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*
c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*
c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2
*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^
3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b
c^5 - 10(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b
^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d - (6*a^7*b^8*c^2 -
 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2
 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e + (2*a^8
*b^7*c^2 - 40*a^9*b^5*c^3 + 224*a^10*b^3*c^4 - 384*a^11*b*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^7 + 20*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c - 112*sqrt(2)*sqrt(b^2 -

```
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*
c^2 + 32*(b^2 - 4*a*c)*a^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f)*arctan
(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c - sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4
*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((
a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^
7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^
3*b^2 - 4*a^4*c)*abs(c)) + 1/3*(6*b*d*x^2 - 3*a*e*x^2 - a*d)/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 36097, normalized size of antiderivative = 62.78

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 +
 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 -
488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d
^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*
e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^
7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f
^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^
2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f -
480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e +
 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*
d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d
*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^
8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e
*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 +
9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640
*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^
2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d
*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2
 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c
^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*
e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)
```

^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f
^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*
f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^1
2*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 2
46*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^
6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*
f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f +
44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b
^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e
*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f -
51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)
^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 9
17504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c
- b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 +
26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*
(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^
3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^
6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(
4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 3024
0*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^
(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 150
4*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d
*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*
(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*
a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e +
39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e -
161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4
*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^
3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*
c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7
*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2
*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^1
0*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b
^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^
3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 157286
4*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b
^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b
^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^

3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e
+ 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 1792
0*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a
^19*b^2*c^7*f - 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*
b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213
*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f
^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c
^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^
5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4
9*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b
^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-
(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*
b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^
2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*
f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^
11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a
^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a
^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d
*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^
4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^
3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*
b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(
1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13
*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*
c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + (x*(204800*a^17*c^9*e^2 - 401408*a^16
*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*
d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*
c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^
12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*
b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b
^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4
*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^
9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4
*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5
*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^1
0*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*
b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^
9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*
b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)
^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880
*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*

c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*
c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c
- b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*
b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*
b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 3
5840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a
*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*
c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2)
 - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132
*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280
*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c
^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*
d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c
^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^
2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c +
240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5
)))^(1/2)*(917504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(25*b^15*d^2 + 9*a^2
*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*
b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3
840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e +
6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 21
9744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^
2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 +
 a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(
1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 -
615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 1
5360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d
*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^
2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4
*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(
4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 4435
2*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9
)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f +
8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a
^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(
1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b
^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 +
 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a

^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^
5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 793
6*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000
*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^1
3*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^
7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2
*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f -
327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f + 851968*a^19*b*c^8*e))*(-(25
*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11
*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*
a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) -
 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4
*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 20
77*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*
a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a
^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6
*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) +
 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^
10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 1
19616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 1
0*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b
^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*
(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a
^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b
^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^
3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 12
80*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/((x*(2
04800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^
14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12
*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 187187
2*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112
*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458
752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a
^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*
a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^
13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^1
3*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a
^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704
*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*
a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520

*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*

a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)
^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a
^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(
1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a
^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*
c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d -
468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d +
2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360
*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a
^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*
c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7
*f - 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(
4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c
*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*
c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35
767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215
040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d
^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2
+ 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^
2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2
 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*
b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^
5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f +
43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2
*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*
d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 27
06*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*
a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(
-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 224
00*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(
-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a
^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a
^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*
a^12*b^2*c^5)))^(1/2) - (x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73
728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^
11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401
280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3
264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 +
365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 -
416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 696
32*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568
*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*

a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 212
1728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39
488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f -
720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22
528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) +
(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^
4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2
- 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(
1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 1169
28*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^
2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 -
44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e
^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 +
3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d
*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(
1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*
a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c
d^2(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d
*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d
*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784
*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^
3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720
*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e
f(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) -
186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^
2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917
504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25
*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 21
3*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*
f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*
c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c
^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*
b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(
-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6
*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d
^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e
*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b
^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*
(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*
a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*
a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9

)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*
d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a
^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c
^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2
*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^
(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^1
3*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4
*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 -
6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^1
9*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^
3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*
d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e -
4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 7782
40*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15
*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^
4*c^6*f + 557056*a^19*b^2*c^7*f + 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*
a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a
^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2
- 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e
+ 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 -
219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c -
b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2
*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^
2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2
- 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f
- 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*
c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246
*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)
^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*
c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*
(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 4
4352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2
)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f
+ 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 5
1*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9
)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-
(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^
3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - 128000*a^15*c^9*e^3 +
476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f^3 - 250880*a^14*c^10*d^2*e - 460
80*a^16*c^8*e*f^2 + 1800*a^9*b^9*c^6*d^3 - 29080*a^10*b^7*c^7*d^3 + 176032*
a^11*b^5*c^8*d^3 - 473216*a^12*b^3*c^9*d^3 - 504*a^11*b^8*c^5*e^3 + 8112*a^
12*b^6*c^6*e^3 - 48704*a^13*b^4*c^7*e^3 + 129280*a^14*b^2*c^8*e^3 + 40*a^13

*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^3 + 2944*a^15*b^3*c^6*f^3 + 215040*a^15*c
^9*d*e*f + 442880*a^14*b*c^9*d*e^2 - 433664*a^14*b*c^9*d^2*f + 109056*a^15*
b*c^8*d*f^2 + 84480*a^15*b*c^8*e^2*f - 1400*a^9*b^10*c^5*d^2*e + 21680*a^10
*b^8*c^6*d^2*e + 1680*a^10*b^9*c^5*d*e^2 - 121648*a^11*b^6*c^7*d^2*e - 2717
6*a^11*b^7*c^6*d*e^2 + 275264*a^12*b^4*c^8*d^2*e + 164448*a^12*b^5*c^7*d*e^
2 - 121088*a^13*b^2*c^9*d^2*e - 441216*a^13*b^3*c^8*d*e^2 + 1000*a^9*b^11*c
^4*d^2*f - 17800*a^10*b^9*c^5*d^2*f + 124280*a^11*b^7*c^6*d^2*f + 400*a^11*
b^9*c^4*d*f^2 - 422944*a^12*b^5*c^7*d^2*f - 6600*a^12*b^7*c^5*d*f^2 + 69491
2*a^13*b^3*c^8*d^2*f + 40416*a^13*b^5*c^6*d*f^2 - 108928*a^14*b^3*c^7*d*f^2
 + 360*a^11*b^9*c^4*e^2*f - 5736*a^12*b^7*c^5*e^2*f - 240*a^12*b^8*c^4*e*f^
2 + 33888*a^13*b^5*c^6*e^2*f + 3792*a^13*b^6*c^5*e*f^2 - 87936*a^14*b^3*c^7
*e^2*f - 21696*a^14*b^4*c^6*e*f^2 + 52992*a^15*b^2*c^7*e*f^2 - 1200*a^10*b^
10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f - 130656*a^12*b^6*c^6*d*e*f + 39436
8*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8*d*e*f))*(-(25*b^15*d^2 + 9*a^2*b
^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*
c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 384
0*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 63
66*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 2197
44*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2
 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a
^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/
2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 61
5*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 153
60*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e
 - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*
b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(
1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d
*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*
a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*
a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^
(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 80
64*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3
*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/
2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^1
2 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3
840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - (d/(3*a) + (x^2*(3*a*e -
 5*b*d))/(3*a^2) + (x^4*(15*b^4*d + 14*a^2*c^2*d + 3*a^2*b^2*f - 9*a*b^3*e
 - 6*a^3*c*f - 62*a*b^2*c*d + 33*a^2*b*c*e))/(6*a^3*(4*a*c - b^2)) + (c*x^6*
(5*b^3*d - 3*a*b^2*e + a^2*b*f + 10*a^2*c*e - 19*a*b*c*d))/(2*a^3*(4*a*c -
b^2)))/(a*x^3 + b*x^5 + c*x^7) + atan(((x*(204800*a^17*c^9*e^2 - 401408*a^1
6*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4
*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6
*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b

^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14
*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*
b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^
4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c
^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^
4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^
5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^
10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15
*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b
^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16
*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2
)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 2688
0*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9
*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3
*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c
 - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6
*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
 - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7
*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f +
35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*
a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b
*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 3913
2*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 16128
0*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*
c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5
*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b
^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*
c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
 - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b
^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*
f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c +
 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^
5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^
2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7
*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 -
3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e +
 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 2
19744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e
^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2
- a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 -
 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f -

15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*
d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a
^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9
)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^
4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-
(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 443
52*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^
9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f +
 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*
a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^
(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4
*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3
+ 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*
a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c
^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 79
36*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 153600
0*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^
13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b
^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^
2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f +
 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f - 851968*a^19*b*c^8*e))*(-(2
5*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^1
1*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27
*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
- 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^
4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4
e^2(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2
077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800
*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*
a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e -
6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2)
+ 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b
^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*
(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e -
119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e -
10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*
b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f
*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*
a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*
b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-
(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a
^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1

280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + (x*(
204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b
^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^1
2*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 18718
72*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 3011
2*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 45
8752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*
a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064
*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b
^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^
13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*
a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 20070
4*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96
*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 10752
0*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13
*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7
*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a
^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*
a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*
a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)
^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 1
0656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*
b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a
*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*
a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e -
258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2
*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2
) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e
+ 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c
 - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6
*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/
2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*
a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^
2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
- 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c -
 b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 +
 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840
*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^20
*c^8*f + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^
(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8
*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c -
 b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*
d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*
d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^

2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*
c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25
*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*
c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840
*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c -
 b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*
d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1
65*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4
*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7
*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d
*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f
 + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9
)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e
*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44
*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9
)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*
a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^
(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*
a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3
*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d
 + 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d -
 2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 4736
0*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*
a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8
*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^
7*f + 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-
(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*
c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5
*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 3
5767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 21
5040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2
 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b
^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^
2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2
*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b
^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f +
 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^
2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5
*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2
706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120
*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*
(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22

400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*
(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*
a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*
a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144
*a^12*b^2*c^5)))^(1/2)*1i)/((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2
- 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 9281
6*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 -
2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2
 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^
2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^
2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 +
 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 23
7568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105
824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e +
 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f
+ 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*
f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f
- 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*
f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6
*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 +
116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 -
9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^
(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^
2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^
2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c
^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^
9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f +
152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b
^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c
^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c
^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 1
4784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^
3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 3
0720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b
*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2
) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^
8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*
(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2
- 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2

- 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*
c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b
^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b
^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*
a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f
^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288
*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13
*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c
^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a
^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7
278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201
600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^
2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*
c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) -
42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b
^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184
*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)
^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096
*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11
*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c
^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040
*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^1
2*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*
c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2
*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e -
778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*
a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^1
8*b^4*c^6*f - 557056*a^19*b^2*c^7*f - 851968*a^19*b*c^8*e))*(-(25*b^15*d^2
+ 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 806
40*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*
f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14
*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d
^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a
*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9
*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^
5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4
*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*
e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b
^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f -
246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c -
b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*

b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f
 + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c -
 b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2
*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f
 + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^
2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*
e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^
6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - (x*(204800*a^17*c^
9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 -
9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2
+ 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^
9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5
*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*
c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f
^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f
- 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e +
11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e
 - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*
d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6
*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3
*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6
*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*
d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3
*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 +
 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d
^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^
2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^
3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c
^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*
c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 +
10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f +
30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c
*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*
a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b
^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b
^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/
2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f -
 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^
2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*
f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a
*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*
c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2)

```
 + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(
25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^
11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 2
7*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a
^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^
4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) +
2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 4480
0*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840
*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e -
 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2)
 + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*
b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2
*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e -
 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e -
 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5
*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*
f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548
*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8
*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(
-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*
a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 -
1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576
*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 -
 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^
12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*
b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*
b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4
*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e
 - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 10240
0*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f + 851968*a
^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^
9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*
a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c
^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c
^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c -
 b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b
^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) -
 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b
^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35
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840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*
c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c
^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*
a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*
a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^
2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d
*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2
)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^
4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) -
44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2
)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 2
40*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)
))^(1/2) - 128000*a^15*c^9*e^3 + 476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f
^3 - 250880*a^14*c^10*d^2*e - 46080*a^16*c^8*e*f^2 + 1800*a^9*b^9*c^6*d^3 -
29080*a^10*b^7*c^7*d^3 + 176032*a^11*b^5*c^8*d^3 - 473216*a^12*b^3*c^9*d^3
- 504*a^11*b^8*c^5*e^3 + 8112*a^12*b^6*c^6*e^3 - 48704*a^13*b^4*c^7*e^3 +
129280*a^14*b^2*c^8*e^3 + 40*a^13*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^3 + 2944
*a^15*b^3*c^6*f^3 + 215040*a^15*c^9*d*e*f + 442880*a^14*b*c^9*d*e^2 - 43366
4*a^14*b*c^9*d^2*f + 109056*a^15*b*c^8*d*f^2 + 84480*a^15*b*c^8*e^2*f - 140
0*a^9*b^10*c^5*d^2*e + 21680*a^10*b^8*c^6*d^2*e + 1680*a^10*b^9*c^5*d*e^2 -
121648*a^11*b^6*c^7*d^2*e - 27176*a^11*b^7*c^6*d*e^2 + 275264*a^12*b^4*c^8
*d^2*e + 164448*a^12*b^5*c^7*d*e^2 - 121088*a^13*b^2*c^9*d^2*e - 441216*a^1
3*b^3*c^8*d*e^2 + 1000*a^9*b^11*c^4*d^2*f - 17800*a^10*b^9*c^5*d^2*f + 1242
80*a^11*b^7*c^6*d^2*f + 400*a^11*b^9*c^4*d*f^2 - 422944*a^12*b^5*c^7*d^2*f
- 6600*a^12*b^7*c^5*d*f^2 + 694912*a^13*b^3*c^8*d^2*f + 40416*a^13*b^5*c^6*
d*f^2 - 108928*a^14*b^3*c^7*d*f^2 + 360*a^11*b^9*c^4*e^2*f - 5736*a^12*b^7*
c^5*e^2*f - 240*a^12*b^8*c^4*e*f^2 + 33888*a^13*b^5*c^6*e^2*f + 3792*a^13*b
^6*c^5*e*f^2 - 87936*a^14*b^3*c^7*e^2*f - 21696*a^14*b^4*c^6*e*f^2 + 52992*
a^15*b^2*c^7*e*f^2 - 1200*a^10*b^10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f -
130656*a^12*b^6*c^6*d*e*f + 394368*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8
*d*e*f))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1
/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b
*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b
^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^
2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^
2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)
^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^
4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a
^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^
3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a
^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b
^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*
f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165

```
*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b
^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b
^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f
 - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f +
6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^
(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f
 + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a
^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^
(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^
9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1
/2)*2i
```

3.74 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 68

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2(2+3x^2+x^4)} + 2\log(1+x^2) + 392\log(2+x^2)$$

```
[Out] -293/2*x^2+49/2*x^4-9/2*x^6+5/8*x^8+1/2*(415*x^2+414)/(x^4+3*x^2+2)+2*ln(x^
2+1)+392*ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1677, 1674, 1671, 646, 31}

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = \frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + 2\log(x^2+1) + 392\log(x^2+2) + \frac{415x^2+414}{2(x^4+3x^2+2)}$$

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8 + (414 + 415*x^2)/(2*(2 +
 3*x^2 + x^4)) + 2*Log[1 + x^2] + 392*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4(4+x+3x^2+5x^3)}{(2+3x+x^2)^2}\,dx, x, x^2\right) \\
&= \frac{414+415x^2}{2\,(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \frac{-206-105x+53x^2-27x^3+12x^4-5x^5}{2+3x+x^2}\,dx, x, x^2\right) \\
&= \frac{414+415x^2}{2\,(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \left(293-98x+27x^2-5x^3-\frac{4(198+197x)}{2+3x+x^2}\right)\,dx, x, x^2\right) \\
&= -\frac{293x^2}{2}+\frac{49x^4}{2}-\frac{9x^6}{2}+\frac{5x^8}{8}+\frac{414+415x^2}{2\,(2+3x^2+x^4)}+2\text{Subst}\left(\int \frac{198+197x}{2+3x+x^2}\,dx, x, x^2\right)
\end{aligned}
$$

$$= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414 + 415x^2}{2(2 + 3x^2 + x^4)} + 2\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) + 392\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right)$$

$$= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414 + 415x^2}{2(2 + 3x^2 + x^4)} + 2\log(1 + x^2) + 392\log(2 + x^2)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x^9(4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^2}\,dx = \frac{1}{8}\left(-1172x^2 + 196x^4 - 36x^6 + 5x^8 + \frac{4(414 + 415x^2)}{2 + 3x^2 + x^4} + 16\log(1 + x^2) + 3136\log(2 + x^2)\right)$$

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-1172*x^2 + 196*x^4 - 36*x^6 + 5*x^8 + (4*(414 + 415*x^2))/(2 + 3*x^2 + x^
4) + 16*Log[1 + x^2] + 3136*Log[2 + x^2])/8
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

method	result
default	$392\ln(x^2+2) + \frac{208}{x^2+2} + \frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + 2\ln(x^2+1) - \frac{1}{2(x^2+1)}$
norman	$\frac{1086x^2 - 82x^6 + \frac{49}{4}x^8 - \frac{21}{8}x^{10} + \frac{5}{8}x^{12} + 988}{x^4+3x^2+2} + 2\ln(x^2+1) + 392\ln(x^2+2)$
risch	$\frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + \frac{\frac{415x^2}{2}+207}{x^4+3x^2+2} + 2\ln(x^2+1) + 392\ln(x^2+2)$
parallelrisch	$\frac{5x^{12} - 21x^{10} + 98x^8 - 656x^6 + 16\ln(x^2+1)x^4 + 3136\ln(x^2+2)x^4 + 7904 + 48\ln(x^2+1)x^2 + 9408\ln(x^2+2)x^2 + 8688x^2 + 32\ln(x^2+}{8x^4+24x^2+16}$

```
[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 392*ln(x^2+2)+208/(x^2+2)+5/8*x^8-9/2*x^6+49/2*x^4-293/2*x^2+2*ln(x^2+1)-1/
2/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5\,x^{12}-21\,x^{10}+98\,x^8-656\,x^6-3124\,x^4-684\,x^2+3136\,(x^4+3\,x^2+2)\log(x^2+2)+16\,(x^4+3\,x^2+2)}{8\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/8*(5*x^12 - 21*x^10 + 98*x^8 - 656*x^6 - 3124*x^4 - 684*x^2 + 3136*(x^4 +
 3*x^2 + 2)*log(x^2 + 2) + 16*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 1656)/(x^4 +
 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{415x^2+414}{2x^4+6x^2+4}+2\log\left(x^2+1\right)+392\log\left(x^2+2\right)$$

```
[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
[Out] 5*x**8/8 - 9*x**6/2 + 49*x**4/2 - 293*x**2/2 + (415*x**2 + 414)/(2*x**4 + 6
*x**2 + 4) + 2*log(x**2 + 1) + 392*log(x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8-\frac{9}{2}\,x^6+\frac{49}{2}\,x^4-\frac{293}{2}\,x^2+\frac{415\,x^2+414}{2\,(x^4+3\,x^2+2)}+392\,\log\left(x^2+2\right)+2\,\log\left(x^2+1\right)$$

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 1/2*(415*x^2 + 414)/(x^4 + 3*x^2
 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8-\frac{9}{2}\,x^6+\frac{49}{2}\,x^4-\frac{293}{2}\,x^2-\frac{394\,x^4+767\,x^2+374}{2\,(x^4+3\,x^2+2)} + 392\,\log\left(x^2+2\right)+2\,\log\left(x^2+1\right)$$

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 - 1/2*(394*x^4 + 767*x^2 + 374)/(x
^4 + 3*x^2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 2\,\ln\left(x^2+1\right)+392\,\ln\left(x^2+2\right)+\frac{\frac{415\,x^2}{2}+207}{x^4+3\,x^2+2} - \frac{293\,x^2}{2}+\frac{49\,x^4}{2}-\frac{9\,x^6}{2}+\frac{5\,x^8}{8}$$

```
[In] int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 2*log(x^2 + 1) + 392*log(x^2 + 2) + ((415*x^2)/2 + 207)/(3*x^2 + x^4 + 2) -
 (293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8
```

3.75 $\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 61

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} - \frac{206+207x^2}{2(2+3x^2+x^4)} - \frac{5}{2}\log\left(1+x^2\right) - 144\log\left(2+x^2\right)$$

```
[Out] 49*x^2-27/4*x^4+5/6*x^6+1/2*(-207*x^2-206)/(x^4+3*x^2+2)-5/2*ln(x^2+1)-144*
ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1677, 1674, 1671, 646, 31}

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = \frac{5x^6}{6} - \frac{27x^4}{4} + 49x^2 - \frac{5}{2}\log\left(x^2+1\right) - 144\log\left(x^2+2\right) - \frac{207x^2+206}{2(x^4+3x^2+2)}$$

```
[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 - (206 + 207*x^2)/(2*(2 + 3*x^2 + x^4)) - (
5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(4+x+3x^2+5x^3)}{(2+3x+x^2)^2}\,dx, x, x^2\right) \\
&= -\frac{206+207x^2}{2\,(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \frac{102+53x-27x^2+12x^3-5x^4}{2+3x+x^2}\,dx, x, x^2\right) \\
&= -\frac{206+207x^2}{2\,(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \left(-98+27x-5x^2+\frac{298+293x}{2+3x+x^2}\right)\,dx, x, x^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= 49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} - \frac{206 + 207x^2}{2(2 + 3x^2 + x^4)} - \frac{1}{2}\text{Subst}\left(\int \frac{298 + 293x}{2 + 3x + x^2}\,dx, x, x^2\right) \\
&= 49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} - \frac{206 + 207x^2}{2(2 + 3x^2 + x^4)} \\
&\quad - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1 + x}\,dx, x, x^2\right) - 144\text{Subst}\left(\int \frac{1}{2 + x}\,dx, x, x^2\right) \\
&= 49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} - \frac{206 + 207x^2}{2(2 + 3x^2 + x^4)} - \frac{5}{2}\log\left(1 + x^2\right) - 144\log\left(2 + x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$
\int \frac{x^7(4 + x^2 + 3x^4 + 5x^6)}{(2 + 3x^2 + x^4)^2}\,dx = 49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} + \frac{-206 - 207x^2}{2(2 + 3x^2 + x^4)} - \frac{5}{2}\log\left(1 + x^2\right) - 144\log\left(2 + x^2\right)
$$

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 + (-206 - 207*x^2)/(2*(2 + 3*x^2 + x^4)) -
(5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

method	result
default	$-144\ln\left(x^2+2\right)-\frac{104}{x^2+2}+\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{5\ln\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$
norman	$\frac{-406x^2+\frac{365}{12}x^6-\frac{17}{4}x^8+\frac{5}{6}x^{10}-370}{x^4+3x^2+2}-\frac{5\ln\left(x^2+1\right)}{2}-144\ln\left(x^2+2\right)$
risch	$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2+\frac{-\frac{207x^2}{2}-103}{x^4+3x^2+2}-\frac{5\ln\left(x^2+1\right)}{2}-144\ln\left(x^2+2\right)$
parallelrisch	$-\frac{-10x^{10}+51x^8-365x^6+30\ln\left(x^2+1\right)x^4+1728\ln\left(x^2+2\right)x^4+4440+90\ln\left(x^2+1\right)x^2+5184\ln\left(x^2+2\right)x^2+4872x^2+60\ln\left(x^2+1\right)+}{12(x^4+3x^2+2)}$

```
[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -144*ln(x^2+2)-104/(x^2+2)+5/6*x^6-27/4*x^4+49*x^2-5/2*ln(x^2+1)+1/2/(x^2+1
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{10\,x^{10}-51\,x^8+365\,x^6+1602\,x^4-66\,x^2-1728\,(x^4+3\,x^2+2)\log(x^2+2)-30\,(x^4+3\,x^2+2)\log(x^2}{12\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(10*x^10 - 51*x^8 + 365*x^6 + 1602*x^4 - 66*x^2 - 1728*(x^4 + 3*x^2 +
2)*log(x^2 + 2) - 30*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 1236)/(x^4 + 3*x^2 +
2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^6}{6} - \frac{27x^4}{4} + 49x^2 + \frac{-207x^2-206}{2x^4+6x^2+4} - \frac{5\log(x^2+1)}{2} - 144\log\left(x^2+2\right)$$

```
[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**6/6 - 27*x**4/4 + 49*x**2 + (-207*x**2 - 206)/(2*x**4 + 6*x**2 + 4) -
5*log(x**2 + 1)/2 - 144*log(x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{27}{4}\,x^4 + 49\,x^2 - \frac{207\,x^2+206}{2\,(x^4+3\,x^2+2)} - 144\,\log\left(x^2+2\right) - \frac{5}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 - 1/2*(207*x^2 + 206)/(x^4 + 3*x^2 + 2) - 144*l
og(x^2 + 2) - 5/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{27}{4}\,x^4 + 49\,x^2 + \frac{293\,x^4 + 465\,x^2 + 174}{4\,(x^4 + 3\,x^2 + 2)} - 144\,\log\left(x^2+2\right) - \frac{5}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 + 1/4*(293*x^4 + 465*x^2 + 174)/(x^4 + 3*x^2 +
2) - 144*log(x^2 + 2) - 5/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 49\,x^2 - 144\,\ln\left(x^2+2\right) - \frac{\frac{207\,x^2}{2} + 103}{x^4 + 3\,x^2 + 2} - \frac{5\,\ln\left(x^2+1\right)}{2} - \frac{27\,x^4}{4} + \frac{5\,x^6}{6}$$

```
[In] int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
[Out] 49*x^2 - 144*log(x^2 + 2) - ((207*x^2)/2 + 103)/(3*x^2 + x^4 + 2) - (5*log(
x^2 + 1))/2 - (27*x^4)/4 + (5*x^6)/6
```

3.76 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 54

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx=-\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2(2+3x^2+x^4)}+3\log\left(1+x^2\right)+46\log\left(2+x^2\right)$$

```
[Out] -27/2*x^2+5/4*x^4+1/2*(103*x^2+102)/(x^4+3*x^2+2)+3*ln(x^2+1)+46*ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1677, 1674, 1671, 646, 31}

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx=\frac{5x^4}{4}-\frac{27x^2}{2}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)+\frac{103x^2+102}{2(x^4+3x^2+2)}$$

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(4+x+3x^2+5x^3)}{(2+3x+x^2)^2}\,dx, x, x^2\right)\\
&= \frac{102+103x^2}{2\,(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \frac{-50-27x+12x^2-5x^3}{2+3x+x^2}\,dx, x, x^2\right)\\
&= \frac{102+103x^2}{2\,(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \left(27-5x-\frac{2(52+49x)}{2+3x+x^2}\right)\,dx, x, x^2\right)\\
&= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\,(2+3x^2+x^4)}+\text{Subst}\left(\int \frac{52+49x}{2+3x+x^2}\,dx, x, x^2\right)\\
&= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\,(2+3x^2+x^4)}\\
&\quad + 3\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)+46\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right)
\end{aligned}$$

$$= -\frac{27x^2}{2} + \frac{5x^4}{4} + \frac{102+103x^2}{2\left(2+3x^2+x^4\right)} + 3\log\left(1+x^2\right) + 46\log\left(2+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{27x^2}{2} + \frac{5x^4}{4} + \frac{102+103x^2}{2\left(2+3x^2+x^4\right)} + 3\log\left(1+x^2\right) + 46\log\left(2+x^2\right)$$

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result
default	$46\ln\left(x^2+2\right)+\frac{52}{x^2+2}+\frac{5x^4}{4}-\frac{27x^2}{2}+3\ln\left(x^2+1\right)-\frac{1}{2(x^2+1)}$
norman	$\frac{\frac{277}{2}x^2-\frac{39}{4}x^6+\frac{5}{4}x^8+127}{x^4+3x^2+2}+3\ln\left(x^2+1\right)+46\ln\left(x^2+2\right)$
risch	$\frac{5x^4}{4}-\frac{27x^2}{2}+\frac{729}{20}+\frac{\frac{103x^2}{2}+51}{x^4+3x^2+2}+3\ln\left(x^2+1\right)+46\ln\left(x^2+2\right)$
parallelrisch	$\frac{5x^8-39x^6+12\ln(x^2+1)x^4+184\ln(x^2+2)x^4+508+36\ln(x^2+1)x^2+552\ln(x^2+2)x^2+554x^2+24\ln(x^2+1)+368\ln(x^2+2)}{4x^4+12x^2+8}$

```
[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] 46*ln(x^2+2)+52/(x^2+2)+5/4*x^4-27/2*x^2+3*ln(x^2+1)-1/2/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5\,x^8 - 39\,x^6 - 152\,x^4 + 98\,x^2 + 184\,(x^4+3\,x^2+2)\log\left(x^2+2\right) + 12\,(x^4+3\,x^2+2)\log\left(x^2+1\right) + 204}{4\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(5*x^8 - 39*x^6 - 152*x^4 + 98*x^2 + 184*(x^4 + 3*x^2 + 2)*log(x^2 + 2)
 + 12*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 204)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^4}{4}-\frac{27x^2}{2}+\frac{103x^2+102}{2x^4+6x^2+4}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)$$

```
[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**4/4 - 27*x**2/2 + (103*x**2 + 102)/(2*x**4 + 6*x**2 + 4) + 3*log(x**2
+ 1) + 46*log(x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4-\frac{27}{2}\,x^2+\frac{103\,x^2+102}{2\,(x^4+3\,x^2+2)} + 46\,\log\left(x^2+2\right)+3\,\log\left(x^2+1\right)$$

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 27/2*x^2 + 1/2*(103*x^2 + 102)/(x^4 + 3*x^2 + 2) + 46*log(x^2 + 2
) + 3*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4-\frac{27}{2}\,x^2-\frac{49\,x^4+44\,x^2-4}{2\,(x^4+3\,x^2+2)} + 46\,\log\left(x^2+2\right)+3\,\log\left(x^2+1\right)$$

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 27/2*x^2 - 1/2*(49*x^4 + 44*x^2 - 4)/(x^4 + 3*x^2 + 2) + 46*log(x
^2 + 2) + 3*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 3\,\ln\left(x^2+1\right)+46\,\ln\left(x^2+2\right)+\frac{\frac{103\,x^2}{2}+51}{x^4+3\,x^2+2}-\frac{27\,x^2}{2}+\frac{5\,x^4}{4}$$

```
[In] int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 3*log(x^2 + 1) + 46*log(x^2 + 2) + ((103*x^2)/2 + 51)/(3*x^2 + x^4 + 2) - (
27*x^2)/2 + (5*x^4)/4
```

3.77 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 49

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2} - \frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{7}{2}\log(1+x^2) - 10\log(2+x^2)$$

```
[Out] 5/2*x^2+1/2*(-51*x^2-50)/(x^4+3*x^2+2)-7/2*ln(x^2+1)-10*ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1677, 1674, 1671, 646, 31}

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2} - \frac{7}{2}\log(x^2+1) - 10\log(x^2+2) - \frac{51x^2+50}{2(x^4+3x^2+2)}$$

```
[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (50 + 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*L
og[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(4+x+3x^2+5x^3)}{(2+3x+x^2)^2}\,dx, x, x^2\right)\\
&= -\frac{50+51x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{24+12x-5x^2}{2+3x+x^2}\,dx, x, x^2\right)\\
&= -\frac{50+51x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \left(-5+\frac{34+27x}{2+3x+x^2}\right)\,dx, x, x^2\right)\\
&= \frac{5x^2}{2} - \frac{50+51x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{34+27x}{2+3x+x^2}\,dx, x, x^2\right)\\
&= \frac{5x^2}{2} - \frac{50+51x^2}{2\left(2+3x^2+x^4\right)} - \frac{7}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - 10\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right)\\
&= \frac{5x^2}{2} - \frac{50+51x^2}{2\left(2+3x^2+x^4\right)} - \frac{7}{2}\log\left(1+x^2\right) - 10\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2}+\frac{-50-51x^2}{2\,(2+3x^2+x^4)}-\frac{7}{2}\log\left(1+x^2\right)-10\log\left(2+x^2\right)$$

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
[Out] (5*x^2)/2 + (-50 - 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*
Log[2 + x^2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$-10\ln\left(x^2+2\right)-\frac{26}{x^2+2}+\frac{5x^2}{2}-\frac{7\ln\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$	41
norman	$\frac{-43x^2+\frac{5}{2}x^6-40}{x^4+3x^2+2}-\frac{7\ln\left(x^2+1\right)}{2}-10\ln\left(x^2+2\right)$	43
risch	$\frac{5x^2}{2}+\frac{-\frac{51x^2}{2}-25}{x^4+3x^2+2}-\frac{7\ln\left(x^2+1\right)}{2}-10\ln\left(x^2+2\right)$	43
parallelrisch	$-\frac{-5x^6+7\ln\left(x^2+1\right)x^4+20\ln\left(x^2+2\right)x^4+80+21\ln\left(x^2+1\right)x^2+60\ln\left(x^2+2\right)x^2+86x^2+14\ln\left(x^2+1\right)+40\ln\left(x^2+2\right)}{2(x^4+3x^2+2)}$	87

```
[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] -10*ln(x^2+2)-26/(x^2+2)+5/2*x^2-7/2*ln(x^2+1)+1/2/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5\,x^6+15\,x^4-41\,x^2-20\,(x^4+3\,x^2+2)\log\left(x^2+2\right)-7\,(x^4+3\,x^2+2)\log\left(x^2+1\right)-50}{2\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/2*(5*x^6 + 15*x^4 - 41*x^2 - 20*(x^4 + 3*x^2 + 2)*log(x^2 + 2) - 7*(x^4 +
 3*x^2 + 2)*log(x^2 + 1) - 50)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2} + \frac{-51x^2-50}{2x^4+6x^2+4} - \frac{7\log(x^2+1)}{2} - 10\log(x^2+2)$$

```
[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**2/2 + (-51*x**2 - 50)/(2*x**4 + 6*x**2 + 4) - 7*log(x**2 + 1)/2 - 10*l
og(x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{51\,x^2+50}{2\,(x^4+3\,x^2+2)} - 10\,\log(x^2+2) - \frac{7}{2}\,\log(x^2+1)$$

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/(x^4 + 3*x^2 + 2) - 10*log(x^2 + 2) - 7/2*log(x
^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{51\,x^2+50}{2\,(x^2+2)(x^2+1)} - 10\,\log(x^2+2) - \frac{7}{2}\,\log(x^2+1)$$

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/((x^2 + 2)*(x^2 + 1)) - 10*log(x^2 + 2) - 7/2*l
og(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx=\frac{5\,x^2}{2}-10\,\ln\left(x^2+2\right)-\frac{\frac{51\,x^2}{2}+25}{x^4+3\,x^2+2}-\frac{7\,\ln\left(x^2+1\right)}{2}$$

```
[In] int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
[Out] (5*x^2)/2 - 10*log(x^2 + 2) - ((51*x^2)/2 + 25)/(3*x^2 + x^4 + 2) - (7*log(
x^2 + 1))/2
```

3.78 $\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 42

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = \frac{24+25x^2}{2(2+3x^2+x^4)} + 4\log(1+x^2) - \frac{3}{2}\log(2+x^2)$$

```
[Out] 1/2*(25*x^2+24)/(x^4+3*x^2+2)+4*ln(x^2+1)-3/2*ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1677, 1674, 646, 31}

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = 4\log(x^2+1) - \frac{3}{2}\log(x^2+2) + \frac{25x^2+24}{2(x^4+3x^2+2)}$$

```
[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{-13-5x}{2+3x+x^2}\,dx,x,x^2\right)\\
&= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^2\right)+4\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)\\
&= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)}+4\log\left(1+x^2\right)-\frac{3}{2}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x(4+x^2+3x^4+5x^6)}{\left(2+3x^2+x^4\right)^2}\,dx=\frac{24+25x^2}{2\left(2+3x^2+x^4\right)}+4\log\left(1+x^2\right)-\frac{3}{2}\log\left(2+x^2\right)$$

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{3\ln(x^2+2)}{2}+\frac{13}{x^2+2}+4\ln\left(x^2+1\right)-\frac{1}{2(x^2+1)}$	36
norman	$\frac{\frac{25x^2}{2}+12}{x^4+3x^2+2}+4\ln\left(x^2+1\right)-\frac{3\ln(x^2+2)}{2}$	38
risch	$\frac{\frac{25x^2}{2}+12}{x^4+3x^2+2}+4\ln\left(x^2+1\right)-\frac{3\ln(x^2+2)}{2}$	38
parallelrisch	$\frac{8\ln(x^2+1)x^4-3\ln(x^2+2)x^4+24+24\ln(x^2+1)x^2-9\ln(x^2+2)x^2+25x^2+16\ln(x^2+1)-6\ln(x^2+2)}{2x^4+6x^2+4}$	82

```
[In] int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] -3/2*ln(x^2+2)+13/(x^2+2)+4*ln(x^2+1)-1/2/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2-3\,(x^4+3\,x^2+2)\log\left(x^2+2\right)+8\,(x^4+3\,x^2+2)\log\left(x^2+1\right)+24}{2\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/2*(25*x^2 - 3*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 8*(x^4 + 3*x^2 + 2)*log(x^
2 + 1) + 24)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25x^2+24}{2x^4+6x^2+4}+4\log\left(x^2+1\right)-\frac{3\log\left(x^2+2\right)}{2}$$

```
[In] integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
[Out] (25*x**2 + 24)/(2*x**4 + 6*x**2 + 4) + 4*log(x**2 + 1) - 3*log(x**2 + 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2+24}{2\,(x^4+3\,x^2+2)} - \frac{3}{2}\,\log\left(x^2+2\right) + 4\,\log\left(x^2+1\right)$$

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
[Out] 1/2*(25*x^2 + 24)/(x^4 + 3*x^2 + 2) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2+24}{2\,(x^2+2)(x^2+1)} - \frac{3}{2}\,\log\left(x^2+2\right) + 4\,\log\left(x^2+1\right)$$

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] 1/2*(25*x^2 + 24)/((x^2 + 2)*(x^2 + 1)) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 4\,\ln\left(x^2+1\right) - \frac{3\,\ln\left(x^2+2\right)}{2} + \frac{\frac{25\,x^2}{2}+12}{x^4+3\,x^2+2}$$

```
[In] int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
[Out] 4*log(x^2 + 1) - (3*log(x^2 + 2))/2 + ((25*x^2)/2 + 12)/(3*x^2 + x^4 + 2)
```

3.79 $\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 44

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)}+\log(x)-\frac{9}{2}\log\left(1+x^2\right)+4\log\left(2+x^2\right)$$

```
[Out] 1/2*(-12*x^2-11)/(x^4+3*x^2+2)+ln(x)-9/2*ln(x^2+1)+4*ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1677, 1660, 814}

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = -\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)}+\log(x)$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/2*(11 + 12*x^2)/(2 + 3*x^2 + x^4) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{-2+7x}{x\left(2+3x+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{9}{1+x}-\frac{8}{2+x}\right)\,dx,x,x^2\right)\\
&= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)}+\log(x)-\frac{9}{2}\log\left(1+x^2\right)+4\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx=\frac{-11-12x^2}{2\left(2+3x^2+x^4\right)}+\log(x)-\frac{9}{2}\log\left(1+x^2\right)+4\log\left(2+x^2\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
[Out] (-11 - 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result
default	$4\ln\left(x^{2}+2\right)-\frac{13}{2\left(x^{2}+2\right)}+\ln\left(x\right)-\frac{9\ln\left(x^{2}+1\right)}{2}+\frac{1}{2x^{2}+2}$
norman	$\frac{-6x^{2}-\frac{11}{2}}{x^{4}+3x^{2}+2}-\frac{9\ln\left(x^{2}+1\right)}{2}+4\ln\left(x^{2}+2\right)+\ln\left(x\right)$
risch	$\frac{-6x^{2}-\frac{11}{2}}{x^{4}+3x^{2}+2}-\frac{9\ln\left(x^{2}+1\right)}{2}+4\ln\left(x^{2}+2\right)+\ln\left(x\right)$
parallelrisch	$\frac{2\ln\left(x\right)x^{4}-9\ln\left(x^{2}+1\right)x^{4}+8\ln\left(x^{2}+2\right)x^{4}-11+6\ln\left(x\right)x^{2}-27\ln\left(x^{2}+1\right)x^{2}+24\ln\left(x^{2}+2\right)x^{2}-12x^{2}+4\ln\left(x\right)-18\ln\left(x^{2}+1\right)+16\ln(}{2x^{4}+6x^{2}+4}$

```
[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] 4*ln(x^2+2)-13/2/(x^2+2)+ln(x)-9/2*ln(x^2+1)+1/2/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = -\frac{12\,x^{2}-8\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+2\right)+9\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+1\right)-2\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x\right)+11}{2\,\left(x^{4}+3\,x^{2}+2\right)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] -1/2*(12*x^2 - 8*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 9*(x^4 + 3*x^2 + 2)*log(x
^2 + 1) - 2*(x^4 + 3*x^2 + 2)*log(x) + 11)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = \frac{-12x^{2}-11}{2x^{4}+6x^{2}+4}+\log\left(x\right)-\frac{9\log\left(x^{2}+1\right)}{2}+4\log\left(x^{2}+2\right)$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)
[Out] (-12*x**2 - 11)/(2*x**4 + 6*x**2 + 4) + log(x) - 9*log(x**2 + 1)/2 + 4*log(
x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = -\frac{12\,x^2+11}{2\,\left(x^4+3\,x^2+2\right)} + 4\,\log\left(x^2+2\right) - \frac{9}{2}\,\log\left(x^2+1\right) + \frac{1}{2}\,\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="maxima")
[Out] -1/2*(12*x^2 + 11)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 + 1) +
1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = \frac{x^4-21\,x^2-20}{4\,\left(x^4+3\,x^2+2\right)} + 4\,\log\left(x^2+2\right) - \frac{9}{2}\,\log\left(x^2+1\right) + \frac{1}{2}\,\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] 1/4*(x^4 - 21*x^2 - 20)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 +
1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = 4\,\ln\left(x^2+2\right) - \frac{9\,\ln\left(x^2+1\right)}{2} + \ln\left(x\right) - \frac{6\,x^2+\frac{11}{2}}{x^4+3\,x^2+2}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(3*x^2 + x^4 + 2)^2),x)
[Out] 4*log(x^2 + 2) - (9*log(x^2 + 1))/2 + log(x) - (6*x^2 + 11/2)/(3*x^2 + x^4
+ 2)
```

3.80 $\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 55

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = -\frac{1}{2x^2}+\frac{9+11x^2}{4\left(2+3x^2+x^4\right)}-\frac{11\log(x)}{4}+5\log\left(1+x^2\right)-\frac{29}{8}\log\left(2+x^2\right)$$

```
[Out] -1/2/x^2+1/4*(11*x^2+9)/(x^4+3*x^2+2)-11/4*ln(x)+5*ln(x^2+1)-29/8*ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1677, 1660, 1642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = -\frac{1}{2x^2}+5\log\left(x^2+1\right)-\frac{29}{8}\log\left(x^2+2\right)+\frac{11x^2+9}{4\left(x^4+3x^2+2\right)}-\frac{11\log(x)}{4}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/2*1/x^2 + (9 + 11*x^2)/(4*(2 + 3*x^2 + x^4)) - (11*Log[x])/4 + 5*Log[1 +
 x^2] - (29*Log[2 + x^2])/8
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^2\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{9+11x^2}{4\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{-2+\frac{5x}{2}-\frac{11x^2}{2}}{x^2\left(2+3x+x^2\right)}\,dx,x,x^2\right)\\
&= \frac{9+11x^2}{4\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{11}{4x}-\frac{10}{1+x}+\frac{29}{4(2+x)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{9+11x^2}{4\left(2+3x^2+x^4\right)}-\frac{11\log(x)}{4}+5\log\left(1+x^2\right)-\frac{29}{8}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{8}\left(-\frac{4}{x^2}+\frac{18+22x^2}{2+3x^2+x^4}-22\log(x)+40\log\left(1+x^2\right)-29\log\left(2+x^2\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
[Out] (-4/x^2 + (18 + 22*x^2)/(2 + 3*x^2 + x^4) - 22*Log[x] + 40*Log[1 + x^2] - 2
9*Log[2 + x^2])/8
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result
default	$-\frac{29\ln(x^2+2)}{8}+\frac{13}{4(x^2+2)}-\frac{1}{2x^2}-\frac{11\ln(x)}{4}+5\ln(x^2+1)-\frac{1}{2(x^2+1)}$
norman	$\frac{-1+\frac{3}{4}x^2+\frac{9}{4}x^4}{x^2(x^4+3x^2+2)}-\frac{11\ln(x)}{4}+5\ln(x^2+1)-\frac{29\ln(x^2+2)}{8}$
risch	$\frac{-1+\frac{3}{4}x^2+\frac{9}{4}x^4}{x^2(x^4+3x^2+2)}-\frac{11\ln(x)}{4}+5\ln(x^2+1)-\frac{29\ln(x^2+2)}{8}$
parallelrisch	$-\frac{22\ln(x)x^6-40\ln(x^2+1)x^6+29\ln(x^2+2)x^6+8+66\ln(x)x^4-120\ln(x^2+1)x^4+87\ln(x^2+2)x^4-18x^4+44\ln(x)x^2-80\ln(x^2\cdots}{8x^2(x^4+3x^2+2)}$

```
[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] -29/8*ln(x^2+2)+13/4/(x^2+2)-1/2/x^2-11/4*ln(x)+5*ln(x^2+1)-1/2/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2}\,dx = \frac{18\,x^4+6\,x^2-29\,(x^6+3\,x^4+2\,x^2)\log(x^2+2)+40\,(x^6+3\,x^4+2\,x^2)\log(x^2+1)-22\,(x^6+3\,x^4+2\,x^2\cdots}{8\,(x^6+3\,x^4+2\,x^2)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/8*(18*x^4 + 6*x^2 - 29*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 2) + 40*(x^6 + 3*x
^4 + 2*x^2)*log(x^2 + 1) - 22*(x^6 + 3*x^4 + 2*x^2)*log(x) - 8)/(x^6 + 3*x^
4 + 2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx=\frac{9x^4+3x^2-4}{4x^6+12x^4+8x^2}-\frac{11\log\left(x\right)}{4}+5\log\left(x^2+1\right)-\frac{29\log\left(x^2+2\right)}{8}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)
```

```
[Out] (9*x**4 + 3*x**2 - 4)/(4*x**6 + 12*x**4 + 8*x**2) - 11*log(x)/4 + 5*log(x**
2 + 1) - 29*log(x**2 + 2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx=\frac{9\,x^4+3\,x^2-4}{4\,\left(x^6+3\,x^4+2\,x^2\right)}-\frac{29}{8}\,\log\left(x^2+2\right)+5\,\log\left(x^2+1\right)-\frac{11}{8}\,\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(x
^2 + 1) - 11/8*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx=\frac{9\,x^4+3\,x^2-4}{4\,\left(x^6+3\,x^4+2\,x^2\right)}-\frac{29}{8}\,\log\left(x^2+2\right)+5\,\log\left(x^2+1\right)-\frac{11}{8}\,\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(x
^2 + 1) - 11/8*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = 5\,\ln\left(x^2+1\right)-\frac{29\,\ln\left(x^2+2\right)}{8}-\frac{11\,\ln\left(x\right)}{4}+\frac{\frac{9\,x^4}{4}+\frac{3\,x^2}{4}-1}{x^6+3\,x^4+2\,x^2}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] 5*log(x^2 + 1) - (29*log(x^2 + 2))/8 - (11*log(x))/4 + ((3*x^2)/4 + (9*x^4)
/4 - 1)/(2*x^2 + 3*x^4 + x^6)
```

3.81 $\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 64

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = -\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}+\frac{23\log(x)}{4} -\frac{11}{2}\log\left(1+x^2\right)+\frac{21}{8}\log\left(2+x^2\right)$$

```
[Out] -1/4/x^4+11/8/x^2+1/8*(-9*x^2-5)/(x^4+3*x^2+2)+23/4*ln(x)-11/2*ln(x^2+1)+21
/8*ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1677, 1660, 1642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = -\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{11}{2}\log\left(x^2+1\right)+\frac{21}{8}\log\left(x^2+2\right) -\frac{9x^2+5}{8\left(x^4+3x^2+2\right)}+\frac{23\log(x)}{4}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/4*1/x^4 + 11/(8*x^2) - (5 + 9*x^2)/(8*(2 + 3*x^2 + x^4)) + (23*Log[x])/4
 - (11*Log[1 + x^2])/2 + (21*Log[2 + x^2])/8
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^3\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{-2+\frac{5x}{2}-\frac{17x^2}{4}+\frac{9x^3}{4}}{x^3\left(2+3x+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^3}+\frac{11}{4x^2}-\frac{23}{4x}+\frac{11}{1+x}-\frac{21}{4(2+x)}\right)\,dx,x,x^2\right)\\
&= -\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}+\frac{23\log(x)}{4}-\frac{11}{2}\log\left(1+x^2\right)+\frac{21}{8}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{8}\left(-\frac{2}{x^4}+\frac{11}{x^2}-\frac{5+9x^2}{2+3x^2+x^4}+46\log(x)-44\log\left(1+x^2\right)+21\log\left(2+x^2\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-2/x^4 + 11/x^2 - (5 + 9*x^2)/(2 + 3*x^2 + x^4) + 46*Log[x] - 44*Log[1 + x
^2] + 21*Log[2 + x^2])/8
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result
default	$\frac{21\ln(x^2+2)}{8}-\frac{13}{8(x^2+2)}-\frac{1}{4x^4}+\frac{11}{8x^2}+\frac{23\ln(x)}{4}-\frac{11\ln(x^2+1)}{2}+\frac{1}{2x^2+2}$
norman	$\frac{-\frac{1}{2}+\frac{1}{4}x^6+\frac{13}{4}x^4+2x^2}{x^4(x^4+3x^2+2)}+\frac{23\ln(x)}{4}-\frac{11\ln(x^2+1)}{2}+\frac{21\ln(x^2+2)}{8}$
risch	$\frac{-\frac{1}{2}+\frac{1}{4}x^6+\frac{13}{4}x^4+2x^2}{x^4(x^4+3x^2+2)}+\frac{23\ln(x)}{4}-\frac{11\ln(x^2+1)}{2}+\frac{21\ln(x^2+2)}{8}$
parallelrisch	$\frac{46\ln(x)x^8-44\ln(x^2+1)x^8+21\ln(x^2+2)x^8-4+138\ln(x)x^6-132\ln(x^2+1)x^6+63\ln(x^2+2)x^6+2x^6+92\ln(x)x^4-88\ln(x^2+1)x}{8x^4(x^4+3x^2+2)}$

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 21/8*ln(x^2+2)-13/8/(x^2+2)-1/4/x^4+11/8/x^2+23/4*ln(x)-11/2*ln(x^2+1)+1/2/
(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2}\,dx = \frac{2\,x^6+26\,x^4+16\,x^2+21\,(x^8+3\,x^6+2\,x^4)\log(x^2+2)-44\,(x^8+3\,x^6+2\,x^4)\log(x^2+1)+46\,(x^8+3\,x^6}{8\,(x^8+3\,x^6+2\,x^4)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^6 + 26*x^4 + 16*x^2 + 21*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 2) - 44*(
x^8 + 3*x^6 + 2*x^4)*log(x^2 + 1) + 46*(x^8 + 3*x^6 + 2*x^4)*log(x) - 4)/(x
^8 + 3*x^6 + 2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{23\log{(x)}}{4} - \frac{11\log{(x^2+1)}}{2} + \frac{21\log{(x^2+2)}}{8} + \frac{x^6+13x^4+8x^2-2}{4x^8+12x^6+8x^4}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)
```

```
[Out] 23*log(x)/4 - 11*log(x**2 + 1)/2 + 21*log(x**2 + 2)/8 + (x**6 + 13*x**4 + 8
*x**2 - 2)/(4*x**8 + 12*x**6 + 8*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{x^6+13\,x^4+8\,x^2-2}{4\,(x^8+3\,x^6+2\,x^4)} + \frac{21}{8}\log\left(x^2+2\right) - \frac{11}{2}\log\left(x^2+1\right) + \frac{23}{8}\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^6 + 13*x^4 + 8*x^2 - 2)/(x^8 + 3*x^6 + 2*x^4) + 21/8*log(x^2 + 2) -
11/2*log(x^2 + 1) + 23/8*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{23\,x^4+51\,x^2+36}{16\,(x^4+3\,x^2+2)} - \frac{69\,x^4-22\,x^2+4}{16\,x^4} + \frac{21}{8}\log\left(x^2+2\right) - \frac{11}{2}\log\left(x^2+1\right) + \frac{23}{8}\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(23*x^4 + 51*x^2 + 36)/(x^4 + 3*x^2 + 2) - 1/16*(69*x^4 - 22*x^2 + 4)/
x^4 + 21/8*log(x^2 + 2) - 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{21\,\ln\left(x^2+2\right)}{8}-\frac{11\,\ln\left(x^2+1\right)}{2}+\frac{23\,\ln\left(x\right)}{4}+\frac{\frac{x^6}{4}+\frac{13\,x^4}{4}+2\,x^2-\frac{1}{2}}{x^8+3\,x^6+2\,x^4}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (21*log(x^2 + 2))/8 - (11*log(x^2 + 1))/2 + (23*log(x))/4 + (2*x^2 + (13*x^
4)/4 + x^6/4 - 1/2)/(2*x^4 + 3*x^6 + x^8)
```

3.82 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 70

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2(2+3x^2+x^4)} + \frac{9\arctan(x)}{2}+340\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -293*x+98/3*x^3-27/5*x^5+5/7*x^7-1/2*x*(207*x^2+206)/(x^4+3*x^2+2)+9/2*arct
an(x)+340*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1682, 1690, 1180, 209}

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = \frac{9\arctan(x)}{2}+340\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{5x^7}{7} -\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{(207x^2+206)x}{2(x^4+3x^2+2)}-293x$$

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 - (x*(206 + 207*x^2))/(2*(2 +
3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(206+207x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \frac{-412-6x^2+212x^4-108x^6+48x^8-20x^{10}}{2+3x^2+x^4}\,dx \\
&= -\frac{x(206+207x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \left(1172-392x^2+108x^4-20x^6-\frac{2(1378+1369x^2)}{2+3x^2+x^4}\right)dx \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2(2+3x^2+x^4)}+\frac{1}{2}\int \frac{1378+1369x^2}{2+3x^2+x^4}\,dx \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2(2+3x^2+x^4)}+\frac{9}{2}\int \frac{1}{1+x^2}\,dx+680\int \frac{1}{2+x^2}\,dx \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2(2+3x^2+x^4)}+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}+\frac{-206x-207x^3}{2\,(2+3x^2+x^4)} + \frac{9\arctan(x)}{2}+340\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 + (-206*x - 207*x^3)/(2*(2 + 3
*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{104x}{x^2+2}+340\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}+\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{x}{2x^2+2}+\frac{9\arctan(x)}{2}$	56
risch	$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{-\frac{207}{2}x^3-103x}{x^4+3x^2+2}+\frac{9\arctan(x)}{2}+340\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	58

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -104*x/(x^2+2)+340*arctan(1/2*x*2^(1/2))*2^(1/2)+5/7*x^7-27/5*x^5+98/3*x^3-
293*x+1/2*x/(x^2+1)+9/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{150\,x^{11}-684\,x^9+3758\,x^7-43218\,x^5-192605\,x^3+71400\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+945\,(x^4+}{210\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/210*(150*x^11 - 684*x^9 + 3758*x^7 - 43218*x^5 - 192605*x^3 + 71400*sqrt(
2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 945*(x^4 + 3*x^2 + 2)*arctan(x
) - 144690*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - 293x + \frac{-207x^3-206x}{2x^4+6x^2+4} + \frac{9\,\mathrm{atan}\,(x)}{2} + 340\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**7/7 - 27*x**5/5 + 98*x**3/3 - 293*x + (-207*x**3 - 206*x)/(2*x**4 + 6*
x**2 + 4) + 9*atan(x)/2 + 340*sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{7}\,x^7 - \frac{27}{5}\,x^5 + \frac{98}{3}\,x^3 + 340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 293\,x - \frac{207\,x^3+206\,x}{2\,(x^4+3\,x^2+2)} + \frac{9}{2}\,\arctan\left(x\right)$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x -
 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{7}\,x^7 - \frac{27}{5}\,x^5 + \frac{98}{3}\,x^3 + 340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 293\,x - \frac{207\,x^3+206\,x}{2\,(x^4+3\,x^2+2)} + \frac{9}{2}\,\arctan\left(x\right)$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x -
 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{9\,\mathrm{atan}(x)}{2} - 293\,x + 340\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{207\,x^3}{2}+103\,x}{x^4+3\,x^2+2} + \frac{98\,x^3}{3} - \frac{27\,x^5}{5} + \frac{5\,x^7}{7}$$

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] (9*atan(x))/2 - 293*x + 340*2^(1/2)*atan((2^(1/2)*x)/2) - (103*x + (207*x^3
)/2)/(3*x^2 + x^4 + 2) + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7
```

3.83 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 57

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 98x-9x^3+x^5+\frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{11\arctan(x)}{2}-118\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 98*x-9*x^3+x^5+1/2*x*(103*x^2+102)/(x^4+3*x^2+2)-11/2*arctan(x)-118*arctan(
1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1682, 1690, 1180, 209}

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{11\arctan(x)}{2}-118\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + x^5-9x^3+\frac{(103x^2+102)\,x}{2(x^4+3x^2+2)}+98x$$

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 98*x - 9*x^3 + x^5 + (x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan
[x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \frac{204+6x^2-108x^4+48x^6-20x^8}{2+3x^2+x^4}\,dx \\
&= \frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \left(-392+108x^2-20x^4+\frac{2(494+483x^2)}{2+3x^2+x^4}\right)dx \\
&= 98x-9x^3+x^5+\frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{1}{2}\int \frac{494+483x^2}{2+3x^2+x^4}\,dx \\
&= 98x-9x^3+x^5+\frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{11}{2}\int \frac{1}{1+x^2}\,dx - 236\int \frac{1}{2+x^2}\,dx \\
&= 98x-9x^3+x^5+\frac{x(102+103x^2)}{2(2+3x^2+x^4)} - \frac{11}{2}\tan^{-1}(x) - 118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 98x-9x^3+x^5+\frac{102x+103x^3}{2(2+3x^2+x^4)} - \frac{11\arctan(x)}{2} - 118\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
[Out] 98*x - 9*x^3 + x^5 + (102*x + 103*x^3)/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan[x
])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{52x}{x^2+2}-118\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}+x^5-9x^3+98x-\frac{x}{2(x^2+1)}-\frac{11\arctan(x)}{2}$	49
risch	$x^5-9x^3+98x+\frac{\frac{103}{2}x^3+51x}{x^4+3x^2+2}-\frac{11\arctan(x)}{2}-118\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	51

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] 52*x/(x^2+2)-118*arctan(1/2*x*2^(1/2))*2^(1/2)+x^5-9*x^3+98*x-1/2*x/(x^2+1)
-11/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{2\,x^9-12\,x^7+146\,x^5+655\,x^3-236\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-11\,(x^4+3\,x^2+2)\arctan(x)+4}{2\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/2*(2*x^9 - 12*x^7 + 146*x^5 + 655*x^3 - 236*sqrt(2)*(x^4 + 3*x^2 + 2)*arc
tan(1/2*sqrt(2)*x) - 11*(x^4 + 3*x^2 + 2)*arctan(x) + 494*x)/(x^4 + 3*x^2 +
 2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5-9x^3+98x+\frac{103x^3+102x}{2x^4+6x^2+4} - \frac{11\,\mathrm{atan}\,(x)}{2} - 118\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] x**5 - 9*x**3 + 98*x + (103*x**3 + 102*x)/(2*x**4 + 6*x**2 + 4) - 11*atan(x
)/2 - 118*sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5-9\,x^3-118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 98\,x+\frac{103\,x^3+102\,x}{2\,(x^4+3\,x^2+2)} - \frac{11}{2}\,\arctan\,(x)$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 102
*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5-9\,x^3-118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 98\,x+\frac{103\,x^3+102\,x}{2\,(x^4+3\,x^2+2)} - \frac{11}{2}\,\arctan\,(x)$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 102
*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 98\,x-\frac{11\,\mathrm{atan}(x)}{2}-118\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)+\frac{\frac{103\,x^3}{2}+51\,x}{x^4+3\,x^2+2}-9\,x^3+x^5$$

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 98*x - (11*atan(x))/2 - 118*2^(1/2)*atan((2^(1/2)*x)/2) + (51*x + (103*x^3)
/2)/(3*x^2 + x^4 + 2) - 9*x^3 + x^5
```

3.84 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = -27x + \frac{5x^3}{3} - \frac{x(50+51x^2)}{2(2+3x^2+x^4)} + \frac{13\arctan(x)}{2} + 33\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -27*x+5/3*x^3-1/2*x*(51*x^2+50)/(x^4+3*x^2+2)+13/2*arctan(x)+33*arctan(1/2*
x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1682, 1690, 1180, 209}

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} \, dx = \frac{13\arctan(x)}{2} + 33\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{5x^3}{3} - \frac{(51x^2+50)x}{2(x^4+3x^2+2)} - 27x$$

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 - (x*(50 + 51*x^2))/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x]
)/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)} - \frac{1}{4}\int \frac{-100-6x^2+48x^4-20x^6}{2+3x^2+x^4}\,dx \\
&= -\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)} - \frac{1}{4}\int \left(108-20x^2-\frac{2(158+145x^2)}{2+3x^2+x^4}\right)dx \\
&= -27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)}+\frac{1}{2}\int \frac{158+145x^2}{2+3x^2+x^4}\,dx \\
&= -27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)}+\frac{13}{2}\int \frac{1}{1+x^2}\,dx+66\int \frac{1}{2+x^2}\,dx \\
&= -27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)}+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -27x+\frac{5x^3}{3}+\frac{-50x-51x^3}{2(2+3x^2+x^4)} + \frac{13\arctan(x)}{2}+33\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 + (-50*x - 51*x^3)/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x])
/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{26x}{x^2+2}+33\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}+\frac{5x^3}{3}-27x+\frac{x}{2x^2+2}+\frac{13\arctan(x)}{2}$	46
risch	$\frac{5x^3}{3}-27x+\frac{-\frac{51}{2}x^3-25x}{x^4+3x^2+2}+\frac{13\arctan(x)}{2}+33\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	48

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -26*x/(x^2+2)+33*arctan(1/2*x*2^(1/2))*2^(1/2)+5/3*x^3-27*x+1/2*x/(x^2+1)+1
3/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{10\,x^7-132\,x^5-619\,x^3+198\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+39\,(x^4+3\,x^2+2)\arctan\left(x\right)-474\,x}{6\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*x^7 - 132*x^5 - 619*x^3 + 198*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*
sqrt(2)*x) + 39*(x^4 + 3*x^2 + 2)*arctan(x) - 474*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^3}{3} - 27x + \frac{-51x^3-50x}{2x^4+6x^2+4} + \frac{13\,\mathrm{atan}\,(x)}{2} + 33\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**3/3 - 27*x + (-51*x**3 - 50*x)/(2*x**4 + 6*x**2 + 4) + 13*atan(x)/2 +
33*sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{3}\,x^3 + 33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 27\,x - \frac{51\,x^3+50\,x}{2\,(x^4+3\,x^2+2)} + \frac{13}{2}\,\arctan\left(x\right)$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x^
4 + 3*x^2 + 2) + 13/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{3}\,x^3 + 33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 27\,x - \frac{51\,x^3+50\,x}{2\,(x^4+3\,x^2+2)} + \frac{13}{2}\,\arctan\left(x\right)$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x^
4 + 3*x^2 + 2) + 13/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{13\,\mathrm{atan}(x)}{2} - 27\,x + 33\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{51\,x^3}{2} + 25\,x}{x^4 + 3\,x^2 + 2} + \frac{5\,x^3}{3}$$

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
[Out] (13*atan(x))/2 - 27*x + 33*2^(1/2)*atan((2^(1/2)*x)/2) - (25*x + (51*x^3)/2
)/(3*x^2 + x^4 + 2) + (5*x^3)/3
```

3.85 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 49

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{15\arctan(x)}{2} - \frac{7\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 5*x+1/2*x*(25*x^2+24)/(x^4+3*x^2+2)-15/2*arctan(x)-7/2*arctan(1/2*x*2^(1/2)
)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1682, 1690, 1180, 209}

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = -\frac{15\arctan(x)}{2} - \frac{7\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{(25x^2+24)x}{2(x^4+3x^2+2)} + 5x$$

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTa
n[x/Sqrt[2]])/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \frac{48-2x^2-20x^4}{2+3x^2+x^4}\,dx \\
&= \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \left(-20+\frac{2(44+29x^2)}{2+3x^2+x^4}\right)dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{2}\int \frac{44+29x^2}{2+3x^2+x^4}\,dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - 7\int \frac{1}{2+x^2}\,dx - \frac{15}{2}\int \frac{1}{1+x^2}\,dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{15}{2}\tan^{-1}(x) - \frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 5x+\frac{24x+25x^3}{2(2+3x^2+x^4)}-\frac{15\arctan(x)}{2}-\frac{7\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (24*x + 25*x^3)/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTan[
x/Sqrt[2]])/Sqrt[2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$5x+\frac{13x}{x^2+2}-\frac{7\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{x}{2(x^2+1)}-\frac{15\arctan(x)}{2}$	41
risch	$5x+\frac{\frac{25}{2}x^3+12x}{x^4+3x^2+2}-\frac{7\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{15\arctan(x)}{2}$	43

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x+13*x/(x^2+2)-7/2*arctan(1/2*x*2^(1/2))*2^(1/2)-1/2*x/(x^2+1)-15/2*arcta
n(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{10\,x^5+55\,x^3-7\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-15\,(x^4+3\,x^2+2)\arctan\left(x\right)+44\,x}{2\,(x^4+3\,x^2+2)}$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(10*x^5 + 55*x^3 - 7*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) -
15*(x^4 + 3*x^2 + 2)*arctan(x) + 44*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 5x+\frac{25x^3+24x}{2x^4+6x^2+4}-\frac{15\,\mathrm{atan}\,(x)}{2}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x + (25*x**3 + 24*x)/(2*x**4 + 6*x**2 + 4) - 15*atan(x)/2 - 7*sqrt(2)*ata
n(sqrt(2)*x/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x+\frac{25\,x^3+24\,x}{2\,(x^4+3\,x^2+2)}-\frac{15}{2}\arctan\left(x\right)$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2
 + 2) - 15/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x+\frac{25\,x^3+24\,x}{2\,(x^4+3\,x^2+2)}-\frac{15}{2}\arctan\left(x\right)$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2
 + 2) - 15/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 5\,x - \frac{15\,\mathrm{atan}(x)}{2} - \frac{7\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} + \frac{\frac{25\,x^3}{2}+12\,x}{x^4+3\,x^2+2}$$

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 5*x - (15*atan(x))/2 - (7*2^(1/2)*atan((2^(1/2)*x)/2))/2 + (12*x + (25*x^3)
/2)/(3*x^2 + x^4 + 2)
```

3.86 $\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 48

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2} \, dx = -\frac{x(11+12x^2)}{2\left(2+3x^2+x^4\right)}+\frac{17\arctan(x)}{2}-\frac{19\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -1/2*x*(12*x^2+11)/(x^4+3*x^2+2)+17/2*arctan(x)-19/4*arctan(1/2*x*2^(1/2))*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1692, 1180, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2} \, dx = \frac{17\arctan(x)}{2}-\frac{19\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{x(12x^2+11)}{2\left(x^4+3x^2+2\right)}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -1/2*(x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + (17*ArcTan[x])/2 - (19*ArcTan[x/
Sqrt[2]])/(2*Sqrt[2])
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(11+12x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \frac{-30+4x^2}{2+3x^2+x^4}\,dx \\
&= -\frac{x(11+12x^2)}{2(2+3x^2+x^4)} + \frac{17}{2}\int \frac{1}{1+x^2}\,dx - \frac{19}{2}\int \frac{1}{2+x^2}\,dx \\
&= -\frac{x(11+12x^2)}{2(2+3x^2+x^4)} + \frac{17}{2}\tan^{-1}(x) - \frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = \frac{1}{4}\left(-\frac{2x(11+12x^2)}{2+3x^2+x^4} + 34\arctan(x) - 19\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
[Out] ((-2*x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + 34*ArcTan[x] - 19*Sqrt[2]*ArcTan[
x/Sqrt[2]])/4
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{13x}{2(x^2+2)}-\frac{19\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{x}{2x^2+2}+\frac{17\arctan(x)}{2}$	38
risch	$\frac{-6x^3-\frac{11}{2}x}{x^4+3x^2+2}+\frac{17\arctan(x)}{2}-\frac{19\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}$	40

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] -13/2*x/(x^2+2)-19/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/2*x/(x^2+1)+17/2*arcta
n(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = -\frac{24\,x^3+19\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-34\,(x^4+3\,x^2+2)\arctan(x)+22\,x}{4\,(x^4+3\,x^2+2)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] -1/4*(24*x^3 + 19*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 34*(x^4
 + 3*x^2 + 2)*arctan(x) + 22*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = \frac{-12x^3-11x}{2x^4+6x^2+4}+\frac{17\,\mathrm{atan}\,(x)}{2}-\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
[Out] (-12*x**3 - 11*x)/(2*x**4 + 6*x**2 + 4) + 17*atan(x)/2 - 19*sqrt(2)*atan(sq
rt(2)*x/2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2}\,dx = -\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{12\,x^3+11\,x}{2\left(x^4+3\,x^2+2\right)}+\frac{17}{2}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2)
 + 17/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2}\,dx = -\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{12\,x^3+11\,x}{2\left(x^4+3\,x^2+2\right)}+\frac{17}{2}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2)
 + 17/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2}\,dx = \frac{17\,\mathrm{atan}(x)}{2}-\frac{19\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4}-\frac{6\,x^3+\frac{11\,x}{2}}{x^4+3\,x^2+2}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^2,x)
[Out] (17*atan(x))/2 - (19*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((11*x)/2 + 6*x^3)/(3
*x^2 + x^4 + 2)
```

3.87 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 53

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} dx = -\frac{1}{x}+\frac{x(9+11x^2)}{4(2+3x^2+x^4)}-\frac{19\arctan(x)}{2}+\frac{45\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/x+1/4*x*(11*x^2+9)/(x^4+3*x^2+2)-19/2*arctan(x)+45/8*arctan(1/2*x*2^(1/2
))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} dx = -\frac{19\arctan(x)}{2}+\frac{45\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{x(11x^2+9)}{4(x^4+3x^2+2)}-\frac{1}{x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -x^(-1) + (x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) - (19*ArcTan[x])/2 + (45*A
rcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-8+19x^2-11x^4}{x^2\left(2+3x^2+x^4\right)}\,dx \\
&= \frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \left(-\frac{4}{x^2}+\frac{38}{1+x^2}-\frac{45}{2+x^2}\right)dx \\
&= -\frac{1}{x}+\frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)} - \frac{19}{2}\int \frac{1}{1+x^2}\,dx + \frac{45}{4}\int \frac{1}{2+x^2}\,dx \\
&= -\frac{1}{x}+\frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)} - \frac{19}{2}\tan^{-1}(x) + \frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{8}\left(-\frac{8}{x}+\frac{2x(9+11x^2)}{2+3x^2+x^4}-76\arctan(x)+45\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
[Out] (-8/x + (2*x*(9 + 11*x^2))/(2 + 3*x^2 + x^4) - 76*ArcTan[x] + 45*Sqrt[2]*Ar
cTan[x/Sqrt[2]])/8
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{13x}{4(x^2+2)}+\frac{45\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{x}-\frac{x}{2(x^2+1)}-\frac{19\arctan(x)}{2}$	43
risch	$\frac{\frac{7}{4}x^4-\frac{3}{4}x^2-2}{x(x^4+3x^2+2)}-\frac{19\arctan(x)}{2}+\frac{45\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}$	46

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] 13/4*x/(x^2+2)+45/8*arctan(1/2*x*2^(1/2))*2^(1/2)-1/x-1/2*x/(x^2+1)-19/2*ar
ctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2}\,dx = \frac{14\,x^4+45\,\sqrt{2}(x^5+3\,x^3+2\,x)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-6\,x^2-76\,(x^5+3\,x^3+2\,x)\arctan(x)-16}{8\,(x^5+3\,x^3+2\,x)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/8*(14*x^4 + 45*sqrt(2)*(x^5 + 3*x^3 + 2*x)*arctan(1/2*sqrt(2)*x) - 6*x^2
- 76*(x^5 + 3*x^3 + 2*x)*arctan(x) - 16)/(x^5 + 3*x^3 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2}\,dx = \frac{7x^4-3x^2-8}{4x^5+12x^3+8x}-\frac{19\,\mathrm{atan}\,(x)}{2}+\frac{45\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)
[Out] (7*x**4 - 3*x**2 - 8)/(4*x**5 + 12*x**3 + 8*x) - 19*atan(x)/2 + 45*sqrt(2)*
atan(sqrt(2)*x/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{7\,x^4-3\,x^2-8}{4\left(x^5+3\,x^3+2\,x\right)} - \frac{19}{2}\,\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 +
 2*x) - 19/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{7\,x^4-3\,x^2-8}{4\left(x^5+3\,x^3+2\,x\right)} - \frac{19}{2}\,\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 +
 2*x) - 19/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8} - \frac{19\,\mathrm{atan}(x)}{2} - \frac{-\frac{7\,x^4}{4}+\frac{3\,x^2}{4}+2}{x^5+3\,x^3+2\,x}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (45*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (19*atan(x))/2 - ((3*x^2)/4 - (7*x^4)/
4 + 2)/(2*x + 3*x^3 + x^5)
```

3.88 $\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 62

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2} dx = -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x(5+9x^2)}{8(2+3x^2+x^4)}+\frac{21\arctan(x)}{2}-\frac{71\arctan\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] -1/3/x^3+11/4/x-1/8*x*(9*x^2+5)/(x^4+3*x^2+2)+21/2*arctan(x)-71/16*arctan(1
/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2} dx = \frac{21\arctan(x)}{2}-\frac{71\arctan\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{1}{3x^3}-\frac{x(9x^2+5)}{8(x^4+3x^2+2)}+\frac{11}{4x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/3*1/x^3 + 11/(4*x) - (x*(5 + 9*x^2))/(8*(2 + 3*x^2 + x^4)) + (21*ArcTan[
x])/2 - (71*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(5+9x^2)}{8(2+3x^2+x^4)} - \frac{1}{4}\int \frac{-8+10x^2-\frac{39x^4}{2}+\frac{9x^6}{2}}{x^4(2+3x^2+x^4)}\,dx \\
&= -\frac{x(5+9x^2)}{8(2+3x^2+x^4)} - \frac{1}{4}\int \left(-\frac{4}{x^4}+\frac{11}{x^2}-\frac{42}{1+x^2}+\frac{71}{2(2+x^2)}\right)dx \\
&= -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x(5+9x^2)}{8(2+3x^2+x^4)}-\frac{71}{8}\int \frac{1}{2+x^2}\,dx+\frac{21}{2}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x(5+9x^2)}{8(2+3x^2+x^4)}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx = \frac{1}{48}\left(-\frac{16}{x^3}+\frac{132}{x}-\frac{6x(5+9x^2)}{2+3x^2+x^4}+504\arctan(x) - 213\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
[Out] (-16/x^3 + 132/x - (6*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + 504*ArcTan[x] - 21
3*Sqrt[2]*ArcTan[x/Sqrt[2]])/48
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{13x}{8(x^2+2)}-\frac{71\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{16}-\frac{1}{3x^3}+\frac{11}{4x}+\frac{x}{2x^2+2}+\frac{21\arctan(x)}{2}$	48
risch	$\frac{\frac{13}{8}x^6+\frac{175}{24}x^4+\frac{9}{2}x^2-\frac{2}{3}}{x^3(x^4+3x^2+2)}-\frac{71\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{16}+\frac{21\arctan(x)}{2}$	51

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -13/8*x/(x^2+2)-71/16*arctan(1/2*x*2^(1/2))*2^(1/2)-1/3/x^3+11/4/x+1/2*x/(x
^2+1)+21/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx$$
$$=\frac{78\,x^6+350\,x^4-213\,\sqrt{2}(x^7+3\,x^5+2\,x^3)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+216\,x^2+504\,(x^7+3\,x^5+2\,x^3)\arctan(x)-}{48\,(x^7+3\,x^5+2\,x^3)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(78*x^6 + 350*x^4 - 213*sqrt(2)*(x^7 + 3*x^5 + 2*x^3)*arctan(1/2*sqrt(
2)*x) + 216*x^2 + 504*(x^7 + 3*x^5 + 2*x^3)*arctan(x) - 32)/(x^7 + 3*x^5 +
2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx=\frac{21\operatorname{atan}(x)}{2}-\frac{71\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}+\frac{39x^6+175x^4+108x^2-16}{24x^7+72x^5+48x^3}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)
```

```
[Out] 21*atan(x)/2 - 71*sqrt(2)*atan(sqrt(2)*x/2)/16 + (39*x**6 + 175*x**4 + 108*
x**2 - 16)/(24*x**7 + 72*x**5 + 48*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx = -\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{39\,x^6+175\,x^4+108\,x^2-16}{24\,(x^7+3\,x^5+2\,x^3)} + \frac{21}{2}\arctan(x)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="maxima")
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/24*(39*x^6 + 175*x^4 + 108*x^2 - 1
6)/(x^7 + 3*x^5 + 2*x^3) + 21/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx = -\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{9\,x^3+5\,x}{8\,(x^4+3\,x^2+2)} + \frac{33\,x^2-4}{12\,x^3} + \frac{21}{2}\arctan(x)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(9*x^3 + 5*x)/(x^4 + 3*x^2 + 2)
+ 1/12*(33*x^2 - 4)/x^3 + 21/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx = \frac{21\,\mathrm{atan}(x)}{2} - \frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16} + \frac{\frac{13\,x^6}{8}+\frac{175\,x^4}{24}+\frac{9\,x^2}{2}-\frac{2}{3}}{x^7+3\,x^5+2\,x^3}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^2),x)
[Out] (21*atan(x))/2 - (71*2^(1/2)*atan((2^(1/2)*x)/2))/16 + ((9*x^2)/2 + (175*x^
4)/24 + (13*x^6)/8 - 2/3)/(2*x^3 + 3*x^5 + x^7)
```

3.89 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 69

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx=-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x(3-5x^2)}{16\left(2+3x^2+x^4\right)}-\frac{23\arctan(x)}{2}+\frac{97\arctan\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -1/5/x^5+11/12/x^3-23/4/x-1/16*x*(-5*x^2+3)/(x^4+3*x^2+2)-23/2*arctan(x)+97
/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx=-\frac{23\arctan(x)}{2}+\frac{97\arctan\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{x(3-5x^2)}{16\left(x^4+3x^2+2\right)}-\frac{23}{4x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/5*1/x^5 + 11/(12*x^3) - 23/(4*x) - (x*(3 - 5*x^2))/(16*(2 + 3*x^2 + x^4)
) - (23*ArcTan[x])/2 + (97*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x(3-5x^2)}{16\,(2+3x^2+x^4)} - \frac{1}{4}\int \frac{-8+10x^2-17x^4+\frac{39x^6}{4}-\frac{5x^8}{4}}{x^6\,(2+3x^2+x^4)}\,dx \\
&= -\frac{x(3-5x^2)}{16\,(2+3x^2+x^4)} - \frac{1}{4}\int \left(-\frac{4}{x^6}+\frac{11}{x^4}-\frac{23}{x^2}+\frac{46}{1+x^2}-\frac{97}{4\,(2+x^2)}\right)dx \\
&= -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x(3-5x^2)}{16\,(2+3x^2+x^4)}+\frac{97}{16}\int \frac{1}{2+x^2}\,dx-\frac{23}{2}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x(3-5x^2)}{16\,(2+3x^2+x^4)}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx=\frac{1}{480}\left(-\frac{96}{x^5}+\frac{440}{x^3}-\frac{2760}{x}+\frac{30x(-3+5x^2)}{2+3x^2+x^4}-5520\arctan(x)+1455\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^5 + 440/x^3 - 2760/x + (30*x*(-3 + 5*x^2))/(2 + 3*x^2 + x^4) - 5520*
ArcTan[x] + 1455*Sqrt[2]*ArcTan[x/Sqrt[2]])/480
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{13x}{16(x^2+2)}+\frac{97\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{32}-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x}{2(x^2+1)}-\frac{23\arctan(x)}{2}$	53
risch	$\frac{-\frac{87}{16}x^8-\frac{793}{48}x^6-\frac{179}{20}x^4+\frac{37}{30}x^2-\frac{2}{5}}{x^5(x^4+3x^2+2)}+\frac{97\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{32}-\frac{23\arctan(x)}{2}$	56

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 13/16*x/(x^2+2)+97/32*arctan(1/2*x*2^(1/2))*2^(1/2)-1/5/x^5+11/12/x^3-23/4/
x-1/2*x/(x^2+1)-23/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx=-\frac{2610\,x^8+7930\,x^6+4296\,x^4-1455\,\sqrt{2}(x^9+3\,x^7+2\,x^5)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-592\,x^2+5520\,(x^9+3\,x^7+}{480\,(x^9+3\,x^7+2\,x^5)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/480*(2610*x^8 + 7930*x^6 + 4296*x^4 - 1455*sqrt(2)*(x^9 + 3*x^7 + 2*x^5)
*arctan(1/2*sqrt(2)*x) - 592*x^2 + 5520*(x^9 + 3*x^7 + 2*x^5)*arctan(x) + 1
92)/(x^9 + 3*x^7 + 2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx = -\frac{23\,\mathrm{atan}\,(x)}{2}+\frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32} + \frac{-1305x^8-3965x^6-2148x^4+296x^2-96}{240x^9+720x^7+480x^5}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)
```

```
[Out] -23*atan(x)/2 + 97*sqrt(2)*atan(sqrt(2)*x/2)/32 + (-1305*x**8 - 3965*x**6 -
 2148*x**4 + 296*x**2 - 96)/(240*x**9 + 720*x**7 + 480*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx = \frac{97}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1305\,x^8+3965\,x^6+2148\,x^4-296\,x^2+96}{240\,(x^9+3\,x^7+2\,x^5)} - \frac{23}{2}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/240*(1305*x^8 + 3965*x^6 + 2148*x^4
 - 296*x^2 + 96)/(x^9 + 3*x^7 + 2*x^5) - 23/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx = \frac{97}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{5\,x^3-3\,x}{16\,(x^4+3\,x^2+2)} - \frac{345\,x^4-55\,x^2+12}{60\,x^5} - \frac{23}{2}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(5*x^3 - 3*x)/(x^4 + 3*x^2 + 2)
- 1/60*(345*x^4 - 55*x^2 + 12)/x^5 - 23/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = \frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}-\frac{23\,\mathrm{atan}(x)}{2}-\frac{\frac{87\,x^8}{16}+\frac{793\,x^6}{48}+\frac{179\,x^4}{20}-\frac{37\,x^2}{30}+\frac{2}{5}}{x^9+3\,x^7+2\,x^5}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (97*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (23*atan(x))/2 - ((179*x^4)/20 - (37*
x^2)/30 + (793*x^6)/48 + (87*x^8)/16 + 2/5)/(2*x^5 + 3*x^7 + x^9)
```

3.90 $\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x(19+3x^2)}{32\left(2+3x^2+x^4\right)}+\frac{25\arctan(x)}{2}-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/7/x^7+11/20/x^5-23/12/x^3+137/16/x+1/32*x*(3*x^2+19)/(x^4+3*x^2+2)+25/2*
arctan(x)-123/64*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = \frac{25\arctan(x)}{2}-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{x(3x^2+19)}{32\left(x^4+3x^2+2\right)}+\frac{137}{16x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/7*1/x^7 + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (x*(19 + 3*x^2))/(32*
(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2]
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(19+3x^2)}{32\,(2+3x^2+x^4)} - \frac{1}{4}\int \frac{-8+10x^2-17x^4+\frac{21x^6}{2}-\frac{39x^8}{8}-\frac{3x^{10}}{8}}{x^8\,(2+3x^2+x^4)}\,dx \\
&= \frac{x(19+3x^2)}{32\,(2+3x^2+x^4)} - \frac{1}{4}\int \left(-\frac{4}{x^8}+\frac{11}{x^6}-\frac{23}{x^4}+\frac{137}{4x^2}-\frac{50}{1+x^2}+\frac{123}{8\,(2+x^2)}\right)dx \\
&= -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x(19+3x^2)}{32\,(2+3x^2+x^4)}-\frac{123}{32}\int \frac{1}{2+x^2}\,dx+\frac{25}{2}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x(19+3x^2)}{32\,(2+3x^2+x^4)}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx = -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{19x+3x^3}{32(2+3x^2+x^4)} + \frac{25\arctan(x)}{2}-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/7*1/x^7 + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (19*x + 3*x^3)/(32*(2
 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{13x}{32(x^2+2)}-\frac{123\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{64}-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x}{2x^2+2}+\frac{25\arctan(x)}{2}$	58
risch	$\frac{\frac{277}{32}x^{10}+\frac{2339}{96}x^8+\frac{477}{40}x^6-\frac{977}{420}x^4+\frac{47}{70}x^2-\frac{2}{7}}{x^7(x^4+3x^2+2)}-\frac{123\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{64}+\frac{25\arctan(x)}{2}$	61

```
[In] int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -13/32*x/(x^2+2)-123/64*arctan(1/2*x*2^(1/2))*2^(1/2)-1/7/x^7+11/20/x^5-23/
12/x^3+137/16/x+1/2*x/(x^2+1)+25/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx = \frac{58170\,x^{10}+163730\,x^8+80136\,x^6-15632\,x^4-12915\,\sqrt{2}(x^{11}+3\,x^9+2\,x^7)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+4512\,x^2+8}{6720\,(x^{11}+3\,x^9+2\,x^7)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(58170*x^10 + 163730*x^8 + 80136*x^6 - 15632*x^4 - 12915*sqrt(2)*(x^
11 + 3*x^9 + 2*x^7)*arctan(1/2*sqrt(2)*x) + 4512*x^2 + 84000*(x^11 + 3*x^9
+ 2*x^7)*arctan(x) - 1920)/(x^11 + 3*x^9 + 2*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx = \frac{25\,\mathrm{atan}\,(x)}{2} - \frac{123\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64} + \frac{29085x^{10}+81865x^8+40068x^6-7816x^4+2256x^2-960}{3360x^{11}+10080x^9+6720x^7}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)
[Out] 25*atan(x)/2 - 123*sqrt(2)*atan(sqrt(2)*x/2)/64 + (29085*x**10 + 81865*x**8
 + 40068*x**6 - 7816*x**4 + 2256*x**2 - 960)/(3360*x**11 + 10080*x**9 + 672
0*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx = -\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{29085\,x^{10}+81865\,x^8+40068\,x^6-7816\,x^4+2256\,x^2-960}{3360\,(x^{11}+3\,x^9+2\,x^7)} + \frac{25}{2}\arctan(x)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="maxima")
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/3360*(29085*x^10 + 81865*x^8 + 40
068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(x^11 + 3*x^9 + 2*x^7) + 25/2*arctan(x
)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx = -\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{3\,x^3+19\,x}{32\,(x^4+3\,x^2+2)} + \frac{14385\,x^6-3220\,x^4+924\,x^2-240}{1680\,x^7} + \frac{25}{2}\arctan(x)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/32*(3*x^3 + 19*x)/(x^4 + 3*x^2 +
2) + 1/1680*(14385*x^6 - 3220*x^4 + 924*x^2 - 240)/x^7 + 25/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx=\frac{25\,\mathrm{atan}(x)}{2}-\frac{123\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64}+\frac{\frac{277\,x^{10}}{32}+\frac{2339\,x^8}{96}+\frac{477\,x^6}{40}-\frac{977\,x^4}{420}+\frac{47\,x^2}{70}-\frac{2}{7}}{x^{11}+3\,x^9+2\,x^7}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^8*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (25*atan(x))/2 - (123*2^(1/2)*atan((2^(1/2)*x)/2))/64 + ((47*x^2)/70 - (977
*x^4)/420 + (477*x^6)/40 + (2339*x^8)/96 + (277*x^10)/32 - 2/7)/(2*x^7 + 3*
x^9 + x^11)
```

3.91 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 81

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx = 214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)} + \frac{477\arctan(x)}{8}-351\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 214*x-14*x^3+x^5+1/4*x*(415*x^2+414)/(x^4+3*x^2+2)^2+1/8*x*(1669*x^2+824)/(
x^4+3*x^2+2)+477/8*arctan(x)-351*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1682, 1692, 1690, 1180, 209}

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx = \frac{477\arctan(x)}{8}-351\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+x^5-14x^3 + \frac{(1669x^2+824)x}{8(x^4+3x^2+2)}+\frac{(415x^2+414)x}{4(x^4+3x^2+2)^2}+214x$$

```
[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 214*x - 14*x^3 + x^5 + (x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(82
4 + 1669*x^2))/(8*(2 + 3*x^2 + x^4)) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcT
an[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

integral

$$=\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}-\frac{1}{8}\int\frac{828-2478x^2-840x^4+424x^6-216x^8+96x^{10}-40x^{12}}{(2+3x^2+x^4)^2}\,dx$$

$$=\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)}+\frac{1}{32}\int\frac{-4952-2700x^2+3136x^4-864x^6+160x^8}{2+3x^2+x^4}\,dx$$

$$=\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)}+\frac{1}{32}\int\left(6848-1344x^2+160x^4-\frac{36(518+571x^2)}{2+3x^2+x^4}\right)dx$$

$$=214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)}-\frac{9}{8}\int\frac{518+571x^2}{2+3x^2+x^4}\,dx$$

$$=214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)}+\frac{477}{8}\int\frac{1}{1+x^2}\,dx-702\int\frac{1}{2+x^2}\,dx$$

$$=214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)}+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int\frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx=\frac{x(9324+26736x^2+26775x^4+10581x^6+1144x^8-64x^{10}+8x^{12})}{8(2+3x^2+x^4)^2}+\frac{477\arctan(x)}{8}-351\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
[Out] (x*(9324 + 26736*x^2 + 26775*x^4 + 10581*x^6 + 1144*x^8 - 64*x^10 + 8*x^12)
)/(8*(2 + 3*x^2 + x^4)^2) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcTan[x/Sqrt[2
]]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

method	result	size
risch	$x^{5}-14x^{3}+214x+\frac{\frac{1669}{8}x^{7}+\frac{5831}{8}x^{5}+830x^{3}+\frac{619}{2}x}{\left(x^{4}+3x^{2}+2\right)^{2}}+\frac{477\arctan\left(x\right)}{8}-351\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	61
default	$-\frac{16\left(-\frac{105}{8}x^{3}-\frac{79}{4}x\right)}{\left(x^{2}+2\right)^{2}}-351\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}+x^{5}-14x^{3}+214x+\frac{-\frac{11}{8}x^{3}-\frac{13}{8}x}{\left(x^{2}+1\right)^{2}}+\frac{477\arctan\left(x\right)}{8}$	64

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x^5-14*x^3+214*x+(1669/8*x^7+5831/8*x^5+830*x^3+619/2*x)/(x^4+3*x^2+2)^2+47
7/8*arctan(x)-351*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$=\frac{8\,x^{13}-64\,x^{11}+1144\,x^{9}+10581\,x^{7}+26775\,x^{5}+26736\,x^{3}-2808\,\sqrt{2}(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4)\,\mathrm{arcta}}{8\,(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4)}$$

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*x^13 - 64*x^11 + 1144*x^9 + 10581*x^7 + 26775*x^5 + 26736*x^3 - 2808
*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 477*(x
^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 9324*x)/(x^8 + 6*x^6 + 13*x^4
 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5-14x^3+214x+\frac{1669x^7+5831x^5+6640x^3+2476x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{477\,\mathrm{atan}\,(x)}{8}-351\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] x**5 - 14*x**3 + 214*x + (1669*x**7 + 5831*x**5 + 6640*x**3 + 2476*x)/(8*x*
*8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 477*atan(x)/8 - 351*sqrt(2)*atan(
sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5 - 14\,x^3 - 351\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 214\,x + \frac{1669\,x^7 + 5831\,x^5 + 6640\,x^3 + 2476\,x}{8\,(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4)} + \frac{477}{8}\,\arctan\left(x\right)$$

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
5831*x^5 + 6640*x^3 + 2476*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 477/8*a
rctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5 - 14\,x^3 - 351\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 214\,x + \frac{1669\,x^7 + 5831\,x^5 + 6640\,x^3 + 2476\,x}{8\,(x^4 + 3\,x^2 + 2)^2} + \frac{477}{8}\,\arctan\left(x\right)$$

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
5831*x^5 + 6640*x^3 + 2476*x)/(x^4 + 3*x^2 + 2)^2 + 477/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 214\,x + \frac{477\,\mathrm{atan}(x)}{8} - 351\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) + \frac{\frac{1669\,x^7}{8} + \frac{5831\,x^5}{8} + 830\,x^3 + \frac{619\,x}{2}}{x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4} - 14\,x^3 + x^5$$

```
[In] int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] 214*x + (477*atan(x))/8 - 351*2^(1/2)*atan((2^(1/2)*x)/2) + ((619*x)/2 + 83
0*x^3 + (5831*x^5)/8 + (1669*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4) -
14*x^3 + x^5
```

3.92 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx = -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}+\frac{x(24-409x^2)}{8(2+3x^2+x^4)} -\frac{449\arctan(x)}{8}+\frac{219\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -42*x+5/3*x^3-1/4*x*(207*x^2+206)/(x^4+3*x^2+2)^2+1/8*x*(-409*x^2+24)/(x^4+
3*x^2+2)-449/8*arctan(x)+219/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1682, 1692, 1690, 1180, 209}

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx = -\frac{449\arctan(x)}{8}+\frac{219\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{5x^3}{3} +\frac{(24-409x^2)x}{8(x^4+3x^2+2)}-\frac{(207x^2+206)x}{4(x^4+3x^2+2)^2}-42x$$

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -42*x + (5*x^3)/3 - (x*(206 + 207*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(24 -
409*x^2))/(8*(2 + 3*x^2 + x^4)) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]
])/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(206+207x^2)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{-412+1230x^2+424x^4-216x^6+96x^8-40x^{10}}{\left(2+3x^2+x^4\right)^2}\,dx \\
&= -\frac{x(206+207x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(24-409x^2)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{728+1500x^2-864x^4+160x^6}{2+3x^2+x^4}\,dx \\
&= -\frac{x(206+207x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(24-409x^2)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \left(-1344+160x^2+\frac{4(854+1303x^2)}{2+3x^2+x^4}\right)dx \\
&= -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(24-409x^2)}{8\left(2+3x^2+x^4\right)}+\frac{1}{8}\int \frac{854+1303x^2}{2+3x^2+x^4}\,dx \\
&= -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(24-409x^2)}{8\left(2+3x^2+x^4\right)}-\frac{449}{8}\int \frac{1}{1+x^2}\,dx+219\int \frac{1}{2+x^2}\,dx \\
&= -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(24-409x^2)}{8\left(2+3x^2+x^4\right)}-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{\left(2+3x^2+x^4\right)^3}\,dx = \frac{x(-5124-15416x^2-16233x^4-6755x^6-768x^8+40x^{10})}{24\left(2+3x^2+x^4\right)^2} - \frac{449\arctan(x)}{8}+\frac{219\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
[Out] (x*(-5124 - 15416*x^2 - 16233*x^4 - 6755*x^6 - 768*x^8 + 40*x^10))/(24*(2 +
 3*x^2 + x^4)^2) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{5x^3}{3}-42x+\frac{-\frac{409}{8}x^7-\frac{1203}{8}x^5-145x^3-\frac{91}{2}x}{\left(x^4+3x^2+2\right)^2}-\frac{449\arctan(x)}{8}+\frac{219\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	58
default	$\frac{-53x^3-54x}{\left(x^2+2\right)^2}+\frac{219\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{5x^3}{3}-42x-\frac{-\frac{15}{8}x^3-\frac{17}{8}x}{\left(x^2+1\right)^2}-\frac{449\arctan(x)}{8}$	62

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/3*x^3-42*x+(-409/8*x^7-1203/8*x^5-145*x^3-91/2*x)/(x^4+3*x^2+2)^2-449/8*a
rctan(x)+219/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{40\,x^{11}-768\,x^9-6755\,x^7-16233\,x^5-15416\,x^3+2628\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right.}{24\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(40*x^11 - 768*x^9 - 6755*x^7 - 16233*x^5 - 15416*x^3 + 2628*sqrt(2)*(
x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1347*(x^8 + 6*x^
6 + 13*x^4 + 12*x^2 + 4)*arctan(x) - 5124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2
 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5x^3}{3} - 42x + \frac{-409x^7-1203x^5-1160x^3-364x}{8x^8+48x^6+104x^4+96x^2+32} - \frac{449\,\operatorname{atan}(x)}{8} + \frac{219\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x**3/3 - 42*x + (-409*x**7 - 1203*x**5 - 1160*x**3 - 364*x)/(8*x**8 + 48*
x**6 + 104*x**4 + 96*x**2 + 32) - 449*atan(x)/8 + 219*sqrt(2)*atan(sqrt(2)*
x/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5}{3}\,x^3 + \frac{219}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 42\,x - \frac{409\,x^7 + 1203\,x^5 + 1160\,x^3 + 364\,x}{8\,(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4)} - \frac{449}{8}\,\arctan\left(x\right)$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*
x^5 + 1160*x^3 + 364*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 449/8*arctan(
x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5}{3}\,x^3 + \frac{219}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 42\,x - \frac{409\,x^7 + 1203\,x^5 + 1160\,x^3 + 364\,x}{8\,(x^4 + 3\,x^2 + 2)^2} - \frac{449}{8}\,\arctan\left(x\right)$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*
x^5 + 1160*x^3 + 364*x)/(x^4 + 3*x^2 + 2)^2 - 449/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{219\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - \frac{449\,\mathrm{atan}(x)}{8} - 42\,x - \frac{\frac{409\,x^7}{8} + \frac{1203\,x^5}{8} + 145\,x^3 + \frac{91\,x}{2}}{x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4} + \frac{5\,x^3}{3}$$

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
[Out] (219*2^(1/2)*atan((2^(1/2)*x)/2))/2 - (449*atan(x))/8 - 42*x - ((91*x)/2 +
145*x^3 + (1203*x^5)/8 + (409*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4) +
 (5*x^3)/3
```

3.93 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx = 5x+\frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8(2+3x^2+x^4)} +\frac{413\arctan(x)}{8}-\frac{191\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 5*x+1/4*x*(103*x^2+102)/(x^4+3*x^2+2)^2-1/8*x*(15*x^2+244)/(x^4+3*x^2+2)+41
3/8*arctan(x)-191/4*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1682, 1692, 1690, 1180, 209}

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx = \frac{413\arctan(x)}{8}-\frac{191\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}} -\frac{(15x^2+244)x}{8(x^4+3x^2+2)}+\frac{(103x^2+102)x}{4(x^4+3x^2+2)^2}+5x$$

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 5*x + (x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(244 + 15*x^2))/(8*(
2 + 3*x^2 + x^4)) + (413*ArcTan[x])/8 - (191*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(102+103x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{204-606x^2-216x^4+96x^6-40x^8}{\left(2+3x^2+x^4\right)^2}\,dx$$
$$= \frac{x(102+103x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{x(244+15x^2)}{8\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{568-924x^2+160x^4}{2+3x^2+x^4}\,dx$$
$$= \frac{x(102+103x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{x(244+15x^2)}{8\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(160+\frac{4(62-351x^2)}{2+3x^2+x^4}\right)dx$$
$$= 5x + \frac{x(102+103x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{x(244+15x^2)}{8\left(2+3x^2+x^4\right)} + \frac{1}{8}\int \frac{62-351x^2}{2+3x^2+x^4}\,dx$$
$$= 5x + \frac{x(102+103x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{x(244+15x^2)}{8\left(2+3x^2+x^4\right)} + \frac{413}{8}\int \frac{1}{1+x^2}\,dx - \frac{191}{2}\int \frac{1}{2+x^2}\,dx$$
$$= 5x + \frac{x(102+103x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{x(244+15x^2)}{8\left(2+3x^2+x^4\right)} + \frac{413}{8}\tan^{-1}(x) - \frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{\left(2+3x^2+x^4\right)^3}\,dx = \frac{1}{8}\left(\frac{x(-124-76x^2+231x^4+225x^6+40x^8)}{\left(2+3x^2+x^4\right)^2} + 413\arctan(x) - 382\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
[Out] ((x*(-124 - 76*x^2 + 231*x^4 + 225*x^6 + 40*x^8))/(2 + 3*x^2 + x^4)^2 + 413
*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

method	result	size
risch	$5x+\frac{-\frac{15}{8}x^7-\frac{289}{8}x^5-\frac{139}{2}x^3-\frac{71}{2}x}{\left(x^4+3x^2+2\right)^2}+\frac{413\arctan(x)}{8}-\frac{191\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}$	53
default	$5x-\frac{16\left(-\frac{1}{32}x^3+\frac{25}{16}x\right)}{\left(x^2+2\right)^2}-\frac{191\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{-\frac{19}{8}x^3-\frac{21}{8}x}{\left(x^2+1\right)^2}+\frac{413\arctan(x)}{8}$	56

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x+(-15/8*x^7-289/8*x^5-139/2*x^3-71/2*x)/(x^4+3*x^2+2)^2+413/8*arctan(x)-
191/4*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{40\,x^9+225\,x^7+231\,x^5-76\,x^3-382\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+413\,(x^8+6\,x^6-}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(40*x^9 + 225*x^7 + 231*x^5 - 76*x^3 - 382*sqrt(2)*(x^8 + 6*x^6 + 13*x^
4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 413*(x^8 + 6*x^6 + 13*x^4 + 12*x^2
+ 4)*arctan(x) - 124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 5x+\frac{-15x^7-289x^5-556x^3-284x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{413\,\mathrm{atan}\,(x)}{8}-\frac{191\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x + (-15*x**7 - 289*x**5 - 556*x**3 - 284*x)/(8*x**8 + 48*x**6 + 104*x**4
 + 96*x**2 + 32) + 413*atan(x)/8 - 191*sqrt(2)*atan(sqrt(2)*x/2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x - \frac{15\,x^7+289\,x^5+556\,x^3+284\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}+\frac{413}{8}\arctan\left(x\right)$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^
3 + 284*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 413/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x - \frac{15\,x^7+289\,x^5+556\,x^3+284\,x}{8\,(x^4+3\,x^2+2)^2}+\frac{413}{8}\arctan\left(x\right)$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^
3 + 284*x)/(x^4 + 3*x^2 + 2)^2 + 413/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 5\,x+\frac{413\,\mathrm{atan}(x)}{8}-\frac{191\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4} - \frac{\frac{15\,x^7}{8}+\frac{289\,x^5}{8}+\frac{139\,x^3}{2}+\frac{71\,x}{2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
[Out] 5*x + (413*atan(x))/8 - (191*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((71*x)/2 + (
139*x^3)/2 + (289*x^5)/8 + (15*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.94 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2}+\frac{x(254+125x^2)}{8(2+3x^2+x^4)}-\frac{369\arctan(x)}{8}+\frac{267\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/4*x*(51*x^2+50)/(x^4+3*x^2+2)^2+1/8*x*(125*x^2+254)/(x^4+3*x^2+2)-369/8*
arctan(x)+267/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1682, 1692, 1180, 209}

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{369\arctan(x)}{8}+\frac{267\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}+\frac{x(125x^2+254)}{8(x^4+3x^2+2)}$$

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -1/4*(x*(50 + 51*x^2))/(2 + 3*x^2 + x^4)^2 + (x*(254 + 125*x^2))/(8*(2 + 3*
x^2 + x^4)) - (369*ArcTan[x])/8 + (267*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(50+51x^2)}{4\,(2+3x^2+x^4)^2}-\frac{1}{8}\int\frac{-100+294x^2+96x^4-40x^6}{(2+3x^2+x^4)^2}\,dx\\
&= -\frac{x(50+51x^2)}{4\,(2+3x^2+x^4)^2}+\frac{x(254+125x^2)}{8\,(2+3x^2+x^4)}+\frac{1}{32}\int\frac{-816+660x^2}{2+3x^2+x^4}\,dx
\end{aligned}$$

$$= -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2} + \frac{x(254+125x^2)}{8(2+3x^2+x^4)} - \frac{369}{8}\int \frac{1}{1+x^2}\,dx + \frac{267}{4}\int \frac{1}{2+x^2}\,dx$$

$$= -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2} + \frac{x(254+125x^2)}{8(2+3x^2+x^4)} - \frac{369}{8}\tan^{-1}(x) + \frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{1}{8}\left(\frac{x(408+910x^2+629x^4+125x^6)}{(2+3x^2+x^4)^2} - 369\arctan(x) + 267\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
[Out] ((x*(408 + 910*x^2 + 629*x^4 + 125*x^6))/(2 + 3*x^2 + x^4)^2 - 369*ArcTan[x
] + 267*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\frac{125}{8}x^7+\frac{629}{8}x^5+\frac{455}{4}x^3+51x}{(x^4+3x^2+2)^2}+\frac{267\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{369\arctan(x)}{8}$	50
default	$\frac{\frac{51}{4}x^3+\frac{77}{2}x}{(x^2+2)^2}+\frac{267\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{-\frac{23}{8}x^3-\frac{25}{8}x}{(x^2+1)^2}-\frac{369\arctan(x)}{8}$	54

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
[Out] (125/8*x^7+629/8*x^5+455/4*x^3+51*x)/(x^4+3*x^2+2)^2+267/8*arctan(1/2*x*2^(
1/2))*2^(1/2)-369/8*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{125\,x^7+629\,x^5+910\,x^3+267\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-369\,(x^8+6\,x^6+13\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 267*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x
^2 + 4)*arctan(1/2*sqrt(2)*x) - 369*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arc
tan(x) + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{125x^7+629x^5+910x^3+408x}{8x^8+48x^6+104x^4+96x^2+32} - \frac{369\operatorname{atan}(x)}{8} + \frac{267\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (125*x**7 + 629*x**5 + 910*x**3 + 408*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96*
x**2 + 32) - 369*atan(x)/8 + 267*sqrt(2)*atan(sqrt(2)*x/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} - \frac{369}{8}\,\arctan(x)$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 40
8*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 369/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\,(x^4+3\,x^2+2)^2} - \frac{369}{8}\arctan(x)$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 40
8*x)/(x^4 + 3*x^2 + 2)^2 - 369/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8} - \frac{369\,\mathrm{atan}(x)}{8} + \frac{\frac{125\,x^7}{8}+\frac{629\,x^5}{8}+\frac{455\,x^3}{4}+51\,x}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
[Out] (267*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (369*atan(x))/8 + (51*x + (455*x^3)/4
 + (629*x^5)/8 + (125*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.95 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2} - \frac{x(211+130x^2)}{8(2+3x^2+x^4)} + \frac{317\arctan(x)}{8} - \frac{447\arctan\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/4*x*(25*x^2+24)/(x^4+3*x^2+2)^2-1/8*x*(130*x^2+211)/(x^4+3*x^2+2)+317/8*a
rctan(x)-447/16*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1682, 1692, 1180, 209}

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{317\arctan(x)}{8} - \frac{447\arctan\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{x(25x^2+24)}{4(x^4+3x^2+2)^2} - \frac{x(130x^2+211)}{8(x^4+3x^2+2)}$$

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(211 + 130*x^2))/(8*(2 + 3*x
^2 + x^4)) + (317*ArcTan[x])/8 - (447*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(24+25x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{48-154x^2-40x^4}{\left(2+3x^2+x^4\right)^2}\,dx \\
&= \frac{x(24+25x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{x(211+130x^2)}{8\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{748-520x^2}{2+3x^2+x^4}\,dx
\end{aligned}
$$

$$= \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2} - \frac{x(211+130x^2)}{8(2+3x^2+x^4)} + \frac{317}{8}\int \frac{1}{1+x^2}\,dx - \frac{447}{8}\int \frac{1}{2+x^2}\,dx$$

$$= \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2} - \frac{x(211+130x^2)}{8(2+3x^2+x^4)} + \frac{317}{8}\tan^{-1}(x) - \frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{1}{16}\left(-\frac{2x(374+843x^2+601x^4+130x^6)}{(2+3x^2+x^4)^2} + 634\arctan(x) - 447\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
[Out] ((-2*x*(374 + 843*x^2 + 601*x^4 + 130*x^6))/(2 + 3*x^2 + x^4)^2 + 634*ArcTa
n[x] - 447*Sqrt[2]*ArcTan[x/Sqrt[2]])/16
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{-\frac{65}{4}x^7-\frac{601}{8}x^5-\frac{843}{8}x^3-\frac{187}{4}x}{(x^4+3x^2+2)^2} - \frac{447\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{16} + \frac{317\arctan(x)}{8}$	50
default	$-\frac{\frac{103}{8}x^3+\frac{129}{4}x}{(x^2+2)^2} - \frac{447\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{16} + \frac{-\frac{27}{8}x^3-\frac{29}{8}x}{(x^2+1)^2} + \frac{317\arctan(x)}{8}$	53

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
[Out] (-65/4*x^7-601/8*x^5-843/8*x^3-187/4*x)/(x^4+3*x^2+2)^2-447/16*arctan(1/2*x
*2^(1/2))*2^(1/2)+317/8*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{260\,x^7+1202\,x^5+1686\,x^3+447\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-634\,(x^8+6\,x^6+\cdots}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(260*x^7 + 1202*x^5 + 1686*x^3 + 447*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 +
12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 634*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
*arctan(x) + 748*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{-130x^7-601x^5-843x^3-374x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{317\,\mathrm{atan}\,(x)}{8} - \frac{447\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}$$

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (-130*x**7 - 601*x**5 - 843*x**3 - 374*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96
*x**2 + 32) + 317*atan(x)/8 - 447*sqrt(2)*atan(sqrt(2)*x/2)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{447}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - \frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} + \frac{317}{8}\,\arctan\left(x\right)$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
374*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 317/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\,(x^4+3\,x^2+2)^2} + \frac{317}{8}\arctan\left(x\right)$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
374*x)/(x^4 + 3*x^2 + 2)^2 + 317/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{317\,\mathrm{atan}(x)}{8} - \frac{447\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16} - \frac{\frac{65\,x^7}{4}+\frac{601\,x^5}{8}+\frac{843\,x^3}{8}+\frac{187\,x}{4}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (317*atan(x))/8 - (447*2^(1/2)*atan((2^(1/2)*x)/2))/16 - ((187*x)/4 + (843*
x^3)/8 + (601*x^5)/8 + (65*x^7)/4)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.96 $\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = -\frac{x(11+12x^2)}{4(2+3x^2+x^4)^2}+\frac{x(335+217x^2)}{16(2+3x^2+x^4)}-\frac{257\arctan(x)}{8}+\frac{731\arctan\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -1/4*x*(12*x^2+11)/(x^4+3*x^2+2)^2+1/16*x*(217*x^2+335)/(x^4+3*x^2+2)-257/8
*arctan(x)+731/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1692, 1192, 1180, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = -\frac{257\arctan(x)}{8}+\frac{731\arctan\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}+\frac{x(217x^2+335)}{16(x^4+3x^2+2)}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -1/4*(x*(11 + 12*x^2))/(2 + 3*x^2 + x^4)^2 + (x*(335 + 217*x^2))/(16*(2 + 3
*x^2 + x^4)) - (257*ArcTan[x])/8 + (731*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(11+12x^2)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int\frac{-38+80x^2}{\left(2+3x^2+x^4\right)^2}\,dx\\
&= -\frac{x(11+12x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(335+217x^2)}{16\left(2+3x^2+x^4\right)}+\frac{1}{32}\int\frac{-594+434x^2}{2+3x^2+x^4}\,dx\\
&= -\frac{x(11+12x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(335+217x^2)}{16\left(2+3x^2+x^4\right)}-\frac{257}{8}\int\frac{1}{1+x^2}\,dx+\frac{731}{16}\int\frac{1}{2+x^2}\,dx\\
&= -\frac{x(11+12x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(335+217x^2)}{16\left(2+3x^2+x^4\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3}\,dx=\frac{1}{32}\left(\frac{2x(626+1391x^2+986x^4+217x^6)}{\left(2+3x^2+x^4\right)^2}-1028\arctan(x)+731\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((2*x*(626 + 1391*x^2 + 986*x^4 + 217*x^6))/(2 + 3*x^2 + x^4)^2 - 1028*ArcT
an[x] + 731*Sqrt[2]*ArcTan[x/Sqrt[2]])/32
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\frac{217}{16}x^7+\frac{493}{8}x^5+\frac{1391}{16}x^3+\frac{313}{8}x}{\left(x^4+3x^2+2\right)^2}-\frac{257\arctan(x)}{8}+\frac{731\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{32}$	50
default	$\frac{\frac{155}{16}x^3+\frac{181}{8}x}{\left(x^2+2\right)^2}+\frac{731\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{32}-\frac{-\frac{31}{8}x^3-\frac{33}{8}x}{\left(x^2+1\right)^2}-\frac{257\arctan(x)}{8}$	53

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (217/16*x^7+493/8*x^5+1391/16*x^3+313/8*x)/(x^4+3*x^2+2)^2-257/8*arctan(x)+
731/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3}\,dx$$
$$=\frac{434\,x^7+1972\,x^5+2782\,x^3+731\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-1028\,(x^8+6\,x^6+13}{32\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(434*x^7 + 1972*x^5 + 2782*x^3 + 731*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 1
2*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1028*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
*arctan(x) + 1252*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{217x^7+986x^5+1391x^3+626x}{16x^8+96x^6+208x^4+192x^2+64} - \frac{257\,\mathrm{atan}\,(x)}{8} + \frac{731\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (217*x**7 + 986*x**5 + 1391*x**3 + 626*x)/(16*x**8 + 96*x**6 + 208*x**4 + 1
92*x**2 + 64) - 257*atan(x)/8 + 731*sqrt(2)*atan(sqrt(2)*x/2)/32
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} - \frac{257}{8}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 +
 626*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 257/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\,(x^4+3\,x^2+2)^2} - \frac{257}{8}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 +
 626*x)/(x^4 + 3*x^2 + 2)^2 - 257/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3}\,dx=\frac{731\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}-\frac{257\,\mathrm{atan}(x)}{8}+\frac{\frac{217\,x^7}{16}+\frac{493\,x^5}{8}+\frac{1391\,x^3}{16}+\frac{313\,x}{8}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (731*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (257*atan(x))/8 + ((313*x)/8 + (1391
*x^3)/16 + (493*x^5)/8 + (217*x^7)/16)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.97 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3}\,dx = -\frac{1}{2x}+\frac{x(9+11x^2)}{8(2+3x^2+x^4)^2}-\frac{x(547+347x^2)}{32(2+3x^2+x^4)} + \frac{189\arctan(x)}{8}-\frac{1119\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/2/x+1/8*x*(11*x^2+9)/(x^4+3*x^2+2)^2-1/32*x*(347*x^2+547)/(x^4+3*x^2+2)+
189/8*arctan(x)-1119/64*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3}\,dx = \frac{189\arctan(x)}{8}-\frac{1119\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}} + \frac{x(11x^2+9)}{8(x^4+3x^2+2)^2}-\frac{x(347x^2+547)}{32(x^4+3x^2+2)}-\frac{1}{2x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/2*1/x + (x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) - (x*(547 + 347*x^2))/(
32*(2 + 3*x^2 + x^4)) + (189*ArcTan[x])/8 - (1119*ArcTan[x/Sqrt[2]])/(32*Sq
rt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{-16+29x^2-55x^4}{x^2\left(2+3x^2+x^4\right)^2}\,dx \\
&= \frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2} - \frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{32+441x^2-347x^4}{x^2\left(2+3x^2+x^4\right)}\,dx \\
&= \frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2} - \frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(\frac{16}{x^2}+\frac{756}{1+x^2}-\frac{1119}{2+x^2}\right)dx \\
&= -\frac{1}{2x} + \frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2} - \frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)} + \frac{189}{8}\int \frac{1}{1+x^2}\,dx - \frac{1119}{32}\int \frac{1}{2+x^2}\,dx \\
&= -\frac{1}{2x} + \frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2} - \frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)} + \frac{189}{8}\tan^{-1}(x) - \frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = \frac{1}{64}\left(-\frac{2(64+1250x^2+2499x^4+1684x^6+363x^8)}{x\left(2+3x^2+x^4\right)^2} + 1512\arctan(x) - 1119\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
[Out] ((-2*(64 + 1250*x^2 + 2499*x^4 + 1684*x^6 + 363*x^8))/(x*(2 + 3*x^2 + x^4)^
2) + 1512*ArcTan[x] - 1119*Sqrt[2]*ArcTan[x/Sqrt[2]])/64
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{-\frac{363}{32}x^8-\frac{421}{8}x^6-\frac{2499}{32}x^4-\frac{625}{16}x^2-2}{x(x^4+3x^2+2)^2}-\frac{1119\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{64}+\frac{189\arctan(x)}{8}$	56
default	$-\frac{\frac{207}{16}x^3+\frac{233}{8}x}{2(x^2+2)^2}-\frac{1119\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{64}-\frac{1}{2x}+\frac{-\frac{35}{8}x^3-\frac{37}{8}x}{(x^2+1)^2}+\frac{189\arctan(x)}{8}$	58

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
[Out] (-363/32*x^8-421/8*x^6-2499/32*x^4-625/16*x^2-2)/x/(x^4+3*x^2+2)^2-1119/64*
arctan(1/2*x*2^(1/2))*2^(1/2)+189/8*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = -\frac{726\,x^8+3368\,x^6+4998\,x^4+1119\,\sqrt{2}(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+2500\,x^2-15}{64\,(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="fricas")
[Out] -1/64*(726*x^8 + 3368*x^6 + 4998*x^4 + 1119*sqrt(2)*(x^9 + 6*x^7 + 13*x^5 +
 12*x^3 + 4*x)*arctan(1/2*sqrt(2)*x) + 2500*x^2 - 1512*(x^9 + 6*x^7 + 13*x^
5 + 12*x^3 + 4*x)*arctan(x) + 128)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = \frac{-363x^8-1684x^6-2499x^4-1250x^2-64}{32x^9+192x^7+416x^5+384x^3+128x} + \frac{189\,\mathrm{atan}\,(x)}{8} - \frac{1119\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)
```

```
[Out] (-363*x**8 - 1684*x**6 - 2499*x**4 - 1250*x**2 - 64)/(32*x**9 + 192*x**7 +
416*x**5 + 384*x**3 + 128*x) + 189*atan(x)/8 - 1119*sqrt(2)*atan(sqrt(2)*x/
2)/64
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = -\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{363\,x^8+1684\,x^6+2499\,x^4+1250\,x^2+64}{32\,(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)} + \frac{189}{8}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(363*x^8 + 1684*x^6 + 2499*x^
4 + 1250*x^2 + 64)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x) + 189/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = -\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{347\,x^7+1588\,x^5+2291\,x^3+1058\,x}{32\,\left(x^4+3\,x^2+2\right)^2} - \frac{1}{2\,x} + \frac{189}{8}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(347*x^7 + 1588*x^5 + 2291*x^
3 + 1058*x)/(x^4 + 3*x^2 + 2)^2 - 1/2/x + 189/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = \frac{189\,\mathrm{atan}(x)}{8} - \frac{1119\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64} - \frac{\frac{363\,x^8}{32}+\frac{421\,x^6}{8}+\frac{2499\,x^4}{32}+\frac{625\,x^2}{16}+2}{x^9+6\,x^7+13\,x^5+12\,x^3+4\,x}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^3),x)
```

```
[Out] (189*atan(x))/8 - (1119*2^(1/2)*atan((2^(1/2)*x)/2))/64 - ((625*x^2)/16 + (
2499*x^4)/32 + (421*x^6)/8 + (363*x^8)/32 + 2)/(4*x + 12*x^3 + 13*x^5 + 6*x
^7 + x^9)
```

3.98 $\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3} dx = -\frac{1}{6x^3}+\frac{17}{8x}-\frac{x(5+9x^2)}{16(2+3x^2+x^4)^2}+\frac{x(951+571x^2)}{64(2+3x^2+x^4)} - \frac{113\arctan(x)}{8}+\frac{1611\arctan\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

```
[Out] -1/6/x^3+17/8/x-1/16*x*(9*x^2+5)/(x^4+3*x^2+2)^2+1/64*x*(571*x^2+951)/(x^4+
3*x^2+2)-113/8*arctan(x)+1611/128*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3} dx = -\frac{113\arctan(x)}{8}+\frac{1611\arctan\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}-\frac{1}{6x^3} - \frac{x(9x^2+5)}{16(x^4+3x^2+2)^2}+\frac{x(571x^2+951)}{64(x^4+3x^2+2)}+\frac{17}{8x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/6*1/x^3 + 17/(8*x) - (x*(5 + 9*x^2))/(16*(2 + 3*x^2 + x^4)^2) + (x*(951
+ 571*x^2))/(64*(2 + 3*x^2 + x^4)) - (113*ArcTan[x])/8 + (1611*ArcTan[x/Sqr
t[2]])/(64*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int\frac{-16+20x^2-\frac{73x^4}{2}+\frac{45x^6}{2}}{x^4\left(2+3x^2+x^4\right)^2}\,dx\\
&= -\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2}+\frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)}+\frac{1}{32}\int\frac{32-88x^2-\frac{573x^4}{2}+\frac{571x^6}{2}}{x^4\left(2+3x^2+x^4\right)}\,dx\\
&= -\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2}+\frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)}+\frac{1}{32}\int\left(\frac{16}{x^4}-\frac{68}{x^2}-\frac{452}{1+x^2}+\frac{1611}{2\left(2+x^2\right)}\right)dx\\
&= -\frac{1}{6x^3}+\frac{17}{8x}-\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2}+\frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)}-\frac{113}{8}\int\frac{1}{1+x^2}\,dx+\frac{1611}{64}\int\frac{1}{2+x^2}\,dx\\
&= -\frac{1}{6x^3}+\frac{17}{8x}-\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2}+\frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)}-\frac{113}{8}\tan^{-1}(x)+\frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = \frac{1}{384}\left(-\frac{64}{x^3}+\frac{816}{x}-\frac{24x(5+9x^2)}{\left(2+3x^2+x^4\right)^2}+\frac{6x(951+571x^2)}{2+3x^2+x^4}-5424\arctan(x)+4833\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
[Out] (-64/x^3 + 816/x - (24*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + (6*x*(951 + 571
*x^2))/(2 + 3*x^2 + x^4) - 5424*ArcTan[x] + 4833*Sqrt[2]*ArcTan[x/Sqrt[2]])
/384
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\frac{707}{64}x^{10}+\frac{1301}{24}x^8+\frac{5663}{64}x^6+\frac{5063}{96}x^4+\frac{13}{2}x^2-\frac{2}{3}}{x^3\left(x^4+3x^2+2\right)^2}+\frac{1611\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{128}-\frac{113\arctan(x)}{8}$	61
default	$\frac{\frac{259}{8}x^3+\frac{285}{4}x}{8\left(x^2+2\right)^2}+\frac{1611\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{128}-\frac{1}{6x^3}+\frac{17}{8x}-\frac{-\frac{39}{8}x^3-\frac{41}{8}x}{\left(x^2+1\right)^2}-\frac{113\arctan(x)}{8}$	64

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
[Out] (707/64*x^10+1301/24*x^8+5663/64*x^6+5063/96*x^4+13/2*x^2-2/3)/x^3/(x^4+3*x
^2+2)^2+1611/128*arctan(1/2*x*2^(1/2))*2^(1/2)-113/8*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = \frac{4242\,x^{10}+20816\,x^8+33978\,x^6+20252\,x^4+4833\,\sqrt{2}(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+}{384\left(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3\right)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="fricas")
[Out] 1/384*(4242*x^10 + 20816*x^8 + 33978*x^6 + 20252*x^4 + 4833*sqrt(2)*(x^11 +
 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(1/2*sqrt(2)*x) + 2496*x^2 - 5424*(
```

```
x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(x) - 256)/(x^11 + 6*x^9 + 13
*x^7 + 12*x^5 + 4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = -\frac{113\,\mathrm{atan}\,(x)}{8}+\frac{1611\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{128} + \frac{2121x^{10}+10408x^8+16989x^6+10126x^4+1248x^2-128}{192x^{11}+1152x^9+2496x^7+2304x^5+768x^3}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)
```

```
[Out] -113*atan(x)/8 + 1611*sqrt(2)*atan(sqrt(2)*x/2)/128 + (2121*x**10 + 10408*x
**8 + 16989*x**6 + 10126*x**4 + 1248*x**2 - 128)/(192*x**11 + 1152*x**9 + 2
496*x**7 + 2304*x**5 + 768*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = \frac{1611}{128}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{2121\,x^{10}+10408\,x^8+16989\,x^6+10126\,x^4+1248\,x^2-128}{192\,\left(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3\right)} - \frac{113}{8}\,\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/192*(2121*x^10 + 10408*x^8 + 169
89*x^6 + 10126*x^4 + 1248*x^2 - 128)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^
3) - 113/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = \frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{571\,x^7+2664\,x^5+3959\,x^3+1882\,x}{64\,\left(x^4+3\,x^2+2\right)^2} + \frac{51\,x^2-4}{24\,x^3} - \frac{113}{8}\arctan\left(x\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="giac")
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/64*(571*x^7 + 2664*x^5 + 3959*x^
3 + 1882*x)/(x^4 + 3*x^2 + 2)^2 + 1/24*(51*x^2 - 4)/x^3 - 113/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = \frac{\frac{707\,x^{10}}{64}+\frac{1301\,x^8}{24}+\frac{5663\,x^6}{64}+\frac{5063\,x^4}{96}+\frac{13\,x^2}{2}-\frac{2}{3}}{x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3} - \frac{113\,\mathrm{atan}(x)}{8} + \frac{1611\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{128}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^3),x)
[Out] ((13*x^2)/2 + (5063*x^4)/96 + (5663*x^6)/64 + (1301*x^8)/24 + (707*x^10)/64
 - 2/3)/(4*x^3 + 12*x^5 + 13*x^7 + 6*x^9 + x^11) - (113*atan(x))/8 + (1611*
2^(1/2)*atan((2^(1/2)*x)/2))/128
```

3.99 $\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 93

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3} dx = -\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x(3-5x^2)}{32(2+3x^2+x^4)^2} - \frac{x(1771+999x^2)}{128(2+3x^2+x^4)}+\frac{29\arctan(x)}{8}-\frac{2207\arctan\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

```
[Out] -1/10/x^5+17/24/x^3-93/16/x-1/32*x*(-5*x^2+3)/(x^4+3*x^2+2)^2-1/128*x*(999*
x^2+1771)/(x^4+3*x^2+2)+29/8*arctan(x)-2207/256*arctan(1/2*x*2^(1/2))*2^(1/
2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3} dx = \frac{29\arctan(x)}{8}-\frac{2207\arctan\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}-\frac{1}{10x^5}+\frac{17}{24x^3} - \frac{x(3-5x^2)}{32(x^4+3x^2+2)^2}-\frac{x(999x^2+1771)}{128(x^4+3x^2+2)}-\frac{93}{16x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/10*1/x^5 + 17/(24*x^3) - 93/(16*x) - (x*(3 - 5*x^2))/(32*(2 + 3*x^2 + x^
4)^2) - (x*(1771 + 999*x^2))/(128*(2 + 3*x^2 + x^4)) + (29*ArcTan[x])/8 - (
2207*ArcTan[x/Sqrt[2]])/(128*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(3-5x^2)}{32(2+3x^2+x^4)^2} - \frac{1}{8}\int \frac{-16+20x^2-34x^4+\frac{81x^6}{4}-\frac{25x^8}{4}}{x^6(2+3x^2+x^4)^2}\,dx \\
&= -\frac{x(3-5x^2)}{32(2+3x^2+x^4)^2} - \frac{x(1771+999x^2)}{128(2+3x^2+x^4)} + \frac{1}{32}\int \frac{32-88x^2+184x^4+\frac{681x^6}{4}-\frac{999x^8}{4}}{x^6(2+3x^2+x^4)}\,dx \\
&= -\frac{x(3-5x^2)}{32(2+3x^2+x^4)^2} - \frac{x(1771+999x^2)}{128(2+3x^2+x^4)} \\
&\quad + \frac{1}{32}\int \left(\frac{16}{x^6}-\frac{68}{x^4}+\frac{186}{x^2}+\frac{116}{1+x^2}-\frac{2207}{4(2+x^2)}\right)dx \\
&= -\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x(3-5x^2)}{32(2+3x^2+x^4)^2} \\
&\quad -\frac{x(1771+999x^2)}{128(2+3x^2+x^4)}+\frac{29}{8}\int \frac{1}{1+x^2}\,dx-\frac{2207}{128}\int \frac{1}{2+x^2}\,dx
\end{aligned}$$

$$= -\frac{1}{10x^5} + \frac{17}{24x^3} - \frac{93}{16x} - \frac{x(3-5x^2)}{32\left(2+3x^2+x^4\right)^2} - \frac{x(1771+999x^2)}{128\left(2+3x^2+x^4\right)} + \frac{29}{8}\tan^{-1}(x) - \frac{2207\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = \frac{-\frac{2(768-3136x^2+30816x^4+170702x^6+246477x^8+137120x^{10}+26145x^{12})}{x^5(2+3x^2+x^4)^2} + 13920\arctan(x) - 33105\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)}{3840}$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(768 - 3136*x^2 + 30816*x^4 + 170702*x^6 + 246477*x^8 + 137120*x^10 +
26145*x^12))/(x^5*(2 + 3*x^2 + x^4)^2) + 13920*ArcTan[x] - 33105*Sqrt[2]*Ar
cTan[x/Sqrt[2]])/3840
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{-\frac{1743}{128}x^{12}-\frac{857}{12}x^{10}-\frac{82159}{640}x^8-\frac{85351}{960}x^6-\frac{321}{20}x^4+\frac{49}{30}x^2-\frac{2}{5}}{x^5(x^4+3x^2+2)^2}+\frac{29\arctan(x)}{8}-\frac{2207\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{256}$	66
default	$-\frac{\frac{311}{8}x^3+\frac{337}{4}x}{16(x^2+2)^2}-\frac{2207\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{256}-\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}+\frac{-\frac{43}{8}x^3-\frac{45}{8}x}{(x^2+1)^2}+\frac{29\arctan(x)}{8}$	68

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1743/128*x^12-857/12*x^10-82159/640*x^8-85351/960*x^6-321/20*x^4+49/30*x^
2-2/5)/x^5/(x^4+3*x^2+2)^2+29/8*arctan(x)-2207/256*arctan(1/2*x*2^(1/2))*2^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.33

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = -\frac{52290\,x^{12}+274240\,x^{10}+492954\,x^{8}+341404\,x^{6}+61632\,x^{4}+33105\,\sqrt{2}(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4}{3840\,(x^{13}+6\,x^{11}+13\,x^{9}-}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/3840*(52290*x^12 + 274240*x^10 + 492954*x^8 + 341404*x^6 + 61632*x^4 + 3
3105*sqrt(2)*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(1/2*sqrt(2)*x
) - 6272*x^2 - 13920*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(x) +
1536)/(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx &= \frac{29\,\mathrm{atan}\,(x)}{8} - \frac{2207\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{256} \\ &+ \frac{-26145x^{12}-137120x^{10}-246477x^{8}-170702x^{6}-30816x^{4}+3136x^{2}-768}{1920x^{13}+11520x^{11}+24960x^{9}+23040x^{7}+7680x^{5}}\end{aligned}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)
```

```
[Out] 29*atan(x)/8 - 2207*sqrt(2)*atan(sqrt(2)*x/2)/256 + (-26145*x**12 - 137120*
x**10 - 246477*x**8 - 170702*x**6 - 30816*x**4 + 3136*x**2 - 768)/(1920*x**
13 + 11520*x**11 + 24960*x**9 + 23040*x**7 + 7680*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx = -\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{26145\,x^{12}+137120\,x^{10}+246477\,x^8+170702\,x^6+30816\,x^4-3136\,x^2+768}{1920\,(x^{13}+6\,x^{11}+13\,x^9+12\,x^7+4\,x^5)} + \frac{29}{8}\arctan(x)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1920*(26145*x^12 + 137120*x^10
+ 246477*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768)/(x^13 + 6*x^11 + 13
*x^9 + 12*x^7 + 4*x^5) + 29/8*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx = -\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{999\,x^7+4768\,x^5+7291\,x^3+3554\,x}{128\,(x^4+3\,x^2+2)^2} - \frac{1395\,x^4-170\,x^2+24}{240\,x^5} + \frac{29}{8}\arctan(x)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/128*(999*x^7 + 4768*x^5 + 7291*
x^3 + 3554*x)/(x^4 + 3*x^2 + 2)^2 - 1/240*(1395*x^4 - 170*x^2 + 24)/x^5 + 2
9/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = \frac{29\,\mathrm{atan}(x)}{8} - \frac{2207\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{256} - \frac{\frac{1743\,x^{12}}{128}+\frac{857\,x^{10}}{12}+\frac{82159\,x^8}{640}+\frac{85351\,x^6}{960}+\frac{321\,x^4}{20}-\frac{49\,x^2}{30}+\frac{2}{5}}{x^{13}+6\,x^{11}+13\,x^9+12\,x^7+4\,x^5}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^3),x)
```

```
[Out] (29*atan(x))/8 - (2207*2^(1/2)*atan((2^(1/2)*x)/2))/256 - ((321*x^4)/20 - (
49*x^2)/30 + (85351*x^6)/960 + (82159*x^8)/640 + (857*x^10)/12 + (1743*x^12
)/128 + 2/5)/(4*x^5 + 12*x^7 + 13*x^9 + 6*x^11 + x^13)
```

3.100 $\int \frac{x^9\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25(15+7x^2)}{8\left(3+2x^2+x^4\right)}+\frac{201\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{183}{4}\log\left(3+2x^2+x^4\right)$$

```
[Out] 19*x^2+19/4*x^4-17/6*x^6+5/8*x^8-25/8*(7*x^2+15)/(x^4+2*x^2+3)-183/4*ln(x^4
+2*x^2+3)+201/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1677, 1674, 1671, 648, 632, 210, 642}

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{201\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{25(7x^2+15)}{8\left(x^4+2x^2+3\right)}-\frac{183}{4}\log\left(x^4+2x^2+3\right)$$

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8 - (25*(15 + 7*x^2))/(8*(3 + 2*
x^2 + x^4)) + (201*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) - (183*Log[3 + 2*
x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
```

```
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(4+x+3x^2+5x^3)}{(3+2x+x^2)^2}\,dx, x, x^2\right)$$
$$= -\frac{25(15+7x^2)}{8(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{-150-400x+200x^2-56x^4+40x^5}{3+2x+x^2}\,dx, x, x^2\right)$$
$$= -\frac{25(15+7x^2)}{8(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(304+152x-136x^2+40x^3-\frac{6(177+244x)}{3+2x+x^2}\right)dx, x, x^2\right)$$
$$= 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8} - \frac{25(15+7x^2)}{8(3+2x^2+x^4)} - \frac{3}{8}\text{Subst}\left(\int \frac{177+244x}{3+2x+x^2}\,dx, x, x^2\right)$$
$$= 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8} - \frac{25(15+7x^2)}{8(3+2x^2+x^4)} + \frac{201}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right) - \frac{183}{4}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)$$
$$= 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8} - \frac{25(15+7x^2)}{8(3+2x^2+x^4)} - \frac{183}{4}\log\left(3+2x^2+x^4\right) - \frac{201}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2(1+x^2)\right)$$
$$= 19x^2 + \frac{19x^4}{4} - \frac{17x^6}{6} + \frac{5x^8}{8} - \frac{25(15+7x^2)}{8(3+2x^2+x^4)} + \frac{201\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{183}{4}\log\left(3+2x^2+x^4\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{48}\left(912x^2+228x^4-136x^6+30x^8-\frac{150(15+7x^2)}{3+2x^2+x^4} + 603\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right) - 2196\log\left(3+2x^2+x^4\right)\right)$$

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
[Out] (912*x^2 + 228*x^4 - 136*x^6 + 30*x^8 - (150*(15 + 7*x^2))/(3 + 2*x^2 + x^4
) + 603*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] - 2196*Log[3 + 2*x^2 + x^4])/48
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{-\frac{175x^2}{8}-\frac{375}{8}}{x^4+2x^2+3}-\frac{183\ln(x^4+2x^2+3)}{4}+\frac{201\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	71
default	$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{\frac{175x^2}{4}+\frac{375}{4}}{2(x^4+2x^2+3)}-\frac{183\ln(x^4+2x^2+3)}{4}+\frac{201\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	74

```
[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
[Out] 5/8*x^8-17/6*x^6+19/4*x^4+19*x^2+(-175/8*x^2-375/8)/(x^4+2*x^2+3)-183/4*ln(
x^4+2*x^2+3)+201/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{30\,x^{12}-76\,x^{10}+46\,x^8+960\,x^6+2508\,x^4+603\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+1686\,x^2-219}{48\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
[Out] 1/48*(30*x^12 - 76*x^10 + 46*x^8 + 960*x^6 + 2508*x^4 + 603*sqrt(2)*(x^4 +
2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1686*x^2 - 2196*(x^4 + 2*x^2 + 3
)*log(x^4 + 2*x^2 + 3) - 2250)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{-175x^2-375}{8x^4+16x^2+24} - \frac{183\log(x^4+2x^2+3)}{4}+\frac{201\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
[Out] 5*x**8/8 - 17*x**6/6 + 19*x**4/4 + 19*x**2 + (-175*x**2 - 375)/(8*x**4 + 16
*x**2 + 24) - 183*log(x**4 + 2*x**2 + 3)/4 + 201*sqrt(2)*atan(sqrt(2)*x**2/
2 + sqrt(2)/2)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{17}{6}\,x^6 + \frac{19}{4}\,x^4 + 19\,x^2 + \frac{201}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(7\,x^2+15)}{8\,(x^4+2\,x^2+3)} - \frac{183}{4}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*
(x^2 + 1)) - 25/8*(7*x^2 + 15)/(x^4 + 2*x^2 + 3) - 183/4*log(x^4 + 2*x^2 +
3)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{17}{6}\,x^6 + \frac{19}{4}\,x^4 + 19\,x^2 + \frac{201}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) + \frac{366\,x^4+557\,x^2+723}{8\,(x^4+2\,x^2+3)} - \frac{183}{4}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*
(x^2 + 1)) + 1/8*(366*x^4 + 557*x^2 + 723)/(x^4 + 2*x^2 + 3) - 183/4*log(x^
4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{201\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{175\,x^2}{8}+\frac{375}{8}}{x^4+2\,x^2+3} - \frac{183\,\ln\left(x^4+2\,x^2+3\right)}{4} + 19\,x^2 + \frac{19\,x^4}{4} - \frac{17\,x^6}{6} + \frac{5\,x^8}{8}$$

```
[In] int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (201*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((175*x^2)/8 + 375/8)/
(2*x^2 + x^4 + 3) - (183*log(2*x^2 + x^4 + 3))/4 + 19*x^2 + (19*x^4)/4 - (1
7*x^6)/6 + (5*x^8)/8
```

3.101 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 81

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx = \frac{19x^2}{2} - \frac{17x^4}{4} + \frac{5x^6}{6} + \frac{25(3+5x^2)}{8(3+2x^2+x^4)} - \frac{455\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{19}{2}\log\left(3+2x^2+x^4\right)$$

```
[Out] 19/2*x^2-17/4*x^4+5/6*x^6+25/8*(5*x^2+3)/(x^4+2*x^2+3)+19/2*ln(x^4+2*x^2+3)
-455/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1677, 1674, 1671, 648, 632, 210, 642}

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx = -\frac{455\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5x^6}{6} - \frac{17x^4}{4} + \frac{19x^2}{2} + \frac{25(5x^2+3)}{8(x^4+2x^2+3)} + \frac{19}{2}\log\left(x^4+2x^2+3\right)$$

```
[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (19*x^2)/2 - (17*x^4)/4 + (5*x^6)/6 + (25*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)
) - (455*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])
/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
```

```
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(4+x+3x^2+5x^3)}{(3+2x+x^2)^2}\,dx, x, x^2\right)\\
&= \frac{25(3+5x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{-150+200x-56x^3+40x^4}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{25(3+5x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(152-136x+40x^2-\frac{2(303-152x)}{3+2x+x^2}\right)\,dx, x, x^2\right)\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8\,(3+2x^2+x^4)}-\frac{1}{8}\text{Subst}\left(\int \frac{303-152x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8\,(3+2x^2+x^4)}+\frac{19}{2}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)\\
&\quad -\frac{455}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8\,(3+2x^2+x^4)}+\frac{19}{2}\log\left(3+2x^2+x^4\right)\\
&\quad +\frac{455}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right)\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8\,(3+2x^2+x^4)}-\frac{455\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{2}\log\left(3+2x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$
\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{48}\left(456x^2-204x^4+40x^6+\frac{150(3+5x^2)}{3+2x^2+x^4}\right.
\left. -1365\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)+456\log\left(3+2x^2+x^4\right)\right)
$$

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
[Out] (456*x^2 - 204*x^4 + 40*x^6 + (150*(3 + 5*x^2))/(3 + 2*x^2 + x^4) - 1365*Sq
rt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 456*Log[3 + 2*x^2 + x^4])/48
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{\frac{125x^2}{8}+\frac{75}{8}}{x^4+2x^2+3}+\frac{19\ln(x^4+2x^2+3)}{2}-\frac{455\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	66
default	$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{\frac{125x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}+\frac{19\ln(x^4+2x^2+3)}{2}-\frac{455\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	69

```
[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/6*x^6-17/4*x^4+19/2*x^2+(125/8*x^2+75/8)/(x^4+2*x^2+3)+19/2*ln(x^4+2*x^2+
3)-455/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{40\,x^{10}-124\,x^8+168\,x^6+300\,x^4-1365\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+2118\,x^2+456\,(x^4+2}{48\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(40*x^10 - 124*x^8 + 168*x^6 + 300*x^4 - 1365*sqrt(2)*(x^4 + 2*x^2 + 3
)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 2118*x^2 + 456*(x^4 + 2*x^2 + 3)*log(x^4
+ 2*x^2 + 3) + 450)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{125x^2+75}{8x^4+16x^2+24} + \frac{19\log(x^4+2x^2+3)}{2}-\frac{455\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**6/6 - 17*x**4/4 + 19*x**2/2 + (125*x**2 + 75)/(8*x**4 + 16*x**2 + 24)
+ 19*log(x**4 + 2*x**2 + 3)/2 - 455*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2
)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{17}{4}\,x^4 + \frac{19}{2}\,x^2 - \frac{455}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) + \frac{25\,(5\,x^2+3)}{8\,(x^4+2\,x^2+3)} + \frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)
) + 25/8*(5*x^2 + 3)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{17}{4}\,x^4 + \frac{19}{2}\,x^2 - \frac{455}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{76\,x^4+27\,x^2+153}{8\,(x^4+2\,x^2+3)} + \frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)
) - 1/8*(76*x^4 + 27*x^2 + 153)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 +
3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{19\,\ln\left(x^4+2\,x^2+3\right)}{2} + \frac{\frac{125\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3} - \frac{455\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} + \frac{19\,x^2}{2} - \frac{17\,x^4}{4} + \frac{5\,x^6}{6}$$

```
[In] int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
[Out] (19*log(2*x^2 + x^4 + 3))/2 + ((125*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (455
*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 + (19*x^2)/2 - (17*x^4)/4 +
(5*x^6)/6
```

3.102 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 74

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx = -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25(3-x^2)}{8(3+2x^2+x^4)} + \frac{203\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{4}\log\left(3+2x^2+x^4\right)$$

```
[Out] -17/2*x^2+5/4*x^4+25/8*(-x^2+3)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)+203/16*a
rctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1677, 1674, 1671, 648, 632, 210, 642}

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx = \frac{203\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5x^4}{4}-\frac{17x^2}{2} + \frac{25(3-x^2)}{8(x^4+2x^2+3)}+\frac{19}{4}\log\left(x^4+2x^2+3\right)$$

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-17*x^2)/2 + (5*x^4)/4 + (25*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (203*ArcTa
n[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(4+x+3x^2+5x^3)}{(3+2x+x^2)^2}\,dx, x, x^2\right)\\
&= \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{150-56x^2+40x^3}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(-136+40x+\frac{2(279+76x)}{3+2x+x^2}\right)\,dx, x, x^2\right)\\
&= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{8}\text{Subst}\left(\int \frac{279+76x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{19}{4}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)\\
&\quad + \frac{203}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{19}{4}\log\left(3+2x^2+x^4\right)\\
&\quad - \frac{203}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right)\\
&= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{203\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{19}{4}\log\left(3+2x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$
\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{16}\left(-136x^2+20x^4-\frac{50(-3+x^2)}{3+2x^2+x^4} + 203\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right) + 76\log\left(3+2x^2+x^4\right)\right)
$$

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
[Out] (-136*x^2 + 20*x^4 - (50*(-3 + x^2))/(3 + 2*x^2 + x^4) + 203*Sqrt[2]*ArcTan
[(1 + x^2)/Sqrt[2]] + 76*Log[3 + 2*x^2 + x^4])/16
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{289}{20}+\frac{-\frac{25x^2}{8}+\frac{75}{8}}{x^4+2x^2+3}+\frac{19\ln\left(x^4+2x^2+3\right)}{4}+\frac{203\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	62
default	$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{-\frac{25x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}+\frac{19\ln\left(x^4+2x^2+3\right)}{4}+\frac{203\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	64

```
[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/4*x^4-17/2*x^2+289/20+(-25/8*x^2+75/8)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)
+203/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$=\frac{20\,x^8-96\,x^6-212\,x^4+203\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-458\,x^2+76\,(x^4+2\,x^2+3)\log(\ldots}{16\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*x^8 - 96*x^6 - 212*x^4 + 203*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*
sqrt(2)*(x^2 + 1)) - 458*x^2 + 76*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) +
150)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx=\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{75-25x^2}{8x^4+16x^2+24}+\frac{19\log\left(x^4+2x^2+3\right)}{4}+\frac{203\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**4/4 - 17*x**2/2 + (75 - 25*x**2)/(8*x**4 + 16*x**2 + 24) + 19*log(x**4
 + 2*x**2 + 3)/4 + 203*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{17}{2}\,x^2 + \frac{203}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(x^2-3)}{8\,(x^4+2\,x^2+3)} + \frac{19}{4}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x
^2 - 3)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{17}{2}\,x^2 + \frac{203}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{38\,x^4+101\,x^2+39}{8\,(x^4+2\,x^2+3)} + \frac{19}{4}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(38
*x^4 + 101*x^2 + 39)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{19\,\ln\left(x^4+2\,x^2+3\right)}{4} - \frac{\frac{25\,x^2}{8}-\frac{75}{8}}{x^4+2\,x^2+3} + \frac{203\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{17\,x^2}{2} + \frac{5\,x^4}{4}$$

```
[In] int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
[Out] (19*log(2*x^2 + x^4 + 3))/4 - ((25*x^2)/8 - 75/8)/(2*x^2 + x^4 + 3) + (203*
2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - (17*x^2)/2 + (5*x^4)/4
```

3.103 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 65

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^2}{2} - \frac{25(3+x^2)}{8(3+2x^2+x^4)} - \frac{17\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{17}{4}\log\left(3+2x^2+x^4\right)$$

```
[Out] 5/2*x^2-25/8*(x^2+3)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*arctan(1/2*(x
^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1677, 1674, 1671, 648, 632, 210, 642}

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{17\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5x^2}{2} - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4}\log\left(x^4+2x^2+3\right)$$

```
[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (25*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (17*ArcTan[(1 + x^2)/Sqr
t[2]])/(8*Sqrt[2]) - (17*Log[3 + 2*x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(4+x+3x^2+5x^3)}{(3+2x+x^2)^2}\,dx, x, x^2\right)\\
&= -\frac{25(3+x^2)}{8(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{-50-56x+40x^2}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{25(3+x^2)}{8(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(40-\frac{34(5+4x)}{3+2x+x^2}\right)dx, x, x^2\right)\\
&= \frac{5x^2}{2} - \frac{25(3+x^2)}{8(3+2x^2+x^4)} - \frac{17}{8}\text{Subst}\left(\int \frac{5+4x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{5x^2}{2} - \frac{25(3+x^2)}{8(3+2x^2+x^4)} - \frac{17}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)\\
&\quad - \frac{17}{4}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{5x^2}{2} - \frac{25(3+x^2)}{8(3+2x^2+x^4)} - \frac{17}{4}\log\left(3+2x^2+x^4\right) + \frac{17}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2(1+x^2)\right)\\
&= \frac{5x^2}{2} - \frac{25(3+x^2)}{8(3+2x^2+x^4)} - \frac{17\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{17}{4}\log\left(3+2x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$
\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{16}\left(40x^2 - \frac{50(3+x^2)}{3+2x^2+x^4} - 17\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right) - 68\log\left(3+2x^2+x^4\right)\right)
$$

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
[Out] (40*x^2 - (50*(3 + x^2))/(3 + 2*x^2 + x^4) - 17*Sqrt[2]*ArcTan[(1 + x^2)/Sq
rt[2]] - 68*Log[3 + 2*x^2 + x^4])/16
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{5x^2}{2}+\frac{-\frac{25x^2}{8}-\frac{75}{8}}{x^4+2x^2+3}-\frac{17\ln\left(x^4+2x^2+3\right)}{4}-\frac{17\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	56
default	$\frac{5x^2}{2}-\frac{\frac{25x^2}{4}+\frac{75}{4}}{2\left(x^4+2x^2+3\right)}-\frac{17\ln\left(x^4+2x^2+3\right)}{4}-\frac{17\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	59

```
[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*x^2+(-25/8*x^2-75/8)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*arctan(1/
2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx =\frac{40\,x^6+80\,x^4-17\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+70\,x^2-68\,(x^4+2\,x^2+3)\log\left(x^4+2\,x^2+3\right.}{16\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(40*x^6 + 80*x^4 - 17*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^
2 + 1)) + 70*x^2 - 68*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 150)/(x^4 +
2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx=\frac{5x^2}{2}+\frac{-25x^2-75}{8x^4+16x^2+24}-\frac{17\log\left(x^4+2x^2+3\right)}{4}-\frac{17\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**2/2 + (-25*x**2 - 75)/(8*x**4 + 16*x**2 + 24) - 17*log(x**4 + 2*x**2 +
 3)/4 - 17*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{17}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(x^2+3)}{8\,(x^4+2\,x^2+3)} - \frac{17}{4}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4
 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{17}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(x^2+3)}{8\,(x^4+2\,x^2+3)} - \frac{17}{4}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4
 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5\,x^2}{2} - \frac{\frac{25\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3} - \frac{17\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{17\,\ln\left(x^4+2\,x^2+3\right)}{4}$$

```
[In] int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (5*x^2)/2 - ((25*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (17*2^(1/2)*atan(2^(1/2
)/2 + (2^(1/2)*x^2)/2))/16 - (17*log(2*x^2 + x^4 + 3))/4
```

3.104 $\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 58

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx = \frac{25(1+x^2)}{8(3+2x^2+x^4)} - \frac{23\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5}{4}\log\left(3+2x^2+x^4\right)$$

```
[Out] 25/8*(x^2+1)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*arctan(1/2*(x^2+1)*2^(
1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1677, 1674, 648, 632, 210, 642}

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx = -\frac{23\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{25(x^2+1)}{8(x^4+2x^2+3)} + \frac{5}{4}\log\left(x^4+2x^2+3\right)$$

```
[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sq
rt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{(3+2x+x^2)^2}\,dx, x, x^2\right)\\ &= \frac{25(1+x^2)}{8(3+2x^2+x^4)}+\frac{1}{16}\text{Subst}\left(\int \frac{-6+40x}{3+2x+x^2}\,dx, x, x^2\right)\\ &= \frac{25(1+x^2)}{8(3+2x^2+x^4)}+\frac{5}{4}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)-\frac{23}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)\end{aligned}$$

$$= \frac{25(1+x^2)}{8(3+2x^2+x^4)} + \frac{5}{4}\log\left(3+2x^2+x^4\right) + \frac{23}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2(1+x^2)\right)$$

$$= \frac{25(1+x^2)}{8(3+2x^2+x^4)} - \frac{23\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5}{4}\log\left(3+2x^2+x^4\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{25(1+x^2)}{8(3+2x^2+x^4)} - \frac{23\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5}{4}\log\left(3+2x^2+x^4\right)$$

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sq
rt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\frac{25x^2}{8}+\frac{25}{8}}{x^4+2x^2+3} + \frac{5\ln(x^4+2x^2+3)}{4} - \frac{23\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	51
default	$\frac{\frac{25x^2}{4}+\frac{25}{4}}{2x^4+4x^2+6} + \frac{5\ln(x^4+2x^2+3)}{4} - \frac{23\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	54

```
[In] int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
[Out] (25/8*x^2+25/8)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*arctan(1/2*(x^2+1)*
2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{23\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-50\,x^2-20\,(x^4+2\,x^2+3)\log\left(x^4+2\,x^2+3\right)-50}{16\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
[Out] -1/16*(23*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 50*x^2
- 20*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 50)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{25x^2+25}{8x^4+16x^2+24}+\frac{5\log\left(x^4+2x^2+3\right)}{4}-\frac{23\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
[Out] (25*x**2 + 25)/(8*x**4 + 16*x**2 + 24) + 5*log(x**4 + 2*x**2 + 3)/4 - 23*sq
rt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{23}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+\frac{25\,(x^2+1)}{8\,(x^4+2\,x^2+3)}+\frac{5}{4}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
+ 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{23}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) + \frac{25\,(x^2+1)}{8\,(x^4+2\,x^2+3)} + \frac{5}{4}\,\log\left(x^4+2\,x^2+3\right)$$

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
+ 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5\,\ln\left(x^4+2\,x^2+3\right)}{4} + \frac{25\,x^2}{8\,\left(x^4+2\,x^2+3\right)} + \frac{25}{8\,\left(x^4+2\,x^2+3\right)} - \frac{23\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
[Out] (5*log(2*x^2 + x^4 + 3))/4 + (25*x^2)/(8*(2*x^2 + x^4 + 3)) + 25/(8*(2*x^2
+ x^4 + 3)) - (23*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16
```

3.105 $\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 66

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx = \frac{25(1-x^2)}{24\left(3+2x^2+x^4\right)} + \frac{89\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{72\sqrt{2}} + \frac{4\log(x)}{9} - \frac{1}{9}\log\left(3+2x^2+x^4\right)$$

```
[Out] 25/24*(-x^2+1)/(x^4+2*x^2+3)+4/9*ln(x)-1/9*ln(x^4+2*x^2+3)+89/144*arctan(1/
2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1677, 1660, 814, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx = \frac{89\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}} + \frac{25(1-x^2)}{24\left(x^4+2x^2+3\right)} - \frac{1}{9}\log\left(x^4+2x^2+3\right) + \frac{4\log(x)}{9}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (25*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + (89*ArcTan[(1 + x^2)/Sqrt[2]])/(72*
Sqrt[2]) + (4*Log[x])/9 - Log[3 + 2*x^2 + x^4]/9
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x\,(3+2x+x^2)^2}\,dx,x,x^2\right)\\
&= \frac{25(1-x^2)}{24\,(3+2x^2+x^4)}+\frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}+\frac{70x}{3}}{x\,(3+2x+x^2)}\,dx,x,x^2\right)\\
&= \frac{25(1-x^2)}{24\,(3+2x^2+x^4)}+\frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x}-\frac{2(-73+16x)}{9\,(3+2x+x^2)}\right)\,dx,x,x^2\right)\\
&= \frac{25(1-x^2)}{24\,(3+2x^2+x^4)}+\frac{4\log(x)}{9}-\frac{1}{72}\text{Subst}\left(\int \frac{-73+16x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25(1-x^2)}{24\,(3+2x^2+x^4)}+\frac{4\log(x)}{9}-\frac{1}{9}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)\\
&\quad+\frac{89}{72}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25(1-x^2)}{24\,(3+2x^2+x^4)}+\frac{4\log(x)}{9}-\frac{1}{9}\log\left(3+2x^2+x^4\right)-\frac{89}{36}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2(1+x^2)\right)\\
&= \frac{25(1-x^2)}{24\,(3+2x^2+x^4)}+\frac{89\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{4\log(x)}{9}-\frac{1}{9}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{4+x^2+3x^4+5x^6}{x\,(3+2x^2+x^4)^2}\,dx = \frac{1}{288}\left(178\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)+128\log(x)+\frac{4(75-75x^2-8(3+2x^2+x^4)\log\left(3+2x^2+x^4\right))}{3+2x^2+x^4}\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
[Out] (178*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 128*Log[x] + (4*(75 - 75*x^2 - 8*(
3 + 2*x^2 + x^4)*Log[3 + 2*x^2 + x^4]))/(3 + 2*x^2 + x^4))/288
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{4\ln(x)}{9}-\frac{\frac{75x^{2}}{4}-\frac{75}{4}}{18(x^{4}+2x^{2}+3)}-\frac{\ln(x^{4}+2x^{2}+3)}{9}+\frac{89\sqrt{2}\,\arctan\left(\frac{(2x^{2}+2)\sqrt{2}}{4}\right)}{144}$	58
risch	$\frac{-\frac{25x^{2}}{24}+\frac{25}{24}}{x^{4}+2x^{2}+3}+\frac{4\ln(x)}{9}-\frac{\ln(7921x^{4}+15842x^{2}+23763)}{9}+\frac{89\sqrt{2}\,\arctan\left(\frac{(89x^{2}+89)\sqrt{2}}{178}\right)}{144}$	59

```
[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/9*ln(x)-1/18*(75/4*x^2-75/4)/(x^4+2*x^2+3)-1/9*ln(x^4+2*x^2+3)+89/144*2^(
1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{4+x^2+3x^4+5x^6}{x\,(3+2x^2+x^4)^2}\,dx = \frac{89\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-150\,x^2-16\,(x^4+2\,x^2+3)\log\left(x^4+2\,x^2+3\right)+64\,(x^4+2\,x}{144\,(x^4+2\,x^2+3)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(89*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 150*x^2
 - 16*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 64*(x^4 + 2*x^2 + 3)*log(x)
+ 150)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{4+x^2+3x^4+5x^6}{x\,(3+2x^2+x^4)^2}\,dx = \frac{25-25x^2}{24x^4+48x^2+72}+\frac{4\log(x)}{9}-\frac{\log(x^4+2x^2+3)}{9}+\frac{89\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)
```

```
[Out] (25 - 25*x**2)/(24*x**4 + 48*x**2 + 72) + 4*log(x)/9 - log(x**4 + 2*x**2 +
3)/9 + 89*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/144
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx = \frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{25\,(x^2-1)}{24\,(x^4+2\,x^2+3)} - \frac{1}{9}\log\left(x^4+2\,x^2+3\right) + \frac{2}{9}\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/24*(x^2 - 1)/(x^4 + 2*x^2
 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx = \frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{8\,x^4-59\,x^2+99}{72\,(x^4+2\,x^2+3)} - \frac{1}{9}\log\left(x^4+2\,x^2+3\right) + \frac{2}{9}\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(8*x^4 - 59*x^2 + 99)/(
x^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx = \frac{4\ln(x)}{9} - \frac{\ln\left(x^4+2\,x^2+3\right)}{9} - \frac{\frac{25\,x^2}{24}-\frac{25}{24}}{x^4+2\,x^2+3} + \frac{89\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(2*x^2 + x^4 + 3)^2),x)
[Out] (4*log(x))/9 - log(2*x^2 + x^4 + 3)/9 - ((25*x^2)/24 - 25/24)/(2*x^2 + x^4
+ 3) + (89*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/144
```

3.106 $\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 71

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx=-\frac{2}{9x^2}-\frac{25(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{71\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\right)$$

```
[Out] -2/9/x^2-25/72*(x^2+5)/(x^4+2*x^2+3)-13/27*ln(x)+13/108*ln(x^4+2*x^2+3)-71/
432*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1677, 1660, 1642, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx=-\frac{71\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{2}{9x^2}-\frac{25(x^2+5)}{72\left(x^4+2x^2+3\right)}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{13\log(x)}{27}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(9*x^2) - (25*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - (71*ArcTan[(1 + x^2)/S
qrt[2]])/(216*Sqrt[2]) - (13*Log[x])/27 + (13*Log[3 + 2*x^2 + x^4])/108
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^2\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{25(5+x^2)}{72\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}-\frac{50x^2}{9}}{x^2\left(3+2x+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{25(5+x^2)}{72\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x^2}-\frac{104}{27x}+\frac{2(-19+52x)}{27\left(3+2x+x^2\right)}\right)dx,x,x^2\right)\\
&= -\frac{2}{9x^2}-\frac{25(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{1}{216}\text{Subst}\left(\int \frac{-19+52x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= -\frac{2}{9x^2}-\frac{25(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{13}{108}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)\\
&\quad -\frac{71}{216}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx,x,x^2\right)\\
&= -\frac{2}{9x^2}-\frac{25(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\right)\\
&\quad +\frac{71}{108}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2\left(1+x^2\right)\right)\\
&= -\frac{2}{9x^2}-\frac{25(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{71\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = \frac{1}{864}\bigg(&-\frac{192}{x^2}-\frac{300(5+x^2)}{3+2x^2+x^4}-416\log(x)\\
&+\sqrt{2}\left(-71i+52\sqrt{2}\right)\log\left(-i+\sqrt{2}-ix^2\right)\\
&+\sqrt{2}\left(71i+52\sqrt{2}\right)\log\left(i+\sqrt{2}+ix^2\right)\bigg)
\end{aligned}$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
[Out] (-192/x^2 - (300*(5 + x^2))/(3 + 2*x^2 + x^4) - 416*Log[x] + Sqrt[2]*(-71*I
 + 52*Sqrt[2])*Log[-I + Sqrt[2] - I*x^2] + Sqrt[2]*(71*I + 52*Sqrt[2])*Log[
I + Sqrt[2] + I*x^2])/864
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2}{9x^2}-\frac{13\ln(x)}{27}+\frac{-\frac{75x^2}{4}-\frac{375}{4}}{54x^4+108x^2+162}+\frac{13\ln(x^4+2x^2+3)}{108}-\frac{71\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432}$	63
risch	$\frac{-\frac{41}{72}x^4-\frac{157}{72}x^2-\frac{2}{3}}{x^2(x^4+2x^2+3)}-\frac{13\ln(x)}{27}+\frac{13\ln(5041x^4+10082x^2+15123)}{108}-\frac{71\sqrt{2}\,\arctan\left(\frac{(71x^2+71)\sqrt{2}}{142}\right)}{432}$	67

```
[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9/x^2-13/27*ln(x)+1/54*(-75/4*x^2-375/4)/(x^4+2*x^2+3)+13/108*ln(x^4+2*x
^2+3)-71/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx = -\frac{246\,x^4+71\,\sqrt{2}(x^6+2\,x^4+3\,x^2)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+942\,x^2-52\,(x^6+2\,x^4+3\,x^2)\log\left(x^4+2\,x^2\right.}{432\,(x^6+2\,x^4+3\,x^2)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(246*x^4 + 71*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*arctan(1/2*sqrt(2)*(x^2
+ 1)) + 942*x^2 - 52*(x^6 + 2*x^4 + 3*x^2)*log(x^4 + 2*x^2 + 3) + 208*(x^6
+ 2*x^4 + 3*x^2)*log(x) + 288)/(x^6 + 2*x^4 + 3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx = \frac{-41x^4-157x^2-48}{72x^6+144x^4+216x^2}-\frac{13\log(x)}{27}+\frac{13\log(x^4+2x^2+3)}{108}-\frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-41*x**4 - 157*x**2 - 48)/(72*x**6 + 144*x**4 + 216*x**2) - 13*log(x)/27 +
 13*log(x**4 + 2*x**2 + 3)/108 - 71*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2
)/432
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = -\frac{71}{432}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-\frac{41\,x^4+157\,x^2+48}{72\,\left(x^6+2\,x^4+3\,x^2\right)}+\frac{13}{108}\,\log\left(x^4+2\,x^2+3\right)-\frac{13}{54}\,\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48
)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = -\frac{71}{432}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-\frac{41\,x^4+157\,x^2+48}{72\,\left(x^6+2\,x^4+3\,x^2\right)}+\frac{13}{108}\,\log\left(x^4+2\,x^2+3\right)-\frac{13}{54}\,\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48
)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = \frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108}-\frac{13\,\ln\left(x\right)}{27}-\frac{\frac{41\,x^4}{72}+\frac{157\,x^2}{72}+\frac{2}{3}}{x^6+2\,x^4+3\,x^2}-\frac{71\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(2*x^2 + x^4 + 3)^2),x)
[Out] (13*log(2*x^2 + x^4 + 3))/108 - (13*log(x))/27 - ((157*x^2)/72 + (41*x^4)/7
2 + 2/3)/(3*x^2 + 2*x^4 + x^6) - (71*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)
/2))/432
```

3.107 $\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx=-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{125\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2+x^4\right)$$

```
[Out] -1/9/x^4+13/54/x^2+25/216*(5*x^2+7)/(x^4+2*x^2+3)+13/27*ln(x)-13/108*ln(x^4
+2*x^2+3)+125/432*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1677, 1660, 1642, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx=\frac{125\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(5x^2+7)}{216\left(x^4+2x^2+3\right)}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{13\log(x)}{27}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -1/9*1/x^4 + 13/(54*x^2) + (25*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) + (125*
ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) + (13*Log[x])/27 - (13*Log[3 + 2*x
^2 + x^4])/108
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^3\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)} + \frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}+\frac{200x^2}{27}+\frac{250x^3}{27}}{x^3\left(3+2x+x^2\right)}\,dx, x, x^2\right)\\
&= \frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)} + \frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x^3}-\frac{104}{27x^2}+\frac{104}{27x}-\frac{2(-73+52x)}{27\left(3+2x+x^2\right)}\right)\,dx, x, x^2\right)\\
&= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}-\frac{1}{216}\text{Subst}\left(\int \frac{-73+52x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}\\
&\quad -\frac{13}{108}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)+\frac{125}{216}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}\\
&\quad -\frac{13}{108}\log\left(3+2x^2+x^4\right)-\frac{125}{108}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right)\\
&= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{125\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2\right.\\
&\qquad \left.+x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx = \frac{1}{864}\left(-\frac{96}{x^4}+\frac{208}{x^2}+\frac{700}{3+2x^2+x^4}+\frac{500x^2}{3+2x^2+x^4}+250\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)+416\log(x)-104\log\left(3+2x^2+x^4\right)\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^4 + 208/x^2 + 700/(3 + 2*x^2 + x^4) + (500*x^2)/(3 + 2*x^2 + x^4) +
250*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 416*Log[x] - 104*Log[3 + 2*x^2 + x^
4])/864
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{13\ln(x)}{27}-\frac{-\frac{125x^2}{4}-\frac{175}{4}}{54(x^4+2x^2+3)}-\frac{13\ln(x^4+2x^2+3)}{108}+\frac{125\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432}$	68
risch	$\frac{\frac{59}{72}x^6+\frac{85}{72}x^4+\frac{1}{2}x^2-\frac{1}{3}}{x^4(x^4+2x^2+3)}+\frac{13\ln(x)}{27}-\frac{13\ln(15625x^4+31250x^2+46875)}{108}+\frac{125\sqrt{2}\,\arctan\left(\frac{(125x^2+125)\sqrt{2}}{250}\right)}{432}$	72

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/x^4+13/54/x^2+13/27*ln(x)-1/54*(-125/4*x^2-175/4)/(x^4+2*x^2+3)-13/108
*ln(x^4+2*x^2+3)+125/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx$$
$$=\frac{354\,x^6+510\,x^4+125\,\sqrt{2}(x^8+2\,x^6+3\,x^4)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+216\,x^2-52\,(x^8+2\,x^6+3\,x^4)\log\left(x^4+\right.}{432\,(x^8+2\,x^6+3\,x^4)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/432*(354*x^6 + 510*x^4 + 125*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*arctan(1/2*sqr
t(2)*(x^2 + 1)) + 216*x^2 - 52*(x^8 + 2*x^6 + 3*x^4)*log(x^4 + 2*x^2 + 3) +
 208*(x^8 + 2*x^6 + 3*x^4)*log(x) - 144)/(x^8 + 2*x^6 + 3*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx=\frac{13\log(x)}{27}-\frac{13\log(x^4+2x^2+3)}{108}+\frac{125\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}+\frac{59x^6+85x^4+36x^2-24}{72x^8+144x^6+216x^4}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)
```

```
[Out] 13*log(x)/27 - 13*log(x**4 + 2*x**2 + 3)/108 + 125*sqrt(2)*atan(sqrt(2)*x**
2/2 + sqrt(2)/2)/432 + (59*x**6 + 85*x**4 + 36*x**2 - 24)/(72*x**8 + 144*x*
*6 + 216*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx = \frac{125}{432}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{59\,x^6+85\,x^4+36\,x^2-24}{72\,\left(x^8+2\,x^6+3\,x^4\right)} - \frac{13}{108}\,\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\,\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(59*x^6 + 85*x^4 + 36*
x^2 - 24)/(x^8 + 2*x^6 + 3*x^4) - 13/108*log(x^4 + 2*x^2 + 3) + 13/54*log(x
^2)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx = \frac{125}{432}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{26\,x^4+177\,x^2+253}{216\,\left(x^4+2\,x^2+3\right)} - \frac{39\,x^4-26\,x^2+12}{108\,x^4}-\frac{13}{108}\,\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\,\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/216*(26*x^4 + 177*x^2 + 2
53)/(x^4 + 2*x^2 + 3) - 1/108*(39*x^4 - 26*x^2 + 12)/x^4 - 13/108*log(x^4 +
 2*x^2 + 3) + 13/54*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx = \frac{13\,\ln\left(x\right)}{27}-\frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108} + \frac{\frac{59\,x^6}{72}+\frac{85\,x^4}{72}+\frac{x^2}{2}-\frac{1}{3}}{x^8+2\,x^6+3\,x^4}+\frac{125\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(2*x^2 + x^4 + 3)^2),x)
[Out] (13*log(x))/27 - (13*log(2*x^2 + x^4 + 3))/108 + (x^2/2 + (85*x^4)/72 + (59
*x^6)/72 - 1/3)/(3*x^4 + 2*x^6 + x^8) + (125*2^(1/2)*atan(2^(1/2)/2 + (2^(1
/2)*x^2)/2))/432
```

3.108 $\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 87

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx=-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648\left(3+2x^2+x^4\right)}-\frac{1237\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}-\frac{61}{972}\log\left(3+2x^2+x^4\right)$$

```
[Out] -2/27/x^6+13/108/x^4-13/54/x^2+25/648*(-7*x^2+1)/(x^4+2*x^2+3)+61/243*ln(x)
-61/972*ln(x^4+2*x^2+3)-1237/3888*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1677, 1660, 1642, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx=-\frac{1237\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}}-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648\left(x^4+2x^2+3\right)}-\frac{61}{972}\log\left(x^4+2x^2+3\right)+\frac{61\log(x)}{243}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(27*x^6) + 13/(108*x^4) - 13/(54*x^2) + (25*(1 - 7*x^2))/(648*(3 + 2*x^2
 + x^4)) - (1237*ArcTan[(1 + x^2)/Sqrt[2]])/(1944*Sqrt[2]) + (61*Log[x])/24
3 - (61*Log[3 + 2*x^2 + x^4])/972
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^4(3+2x+x^2)^2}\,dx, x, x^2\right)$$
$$= \frac{25(1-7x^2)}{648(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}+\frac{200x^2}{27}+\frac{800x^3}{81}-\frac{350x^4}{81}}{x^4(3+2x+x^2)}\,dx, x, x^2\right)$$
$$= \frac{25(1-7x^2)}{648(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x^4}-\frac{104}{27x^3}+\frac{104}{27x^2}+\frac{488}{243x}-\frac{2(1481+244x)}{243(3+2x+x^2)}\right)dx, x, x^2\right)$$
$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)}+\frac{61\log(x)}{243}-\frac{\text{Subst}\left(\int \frac{1481+244x}{3+2x+x^2}\,dx, x, x^2\right)}{1944}$$
$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)}+\frac{61\log(x)}{243} - \frac{61}{972}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right) - \frac{1237\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)}{1944}$$
$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)}+\frac{61\log(x)}{243} - \frac{61}{972}\log\left(3+2x^2+x^4\right)+\frac{1237}{972}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right)$$
$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)} - \frac{1237\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}-\frac{61}{972}\log\left(3+2x^2+x^4\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx$$
$$= \frac{-\frac{576}{x^6}+\frac{936}{x^4}-\frac{1872}{x^2}-\frac{300(-1+7x^2)}{3+2x^2+x^4}+1952\log(x)-\sqrt{2}(1237i+244\sqrt{2})\log\left(-i+\sqrt{2}-ix^2\right)+\sqrt{2}(1237i-2}{7776}$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-576/x^6 + 936/x^4 - 1872/x^2 - (300*(-1 + 7*x^2))/(3 + 2*x^2 + x^4) + 195
2*Log[x] - Sqrt[2]*(1237*I + 244*Sqrt[2])*Log[-I + Sqrt[2] - I*x^2] + Sqrt[
2]*(1237*I - 244*Sqrt[2])*Log[I + Sqrt[2] + I*x^2])/7776
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

method	result
default	$-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{61\ln(x)}{243}-\frac{\frac{525x^2}{4}-\frac{75}{4}}{486(x^4+2x^2+3)}-\frac{61\ln(x^4+2x^2+3)}{972}-\frac{1237\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{3888}$
risch	$\frac{-\frac{331}{648}x^8-\frac{209}{648}x^6-\frac{5}{9}x^4+\frac{23}{108}x^2-\frac{2}{9}}{x^6(x^4+2x^2+3)}+\frac{61\ln(x)}{243}-\frac{61\ln(1530169x^4+3060338x^2+4590507)}{972}-\frac{1237\sqrt{2}\,\arctan\left(\frac{(1237x^2+1237)\sqrt{2}}{2474}\right)}{3888}$

```
[In] int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/27/x^6+13/108/x^4-13/54/x^2+61/243*ln(x)-1/486*(525/4*x^2-75/4)/(x^4+2*x
^2+3)-61/972*ln(x^4+2*x^2+3)-1237/3888*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.32

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = -\frac{1986\,x^8+1254\,x^6+2160\,x^4+1237\,\sqrt{2}(x^{10}+2\,x^8+3\,x^6)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-828\,x^2+244\,(x^{10}+}{3888\,(x^{10}+2\,x^8+3\,x^6)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/3888*(1986*x^8 + 1254*x^6 + 2160*x^4 + 1237*sqrt(2)*(x^10 + 2*x^8 + 3*x^
6)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 828*x^2 + 244*(x^10 + 2*x^8 + 3*x^6)*log
(x^4 + 2*x^2 + 3) - 976*(x^10 + 2*x^8 + 3*x^6)*log(x) + 864)/(x^10 + 2*x^8
+ 3*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = \frac{61\log(x)}{243} - \frac{61\log\left(x^4+2x^2+3\right)}{972} - \frac{1237\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888} + \frac{-331x^8-209x^6-360x^4+138x^2-144}{648x^{10}+1296x^8+1944x^6}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)
[Out] 61*log(x)/243 - 61*log(x**4 + 2*x**2 + 3)/972 - 1237*sqrt(2)*atan(sqrt(2)*x
**2/2 + sqrt(2)/2)/3888 + (-331*x**8 - 209*x**6 - 360*x**4 + 138*x**2 - 144
)/(648*x**10 + 1296*x**8 + 1944*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = -\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{331\,x^8+209\,x^6+360\,x^4-138\,x^2+144}{648\,(x^{10}+2\,x^8+3\,x^6)} - \frac{61}{972}\log\left(x^4+2\,x^2+3\right) + \frac{61}{486}\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/648*(331*x^8 + 209*x^6
 + 360*x^4 - 138*x^2 + 144)/(x^10 + 2*x^8 + 3*x^6) - 61/972*log(x^4 + 2*x^2
 + 3) + 61/486*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = -\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{122\,x^4-281\,x^2+441}{1944\,(x^4+2\,x^2+3)} - \frac{671\,x^6+702\,x^4-351\,x^2+216}{2916\,x^6} - \frac{61}{972}\log\left(x^4+2\,x^2+3\right) + \frac{61}{486}\log\left(x^2\right)$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/1944*(122*x^4 - 281*x^
2 + 441)/(x^4 + 2*x^2 + 3) - 1/2916*(671*x^6 + 702*x^4 - 351*x^2 + 216)/x^6
 - 61/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = \frac{61\,\ln\left(x\right)}{243}-\frac{61\,\ln\left(x^4+2\,x^2+3\right)}{972} - \frac{\frac{331\,x^8}{648}+\frac{209\,x^6}{648}+\frac{5\,x^4}{9}-\frac{23\,x^2}{108}+\frac{2}{9}}{x^{10}+2\,x^8+3\,x^6}-\frac{1237\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^7*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (61*log(x))/243 - (61*log(2*x^2 + x^4 + 3))/972 - ((5*x^4)/9 - (23*x^2)/108
 + (209*x^6)/648 + (331*x^8)/648 + 2/9)/(3*x^6 + 2*x^8 + x^10) - (1237*2^(1
/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/3888
```

3.109 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 248

$$
\begin{aligned}
&\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx \\
&= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8\,(3+2x^2+x^4)} \\
&\quad+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad+\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}
$$

```
[Out] 38*x+19/3*x^3-17/5*x^5+5/7*x^7+25/8*x*(5*x^2+3)/(x^4+2*x^2+3)-1/64*ln(x^2+3
^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-525542+1236582*3^(1/2))^(1/2)+1/64*ln(x^2+
3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-525542+1236582*3^(1/2))^(1/2)+1/32*arctan
((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(525542+1236582*3^(1/2))^
(1/2)-1/32*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(525542+1
236582*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1682, 1690, 1183, 648, 632, 210, 642}

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$
$$-\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{5x^7}{7}-\frac{17x^5}{5}$$
$$+\frac{19x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)$$
$$+\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)$$
$$+\frac{25(5x^2+3)\,x}{8\,(x^4+2x^2+3)}+38x$$

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(262771 + 618291*Sqrt[3])/2]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-
262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32 + (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3]
)]*x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{25x(3+5x^2)}{8\,(3+2x^2+x^4)}+\frac{1}{48}\int\frac{-450-1650x^2+1200x^4-336x^8+240x^{10}}{3+2x^2+x^4}\,dx\\ &= \frac{25x(3+5x^2)}{8\,(3+2x^2+x^4)}+\frac{1}{48}\int\left(1824+912x^2-816x^4+240x^6-\frac{6(987+1339x^2)}{3+2x^2+x^4}\right)dx\end{aligned}$$

$$= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3+5x^2)}{8(3+2x^2+x^4)} - \frac{1}{8}\int \frac{987+1339x^2}{3+2x^2+x^4}\,dx$$

$$= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3+5x^2)}{8(3+2x^2+x^4)} - \frac{\int \frac{987\sqrt{2\left(-1+\sqrt{3}\right)}-\left(987-1339\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}} - \frac{\int \frac{987\sqrt{2\left(-1+\sqrt{3}\right)}+\left(987-1339\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3+5x^2)}{8(3+2x^2+x^4)} - \frac{1}{32}\left(1339+329\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx - \frac{1}{32}\left(1339+329\sqrt{3}\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx - \frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx + \frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$

$$= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3+5x^2)}{8(3+2x^2+x^4)} - \frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) + \frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{16}\left(-1339-329\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right) - \frac{1}{16}\left(-1339-329\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$= 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3+5x^2)}{8(3+2x^2+x^4)}$$
$$+ \frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right)$$
$$- \frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right)$$
$$- \frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$+ \frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.58

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 38x + \frac{19x^3}{3} - \frac{17x^5}{5} + \frac{5x^7}{7} + \frac{25x(3+5x^2)}{8(3+2x^2+x^4)} - \frac{\left(352i+1339\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} - \frac{\left(-352i+1339\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) - ((352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqr
t[2 - (2*I)*Sqrt[2]]) - ((-352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2
]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.32

method	result
risch	$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x+\frac{\frac{125}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-1339\_R^2-987\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x-\frac{-\frac{125}{8}x^3-\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(-505\sqrt{-2+2\sqrt{3}}\sqrt{3}-176\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{64}+\frac{\left(-6\right.}{}$

[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)

[Out] 5/7*x^7-17/5*x^5+19/3*x^3+38*x+(125/8*x^3+75/8*x)/(x^4+2*x^2+3)+1/32*sum((-1339*_R^2-987)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.87

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$=\frac{2400\,x^{11}-6624\,x^9+5632\,x^7+135968\,x^5+371700\,x^3-105\,(x^4+2\,x^2+3)\sqrt{734099i\,\sqrt{2}-262771}\log\left(\right.}{}$$

[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/3360*(2400*x^11 - 6624*x^9 + 5632*x^7 + 135968*x^5 + 371700*x^3 - 105*(x^4 + 2*x^2 + 3)*sqrt(734099*I*sqrt(2) - 262771)*log(sqrt(734099*I*sqrt(2) - 262771)*(505*I*sqrt(2) + 329) + 618291*x) + 105*(x^4 + 2*x^2 + 3)*sqrt(734099*I*sqrt(2) - 262771)*log(sqrt(734099*I*sqrt(2) - 262771)*(-505*I*sqrt(2) - 329) + 618291*x) + 105*(x^4 + 2*x^2 + 3)*sqrt(-734099*I*sqrt(2) - 262771)*log((505*I*sqrt(2) - 329)*sqrt(-734099*I*sqrt(2) - 262771) + 618291*x) - 105*(x^4 + 2*x^2 + 3)*sqrt(-734099*I*sqrt(2) - 262771)*log((-505*I*sqrt(2) + 329)*sqrt(-734099*I*sqrt(2) - 262771) + 618291*x) + 414540*x)/(x^4 + 2*x^2 + 3)

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.29

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + 38x + \frac{125x^3+75x}{8x^4+16x^2+24} + \mathrm{RootSum}\left(1048576t^4 + 538155008t^2 + 1146851282043, \left(t \mapsto t\log\left(-\frac{16547840t^3}{453886804809} - \frac{11974973632t}{453886804809}\right.\right.\right.$$

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**7/7 - 17*x**5/5 + 19*x**3/3 + 38*x + (125*x**3 + 75*x)/(8*x**4 + 16*x*
*2 + 24) + RootSum(1048576*_t**4 + 538155008*_t**2 + 1146851282043, Lambda(
_t, _t*log(-16547840*_t**3/453886804809 - 11974973632*_t/453886804809 + x))
)
```

Maxima [F]

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^8}{(x^4+2\,x^2+3)^2}\,dx$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3)
 - 1/8*integrate((1339*x^2 + 987)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(175) = 350$.

Time = 0.59 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.36

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{7}\,x^7 - \frac{17}{5}\,x^5 + \frac{19}{3}\,x^3$$

$$+\frac{1}{20736}\,\sqrt{2}\left(1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}} + 24102\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right) - 24102\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{}\right.$$

$$+\frac{1}{20736}\,\sqrt{2}\left(1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}} + 24102\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right) - 24102\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{}\right.$$

$$+\frac{1}{41472}\,\sqrt{2}\left(24102\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+18} - 1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\,\sqrt{3}+18\right)^{\frac{3}{2}} + 1339\cdot 3^{\frac{3}{4}}\left(6\,\sqrt{3}+1\right.\right.$$

$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\,\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right)$$

$$-\frac{1}{41472}\,\sqrt{2}\left(24102\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+18} - 1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\,\sqrt{3}+18\right)^{\frac{3}{2}} + 1339\cdot 3^{\frac{3}{4}}\left(6\,\sqrt{3}+1\right.\right.$$

$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\,\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right) + 38\,x + \frac{25\,(5\,x^3+3\,x)}{8\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*(6*sq
rt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3
) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(-6*sq
rt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532*3^(1
/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3
) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*
(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3
) - 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(
-6*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532
*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*s
qrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/41472*sqrt(2)*(24102*3^(3/4)*sq
rt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3
) + 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*
sqrt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) -
 35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3
```

```
) + 1/2) + sqrt(3)) - 1/41472*sqrt(2)*(24102*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*
sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 1339
*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(
3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 35532*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
+ 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.69

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 38\,x+\frac{\frac{125\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3}+\frac{19\,x^3}{3}-\frac{17\,x^5}{5}+\frac{5\,x^7}{7}$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}\,734099\mathrm{i}}{64\,\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}+\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}}{128\,\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}\right)\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}\,1\mathrm{i}}{16}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}\,734099\mathrm{i}}{64\,\left(\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}-\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}}{128\,\left(\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}\right)\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}\,1\mathrm{i}}{16}$$

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 38*x + (atan((x*(- 2^(1/2)*734099i - 262771)^(1/2)*734099i)/(64*((2^(1/2)*7
24555713i)/128 - 1112159985/64)) + (734099*2^(1/2)*x*(- 2^(1/2)*734099i - 2
62771)^(1/2))/(128*((2^(1/2)*724555713i)/128 - 1112159985/64)))*(- 2^(1/2)*
734099i - 262771)^(1/2)*1i)/16 - (atan((x*(2^(1/2)*734099i - 262771)^(1/2)*
734099i)/(64*((2^(1/2)*724555713i)/128 + 1112159985/64)) - (734099*2^(1/2)*
x*(2^(1/2)*734099i - 262771)^(1/2))/(128*((2^(1/2)*724555713i)/128 + 111215
9985/64)))*(2^(1/2)*734099i - 262771)^(1/2)*1i)/16 + ((75*x)/8 + (125*x^3)/
8)/(2*x^2 + x^4 + 3) + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7
```

3.110 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 237

$$
\begin{aligned}
&\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx\\
&= 19x-\frac{17x^3}{3}+x^5+\frac{25x(3-x^2)}{8(3+2x^2+x^4)}\\
&\quad+\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\
&\quad-\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)\\
&\quad+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\\
&\quad-\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}
$$

```
[Out] 19*x-17/3*x^3+x^5+25/8*x*(-x^2+3)/(x^4+2*x^2+3)+3/32*arctan((-2*x+(-2+2*3^(
1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-52014+30066*3^(1/2))^(1/2)-3/32*arctan(
(2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-52014+30066*3^(1/2))^(1/2
)+3/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(52014+30066*3^(1/2))^(1/2)-3
/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(52014+30066*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1682, 1690, 1183, 648, 632, 210, 642}

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{3}{16}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{3}{16}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + x^5 - \frac{17x^3}{3} + \frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2 - \sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) - \frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{25(3-x^2)\,x}{8\left(x^4+2x^2+3\right)} + 19x$$

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 + (25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (3*Sqrt[
(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 + (3*Sqrt[(3*(8669 + 5011
*Sqrt[3]))/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 - (3*Sqrt[(
3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)}+\frac{1}{48}\int\frac{-450+1050x^2-336x^6+240x^8}{3+2x^2+x^4}\,dx\\ &= \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)}+\frac{1}{48}\int\left(912-816x^2+240x^4-\frac{54(59-31x^2)}{3+2x^2+x^4}\right)\,dx\end{aligned}$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\left(3+2x^2+x^4\right)} - \frac{9}{8}\int \frac{59-31x^2}{3+2x^2+x^4}\,dx$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\left(3+2x^2+x^4\right)}$$
$$- \frac{1}{32}\left(3\sqrt{3\left(1+\sqrt{3}\right)}\right)\int \frac{59\sqrt{2\left(-1+\sqrt{3}\right)} - \left(59+31\sqrt{3}\right)x}{\sqrt{3} - \sqrt{2\left(-1+\sqrt{3}\right)}x + x^2}\,dx$$
$$- \frac{1}{32}\left(3\sqrt{3\left(1+\sqrt{3}\right)}\right)\int \frac{59\sqrt{2\left(-1+\sqrt{3}\right)} + \left(59+31\sqrt{3}\right)x}{\sqrt{3} + \sqrt{2\left(-1+\sqrt{3}\right)}x + x^2}\,dx$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\left(3+2x^2+x^4\right)}$$
$$- \frac{1}{16}\left(3\sqrt{\frac{3}{2}\left(3182-1829\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{3} - \sqrt{2\left(-1+\sqrt{3}\right)}x + x^2}\,dx$$
$$- \frac{1}{16}\left(3\sqrt{\frac{3}{2}\left(3182-1829\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{3} + \sqrt{2\left(-1+\sqrt{3}\right)}x + x^2}\,dx$$
$$+ \frac{1}{32}\left(3\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\right)\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)} + 2x}{\sqrt{3} - \sqrt{2\left(-1+\sqrt{3}\right)}x + x^2}\,dx$$
$$- \frac{1}{32}\left(3\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)} + 2x}{\sqrt{3} + \sqrt{2\left(-1+\sqrt{3}\right)}x + x^2}\,dx$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) + \frac{1}{8}\left(3\sqrt{\frac{3}{2}\left(3182-1829\sqrt{3}\right)}\right)\text{Subst}\left(\int\frac{1}{-2\,(1+\sqrt{3})-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right) + \frac{1}{8}\left(3\sqrt{\frac{3}{2}\left(3182-1829\sqrt{3}\right)}\right)\text{Subst}\left(\int\frac{1}{-2\,(1+\sqrt{3})-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right) - \frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right) + \frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.56

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 19x-\frac{17x^3}{3}+x^5-\frac{25x(-3+x^2)}{8\,(3+2x^2+x^4)} + \frac{9(90i+31\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{9(-90i+31\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 - (25*x*(-3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (9*(90*
I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]])
 + (9*(-90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I
)*Sqrt[2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.30

method	result
risch	$x^5-\frac{17x^3}{3}+19x+\frac{-\frac{25}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{9\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(31\_R^2-59\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$x^5-\frac{17x^3}{3}+19x+\frac{-\frac{25}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{3\left(76\sqrt{-2+2\sqrt{3}}\sqrt{3}+135\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{64}+\frac{3\left(-118\sqrt{3}+\frac{(76\sqrt{-}}{}\right.}{}$

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^5-17/3*x^3+19*x+(-25/8*x^3+75/8*x)/(x^4+2*x^2+3)+9/32*sum((31*_R^2-59)/(_
R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.89

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{96\,x^9 - 352\,x^7 + 1024\,x^5 + 1716\,x^3 - 3\,(x^4+2\,x^2+3)\sqrt{8073i\,\sqrt{2}+234063}\log\left(\sqrt{8073i\,\sqrt{2}+234063}(7\right.}{}$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(96*x^9 - 352*x^7 + 1024*x^5 + 1716*x^3 - 3*(x^4 + 2*x^2 + 3)*sqrt(807
3*I*sqrt(2) + 234063)*log(sqrt(8073*I*sqrt(2) + 234063)*(76*I*sqrt(2) + 59)
 + 45099*x) + 3*(x^4 + 2*x^2 + 3)*sqrt(8073*I*sqrt(2) + 234063)*log(sqrt(80
73*I*sqrt(2) + 234063)*(-76*I*sqrt(2) - 59) + 45099*x) + 3*(x^4 + 2*x^2 + 3
)*sqrt(-8073*I*sqrt(2) + 234063)*log((76*I*sqrt(2) - 59)*sqrt(-8073*I*sqrt(
2) + 234063) + 45099*x) - 3*(x^4 + 2*x^2 + 3)*sqrt(-8073*I*sqrt(2) + 234063
)*log((-76*I*sqrt(2) + 59)*sqrt(-8073*I*sqrt(2) + 234063) + 45099*x) + 6372
*x)/(x^4 + 2*x^2 + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. 2(199) = 398.

Time = 0.74 (sec) , antiderivative size = 1205, normalized size of antiderivative = 5.08

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] x**5 - 17*x**3/3 + 19*x + (-25*x**3 + 75*x)/(8*x**4 + 16*x**2 + 24) - 3*sqr
t(26007/2048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-304*sqrt(2)*sqrt(8669 + 5
011*sqrt(3))/299 - 433349*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/1498289 + 152*s
qrt(3)*sqrt(8669 + 5011*sqrt(3))*sqrt(43440359*sqrt(3) + 75240962)/1498289)
 - 2882918249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/2244869927521 -
993398584*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 4993637694
9404567/2244869927521 + 17261871038090*sqrt(3)/1343965233) + 3*sqrt(26007/2
048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-152*sqrt(3)*sqrt(8669 + 5011*sqrt(
3))*sqrt(43440359*sqrt(3) + 75240962)/1498289 + 433349*sqrt(6)*sqrt(8669 +
5011*sqrt(3))/1498289 + 304*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/299) - 288291
8249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/2244869927521 - 993398584
*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 49936376949404567/2
```

```
244869927521 + 17261871038090*sqrt(3)/1343965233) - 2*sqrt(-27*sqrt(2)*sqrt
(43440359*sqrt(3) + 75240962)/1024 + 234063/2048 + 405891*sqrt(3)/2048)*ata
n(2996578*sqrt(3)*x/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) +
75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*s
qrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))) -
 1523344*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/(17641*sqrt(2)*sqrt(-2*sqrt(2)*s
qrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(4344035
9*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8
669 + 15033*sqrt(3))) - 1300047*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/(17641*sq
rt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt
(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*
sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))) + 456*sqrt(8669 + 5011*sqrt(3)
)*sqrt(43440359*sqrt(3) + 75240962)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(434
40359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(
3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 1
5033*sqrt(3)))) - 2*sqrt(-27*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/1024
 + 234063/2048 + 405891*sqrt(3)/2048)*atan(2996578*sqrt(3)*x/(17641*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))
+ 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(
3) + 75240962) + 8669 + 15033*sqrt(3))) - 456*sqrt(8669 + 5011*sqrt(3))*sqr
t(43440359*sqrt(3) + 75240962)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359
*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) +
75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*
sqrt(3))) + 1300047*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/(17641*sqrt(2)*sqrt(-
2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*s
qrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75
240962) + 8669 + 15033*sqrt(3))) + 1523344*sqrt(6)*sqrt(8669 + 5011*sqrt(3)
)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 +
 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqr
t(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))))
```

Maxima [F]

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^6}{(x^4+2\,x^2+3)^2}\,dx$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 17/3*x^3 + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3) + 9/8*integrate(
(31*x^2 - 59)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(166) = 332.

Time = 0.63 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.43

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = x^5 - \frac{17}{3}\,x^3$$

$$-\frac{1}{2304}\sqrt{2}\left(31\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 558\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 558\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\cdots}\right.$$

$$-\frac{1}{2304}\sqrt{2}\left(31\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 558\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 558\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\cdots}\right.$$

$$-\frac{1}{4608}\sqrt{2}\left(558\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 31\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 31\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 5\cdots\right.$$
$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right)$$

$$+\frac{1}{4608}\sqrt{2}\left(558\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 31\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 31\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 5\cdots\right.$$
$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right) + 19\,x - \frac{25\,(x^3-3\,x)}{8\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] x^5 - 17/3*x^3 - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
+ 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sqr
t(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2124
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 558*
3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sqrt(3) +
 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2124*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arcta
n(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2
)) - 1/4608*sqrt(2)*(558*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 31*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 31*3^(3/4)*(6*sqrt(3) + 18
)^(3/2) + 558*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 2124*3^(1/4)*sqr
t(2)*sqrt(-6*sqrt(3) + 18) + 2124*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2
*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/4608*sqrt(2)*(558*3^(3/4
```

```
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 31*3^(3/4)*sqrt(2)*(-6*sqrt
(3) + 18)^(3/2) + 31*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 558*3^(3/4)*sqrt(6*sq
rt(3) + 18)*(sqrt(3) - 3) + 2124*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 21
24*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
1/2) + sqrt(3)) + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.69

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= 19\,x + \frac{\frac{75\,x}{8} - \frac{25\,x^3}{8}}{x^4+2\,x^2+3} - \frac{17\,x^3}{3} + x^5$$
$$- \frac{\operatorname{atan}\left(\frac{x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)} - \frac{24219\,\sqrt{2}\,x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}}{128\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}}{16}$$
$$+ \frac{\operatorname{atan}\left(\frac{x\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\left(\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)} + \frac{24219\,\sqrt{2}\,x\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}}{128\left(\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\sqrt{26007+\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}}{16}$$

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 19*x + ((75*x)/8 - (25*x^3)/8)/(2*x^2 + x^4 + 3) - (atan((x*(26007 - 2^(1/2
)*897i)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 - 1380483/16)) - (24219*2
^(1/2)*x*(26007 - 2^(1/2)*897i)^(1/2))/(128*((2^(1/2)*4286763i)/128 - 13804
83/16)))*(26007 - 2^(1/2)*897i)^(1/2)*3i)/16 + (atan((x*(2^(1/2)*897i + 260
07)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 + 1380483/16)) + (24219*2^(1/
2)*x*(2^(1/2)*897i + 26007)^(1/2))/(128*((2^(1/2)*4286763i)/128 + 1380483/1
6)))*(2^(1/2)*897i + 26007)^(1/2)*3i)/16 - (17*x^3)/3 + x^5
```

3.111 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 232

$$\begin{aligned}
\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8\,(3+2x^2+x^4)} \\
&\quad -\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad -\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad +\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}$$

```
[Out] -17*x+5/3*x^3-25/8*x*(x^2+3)/(x^4+2*x^2+3)-1/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1
/2))^(1/2))*(-28790+52998*3^(1/2))^(1/2)+1/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2
))^(1/2))*(-28790+52998*3^(1/2))^(1/2)-1/32*arctan((-2*x+(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))*(28790+52998*3^(1/2))^(1/2)+1/32*arctan((2*x+(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(28790+52998*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1682, 1690, 1183, 648, 632, 210, 642}

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{5x^3}{3} - \frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) - \frac{25(x^2+3)\,x}{8\left(x^4+2x^2+3\right)} - 17x$$

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (Sqrt[(14395 +
 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/16 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[S
qrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-14395 + 26499*Sqrt[3
])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\text{integral} = -\frac{25x(3+x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{48}\int \frac{450-150x^2-336x^4+240x^6}{3+2x^2+x^4}\,dx$$

$$= -\frac{25x(3+x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \left(-816+240x^2+\frac{6(483+127x^2)}{3+2x^2+x^4}\right) dx$$

$$= -17x + \frac{5x^3}{3} - \frac{25x(3+x^2)}{8(3+2x^2+x^4)} + \frac{1}{8}\int \frac{483+127x^2}{3+2x^2+x^4}\, dx$$

$$= -17x + \frac{5x^3}{3} - \frac{25x(3+x^2)}{8(3+2x^2+x^4)} + \frac{\int \frac{483\sqrt{2\left(-1+\sqrt{3}\right)}-\left(483-127\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\, dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$+ \frac{\int \frac{483\sqrt{2\left(-1+\sqrt{3}\right)}+\left(483-127\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\, dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -17x + \frac{5x^3}{3} - \frac{25x(3+x^2)}{8(3+2x^2+x^4)}$$

$$+ \frac{1}{32}\left(127+161\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\, dx$$

$$+ \frac{1}{32}\left(127+161\sqrt{3}\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\, dx$$

$$- \frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\, dx$$

$$+ \frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\, dx$$

$$= -17x + \frac{5x^3}{3} - \frac{25x(3+x^2)}{8(3+2x^2+x^4)}$$

$$- \frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

$$+ \frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

$$+ \frac{1}{16}\left(-127-161\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\, dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$+ \frac{1}{16}\left(-127-161\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\, dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$
\begin{aligned}
&= -17x + \frac{5x^3}{3} - \frac{25x(3+x^2)}{8\,(3+2x^2+x^4)} \\
&\quad - \frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad + \frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad - \frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad + \frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.56

$$
\begin{aligned}
\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= -17x + \frac{5x^3}{3} - \frac{25x(3+x^2)}{8\,(3+2x^2+x^4)} \\
&\quad + \frac{(-356i+127\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} \\
&\quad + \frac{(356i+127\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}
\end{aligned}
$$

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + ((-356*I + 127
*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) + ((3
56*I + 127*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[
2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.30

method	result
risch	$\frac{5x^3}{3} - 17x + \frac{-\frac{25}{8}x^3 - \frac{75}{8}x}{x^4+2x^2+3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)} \frac{(127\_R^2+483)\ln(x-\_R)}{\_R^3+\_R}\right)}{32}$
default	$\frac{5x^3}{3} - 17x + \frac{-\frac{25}{8}x^3 - \frac{75}{8}x}{x^4+2x^2+3} + \frac{\left(-17\sqrt{-2+2\sqrt{3}}\sqrt{3} - 178\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{64} + \frac{\left(322\sqrt{3}+\frac{(-17\sqrt{-2+2\sqrt{3}}\sqrt{3}\right.}{}}{}$

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/3*x^3-17*x+(-25/8*x^3-75/8*x)/(x^4+2*x^2+3)+1/32*sum((127*_R^2+483)/(_R^3
+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.88

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{160\,x^7 - 1312\,x^5 - 3084\,x^3 + 3\,(x^4+2\,x^2+3)\sqrt{30817i\,\sqrt{2}-14395}\log\left(\sqrt{30817i\,\sqrt{2}-14395}(17i\,\sqrt{2}+1\right.}{}$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(160*x^7 - 1312*x^5 - 3084*x^3 + 3*(x^4 + 2*x^2 + 3)*sqrt(30817*I*sqrt
(2) - 14395)*log(sqrt(30817*I*sqrt(2) - 14395)*(17*I*sqrt(2) + 161) + 26499
*x) - 3*(x^4 + 2*x^2 + 3)*sqrt(30817*I*sqrt(2) - 14395)*log(sqrt(30817*I*sq
rt(2) - 14395)*(-17*I*sqrt(2) - 161) + 26499*x) - 3*(x^4 + 2*x^2 + 3)*sqrt(
-30817*I*sqrt(2) - 14395)*log((17*I*sqrt(2) - 161)*sqrt(-30817*I*sqrt(2) -
14395) + 26499*x) + 3*(x^4 + 2*x^2 + 3)*sqrt(-30817*I*sqrt(2) - 14395)*log(
(-17*I*sqrt(2) + 161)*sqrt(-30817*I*sqrt(2) - 14395) + 26499*x) - 5796*x)/(
x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.26

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^3}{3} - 17x + \frac{-25x^3-75x}{8x^4+16x^2+24} + \operatorname{RootSum}\left(1048576t^4+29480960t^2+2106591003, \left(t \mapsto t\log\left(\frac{557056t^3}{816619683}+\frac{166600064t}{816619683}+x\right)\right)\right)$$

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**3/3 - 17*x + (-25*x**3 - 75*x)/(8*x**4 + 16*x**2 + 24) + RootSum(10485
76*_t**4 + 29480960*_t**2 + 2106591003, Lambda(_t, _t*log(557056*_t**3/8166
19683 + 166600064*_t/816619683 + x)))
```

Maxima [F]

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^4}{(x^4+2\,x^2+3)^2}\,dx$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2 + 3) + 1/8*integrate((127*x^
2 + 483)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(163) = 326.

Time = 0.59 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.47

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{3}x^3$$

$$-\frac{1}{20736}\sqrt{2}\Big(127\cdot 3^{\frac{3}{4}}\sqrt{2}\Big(6\sqrt{3}+18\Big)^{\frac{3}{2}}+2286\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\Big(\sqrt{3}-3\Big)-2286\cdot 3^{\frac{3}{4}}\Big(\sqrt{3}+3\Big)\sqrt{-6\sqrt{}}$$

$$-\frac{1}{20736}\sqrt{2}\Big(127\cdot 3^{\frac{3}{4}}\sqrt{2}\Big(6\sqrt{3}+18\Big)^{\frac{3}{2}}+2286\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\Big(\sqrt{3}-3\Big)-2286\cdot 3^{\frac{3}{4}}\Big(\sqrt{3}+3\Big)\sqrt{-6\sqrt{}}$$

$$-\frac{1}{41472}\sqrt{2}\Big(2286\cdot 3^{\frac{3}{4}}\sqrt{2}\Big(\sqrt{3}+3\Big)\sqrt{-6\sqrt{3}+18}-127\cdot 3^{\frac{3}{4}}\sqrt{2}\Big(-6\sqrt{3}+18\Big)^{\frac{3}{2}}+127\cdot 3^{\frac{3}{4}}\Big(6\sqrt{3}+18\Big)^{\frac{3}{2}}$$

$$+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\Big)$$

$$+\frac{1}{41472}\sqrt{2}\Big(2286\cdot 3^{\frac{3}{4}}\sqrt{2}\Big(\sqrt{3}+3\Big)\sqrt{-6\sqrt{3}+18}-127\cdot 3^{\frac{3}{4}}\sqrt{2}\Big(-6\sqrt{3}+18\Big)^{\frac{3}{2}}+127\cdot 3^{\frac{3}{4}}\Big(6\sqrt{3}+18\Big)^{\frac{3}{2}}$$

$$-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\Big)-17\,x-\frac{25\,(x^3+3\,x)}{8\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/3*x^3 - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 228
6*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 17388*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 22
86*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 17388
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) + 18)
)*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3
) + 1/2)) - 1/41472*sqrt(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqr
t(3) + 18) - 127*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*3^(3/4)*(6*s
qrt(3) + 18)^(3/2) + 2286*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1738
8*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(6*sqrt(3) + 18
))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/41472*sqrt
```

```
(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 127*3^(3/4)
*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 228
6*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 17388*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x
*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2
 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.70

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{5\,x^3}{3} - \frac{\frac{25\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3} - 17\,x$$
$$+ \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)} - \frac{30817\,\sqrt{2}\,x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}}{128\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}\right)\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,1\mathrm{i}}{16}$$
$$- \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\left(\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)} + \frac{30817\,\sqrt{2}\,x\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}}{128\left(\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}\right)\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}\,1\mathrm{i}}{16}$$

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (atan((x*(- 2^(1/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)/
128 - 1571667/64)) - (30817*2^(1/2)*x*(- 2^(1/2)*30817i - 14395)^(1/2))/(12
8*((2^(1/2)*14884611i)/128 - 1571667/64)))*(- 2^(1/2)*30817i - 14395)^(1/2)
*1i)/16 - ((75*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) - 17*x - (atan((x*(2^(1
/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)/128 + 1571667/64
)) + (30817*2^(1/2)*x*(2^(1/2)*30817i - 14395)^(1/2))/(128*((2^(1/2)*148846
11i)/128 + 1571667/64)))*(2^(1/2)*30817i - 14395)^(1/2)*1i)/16 + (5*x^3)/3
```

3.112 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 225

$$\begin{aligned}
&\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx \\
&= 5x+\frac{25x(1+x^2)}{8(3+2x^2+x^4)}+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad+\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}$$

```
[Out] 5*x+25/8*x*(x^2+1)/(x^4+2*x^2+3)-1/192*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2
))*(-115746+77394*3^(1/2))^(1/2)+1/192*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2
))*(-115746+77394*3^(1/2))^(1/2)+1/96*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))*(115746+77394*3^(1/2))^(1/2)-1/96*arctan((2*x+(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(115746+77394*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1682, 1690, 1183, 648, 632, 210, 642}

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{25(x^2+1)\,x}{8\left(x^4+2x^2+3\right)} + 5x$$

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(19291 + 12899*Sqrt[3]
)/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sq
rt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt
[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{25x(1+x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \frac{-150-186x^2+240x^4}{3+2x^2+x^4}\,dx \\
&= \frac{25x(1+x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \left(240-\frac{6(145+111x^2)}{3+2x^2+x^4}\right)dx \\
&= 5x + \frac{25x(1+x^2)}{8(3+2x^2+x^4)} - \frac{1}{8}\int \frac{145+111x^2}{3+2x^2+x^4}\,dx
\end{aligned}$$

$$=5x+\frac{25x(1+x^2)}{8(3+2x^2+x^4)}-\frac{\int\frac{145\sqrt{2\left(-1+\sqrt{3}\right)}-\left(145-111\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int\frac{145\sqrt{2\left(-1+\sqrt{3}\right)}+\left(145-111\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$=5x+\frac{25x(1+x^2)}{8(3+2x^2+x^4)}-\frac{1}{96}\left(333+145\sqrt{3}\right)\int\frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$
$$-\frac{1}{96}\left(333+145\sqrt{3}\right)\int\frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$
$$-\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\int\frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$
$$+\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$

$$=5x+\frac{25x(1+x^2)}{8(3+2x^2+x^4)}-\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x\right.$$
$$\left.+x^2\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$+\frac{1}{48}\left(333+145\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx,x,-\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$
$$+\frac{1}{48}\left(333+145\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx,x,\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$=5x+\frac{25x(1+x^2)}{8(3+2x^2+x^4)}+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$
$$-\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$
$$-\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$+\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.54

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 5x+\frac{25(x+x^3)}{8(3+2x^2+x^4)}-\frac{(-34i+111\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} - \frac{(34i+111\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*(x + x^3))/(8*(3 + 2*x^2 + x^4)) - ((-34*I + 111*Sqrt[2])*ArcTan[
x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) - ((34*I + 111*Sqrt[2]
)*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.28

method	result
risch	$5x+\frac{\frac{25}{8}x^3+\frac{25}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-111\_R^2-145\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$5x-\frac{-\frac{25}{8}x^3-\frac{25}{8}x}{x^4+2x^2+3}-\frac{\left(94\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-51\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{192}-\frac{\left(290\sqrt{3}+\frac{\left(94\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-51\sqrt{-2+2\sqrt{3}}\right)}{2}\right.}{48\sqrt{2+}}$

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x+(25/8*x^3+25/8*x)/(x^4+2*x^2+3)+1/32*sum((-111*_R^2-145)/(_R^3+_R)*ln(x
-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{480\,x^5+1260\,x^3-\sqrt{3}(x^4+2\,x^2+3)\sqrt{7969i\,\sqrt{2}-19291}\log\left(\sqrt{3}\sqrt{7969i\,\sqrt{2}-19291}(94i\,\sqrt{2}+145)+\right.}{}$$

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/96*(480*x^5 + 1260*x^3 - sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(7969*I*sqrt(2) - 19291)*log(sqrt(3)*sqrt(7969*I*sqrt(2) - 19291)*(94*I*sqrt(2) + 145) + 38697*x) + sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(7969*I*sqrt(2) - 19291)*log(sqrt(3)*sqrt(7969*I*sqrt(2) - 19291)*(-94*I*sqrt(2) - 145) + 38697*x) + sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(-7969*I*sqrt(2) - 19291)*log(sqrt(3)*(94*I*sqrt(2) - 145)*sqrt(-7969*I*sqrt(2) - 19291) + 38697*x) - sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(-7969*I*sqrt(2) - 19291)*log(sqrt(3)*(-94*I*sqrt(2) + 145)*sqrt(-7969*I*sqrt(2) - 19291) + 38697*x) + 1740*x)/(x^4 + 2*x^2 + 3)

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.23

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 5x+\frac{25x^3+25x}{8x^4+16x^2+24} + \operatorname{RootSum}\left(3145728t^4+39507968t^2+166384201,\left(t\mapsto t\log\left(-\frac{9240576t^3}{102792131}-\frac{95003488t}{102792131}+x\right)\right)\right)$$

[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)

[Out] 5*x + (25*x**3 + 25*x)/(8*x**4 + 16*x**2 + 24) + RootSum(3145728*_t**4 + 39507968*_t**2 + 166384201, Lambda(_t, _t*log(-9240576*_t**3/102792131 - 95003488*_t/102792131 + x)))

Maxima [F]

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^2}{(x^4+2\,x^2+3)^2}\,dx$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((111*x^2 + 145)/(x^4
 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(158) = 316.

Time = 0.61 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.52

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$

$$= \frac{1}{6912}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}\right.$$

$$+\frac{1}{6912}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\right.$$

$$+\frac{1}{13824}\sqrt{2}\left(666\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 37\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 66\right.$$

$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right)$$

$$-\frac{1}{13824}\sqrt{2}\left(666\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 37\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 66\right.$$

$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\right) + 5\,x + \frac{25\,(x^3+x)}{8\,(x^4+2\,x^2+3)}$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-6
*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2)*
sqrt(6*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/
```

```
4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/6912
*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(2)*s
qrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(
3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x
- 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/13824*sqrt
(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*s
qrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^
(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqr
t(3) + 18) - 1740*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(
-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/13824*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2)
 + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sq
rt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 1740*3^(1/4)*sqrt
(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
 + 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.69

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= 5\,x+\frac{\frac{25\,x^3}{8}+\frac{25\,x}{8}}{x^4+2\,x^2+3} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}\,7969\mathrm{i}}{576\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}+\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}}{1152\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}\,1\mathrm{i}}{48} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}\,7969\mathrm{i}}{576\,\left(\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}-\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}}{1152\,\left(\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}\,1\mathrm{i}}{48}$$

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 5*x + ((25*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) + (atan((x*(- 2^(1/2)*23907
i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 - 374543/96)) + (7969*
2^(1/2)*x*(- 2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/384 -
 374543/96)))*(- 2^(1/2)*23907i - 57873)^(1/2)*1i)/48 - (atan((x*(2^(1/2)*2
3907i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 + 374543/96)) - (7
969*2^(1/2)*x*(2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/384
 + 374543/96)))*(2^(1/2)*23907i - 57873)^(1/2)*1i)/48
```

3.113 $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 224

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2} dx = \frac{25x(1-x^2)}{24(3+2x^2+x^4)} - \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x + x^2\right) - \frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(\sqrt{3} + \sqrt{2\left(-1+\sqrt{3}\right)}x + x^2\right)$$

```
[Out] 25/24*x*(-x^2+1)/(x^4+2*x^2+3)-1/288*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+
2*3^(1/2))^(1/2))*(-69402+77382*3^(1/2))^(1/2)+1/288*arctan((2*x+(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-69402+77382*3^(1/2))^(1/2)+1/576*ln(x^2+
3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(69402+77382*3^(1/2))^(1/2)-1/576*ln(x^2+3^
(1/2)+x*(-2+2*3^(1/2))^(1/2))*(69402+77382*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1692, 1183, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = -\frac{1}{48}\sqrt{\frac{1}{6}\left(12897\sqrt{3}-11567\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{48}\sqrt{\frac{1}{6}\left(12897\sqrt{3}-11567\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x(1-x^2)}{24\left(x^4+2x^2+3\right)}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) - (Sqrt[(-11567 + 12897*Sqrt[3])/6]
*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 + (Sqrt[(
-11567 + 12897*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/48 + (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/96 - (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] +
 Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{25x(1-x^2)}{24\,(3+2x^2+x^4)} + \frac{1}{48}\int \frac{14+190x^2}{3+2x^2+x^4}\,dx$$

$$= \frac{25x(1-x^2)}{24\,(3+2x^2+x^4)} + \frac{\int \frac{14\sqrt{2\left(-1+\sqrt{3}\right)}-\left(14-190\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}} + \frac{\int \frac{14\sqrt{2\left(-1+\sqrt{3}\right)}+\left(14-190\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= \frac{25x(1-x^2)}{24\left(3+2x^2+x^4\right)} + \frac{\left(7-95\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}}$$
$$+ \frac{1}{288}\left(285+7\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$
$$+ \frac{1}{288}\left(285+7\sqrt{3}\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$
$$+ \frac{\left(-7+95\sqrt{3}\right)\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= \frac{25x(1-x^2)}{24\left(3+2x^2+x^4\right)} + \frac{1}{96}\sqrt{\frac{11567}{6}+\frac{4299\sqrt{3}}{2}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$- \frac{1}{96}\sqrt{\frac{11567}{6}+\frac{4299\sqrt{3}}{2}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$- \frac{1}{144}\left(285+7\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$
$$- \frac{1}{144}\left(285+7\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$= \frac{25x(1-x^2)}{24\left(3+2x^2+x^4\right)} - \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$
$$+ \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$
$$+ \frac{1}{96}\sqrt{\frac{11567}{6}+\frac{4299\sqrt{3}}{2}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$- \frac{1}{96}\sqrt{\frac{11567}{6}+\frac{4299\sqrt{3}}{2}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.51

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \frac{1}{48}\left(-\frac{50x(-1+x^2)}{3+2x^2+x^4}+\frac{(95+44i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{(95-44i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] ((-50*x*(-1 + x^2))/(3 + 2*x^2 + x^4) + ((95 + (44*I)*Sqrt[2])*ArcTan[x/Sqr
t[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((95 - (44*I)*Sqrt[2])*ArcTan[x/Sq
rt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/48
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.27

method	result
risch	$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(95\_R^2+7)\ln(x-\_R)}{\_R^3+\_R}\right)}{96}$
default	$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}+\frac{\left(139\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+132\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{576}+\frac{\left(14\sqrt{3}+\frac{\left(139\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+132\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2}}{2}\right.}{144\sqrt{2+2\sqrt{3}}}$

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-25/24*x^3+25/24*x)/(x^4+2*x^2+3)+1/96*sum((95*_R^2+7)/(_R^3+_R)*ln(x-_R),
_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = -\frac{300\,x^3-\sqrt{3}(x^4+2\,x^2+3)\sqrt{13513i\,\sqrt{2}+11567}\log\left(\sqrt{3}\sqrt{13513i\,\sqrt{2}+11567}(139i\,\sqrt{2}+7)+38691\right.}{}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/288*(300*x^3 - sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(13513*I*sqrt(2) + 11567)*l
og(sqrt(3)*sqrt(13513*I*sqrt(2) + 11567)*(139*I*sqrt(2) + 7) + 38691*x) + s
qrt(3)*(x^4 + 2*x^2 + 3)*sqrt(13513*I*sqrt(2) + 11567)*log(sqrt(3)*sqrt(135
13*I*sqrt(2) + 11567)*(-139*I*sqrt(2) - 7) + 38691*x) + sqrt(3)*(x^4 + 2*x^
2 + 3)*sqrt(-13513*I*sqrt(2) + 11567)*log(sqrt(3)*(139*I*sqrt(2) - 7)*sqrt(
-13513*I*sqrt(2) + 11567) + 38691*x) - sqrt(3)*(x^4 + 2*x^2 + 3)*sqrt(-1351
3*I*sqrt(2) + 11567)*log(sqrt(3)*(-139*I*sqrt(2) + 7)*sqrt(-13513*I*sqrt(2)
 + 11567) + 38691*x) - 300*x)/(x^4 + 2*x^2 + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(178) = 356.

Time = 0.69 (sec) , antiderivative size = 1185, normalized size of antiderivative = 5.29

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-25*x**3 + 25*x)/(24*x**4 + 48*x**2 + 72) + sqrt(11567/55296 + 1433*sqrt(3
)/6144)*log(x**2 + x*(-556*sqrt(2)*sqrt(11567 + 12897*sqrt(3))/13513 - 1040
345*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 278*sqrt(3)*sqrt(11567
+ 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161) - 476102762
00401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)/30372528846219921 - 43908
31246*sqrt(6)*sqrt(149179599*sqrt(3) + 316396658)/7065021829779 + 128104648
1635939181/30372528846219921 + 200684595453464*sqrt(3)/7065021829779) - sqr
t(11567/55296 + 1433*sqrt(3)/6144)*log(x**2 + x*(-278*sqrt(3)*sqrt(11567 +
12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161 + 1040345*sqrt
(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 556*sqrt(2)*sqrt(11567 + 12897*
sqrt(3))/13513) - 47610276200401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658
)/30372528846219921 - 4390831246*sqrt(6)*sqrt(149179599*sqrt(3) + 316396658
```

```
)/7065021829779 + 1281046481635939181/30372528846219921 + 200684595453464*s
qrt(3)/7065021829779) + 2*sqrt(-sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)
/27648 + 11567/55296 + 1433*sqrt(3)/2048)*atan(348554322*sqrt(3)*x/(94591*s
qrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*
sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149
179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))) - 7170732*sqrt(6)*sqr
t(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt
(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 31
6396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 3869
1*sqrt(3))) - 3121035*sqrt(2)*sqrt(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sq
rt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))
+ 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sq
rt(3) + 316396658) + 11567 + 38691*sqrt(3))) + 834*sqrt(11567 + 12897*sqrt(
3))*sqrt(149179599*sqrt(3) + 316396658)/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt
(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(1491795
99*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)
 + 11567 + 38691*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(149179599*sqrt(3) + 3163
96658)/27648 + 11567/55296 + 1433*sqrt(3)/2048)*atan(348554322*sqrt(3)*x/(9
4591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 +
38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sq
rt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))) - 834*sqrt(1156
7 + 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/(94591*sqrt(2)*sqrt(
-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 2
78*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(
3) + 316396658) + 11567 + 38691*sqrt(3))) + 3121035*sqrt(2)*sqrt(11567 + 12
897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396
658) + 11567 + 38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqr
t(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)))
+ 7170732*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2
)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(1
49179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 3163
96658) + 11567 + 38691*sqrt(3))))
```

Maxima [F]

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{(3 + 2x^2 + x^4)^2} \, dx = \int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{(x^4 + 2\,x^2 + 3)^2} \, dx$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -25/24*(x^3 - x)/(x^4 + 2*x^2 + 3) + 1/24*integrate((95*x^2 + 7)/(x^4 + 2*x
^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(155) = 310$.

Time = 0.58 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.52

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx =$$

$$-\frac{1}{62208}\sqrt{2}\left(95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1710\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1710\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{}}\right.$$

$$-\frac{1}{62208}\sqrt{2}\left(95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1710\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1710\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{}}\right.$$

$$-\frac{1}{124416}\sqrt{2}\left(1710\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+95\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{124416}\sqrt{2}\left(1710\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+95\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{25\,(x^3-x)}{24\,(x^4+2\,x^2+3)}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1/62208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/62
208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*
sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sqrt(2)*sq
rt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/124416*
sqrt(2)*(1710*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3
/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1
710*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/124416*sqrt(2)*(1710*3^(3/4)*sqrt(2)
```

```
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)
^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*sqrt(6*sqrt(3) +
18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 252*3^(1/4)
*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqr
t(3)) - 25/24*(x^3 - x)/(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{\frac{25\,x}{24}-\frac{25\,x^3}{24}}{x^4+2\,x^2+3} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}\,13513\mathrm{i}}{15552\,\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}+\frac{13513\,\sqrt{2}\,x\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}}{31104\,\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}\right)\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}\,1\mathrm{i}}{144} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}\,13513\mathrm{i}}{15552\,\left(\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}-\frac{13513\,\sqrt{2}\,x\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}}{31104\,\left(\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}\right)\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}\,1\mathrm{i}}{144}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] ((25*x)/24 - (25*x^3)/24)/(2*x^2 + x^4 + 3) - (atan((x*(34701 - 2^(1/2)*405
39i)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 - 1878307/5184)) + (13513
*2^(1/2)*x*(34701 - 2^(1/2)*40539i)^(1/2))/(31104*((2^(1/2)*94591i)/10368 -
 1878307/5184)))*(34701 - 2^(1/2)*40539i)^(1/2)*1i)/144 + (atan((x*(2^(1/2)
*40539i + 34701)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 + 1878307/518
4)) - (13513*2^(1/2)*x*(2^(1/2)*40539i + 34701)^(1/2))/(31104*((2^(1/2)*945
91i)/10368 + 1878307/5184)))*(2^(1/2)*40539i + 34701)^(1/2)*1i)/144
```

3.114 $\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 229

$$
\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx = & -\frac{4}{9x}-\frac{25x(5+x^2)}{72(3+2x^2+x^4)} \\
& +\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
& -\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
& -\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
& +\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}
$$

```
[Out] -4/9/x-25/72*x*(x^2+5)/(x^4+2*x^2+3)+1/288*arctan((-2*x+(-2+2*3^(1/2))^(1/2
))/(2+2*3^(1/2))^(1/2))*(-5790+4194*3^(1/2))^(1/2)-1/288*arctan((2*x+(-2+2*
3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-5790+4194*3^(1/2))^(1/2)-1/576*ln(x^
2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(5790+4194*3^(1/2))^(1/2)+1/576*ln(x^2+3^
(1/2)+x*(-2+2*3^(1/2))^(1/2))*(5790+4194*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx = \frac{1}{48}\sqrt{\frac{1}{6}\left(699\sqrt{3}-965\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{1}{48}\sqrt{\frac{1}{6}\left(699\sqrt{3}-965\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) - \frac{25x(x^2+5)}{72\left(x^4+2x^2+3\right)} - \frac{4}{9x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (Sqrt[(-965 + 699*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 -
(Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/48 - (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/96 + (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\int\frac{64+\frac{170x^2}{3}-\frac{50x^4}{3}}{x^2\left(3+2x^2+x^4\right)}\,dx\\
&= -\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\int\left(\frac{64}{3x^2}-\frac{2(-7+19x^2)}{3+2x^2+x^4}\right)dx\\
&= -\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{1}{24}\int\frac{-7+19x^2}{3+2x^2+x^4}\,dx
\end{aligned}
$$

$$= -\frac{4}{9x} - \frac{25x(5+x^2)}{72(3+2x^2+x^4)} - \frac{\int \frac{-7\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-7-19\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)x+x^2}}\,dx}{48\sqrt{6\left(-1+\sqrt{3}\right)}} - \frac{\int \frac{-7\sqrt{2\left(-1+\sqrt{3}\right)}+\left(-7-19\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)x+x^2}}\,dx}{48\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{4}{9x} - \frac{25x(5+x^2)}{72(3+2x^2+x^4)}$$
$$- \frac{1}{48}\sqrt{\frac{1}{6}\left(566-133\sqrt{3}\right)} \int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)x+x^2}}\,dx$$
$$- \frac{1}{48}\sqrt{\frac{1}{6}\left(566-133\sqrt{3}\right)} \int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)x+x^2}}\,dx$$
$$- \frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)} \int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)x+x^2}}\,dx$$
$$+ \frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)} \int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)x+x^2}}\,dx$$

$$= -\frac{4}{9x} - \frac{25x(5+x^2)}{72(3+2x^2+x^4)}$$
$$- \frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)} \log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$+ \frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)} \log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$+ \frac{1}{24}\sqrt{\frac{1}{6}\left(566-133\sqrt{3}\right)} \text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)} + 2x\right)$$
$$+ \frac{1}{24}\sqrt{\frac{1}{6}\left(566-133\sqrt{3}\right)} \text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)} + 2x\right)$$

$$
\begin{aligned}
&= -\frac{4}{9x} - \frac{25x(5+x^2)}{72\,(3+2x^2+x^4)} \\
&+ \frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&- \frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&- \frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&+ \frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.55

$$
\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^2\,(3+2x^2+x^4)^2}\,dx &= -\frac{4}{9x} - \frac{25x(5+x^2)}{72\,(3+2x^2+x^4)} - \frac{(26i+19\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{48\sqrt{2-2i\sqrt{2}}} \\
&\quad - \frac{(-26i+19\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{48\sqrt{2+2i\sqrt{2}}}
\end{aligned}
$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - ((26*I + 19*Sqrt[2])*A
rcTan[x/Sqrt[1 - I*Sqrt[2]]])/(48*Sqrt[2 - (2*I)*Sqrt[2]]) - ((-26*I + 19*S
qrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(48*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.28

method	result
risch	$\frac{-\frac{19}{24}x^4-\frac{21}{8}x^2-\frac{4}{3}}{x(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4-1930\_Z^2+488601\right)}\_R\ln\left(-96\_R^3+34499\_R+361383x\right)\right)}{96}$
default	$-\frac{4}{9x}-\frac{\frac{25}{8}x^3+\frac{125}{8}x}{9(x^4+2x^2+3)}-\frac{\left(32\sqrt{-2+2\sqrt{3}}\sqrt{3}+39\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{576}-\frac{\left(-14\sqrt{3}+\frac{\left(32\sqrt{-2+2\sqrt{3}}\sqrt{3}+39\sqrt{-2+2}\right.}{2}\right.}{144\sqrt{}}$

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-19/24*x^4-21/8*x^2-4/3)/x/(x^4+2*x^2+3)+1/96*sum(_R*ln(-96*_R^3+34499*_R+
361383*x),_R=RootOf(3*_Z^4-1930*_Z^2+488601))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx = -\frac{228\,x^4+\sqrt{3}(x^5+2\,x^3+3\,x)\sqrt{517i\,\sqrt{2}+965}\log\left(\sqrt{3}\sqrt{517i\,\sqrt{2}+965}(32i\,\sqrt{2}-7)+2097\,x\right)-\sqrt{3}(x^5}{}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/288*(228*x^4 + sqrt(3)*(x^5 + 2*x^3 + 3*x)*sqrt(517*I*sqrt(2) + 965)*log
(sqrt(3)*sqrt(517*I*sqrt(2) + 965)*(32*I*sqrt(2) - 7) + 2097*x) - sqrt(3)*(
x^5 + 2*x^3 + 3*x)*sqrt(517*I*sqrt(2) + 965)*log(sqrt(3)*sqrt(517*I*sqrt(2)
 + 965)*(-32*I*sqrt(2) + 7) + 2097*x) - sqrt(3)*(x^5 + 2*x^3 + 3*x)*sqrt(-5
17*I*sqrt(2) + 965)*log(sqrt(3)*(32*I*sqrt(2) + 7)*sqrt(-517*I*sqrt(2) + 96
5) + 2097*x) + sqrt(3)*(x^5 + 2*x^3 + 3*x)*sqrt(-517*I*sqrt(2) + 965)*log(s
qrt(3)*(-32*I*sqrt(2) - 7)*sqrt(-517*I*sqrt(2) + 965) + 2097*x) + 756*x^2 +
 384)/(x^5 + 2*x^3 + 3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. 2(184) = 368.

Time = 0.76 (sec) , antiderivative size = 1192, normalized size of antiderivative = 5.21

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-19*x**4 - 63*x**2 - 32)/(24*x**5 + 48*x**3 + 72*x) - sqrt(965/55296 + 233
*sqrt(3)/18432)*log(x**2 + x*(-128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517 - 21
793*sqrt(6)*sqrt(965 + 699*sqrt(3))/361383 + 64*sqrt(3)*sqrt(965 + 699*sqrt
(3))*sqrt(674535*sqrt(3) + 1198514)/361383) - 8882635459*sqrt(2)*sqrt(67453
5*sqrt(3) + 1198514)/130597672689 - 20458048*sqrt(6)*sqrt(674535*sqrt(3) +
1198514)/560505033 + 18567565928783/130597672689 + 46950427730*sqrt(3)/5605
05033) + sqrt(965/55296 + 233*sqrt(3)/18432)*log(x**2 + x*(-64*sqrt(3)*sqrt
(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/361383 + 21793*sqrt(6)*s
qrt(965 + 699*sqrt(3))/361383 + 128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517) -
8882635459*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/130597672689 - 20458048*s
qrt(6)*sqrt(674535*sqrt(3) + 1198514)/560505033 + 18567565928783/1305976726
89 + 46950427730*sqrt(3)/560505033) + 2*sqrt(-sqrt(2)*sqrt(674535*sqrt(3) +
 1198514)/27648 + 965/55296 + 233*sqrt(3)/6144)*atan(722766*sqrt(3)*x/(-64*
sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 119851
4) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3)
 + 1198514) + 965 + 2097*sqrt(3))) + 89472*sqrt(6)*sqrt(965 + 699*sqrt(3))/
(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1
198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sq
rt(3) + 1198514) + 965 + 2097*sqrt(3))) + 65379*sqrt(2)*sqrt(965 + 699*sqrt
(3))/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3
) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(6745
35*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) - 192*sqrt(965 + 699*sqrt(3))*
sqrt(674535*sqrt(3) + 1198514)/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*
sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)))) + 2
*sqrt(-sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/27648 + 965/55296 + 233*sqrt(
3)/6144)*atan(722766*sqrt(3)*x/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*
sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 19
2*sqrt(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/(-64*sqrt(674535*s
qrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 20
97*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) +
 965 + 2097*sqrt(3))) - 65379*sqrt(2)*sqrt(965 + 699*sqrt(3))/(-64*sqrt(674
535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965
 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 11985
14) + 965 + 2097*sqrt(3))) - 89472*sqrt(6)*sqrt(965 + 699*sqrt(3))/(-64*sqr
t(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)
+ 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) +
1198514) + 965 + 2097*sqrt(3))))
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx=\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^2x^2}\,dx$$

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] -1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x) - 1/24*integrate((19*x^2 - 7)/(x^4 + 2*x^2 + 3), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(160) = 320.

Time = 0.58 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.50

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx$$

$$=\frac{1}{62208}\sqrt{2}\left(19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+342\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-342\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}\right.$$

$$+\frac{1}{62208}\sqrt{2}\left(19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+342\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-342\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+}\right.$$

$$+\frac{1}{124416}\sqrt{2}\left(342\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+19\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$-\frac{1}{124416}\sqrt{2}\left(342\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+19\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{19\,x^4+63\,x^2+32}{24\left(x^5+2\,x^3+3\,x\right)}$$

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 1/62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4

```
)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/62208
*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(2)*s
qrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(
3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)*sqrt(6*
sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x -
3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/124416*sqrt(
2)*(342*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sq
rt(2)*(-6*sqrt(3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(
3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(
3) + 18) + 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/
6*sqrt(3) + 1/2) + sqrt(3)) - 1/124416*sqrt(2)*(342*3^(3/4)*sqrt(2)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) +
 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt
(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(6*s
qrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1
/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.69

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx = -\frac{\frac{19\,x^4}{24}+\frac{21\,x^2}{8}+\frac{4}{3}}{x^5+2\,x^3+3\,x} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}\,517\mathrm{i}}{15552\,\left(\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}+\frac{517\,\sqrt{2}\,x\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}}{31104\,\left(\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}\right)\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}\,1\mathrm{i}}{144} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}\,517\mathrm{i}}{15552\,\left(-\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}-\frac{517\,\sqrt{2}\,x\,\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}}{31104\,\left(-\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}\right)\,\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}\,1\mathrm{i}}{144}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (atan((x*(2^(1/2)*1551i + 2895)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 -
 517/162)) - (517*2^(1/2)*x*(2^(1/2)*1551i + 2895)^(1/2))/(31104*((2^(1/2)*
3619i)/10368 - 517/162)))*(2^(1/2)*1551i + 2895)^(1/2)*1i)/144 - (atan((x*(
2895 - 2^(1/2)*1551i)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 + 517/162))
 + (517*2^(1/2)*x*(2895 - 2^(1/2)*1551i)^(1/2))/(31104*((2^(1/2)*3619i)/103
68 + 517/162)))*(2895 - 2^(1/2)*1551i)^(1/2)*1i)/144 - ((21*x^2)/8 + (19*x^
4)/24 + 4/3)/(3*x + 2*x^3 + x^5)
```

3.115 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 238

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx = -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216(3+2x^2+x^4)} - \frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

```
[Out] -4/27/x^3+13/27/x+25/216*x*(5*x^2+7)/(x^4+2*x^2+3)+1/5184*ln(x^2+3^(1/2)-x*
(-2+2*3^(1/2))^(1/2))*(-36438+340038*3^(1/2))^(1/2)-1/5184*ln(x^2+3^(1/2)+x
*(-2+2*3^(1/2))^(1/2))*(-36438+340038*3^(1/2))^(1/2)-1/2592*arctan((-2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(36438+340038*3^(1/2))^(1/2)+1/259
2*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(36438+340038*3^(1
/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx = -\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{4}{27x^3} + \frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) - \frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{25x(5x^2+7)}{216\left(x^4+2x^2+3\right)} + \frac{13}{27x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(27*x^3) + 13/(27*x) + (25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) - (Sqr
t[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1
 + Sqrt[3])]])/432 + (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + S
qrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(-6073 + 56673*Sqrt[3])
/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864 - (Sqrt[(-6073 + 566
73*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{25x(7+5x^2)}{216\,(3+2x^2+x^4)} + \frac{1}{48}\int \frac{64-\frac{80x^2}{3}+\frac{50x^4}{9}+\frac{250x^6}{9}}{x^4\,(3+2x^2+x^4)}\,dx \\ &= \frac{25x(7+5x^2)}{216\,(3+2x^2+x^4)} + \frac{1}{48}\int \left(\frac{64}{3x^4}-\frac{208}{9x^2}+\frac{2(137+229x^2)}{9\,(3+2x^2+x^4)}\right)dx\end{aligned}$$

$$= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7+5x^2)}{216\,(3+2x^2+x^4)} + \frac{1}{216}\int \frac{137+229x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7+5x^2)}{216\,(3+2x^2+x^4)} + \frac{\int \frac{137\sqrt{2\left(-1+\sqrt{3}\right)}-\left(137-229\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{432\sqrt{6\,(-1+\sqrt{3})}} + \frac{\int \frac{137\sqrt{2\left(-1+\sqrt{3}\right)}+\left(137-229\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{432\sqrt{6\,(-1+\sqrt{3})}}$$

$$= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7+5x^2)}{216\,(3+2x^2+x^4)} + \frac{1}{432}\sqrt{\frac{1}{6}\left(88046+31373\sqrt{3}\right)}\int \frac{1}{\sqrt{3}-\sqrt{2\,(-1+\sqrt{3})}x+x^2}\,dx + \frac{1}{432}\sqrt{\frac{1}{6}\left(88046+31373\sqrt{3}\right)}\int \frac{1}{\sqrt{3}+\sqrt{2\,(-1+\sqrt{3})}x+x^2}\,dx + \frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\int \frac{-\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{3}-\sqrt{2\,(-1+\sqrt{3})}x+x^2}\,dx - \frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\int \frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{3}+\sqrt{2\,(-1+\sqrt{3})}x+x^2}\,dx$$

$$= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7+5x^2)}{216\,(3+2x^2+x^4)} + \frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{216}\sqrt{\frac{1}{6}\left(88046+31373\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2\,(1+\sqrt{3})-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right) - \frac{1}{216}\sqrt{\frac{1}{6}\left(88046+31373\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2\,(1+\sqrt{3})-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$
\begin{aligned}
&= -\frac{4}{27x^3} + \frac{13}{27x} + \frac{25x(7+5x^2)}{216\,(3+2x^2+x^4)} \\
&\quad - \frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad + \frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad + \frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad - \frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$
\int \frac{4+x^2+3x^4+5x^6}{x^4\,(3+2x^2+x^4)^2}\,dx = \frac{1}{864}\left(\frac{4(-96+248x^2+351x^4+229x^6)}{x^3\,(3+2x^2+x^4)} + \frac{2\left(229+46i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{2\left(229-46i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)
$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((4*(-96 + 248*x^2 + 351*x^4 + 229*x^6))/(x^3*(3 + 2*x^2 + x^4)) + (2*(229
+ (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (2*(
229 - (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/8
64
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\frac{229}{216}x^6+\frac{13}{8}x^4+\frac{31}{27}x^2-\frac{4}{9}}{x^3(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+12146\_Z^2+3211828929\right)}\_R\ln\left(825\_R^3+11161024\_R+3926135421x\right)\right)}{864}$
default	$-\frac{4}{27x^3}+\frac{13}{27x}+\frac{\frac{125}{8}x^3+\frac{175}{8}x}{27x^4+54x^2+81}+\frac{\left(275\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+138\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{5184}+\frac{\left(274\sqrt{3}+\frac{(275\sqrt{-2+}\right.}{}$

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (229/216*x^6+13/8*x^4+31/27*x^2-4/9)/x^3/(x^4+2*x^2+3)+1/864*sum(_R*ln(825*
_R^3+11161024*_R+3926135421*x),_R=RootOf(3*_Z^4+12146*_Z^2+3211828929))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.03

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx = \frac{2748\,x^6+4212\,x^4+\sqrt{3}(x^7+2\,x^5+3\,x^3)\sqrt{69277i\,\sqrt{2}-6073}\log\left(\sqrt{3}\sqrt{69277i\,\sqrt{2}-6073}(275i\,\sqrt{2}+13\right.}{}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2592*(2748*x^6 + 4212*x^4 + sqrt(3)*(x^7 + 2*x^5 + 3*x^3)*sqrt(69277*I*sq
rt(2) - 6073)*log(sqrt(3)*sqrt(69277*I*sqrt(2) - 6073)*(275*I*sqrt(2) + 137
) + 170019*x) - sqrt(3)*(x^7 + 2*x^5 + 3*x^3)*sqrt(69277*I*sqrt(2) - 6073)*
log(sqrt(3)*sqrt(69277*I*sqrt(2) - 6073)*(-275*I*sqrt(2) - 137) + 170019*x)
 - sqrt(3)*(x^7 + 2*x^5 + 3*x^3)*sqrt(-69277*I*sqrt(2) - 6073)*log(sqrt(3)*
(275*I*sqrt(2) - 137)*sqrt(-69277*I*sqrt(2) - 6073) + 170019*x) + sqrt(3)*(
x^7 + 2*x^5 + 3*x^3)*sqrt(-69277*I*sqrt(2) - 6073)*log(sqrt(3)*(-275*I*sqrt
(2) + 137)*sqrt(-69277*I*sqrt(2) - 6073) + 170019*x) + 2976*x^2 - 1152)/(x^
7 + 2*x^5 + 3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.25

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx$$
$$=\operatorname{RootSum}\left(2293235712t^4+12437504t^2+4405801,\left(t\mapsto t\log\left(\frac{19707494400t^3}{145412423}+\frac{357152768t}{145412423}+x\right)\right)\right)$$
$$+\frac{229x^6+351x^4+248x^2-96}{216x^7+432x^5+648x^3}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)
```

```
[Out] RootSum(2293235712*_t**4 + 12437504*_t**2 + 4405801, Lambda(_t, _t*log(1970
7494400*_t**3/145412423 + 357152768*_t/145412423 + x))) + (229*x**6 + 351*x
**4 + 248*x**2 - 96)/(216*x**7 + 432*x**5 + 648*x**3)
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx=\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^2x^4}\,dx$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(229*x^6 + 351*x^4 + 248*x^2 - 96)/(x^7 + 2*x^5 + 3*x^3) + 1/216*inte
grate((229*x^2 + 137)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(167) = 334$.

Time = 0.60 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.43

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx =$$

$$-\frac{1}{559872}\sqrt{2}\left(229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+4122\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-4122\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\right.$$

$$-\frac{1}{559872}\sqrt{2}\left(229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+4122\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-4122\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\right.$$

$$-\frac{1}{1119744}\sqrt{2}\left(4122\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+229\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right.\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{1119744}\sqrt{2}\left(4122\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+229\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right.\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)+\frac{25\left(5x^3+7x\right)}{216\left(x^4+2x^2+3\right)}+\frac{13x^2-4}{27x^3}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/
3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/
3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/1119744*sqrt(2)*(4122*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 229*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(6*sqrt(3) +
18)^(3/2) + 4122*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4932*3^(1/4)*
sqrt(2)*sqrt(-6*sqrt(3) + 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2
+ 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/1119744*sqrt(2)*(4122
```

```
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 229*3^(3/4)*sqrt(2)*
(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4)
*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4932*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3)
+ 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*
sqrt(3) + 1/2) + sqrt(3)) + 25/216*(5*x^3 + 7*x)/(x^4 + 2*x^2 + 3) + 1/27*(
13*x^2 - 4)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx = \frac{\frac{229\,x^6}{216}+\frac{13\,x^4}{8}+\frac{31\,x^2}{27}-\frac{4}{9}}{x^7+2\,x^5+3\,x^3} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}\,69277\mathrm{i}}{11337408\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}+\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}}{22674816\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}\right)\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}\,1\mathrm{i}}{1296} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}\,69277\mathrm{i}}{11337408\left(\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}-\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}}{22674816\left(\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}\right)\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}\,1\mathrm{i}}{1296}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] ((31*x^2)/27 + (13*x^4)/8 + (229*x^6)/216 - 4/9)/(3*x^3 + 2*x^5 + x^7) - (a
tan((x*(- 2^(1/2)*207831i - 18219)^(1/2)*69277i)/(11337408*((2^(1/2)*949094
9i)/7558272 - 19051175/3779136)) + (69277*2^(1/2)*x*(- 2^(1/2)*207831i - 18
219)^(1/2))/(22674816*((2^(1/2)*9490949i)/7558272 - 19051175/3779136)))*(-
2^(1/2)*207831i - 18219)^(1/2)*1i)/1296 + (atan((x*(2^(1/2)*207831i - 18219
)^(1/2)*69277i)/(11337408*((2^(1/2)*9490949i)/7558272 + 19051175/3779136))
- (69277*2^(1/2)*x*(2^(1/2)*207831i - 18219)^(1/2))/(22674816*((2^(1/2)*949
0949i)/7558272 + 19051175/3779136)))*(2^(1/2)*207831i - 18219)^(1/2)*1i)/12
96
```

3.116 $\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 245

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx
&= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648(3+2x^2+x^4)} \\
&\quad+\frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296} \\
&\quad-\frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296} \\
&\quad-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{2592} \\
&\quad+\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{2592}
\end{aligned}$$

```
[Out] -4/45/x^5+13/81/x^3-13/27/x+25/648*x*(-7*x^2+1)/(x^4+2*x^2+3)+1/7776*arctan
((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-6836286+4130514*3^(1/2)
)^(1/2)-1/7776*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-683
6286+4130514*3^(1/2))^(1/2)-1/15552*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*
(6836286+4130514*3^(1/2))^(1/2)+1/15552*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/
2))*(6836286+4130514*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = \frac{\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296} - \frac{\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296} - \frac{4}{45x^5} + \frac{13}{81x^3} - \frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592} + \frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592} + \frac{25x(1-7x^2)}{648\left(x^4+2x^2+3\right)} - \frac{13}{27x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(45*x^5) + 13/(81*x^3) - 13/(27*x) + (25*x*(1 - 7*x^2))/(648*(3 + 2*x^2
+ x^4)) + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]
)] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(-1139381 + 688419*Sqrt[3])/
6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sq
rt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x
^2])/2592 + (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 +
Sqrt[3])]*x + x^2])/2592
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{25x(1-7x^2)}{648\,(3+2x^2+x^4)} + \frac{1}{48}\int \frac{64 - \frac{80x^2}{3} + \frac{400x^4}{9} + \frac{1550x^6}{27} - \frac{350x^8}{27}}{x^6\,(3+2x^2+x^4)}\,dx \\ &= \frac{25x(1-7x^2)}{648\,(3+2x^2+x^4)} + \frac{1}{48}\int \left(\frac{64}{3x^6} - \frac{208}{9x^4} + \frac{208}{9x^2} - \frac{2(-463+487x^2)}{27\,(3+2x^2+x^4)}\right)\,dx\end{aligned}$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1-7x^2)}{648\,(3+2x^2+x^4)} - \frac{1}{648}\int \frac{-463+487x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1-7x^2)}{648\,(3+2x^2+x^4)}$$
$$-\frac{\int \frac{-463\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-463-487\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{1296\sqrt{6\,(-1+\sqrt{3})}} - \frac{\int \frac{-463\sqrt{2\left(-1+\sqrt{3}\right)}+\left(-463-487\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{1296\sqrt{6\,(-1+\sqrt{3})}}$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1-7x^2)}{648\,(3+2x^2+x^4)}$$
$$-\frac{\left(1461-463\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{7776}$$
$$+\frac{\left(-1461+463\sqrt{3}\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{7776}$$
$$-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{2592}$$
$$+\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{2592}$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1-7x^2)}{648\,(3+2x^2+x^4)}$$
$$-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\,(-1+\sqrt{3})}x+x^2\right)}{2592}$$
$$+\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\,(-1+\sqrt{3})}x+x^2\right)}{2592}$$
$$+\frac{\left(1461-463\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, \sqrt{2\,(-1+\sqrt{3})}+2x\right)}{3888}$$
$$-\frac{\left(-1461+463\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, -\sqrt{2\,(-1+\sqrt{3})}+2x\right)}{3888}$$

$$= -\frac{4}{45x^5} + \frac{13}{81x^3} - \frac{13}{27x} + \frac{25x(1-7x^2)}{648\,(3+2x^2+x^4)}$$
$$+ \frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}$$
$$- \frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}$$
$$- \frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{2592}$$
$$+ \frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{2592}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.57

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\,(3+2x^2+x^4)^2}\,dx$$
$$= \frac{-\frac{4(864-984x^2+3928x^4+2475x^6+2435x^8)}{x^5(3+2x^2+x^4)} - \frac{10i\left(-487i+475\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{10i\left(487i+475\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{12960}$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
[Out] ((-4*(864 - 984*x^2 + 3928*x^4 + 2475*x^6 + 2435*x^8))/(x^5*(3 + 2*x^2 + x^
4)) - ((10*I)*(-487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1
- I*Sqrt[2]] + ((10*I)*(487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])
/Sqrt[1 + I*Sqrt[2]])/12960
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.30

method	result
risch	$\frac{-\frac{487}{648}x^8-\frac{55}{72}x^6-\frac{491}{405}x^4+\frac{41}{135}x^2-\frac{4}{15}}{x^5(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4-2278762\_Z^2+473920719561\right)}\_R\ln\left(-2886\_R^3+1211171969\_R+171\right.\right.}{2592}$
default	$-\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}-\frac{\frac{175}{24}x^3-\frac{25}{24}x}{27(x^4+2x^2+3)}-\frac{\left(962\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+1425\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{15552}-\frac{\left(-926\sqrt{3}+\right.}{}$

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-487/648*x^8-55/72*x^6-491/405*x^4+41/135*x^2-4/15)/x^5/(x^4+2*x^2+3)+1/25
92*sum(_R*ln(-2886*_R^3+1211171969*_R+171119622411*x),_R=RootOf(3*_Z^4-2278
762*_Z^2+473920719561))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.03

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx =$$
$$-\frac{29220\,x^8+29700\,x^6+47136\,x^4+5\,\sqrt{3}(x^9+2\,x^7+3\,x^5)\sqrt{248569i\,\sqrt{2}+1139381}\log\left(\sqrt{3}\sqrt{248569i\,\sqrt{2}\right.}{}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/38880*(29220*x^8 + 29700*x^6 + 47136*x^4 + 5*sqrt(3)*(x^9 + 2*x^7 + 3*x^
5)*sqrt(248569*I*sqrt(2) + 1139381)*log(sqrt(3)*sqrt(248569*I*sqrt(2) + 113
9381)*(962*I*sqrt(2) - 463) + 2065257*x) - 5*sqrt(3)*(x^9 + 2*x^7 + 3*x^5)*
sqrt(248569*I*sqrt(2) + 1139381)*log(sqrt(3)*sqrt(248569*I*sqrt(2) + 113938
1)*(-962*I*sqrt(2) + 463) + 2065257*x) - 5*sqrt(3)*(x^9 + 2*x^7 + 3*x^5)*sq
rt(-248569*I*sqrt(2) + 1139381)*log(sqrt(3)*(962*I*sqrt(2) + 463)*sqrt(-248
569*I*sqrt(2) + 1139381) + 2065257*x) + 5*sqrt(3)*(x^9 + 2*x^7 + 3*x^5)*sqr
t(-248569*I*sqrt(2) + 1139381)*log(sqrt(3)*(-962*I*sqrt(2) - 463)*sqrt(-248
569*I*sqrt(2) + 1139381) + 2065257*x) - 11808*x^2 + 10368)/(x^9 + 2*x^7 + 3
*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(199) = 398$.

Time = 0.77 (sec) , antiderivative size = 1202, normalized size of antiderivative = 4.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

```
[Out] -sqrt(1139381/40310784 + 2833*sqrt(3)/165888)*log(x**2 + x*(-3848*sqrt(2)*s
qrt(1139381 + 688419*sqrt(3))/248569 - 769085497*sqrt(6)*sqrt(1139381 + 688
419*sqrt(3))/171119622411 + 1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*sqr
t(784371528639*sqrt(3) + 1359975610922)/171119622411) - 8677510907569510603
*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/29281925174083213452921
 - 21752950947364*sqrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/127605
100269239577 + 20196165220927340076543947/29281925174083213452921 + 5094503
6826336313070*sqrt(3)/127605100269239577) + sqrt(1139381/40310784 + 2833*sq
rt(3)/165888)*log(x**2 + x*(-1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*sq
rt(784371528639*sqrt(3) + 1359975610922)/171119622411 + 769085497*sqrt(6)*s
qrt(1139381 + 688419*sqrt(3))/171119622411 + 3848*sqrt(2)*sqrt(1139381 + 68
8419*sqrt(3))/248569) - 8677510907569510603*sqrt(2)*sqrt(784371528639*sqrt(
3) + 1359975610922)/29281925174083213452921 - 21752950947364*sqrt(6)*sqrt(7
84371528639*sqrt(3) + 1359975610922)/127605100269239577 + 20196165220927340
076543947/29281925174083213452921 + 50945036826336313070*sqrt(3)/1276051002
69239577) + 2*sqrt(-sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/2015
5392 + 1139381/40310784 + 2833*sqrt(3)/55296)*atan(342239244822*sqrt(3)*x/(
-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(78437
1528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*s
qrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381
 + 2065257*sqrt(3))) + 2649036312*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/(-
1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371
528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sq
rt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381
+ 2065257*sqrt(3))) + 2307256491*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/(-1
924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(7843715
28639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqr
t(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 +
 2065257*sqrt(3))) - 5772*sqrt(1139381 + 688419*sqrt(3))*sqrt(784371528639*
sqrt(3) + 1359975610922)/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*
sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065
257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3)
+ 1359975610922) + 1139381 + 2065257*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(7843
71528639*sqrt(3) + 1359975610922)/20155392 + 1139381/40310784 + 2833*sqrt(3
)/55296)*atan(342239244822*sqrt(3)*x/(-1924*sqrt(784371528639*sqrt(3) + 135
```

```
9975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 11
39381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528
639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) + 5772*sqrt(1139
381 + 688419*sqrt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/(-1924*sqr
t(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*s
qrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sq
rt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 206525
7*sqrt(3))) - 2307256491*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt
(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sq
rt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqr
t(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257
*sqrt(3))) - 2649036312*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt(
784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqr
t(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt
(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*
sqrt(3)))) + (-2435*x**8 - 2475*x**6 - 3928*x**4 + 984*x**2 - 864)/(3240*x*
*9 + 6480*x**7 + 9720*x**5)
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx=\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^2x^6}\,dx$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1/3240*(2435*x^8 + 2475*x^6 + 3928*x^4 - 984*x^2 + 864)/(x^9 + 2*x^7 + 3*x
^5) - 1/648*integrate((487*x^2 - 463)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(172) = 344.

Time = 0.63 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.38

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx$$

$$=\frac{1}{1679616}\sqrt{2}\left(487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+8766\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-8766\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\right.$$

$$+\frac{1}{1679616}\sqrt{2}\left(487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+8766\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-8766\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-\right.$$

$$+\frac{1}{3359232}\sqrt{2}\left(8766\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+487\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right.\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$-\frac{1}{3359232}\sqrt{2}\left(8766\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+487\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right.\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{25\left(7x^3-x\right)}{648\left(x^4+2x^2+3\right)}-\frac{195x^4-65x^2+36}{405x^5}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 16668*3^(1/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(
1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
 + 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(
3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 16668*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arct
an(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/3359232*sqrt(2)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
 + 18) - 487*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 8766*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 16668*3^
(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sqrt(3) + 18))*l
og(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/3359232*sqrt(2
```

```
)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 487*3^(3/4)*s
qrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 8766*
3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 16668*3^(1/4)*sqrt(2)*sqrt(-6*
sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 25/648*(7*x^3 - x)/(x^4 + 2*x^2 + 3) -
 1/405*(195*x^4 - 65*x^2 + 36)/x^5
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.70

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = -\frac{\frac{487\,x^8}{648}+\frac{55\,x^6}{72}+\frac{491\,x^4}{405}-\frac{41\,x^2}{135}+\frac{4}{15}}{x^9+2\,x^7+3\,x^5} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}\,248569\mathrm{i}}{306110016\,\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}+\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}}{612220032\,\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}\right)\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}\,1\mathrm{i}}{3888} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}\,248569\mathrm{i}}{306110016\,\left(-\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}-\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}}{612220032\,\left(-\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}\right)\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}\,1\mathrm{i}}{3888}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (atan((x*(2^(1/2)*745707i + 3418143)^(1/2)*248569i)/(306110016*((2^(1/2)*11
5087447i)/204073344 - 119561689/51018336)) - (248569*2^(1/2)*x*(2^(1/2)*745
707i + 3418143)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 - 1195616
89/51018336)))*(2^(1/2)*745707i + 3418143)^(1/2)*1i)/3888 - (atan((x*(34181
43 - 2^(1/2)*745707i)^(1/2)*248569i)/(306110016*((2^(1/2)*115087447i)/20407
3344 + 119561689/51018336)) + (248569*2^(1/2)*x*(3418143 - 2^(1/2)*745707i)
^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 + 119561689/51018336)))*
(3418143 - 2^(1/2)*745707i)^(1/2)*1i)/3888 - ((491*x^4)/405 - (41*x^2)/135
+ (55*x^6)/72 + (487*x^8)/648 + 4/15)/(3*x^5 + 2*x^7 + x^9)
```

3.117 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 243

$$\begin{aligned}\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx &= 58x-9x^3+x^5-\frac{25x(15+7x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{x(3305+252x^2)}{64\left(3+2x^2+x^4\right)} \\ &+\frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ &-\frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ &+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\ &-\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

```
[Out] 58*x-9*x^3+x^5-25/16*x*(7*x^2+15)/(x^4+2*x^2+3)^2+1/64*x*(252*x^2+3305)/(x^
4+2*x^2+3)+3/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-
8595619+7678611*3^(1/2))^(1/2)-3/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))*(-8595619+7678611*3^(1/2))^(1/2)+3/512*ln(x^2+3^(1/2)-x*(-
2+2*3^(1/2))^(1/2))*(8595619+7678611*3^(1/2))^(1/2)-3/512*ln(x^2+3^(1/2)+x*
(-2+2*3^(1/2))^(1/2))*(8595619+7678611*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1682, 1692, 1690, 1183, 648, 632, 210, 642}

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$= \frac{3}{256}\sqrt{7678611\sqrt{3}-8595619}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$
$$-\frac{3}{256}\sqrt{7678611\sqrt{3}-8595619}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+x^5$$
$$-9x^3+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)$$
$$-\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)$$
$$+\frac{(252x^2+3305)\,x}{64\,(x^4+2x^2+3)}-\frac{25(7x^2+15)\,x}{16\,(x^4+2x^2+3)^2}+58x$$

```
[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (3*Sqrt
[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(
1 + Sqrt[3])]])/256 + (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] - Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/512 - (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[S
qrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
```

```
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96}\int \frac{2250-2850x^2-4800x^4+2400x^6-672x^{10}+480x^{12}}{(3+2x^2+x^4)^2}\,dx \\
&= -\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{-201960+193248x^2+87552x^4-78336x^6+23040x^8}{3+2x^2+x^4}\,dx}{4608} \\
&= -\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{\int \left(267264-124416x^2+23040x^4-\frac{216(4647-148x^2)}{3+2x^2+x^4}\right)dx}{4608} \\
&= 58x-9x^3+x^5-\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} - \frac{3}{64}\int \frac{4647-148x^2}{3+2x^2+x^4}\,dx \\
&= 58x-9x^3+x^5-\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} \\
&\quad - \frac{1}{256}\sqrt{3\left(1+\sqrt{3}\right)}\int \frac{4647\sqrt{2\left(-1+\sqrt{3}\right)}-\left(4647+148\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx \\
&\quad - \frac{1}{256}\sqrt{3\left(1+\sqrt{3}\right)}\int \frac{4647\sqrt{2\left(-1+\sqrt{3}\right)}+\left(4647+148\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= 58x - 9x^3 + x^5 - \frac{25x(15+7x^2)}{16\left(3+2x^2+x^4\right)^2} + \frac{x(3305+252x^2)}{64\left(3+2x^2+x^4\right)} \\
&\quad - \frac{1}{256}\left(3\sqrt{7220107-458504\sqrt{3}}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx \\
&\quad - \frac{1}{256}\left(3\sqrt{7220107-458504\sqrt{3}}\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx \\
&\quad + \frac{1}{512}\left(3\sqrt{8595619+7678611\sqrt{3}}\right)\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx \\
&\quad - \frac{1}{512}\left(3\sqrt{8595619+7678611\sqrt{3}}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx \\
&= 58x - 9x^3 + x^5 - \frac{25x(15+7x^2)}{16\left(3+2x^2+x^4\right)^2} + \frac{x(3305+252x^2)}{64\left(3+2x^2+x^4\right)} \\
&\quad + \frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad - \frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad + \frac{1}{128}\left(3\sqrt{7220107-458504\sqrt{3}}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right) \\
&\quad + \frac{1}{128}\left(3\sqrt{7220107-458504\sqrt{3}}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)
\end{aligned}
$$

$$= 58x - 9x^3 + x^5 - \frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)}$$
$$+ \frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right)$$
$$- \frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right)$$
$$+ \frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$- \frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 58x - 9x^3 + x^5 - \frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{3\left(4795i+148\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{3\left(-4795i+148\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*(4795*I + 148*Sqrt[2])*ArcTan[x/S
qrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (3*(-4795*I + 148*Sqrt
[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.34

method	result
risch	$x^5-9x^3+58x+\frac{\frac{63}{16}x^7+\frac{3809}{64}x^5+\frac{3333}{32}x^3+\frac{8415}{64}x}{(x^4+2x^2+3)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(148\_R^2-4647)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$x^5-9x^3+58x+\frac{\frac{63}{16}x^7+\frac{3809}{64}x^5+\frac{3333}{32}x^3+\frac{8415}{64}x}{(x^4+2x^2+3)^2}+\frac{3\left(1697\sqrt{-2+2\sqrt{3}}\sqrt{3}+4795\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024}$

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x^5-9*x^3+58*x+(63/16*x^7+3809/64*x^5+3333/32*x^3+8415/64*x)/(x^4+2*x^2+3)^
2+3/256*sum((148*_R^2-4647)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.16

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$=\frac{512\,x^{13}-2560\,x^{11}+16384\,x^9+80864\,x^7+276744\,x^5+368208\,x^3-\sqrt{2}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{}}{}$$

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(512*x^13 - 2560*x^11 + 16384*x^9 + 80864*x^7 + 276744*x^5 + 368208*x
^3 - sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(64586691*I*sqrt(2) +
77360571)*log((1549*sqrt(2) + 1697*I)*sqrt(64586691*I*sqrt(2) + 77360571) +
 23035833*x) + sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(64586691*I*
sqrt(2) + 77360571)*log(-(1549*sqrt(2) + 1697*I)*sqrt(64586691*I*sqrt(2) +
77360571) + 23035833*x) - sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(
-64586691*I*sqrt(2) + 77360571)*log((1549*sqrt(2) - 1697*I)*sqrt(-64586691*
I*sqrt(2) + 77360571) + 23035833*x) + sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^
2 + 9)*sqrt(-64586691*I*sqrt(2) + 77360571)*log(-(1549*sqrt(2) - 1697*I)*sq
rt(-64586691*I*sqrt(2) + 77360571) + 23035833*x) + 334584*x)/(x^8 + 4*x^6 +
 10*x^4 + 12*x^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(221) = 442$.

Time = 0.75 (sec) , antiderivative size = 1204, normalized size of antiderivative = 4.95

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x**5 - 9*x**3 + 58*x + (252*x**7 + 3809*x**5 + 6666*x**3 + 8415*x)/(64*x**8
 + 256*x**6 + 640*x**4 + 768*x**2 + 576) - 3*sqrt(8595619/262144 + 7678611*
sqrt(3)/262144)*log(x**2 + x*(-6788*sqrt(3)*sqrt(8595619 + 7678611*sqrt(3))
/7176299 - 2313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 + 1697
*sqrt(2)*sqrt(8595619 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) + 1253
83933330562)/18368002813563) - 12180952402524688792 79*sqrt(2)*sqrt(66002414
605209*sqrt(3) + 125383933330562)/1012150582077174852410264907 - 1343534101
96228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383933330562)/39544284066890
8030011 + 18391902996311867463806959889/1012150582077174852410264907 + 5204
579286823805792980*sqrt(3)/395442840668908030011) + 3*sqrt(8595619/262144 +
 7678611*sqrt(3)/262144)*log(x**2 + x*(-1697*sqrt(2)*sqrt(8595619 + 7678611
*sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/18368002813563 + 2
313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 + 6788*sqrt(3)*sqr
t(8595619 + 7678611*sqrt(3))/7176299) - 1218095240252468879279*sqrt(2)*sqrt
(66002414605209*sqrt(3) + 125383933330562)/1012150582077174852410264907 - 1
34353410196228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383933330562)/39544
2840668908030011 + 18391902996311867463806959889/10121505820771748524102649
07 + 5204579286823805792980*sqrt(3)/395442840668908030011) - 2*sqrt(-9*sqrt
(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)/131072 + 77360571/262144
 + 207322497*sqrt(3)/262144)*atan(110208016881378*x/(22232174302*sqrt(-2*sq
rt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 23035833*s
qrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)*sqrt(
-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 23035
833*sqrt(3))) - 52122411468*sqrt(3)*sqrt(8595619 + 7678611*sqrt(3))/(222321
74302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595
619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) +
 8595619 + 23035833*sqrt(3))) - 6941356584*sqrt(8595619 + 7678611*sqrt(3))/
(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)
 + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 12538393333
0562) + 8595619 + 23035833*sqrt(3))) + 5091*sqrt(2)*sqrt(8595619 + 7678611*
sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/(22232174302*sqrt(-
2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 230358
33*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)*s
```

```
qrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 2
3035833*sqrt(3)))) - 2*sqrt(-9*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)/131072 + 77360571/262144 + 207322497*sqrt(3)/262144)*atan(110208
016881378*x/(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 1253
83933330562) + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(660024146052
09*sqrt(3) + 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562) + 8595619 + 23035833*sqrt(3))) - 5091*sqrt(2)*sqrt(859561
9 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/(222321
74302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595
619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) +
 8595619 + 23035833*sqrt(3))) + 6941356584*sqrt(8595619 + 7678611*sqrt(3))/
(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)
 + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 12538393333
0562) + 8595619 + 23035833*sqrt(3))) + 52122411468*sqrt(3)*sqrt(8595619 + 7
678611*sqrt(3))/(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) +
125383933330562) + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414
605209*sqrt(3) + 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(
3) + 125383933330562) + 8595619 + 23035833*sqrt(3))))
```

Maxima [F]

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^{10}}{(x^4+2\,x^2+3)^3}\,dx$$

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x)/(x^8 + 4
*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*integrate((148*x^2 - 4647)/(x^4 + 2*x^2
+ 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(190) = 380.

Time = 0.73 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.42

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = x^5-9\,x^3$$

$$-\frac{1}{13824}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\right.$$

$$-\frac{1}{13824}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\right.$$

$$-\frac{1}{27648}\sqrt{2}\left(666\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+37\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+66\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{27648}\sqrt{2}\left(666\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+37\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+66\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)+58\,x+\frac{252\,x^7+3809\,x^5+6666\,x^3+8415\,x}{64\,(x^4+2\,x^2+3)^2}$$

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] x^5 - 9*x^3 - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 41823*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666
*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3)
+ 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 41823*3^(
1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) + 18))*ar
ctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
1/2)) - 1/27648*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3)
+ 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4
)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x
^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/27648*sqrt(2)*(666
```

```
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(
-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sq
rt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) +
18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*s
qrt(3) + 1/2) + sqrt(3)) + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 841
5*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.76

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 58\,x+\frac{\frac{63\,x^7}{16}+\frac{3809\,x^5}{64}+\frac{3333\,x^3}{32}+\frac{8415\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}-9\,x^3+x^5 - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}\,193760073\mathrm{i}}{131072\,\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}-\frac{193760073\,\sqrt{2}\,x\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}}{262144\,\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}\right)\,\sqrt{17191238-\sqrt{2}\,1435259}}{256} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}\,193760073\mathrm{i}}{131072\,\left(\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}+\frac{193760073\,\sqrt{2}\,x\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}}{262144\,\left(\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}\right)\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}}{256}$$

```
[In] int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] 58*x - (atan((x*(17191238 - 2^(1/2)*14352598i)^(1/2)*193760073i)/(131072*((
2^(1/2)*900403059231i)/131072 - 986432531643/131072)) - (193760073*2^(1/2)*
x*(17191238 - 2^(1/2)*14352598i)^(1/2))/(262144*((2^(1/2)*900403059231i)/13
1072 - 986432531643/131072)))*(17191238 - 2^(1/2)*14352598i)^(1/2)*3i)/256
+ (atan((x*(2^(1/2)*14352598i + 17191238)^(1/2)*193760073i)/(131072*((2^(1/
2)*900403059231i)/131072 + 986432531643/131072)) + (193760073*2^(1/2)*x*(2^
(1/2)*14352598i + 17191238)^(1/2))/(262144*((2^(1/2)*900403059231i)/131072
+ 986432531643/131072)))*(2^(1/2)*14352598i + 17191238)^(1/2)*3i)/256 + ((8
415*x)/64 + (3333*x^3)/32 + (3809*x^5)/64 + (63*x^7)/16)/(12*x^2 + 10*x^4 +
 4*x^6 + x^8 + 9) - 9*x^3 + x^5
```

3.118 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 242

$$\begin{aligned}\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx &= -27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64(3+2x^2+x^4)} \\ &\quad -\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right) \\ &\quad +\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right) \\ &\quad -\frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x \right.\\ &\quad \left.+x^2\right)+\frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\sqrt{3}\right. \\ &\quad \left.+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)\end{aligned}$$

```
[Out] -27*x+5/3*x^3+25/16*x*(5*x^2+3)/(x^4+2*x^2+3)^2-1/64*x*(835*x^2+1468)/(x^4+
2*x^2+3)-21/512*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-34271+22721*3^(1/2
))^(1/2)+21/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-34271+22721*3^(1/2
))^(1/2)-21/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(34
271+22721*3^(1/2))^(1/2)+21/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1
/2))^(1/2))*(34271+22721*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1682, 1692, 1690, 1183, 648, 632, 210, 642}

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{5x^3}{3} - \frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) - \frac{(835x^2+1468)\,x}{64\,(x^4+2x^2+3)} + \frac{25(5x^2+3)\,x}{16\,(x^4+2x^2+3)^2} - 27x$$

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) - (21*Sqrt[34271 + 22721*Sqrt[3]]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (21*Sqrt[3427
1 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/256 - (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqr
t[3])]*x + x^2])/512 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] + Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
```

```
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96}\int \frac{-450-1050x^2+2400x^4-672x^8+480x^{10}}{(3+2x^2+x^4)^2}\,dx$$

$$= \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{98496+27432x^2-78336x^4+23040x^6}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{\int \left(-124416+23040x^2+\frac{1512(312+137x^2)}{3+2x^2+x^4}\right)dx}{4608}$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{21}{64}\int \frac{312+137x^2}{3+2x^2+x^4}\,dx$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{1}{256}\left(7\sqrt{3\left(1+\sqrt{3}\right)}\right)\int \frac{312\sqrt{2\left(-1+\sqrt{3}\right)}-\left(312-137\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx + \frac{1}{256}\left(7\sqrt{3\left(1+\sqrt{3}\right)}\right)\int \frac{312\sqrt{2\left(-1+\sqrt{3}\right)}+\left(312-137\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} - \frac{1}{512}\left(21\sqrt{-34271+22721\sqrt{3}}\right)\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx + \frac{1}{512}\left(21\sqrt{-34271+22721\sqrt{3}}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx + \frac{1}{256}\left(21\sqrt{51217+28496\sqrt{3}}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx + \frac{1}{256}\left(21\sqrt{51217+28496\sqrt{3}}\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16\left(3+2x^2+x^4\right)^2} - \frac{x(1468+835x^2)}{64\left(3+2x^2+x^4\right)} - \frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) + \frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{128}\left(21\sqrt{51217+28496\sqrt{3}}\right)\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right) - \frac{1}{128}\left(21\sqrt{51217+28496\sqrt{3}}\right)\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16\left(3+2x^2+x^4\right)^2} - \frac{x(1468+835x^2)}{64\left(3+2x^2+x^4\right)} - \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) + \frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64\,(3+2x^2+x^4)} + \frac{21\left(-175i+137\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{21\left(175i+137\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) + (21*(-175*I + 137*Sqrt[2])*ArcTan[x/Sq
rt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (21*(175*I + 137*Sqrt[2
])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.33

method	result
risch	$\frac{5x^3}{3}-27x+\frac{-\frac{835}{64}x^7-\frac{1569}{32}x^5-\frac{4941}{64}x^3-\frac{513}{8}x}{(x^4+2x^2+3)^2}+\frac{21\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(137\_R^2+312\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{256}$
default	$\frac{5x^3}{3}-27x+\frac{-\frac{835}{64}x^7-\frac{1569}{32}x^5-\frac{4941}{64}x^3-\frac{513}{8}x}{(x^4+2x^2+3)^2}+\frac{21\left(33\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-175\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024}+\frac{21\left(4\right.}{}$

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/3*x^3-27*x+(-835/64*x^7-1569/32*x^5-4941/64*x^3-513/8*x)/(x^4+2*x^2+3)^2+
21/256*sum((137*_R^2+312)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.15

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{2560\,x^{11} - 31232\,x^9 - 160328\,x^7 - 459312\,x^5 - 593208\,x^3 + 3\,\sqrt{2}(x^8 + 4\,x^6 + 10\,x^4 + 12\,x^2 + 9)\sqrt{6032439}}{}$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/1536*(2560*x^11 - 31232*x^9 - 160328*x^7 - 459312*x^5 - 593208*x^3 + 3*sq
rt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(6032439*I*sqrt(2) - 15113511
)*log((104*sqrt(2) - 33*I)*sqrt(6032439*I*sqrt(2) - 15113511) + 477141*x) -
 3*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(6032439*I*sqrt(2) - 151
13511)*log(-(104*sqrt(2) - 33*I)*sqrt(6032439*I*sqrt(2) - 15113511) + 47714
1*x) + 3*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-6032439*I*sqrt(2
) - 15113511)*log((104*sqrt(2) + 33*I)*sqrt(-6032439*I*sqrt(2) - 15113511)
+ 477141*x) - 3*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-6032439*I
*sqrt(2) - 15113511)*log(-(104*sqrt(2) + 33*I)*sqrt(-6032439*I*sqrt(2) - 15
113511) + 477141*x) - 471744*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.34

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{5x^3}{3} - 27x + \frac{-835x^7 - 3138x^5 - 4941x^3 - 4104x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + 21\,\mathrm{RootSum}\left(17179869184t^4 + 8983937024t^2 + 1548731523, \left(t \mapsto t\log\left(-\frac{1107296256t^3}{310800559} + \frac{438857984}{310800559}\right.\right.\right.$$

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x**3/3 - 27*x + (-835*x**7 - 3138*x**5 - 4941*x**3 - 4104*x)/(64*x**8 + 2
56*x**6 + 640*x**4 + 768*x**2 + 576) + 21*RootSum(17179869184*_t**4 + 89839
37024*_t**2 + 1548731523, Lambda(_t, _t*log(-1107296256*_t**3/310800559 + 4
38857984*_t/310800559 + x)))
```

Maxima [F]

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^8}{(x^4+2\,x^2+3)^3}\,dx$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 27*x - 1/64*(835*x^7 + 3138*x^5 + 4941*x^3 + 4104*x)/(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9) + 21/64*integrate((137*x^2 + 312)/(x^4 + 2*x^2 + 3)
, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(187) = 374$.

Time = 0.75 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.42

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{5}{3}\,x^3$$

$$-\frac{7}{55296}\,\sqrt{2}\left(137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}}+2466\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right)-2466\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\right.$$

$$-\frac{7}{55296}\,\sqrt{2}\left(137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}}+2466\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right)-2466\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\right.$$

$$-\frac{7}{110592}\,\sqrt{2}\left(2466\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+18}-137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\,\sqrt{3}+18\right)^{\frac{3}{2}}+137\cdot 3^{\frac{3}{4}}\left(6\,\sqrt{3}+18\right)\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\,\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{7}{110592}\,\sqrt{2}\left(2466\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+18}-137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\,\sqrt{3}+18\right)^{\frac{3}{2}}+137\cdot 3^{\frac{3}{4}}\left(6\,\sqrt{3}+18\right)\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\,\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-27\,x-\frac{835\,x^7+3138\,x^5+4941\,x^3+4104\,x}{64\,\left(x^4+2\,x^2+3\right)^2}$$

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 5/3*x^3 - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 246
6*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 11232*
```

```
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 24
66*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 11232
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) + 18)
)*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3
) + 1/2)) - 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sq
rt(3) + 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*
sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 112
32*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 1
8))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 7/110592*sq
rt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 137*3^(3/
4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2
466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11232*3^(1/4)*sqrt(2)*sqrt
(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)
*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 27*x - 1/64*(835*x^7 + 3138*x^5 +
4941*x^3 + 4104*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.75

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{5\,x^3}{3} - \frac{\frac{835\,x^7}{64}+\frac{1569\,x^5}{32}+\frac{4941\,x^3}{64}+\frac{513\,x}{8}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} - 27\,x + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}\,126681219\mathrm{i}}{131072\,\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}-\frac{126681219\,\sqrt{2}\,x\,\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}}{262144\,\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}\right)\,\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}\,21\mathrm{i}}{256} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}\,126681219\mathrm{i}}{131072\,\left(-\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}+\frac{126681219\,\sqrt{2}\,x\,\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}}{262144\,\left(-\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}\right)\,\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}\,21\mathrm{i}}{256}$$

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] (atan((x*(- 2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^(1/2)*494
0567541i)/16384 + 12541440681/131072)) - (126681219*2^(1/2)*x*(- 2^(1/2)*27
358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 + 12541440681/131
072)))*(- 2^(1/2)*27358i - 68542)^(1/2)*21i)/256 - ((513*x)/8 + (4941*x^3)/
64 + (1569*x^5)/32 + (835*x^7)/64)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - 27
*x - (atan((x*(2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^(1/2)*
4940567541i)/16384 - 12541440681/131072)) + (126681219*2^(1/2)*x*(2^(1/2)*2
7358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 - 12541440681/13
1072)))*(2^(1/2)*27358i - 68542)^(1/2)*21i)/256 + (5*x^3)/3
```

3.119 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 235

$$\begin{aligned}
&\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx \\
&= 5x+\frac{25x(3-x^2)}{16\,(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64\,(3+2x^2+x^4)} \\
&\quad+\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad-\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad-\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad+\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}$$

```
[Out] 5*x+25/16*x*(-x^2+3)/(x^4+2*x^2+3)^2+7/64*x*(58*x^2+11)/(x^4+2*x^2+3)-1/512
*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-827621+1176531*3^(1/2))^(1/2)+1/5
12*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-827621+1176531*3^(1/2))^(1/2)+1
/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(827621+117653
1*3^(1/2))^(1/2)-1/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2
))*(827621+1176531*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1682, 1692, 1690, 1183, 648, 632, 210, 642}

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{7(58x^2+11)\,x}{64\,(x^4+2x^2+3)} + \frac{25(3-x^2)\,x}{16\,(x^4+2x^2+3)^2} + 5x$$

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 5*x + (25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + (7*x*(11 + 58*x^2))/(64*(
3 + 2*x^2 + x^4)) + (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sq
rt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[827621 + 1176531*Sqrt[3]
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt
[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512 + (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]
*x + x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
```

```
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{25x(3-x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int\frac{-450+1650x^2-672x^6+480x^8}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$=\frac{25x(3-x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x(11+58x^2)}{64\left(3+2x^2+x^4\right)}+\frac{\int\frac{-12744-49104x^2+23040x^4}{3+2x^2+x^4}\,dx}{4608}$$

$$=\frac{25x(3-x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x(11+58x^2)}{64\left(3+2x^2+x^4\right)}+\frac{\int\left(23040-\frac{72(1137+1322x^2)}{3+2x^2+x^4}\right)dx}{4608}$$

$$=5x+\frac{25x(3-x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x(11+58x^2)}{64\left(3+2x^2+x^4\right)}-\frac{1}{64}\int\frac{1137+1322x^2}{3+2x^2+x^4}\,dx$$

$$=5x+\frac{25x(3-x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x(11+58x^2)}{64\left(3+2x^2+x^4\right)}-\frac{\int\frac{1137\sqrt{2\left(-1+\sqrt{3}\right)}-\left(1137-1322\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{128\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int\frac{1137\sqrt{2\left(-1+\sqrt{3}\right)}+\left(1137-1322\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{128\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$=5x+\frac{25x(3-x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x(11+58x^2)}{64\left(3+2x^2+x^4\right)}-\frac{1}{256}\left(1322+379\sqrt{3}\right)\int\frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx-\frac{1}{256}\left(1322+379\sqrt{3}\right)\int\frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx-\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\int\frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx+\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$

$$= 5x + \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} - \frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) + \frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{128}\left(-1322-379\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{-2(1+\sqrt{3})-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right) - \frac{1}{128}\left(-1322-379\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{-2(1+\sqrt{3})-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$= 5x + \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2} + \frac{7x(11+58x^2)}{64(3+2x^2+x^4)} + \frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) + \frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.59

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{256}\left(\frac{4x(3411+5112x^2+4089x^4+1686x^6+320x^8)}{(3+2x^2+x^4)^2} - \frac{i(-2644i+185\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{i(2644i+185\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(3411 + 5112*x^2 + 4089*x^4 + 1686*x^6 + 320*x^8))/(3 + 2*x^2 + x^4)^
2 - (I*(-2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sq
rt[2]] + (I*(2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 +
I*Sqrt[2]])/256
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

method	result
risch	$5x+\frac{\frac{203}{32}x^7+\frac{889}{64}x^5+\frac{159}{8}x^3+\frac{531}{64}x}{(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(-1322\_R^2-1137)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$5x-\frac{-\frac{203}{32}x^7-\frac{889}{64}x^5-\frac{159}{8}x^3-\frac{531}{64}x}{(x^4+2x^2+3)^2}-\frac{\left(943\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+185\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024}-\frac{\left(1516\sqrt{3}+\frac{(943\sqrt{-2}}{}\right.}{}$

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x+(203/32*x^7+889/64*x^5+159/8*x^3+531/64*x)/(x^4+2*x^2+3)^2+1/256*sum((-
1322*_R^2-1137)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.16

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$

$$=\frac{2560\,x^9+13488\,x^7+32712\,x^5+40896\,x^3-\sqrt{2}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{1316761i\,\sqrt{2}-827621}\,\mathrm{lo}}{}$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(2560*x^9 + 13488*x^7 + 32712*x^5 + 40896*x^3 - sqrt(2)*(x^8 + 4*x^6
+ 10*x^4 + 12*x^2 + 9)*sqrt(1316761*I*sqrt(2) - 827621)*log((379*sqrt(2) +
943*I)*sqrt(1316761*I*sqrt(2) - 827621) + 1176531*x) + sqrt(2)*(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9)*sqrt(1316761*I*sqrt(2) - 827621)*log(-(379*sqrt(2)
+ 943*I)*sqrt(1316761*I*sqrt(2) - 827621) + 1176531*x) - sqrt(2)*(x^8 + 4*x
^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1316761*I*sqrt(2) - 827621)*log((379*sqrt(2
) - 943*I)*sqrt(-1316761*I*sqrt(2) - 827621) + 1176531*x) + sqrt(2)*(x^8 +
```

```
4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1316761*I*sqrt(2) - 827621)*log(-(379*sq
rt(2) - 943*I)*sqrt(-1316761*I*sqrt(2) - 827621) + 1176531*x) + 27288*x)/(x
^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 5x + \frac{406x^7+889x^5+1272x^3+531x}{64x^8+256x^6+640x^4+768x^2+576} + \operatorname{RootSum}\left(17179869184t^4+216955879424t^2+4152675581883, \left(t\mapsto t\log\left(-\frac{31641829376t^3}{1549210136091} - \frac{4553}{1549}\right.\right.\right.$$

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x + (406*x**7 + 889*x**5 + 1272*x**3 + 531*x)/(64*x**8 + 256*x**6 + 640*x
**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 + 216955879424*_t**2 + 41
52675581883, Lambda(_t, _t*log(-31641829376*_t**3/1549210136091 - 455309168
896*_t/1549210136091 + x)))
```

Maxima [F]

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^6}{(x^4+2\,x^2+3)^3}\,dx$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 5*x + 1/64*(406*x^7 + 889*x^5 + 1272*x^3 + 531*x)/(x^8 + 4*x^6 + 10*x^4 + 1
2*x^2 + 9) - 1/64*integrate((1322*x^2 + 1137)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(180) = 360.

Time = 0.72 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.47

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$

$$= \frac{1}{82944}\sqrt{2}\Big(661\cdot 3^{\frac{3}{4}}\sqrt{2}\big(6\sqrt{3}+18\big)^{\frac{3}{2}} + 11898\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\big(\sqrt{3}-3\big) - 11898\cdot 3^{\frac{3}{4}}\big(\sqrt{3}+3\big)\sqrt{-6\sqrt{}}$$

$$+ \frac{1}{82944}\sqrt{2}\Big(661\cdot 3^{\frac{3}{4}}\sqrt{2}\big(6\sqrt{3}+18\big)^{\frac{3}{2}} + 11898\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\big(\sqrt{3}-3\big) - 11898\cdot 3^{\frac{3}{4}}\big(\sqrt{3}+3\big)\sqrt{-6}$$

$$+ \frac{1}{165888}\sqrt{2}\Big(11898\cdot 3^{\frac{3}{4}}\sqrt{2}\big(\sqrt{3}+3\big)\sqrt{-6\sqrt{3}+18} - 661\cdot 3^{\frac{3}{4}}\sqrt{2}\big(-6\sqrt{3}+18\big)^{\frac{3}{2}} + 661\cdot 3^{\frac{3}{4}}\big(6\sqrt{3}+18\big)$$

$$+ 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\Big)$$

$$- \frac{1}{165888}\sqrt{2}\Big(11898\cdot 3^{\frac{3}{4}}\sqrt{2}\big(\sqrt{3}+3\big)\sqrt{-6\sqrt{3}+18} - 661\cdot 3^{\frac{3}{4}}\sqrt{2}\big(-6\sqrt{3}+18\big)^{\frac{3}{2}} + 661\cdot 3^{\frac{3}{4}}\big(6\sqrt{3}+18\big)$$

$$- 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}} + \sqrt{3}\Big) + 5\,x + \frac{406\,x^7+889\,x^5+1272\,x^3+531\,x}{64\,(x^4+2\,x^2+3)^2}$$

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(
1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
 + 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3
/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arct
an(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/165888*sqrt(2)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
 + 18) - 661*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 11898*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3
^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*
log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/165888*sqrt(2
)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 661*3^(3/4)*
sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1189
8*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3^(1/4)*sqrt(2)*sqrt(-
```

```
6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x
*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 5*x + 1/64*(406*x^7 + 889*x^5 + 1272
*x^3 + 531*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.75

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 5\,x+\frac{\frac{203\,x^7}{32}+\frac{889\,x^5}{64}+\frac{159\,x^3}{8}+\frac{531\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\,\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}+\frac{1316761\,\sqrt{2}\,x\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}}{262144\,\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}\right)\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1\mathrm{i}}{256} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\,\left(\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}-\frac{1316761\,\sqrt{2}\,x\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}}{262144\,\left(\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}\right)\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}\,1\mathrm{i}}{256}$$

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] 5*x + (atan((x*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^(
1/2)*1497157257i)/131072 - 3725116869/131072)) + (1316761*2^(1/2)*x*(- 2^(1
/2)*2633522i - 1655242)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 - 3725
116869/131072)))*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1i)/256 - (atan((x*(2
^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^(1/2)*1497157257i)/1
31072 + 3725116869/131072)) - (1316761*2^(1/2)*x*(2^(1/2)*2633522i - 165524
2)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 + 3725116869/131072)))*(2^(
1/2)*2633522i - 1655242)^(1/2)*1i)/256 + ((531*x)/64 + (159*x^3)/8 + (889*x
^5)/64 + (203*x^7)/32)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9)
```

3.120 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 238

$$
\begin{aligned}
&\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx \\
&= -\frac{25x(3+x^2)}{16\,(3+2x^2+x^4)^2}+\frac{x(238-59x^2)}{64\,(3+2x^2+x^4)} \\
&\quad -\frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad -\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}
$$

```
[Out] -25/16*x*(x^2+3)/(x^4+2*x^2+3)^2+1/64*x*(-59*x^2+238)/(x^4+2*x^2+3)-1/256*a
rctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-146505+98481*3^(1/
2))^(1/2)+1/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-14
6505+98481*3^(1/2))^(1/2)+1/512*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(146
505+98481*3^(1/2))^(1/2)-1/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(1465
05+98481*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1682, 1692, 1183, 648, 632, 210, 642}

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$= -\frac{1}{256}\sqrt{3\left(32827\sqrt{3}-48835\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$
$$+\frac{1}{256}\sqrt{3\left(32827\sqrt{3}-48835\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$
$$+\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)$$
$$-\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)$$
$$+\frac{x(238-59x^2)}{64\left(x^4+2x^2+3\right)}-\frac{25x(x^2+3)}{16\left(x^4+2x^2+3\right)^2}$$

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(238 - 59*x^2))/(64*(3 + 2*
x^2 + x^4)) - (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(-48835 + 32827*Sqrt[3])]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3
*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/51
2 - (Sqrt[3*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
 + x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96}\int \frac{450-750x^2-672x^4+480x^6}{(3+2x^2+x^4)^2}\,dx$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{-9936+18792x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{-9936\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-9936-18792\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{9216\sqrt{6\left(-1+\sqrt{3}\right)}} + \frac{\int \frac{-9936\sqrt{2\left(-1+\sqrt{3}\right)}+\left(-9936-18792\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{9216\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} + \frac{1}{256}\left(261-46\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx + \frac{1}{256}\left(261-46\sqrt{3}\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx + \frac{1}{256}\left(\sqrt{\frac{3}{2\left(-1+\sqrt{3}\right)}}\left(46+87\sqrt{3}\right)\right)\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx - \frac{1}{256}\left(\sqrt{\frac{3}{2\left(-1+\sqrt{3}\right)}}\left(46+87\sqrt{3}\right)\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} + \frac{1}{512}\sqrt{146505+98481\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{512}\sqrt{146505+98481\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) + \frac{1}{128}\left(-261+46\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right) + \frac{1}{128}\left(-261+46\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$= -\frac{25x(3+x^2)}{16\left(3+2x^2+x^4\right)^2} + \frac{x(238-59x^2)}{64\left(3+2x^2+x^4\right)} - \frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{512}\sqrt{146505+98481\sqrt{3}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{512}\sqrt{146505+98481\sqrt{3}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.54

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{256}\left(\frac{4x(414+199x^2+120x^4-59x^6)}{(3+2x^2+x^4)^2} + \frac{3\left(174+133i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3\left(174-133i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(414 + 199*x^2 + 120*x^4 - 59*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(174 + (
133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(17
4 - (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/25
6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

method	result
risch	$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(87\_R^2-46)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}+\frac{\left(307\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+399\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024}+\Big(-184\sqrt{3}+\frac{(307\sqrt{-2+2\sqrt{3}}\,\sqrt{}}{}$

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-59/64*x^7+15/8*x^5+199/64*x^3+207/32*x)/(x^4+2*x^2+3)^2+3/256*sum((87*_R^
2-46)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.12

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx =$$
$$-\frac{472\,x^7-960\,x^5-1592\,x^3+\sqrt{2}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{61773i\,\sqrt{2}+146505}\log\left(\left(46\sqrt{2}-30\right.\right.}{}$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/512*(472*x^7 - 960*x^5 - 1592*x^3 + sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x
^2 + 9)*sqrt(61773*I*sqrt(2) + 146505)*log((46*sqrt(2) - 307*I)*sqrt(61773*
I*sqrt(2) + 146505) + 98481*x) - sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9
)*sqrt(61773*I*sqrt(2) + 146505)*log(-(46*sqrt(2) - 307*I)*sqrt(61773*I*sqr
t(2) + 146505) + 98481*x) + sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqr
t(-61773*I*sqrt(2) + 146505)*log((46*sqrt(2) + 307*I)*sqrt(-61773*I*sqrt(2)
 + 146505) + 98481*x) - sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-6
1773*I*sqrt(2) + 146505)*log(-(46*sqrt(2) + 307*I)*sqrt(-61773*I*sqrt(2) +
146505) + 98481*x) - 3312*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(201) = 402$.

Time = 0.70 (sec) , antiderivative size = 1198, normalized size of antiderivative = 5.03

$$\int \frac{x^4(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^3} dx = \text{Too large to display}$$

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-59*x**7 + 120*x**5 + 199*x**3 + 414*x)/(64*x**8 + 256*x**6 + 640*x**4 + 7
68*x**2 + 576) - sqrt(146505/262144 + 98481*sqrt(3)/262144)*log(x**2 + x*(-
307*sqrt(6)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884650
6)/675940757 + 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 122
8*sqrt(48835 + 32827*sqrt(3))/20591) - 941929306825573*sqrt(2)*sqrt(1603106
545*sqrt(3) + 2808846506)/456895906973733049 - 47771215762*sqrt(6)*sqrt(160
3106545*sqrt(3) + 2808846506)/41754888382161 + 974779496667908823353/4568959
06973733049 + 5200450130596150*sqrt(3)/41754888382161) + sqrt(146505/262144
 + 98481*sqrt(3)/262144)*log(x**2 + x*(-1228*sqrt(48835 + 32827*sqrt(3))/20
591 - 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 307*sqrt(6)*
sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506)/675940757
) - 941929306825573*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)/456895906
973733049 - 47771215762*sqrt(6)*sqrt(1603106545*sqrt(3) + 2808846506)/41754
888382161 + 974779496667908823353/456895906973733049 + 5200450130596150*sqrt
(3)/41754888382161) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt(3) + 280884650
6)/131072 + 146505/262144 + 295443*sqrt(3)/262144)*atan(1351881514*sqrt(3)*
x/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2
*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) -
40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt
(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sq
rt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3)
 + 2808846506) + 48835 + 98481*sqrt(3))) - 31879062*sqrt(48835 + 32827*sqrt
(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 4883
5 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt
(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)))
 + 921*sqrt(2)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884
6506)/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 488
35 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqr
t(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))
)) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)/131072 + 14650
5/262144 + 295443*sqrt(3)/262144)*atan(1351881514*sqrt(3)*x/(-1894372*sqrt(
-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)) +
 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(160
3106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) - 921*sqrt(2)*sqrt(
```

```
48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506)/(-1894372*sqrt
(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))
+ 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(16
03106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) + 31879062*sqrt(48
835 + 32827*sqrt(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 28
08846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) +
2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
 98481*sqrt(3))) + 40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-1894372*s
qrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3
)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt
(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)))
```

Maxima [F]

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^4}{(x^4+2\,x^2+3)^3}\,dx$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) + 3/64*integrate((87*x^2 - 46)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(177) = 354$.

Time = 0.87 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.42

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx =$$

$$-\frac{1}{18432}\sqrt{2}\left(29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+522\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-522\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\right.$$

$$-\frac{1}{18432}\sqrt{2}\left(29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+522\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-522\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\right.$$

$$-\frac{1}{36864}\sqrt{2}\left(522\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+29\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+52\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{36864}\sqrt{2}\left(522\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+29\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+52\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{59\,x^7-120\,x^5-199\,x^3-414\,x}{64\,(x^4+2\,x^2+3)^2}$$

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2)
*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/
4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/1843
2*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(2)*
sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt
(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x -
 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/36864*sqrt(
2)*(522*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sq
rt(2)*(-6*sqrt(3) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(
3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(
3) + 18) + 552*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/
6*sqrt(3) + 1/2) + sqrt(3)) + 1/36864*sqrt(2)*(522*3^(3/4)*sqrt(2)*(sqrt(3)
 + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) +
29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(
3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 552*3^(1/4)*sqrt(6*sq
```

```
rt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/
64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.73

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{-\frac{59\,x^7}{64}+\frac{15\,x^5}{8}+\frac{199\,x^3}{64}+\frac{207\,x}{32}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$
$$+\frac{\operatorname{atan}\left(\frac{x\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}\,61773\mathrm{i}}{131072\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}+\frac{61773\,\sqrt{2}\,x\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}}{262144\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}\right)\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}\,1\mathrm{i}}{256}$$
$$-\frac{\operatorname{atan}\left(\frac{x\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}\,61773\mathrm{i}}{131072\left(-\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}-\frac{61773\,\sqrt{2}\,x\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}}{262144\left(-\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}\right)\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}\,1\mathrm{i}}{256}$$

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] ((207*x)/32 + (199*x^3)/64 + (15*x^5)/8 - (59*x^7)/64)/(12*x^2 + 10*x^4 + 4
*x^6 + x^8 + 9) + (atan((x*(293010 - 2^(1/2)*123546i)^(1/2)*61773i)/(131072
*((2^(1/2)*4262337i)/65536 + 56892933/131072)) + (61773*2^(1/2)*x*(293010 -
 2^(1/2)*123546i)^(1/2))/(262144*((2^(1/2)*4262337i)/65536 + 56892933/13107
2)))*(293010 - 2^(1/2)*123546i)^(1/2)*1i)/256 - (atan((x*(2^(1/2)*123546i +
 293010)^(1/2)*61773i)/(131072*((2^(1/2)*4262337i)/65536 - 56892933/131072)
) - (61773*2^(1/2)*x*(2^(1/2)*123546i + 293010)^(1/2))/(262144*((2^(1/2)*42
62337i)/65536 - 56892933/131072)))*(2^(1/2)*123546i + 293010)^(1/2)*1i)/256
```

3.121 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 246

$$\begin{aligned}
&\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} \, dx \\
&= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} \\
&\quad - \frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)} \arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad + \frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)} \arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad - \frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)} \log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{1536} \\
&\quad + \frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)} \log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{1536}
\end{aligned}$$

```
[Out] 25/16*x*(x^2+1)/(x^4+2*x^2+3)^2-1/192*x*(88*x^2+353)/(x^4+2*x^2+3)-11/2304*
arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-5475+3267*3^(1/2)
)^(1/2)+11/2304*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-54
75+3267*3^(1/2))^(1/2)-11/4608*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(5475
+3267*3^(1/2))^(1/2)+11/4608*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(5475+3
267*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1682, 1692, 1183, 648, 632, 210, 642}

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -\frac{11}{768}\sqrt{\frac{1}{3}\left(1089\sqrt{3}-1825\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{11}{768}\sqrt{\frac{1}{3}\left(1089\sqrt{3}-1825\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536} + \frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536} + \frac{25x(x^2+1)}{16\left(x^4+2x^2+3\right)^2} - \frac{x(88x^2+353)}{192\left(x^4+2x^2+3\right)}$$

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(353 + 88*x^2))/(192*(3 + 2*
x^2 + x^4)) - (11*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[
3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 + (11*Sqrt[(-1825 + 1089*Sqrt[3])/3
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 - (11*S
qrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])
/1536 + (11*Sqrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/1536
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{25x(1+x^2)}{16\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{-150+78x^2+480x^4}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x(1+x^2)}{16\left(3+2x^2+x^4\right)^2} - \frac{x(353+88x^2)}{192\left(3+2x^2+x^4\right)} + \frac{\int \frac{6072-2112x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x(1+x^2)}{16\left(3+2x^2+x^4\right)^2} - \frac{x(353+88x^2)}{192\left(3+2x^2+x^4\right)} + \frac{\int \frac{6072\sqrt{2\left(-1+\sqrt{3}\right)}-\left(6072+2112\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{9216\sqrt{6\left(-1+\sqrt{3}\right)}} + \frac{\int \frac{6072\sqrt{2\left(-1+\sqrt{3}\right)}+\left(6072+2112\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{9216\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= \frac{25x(1+x^2)}{16\left(3+2x^2+x^4\right)^2} - \frac{x(353+88x^2)}{192\left(3+2x^2+x^4\right)} - \frac{\left(11\left(24-23\sqrt{3}\right)\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{2304} - \frac{\left(11\left(24-23\sqrt{3}\right)\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{2304} - \frac{\left(11\left(23+8\sqrt{3}\right)\right)\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{768\sqrt{6\left(-1+\sqrt{3}\right)}} + \frac{\left(11\left(23+8\sqrt{3}\right)\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{768\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$=\frac{25x(1+x^2)}{16(3+2x^2+x^4)^2}-\frac{x(353+88x^2)}{192(3+2x^2+x^4)}-\frac{11}{768}\sqrt{\frac{1825}{12}+\frac{363\sqrt{3}}{4}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)+\frac{11}{768}\sqrt{\frac{1825}{12}+\frac{363\sqrt{3}}{4}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)+\frac{\left(11\left(24-23\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}dx,x,-\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)}{1152}+\frac{\left(11\left(24-23\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}dx,x,\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)}{1152}$$

$$=\frac{25x(1+x^2)}{16(3+2x^2+x^4)^2}-\frac{x(353+88x^2)}{192(3+2x^2+x^4)}-\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{11}{768}\sqrt{\frac{1825}{12}+\frac{363\sqrt{3}}{4}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)+\frac{11}{768}\sqrt{\frac{1825}{12}+\frac{363\sqrt{3}}{4}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.54

$$\int\frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx=\frac{1}{768}\left(-\frac{4x(759+670x^2+529x^4+88x^6)}{(3+2x^2+x^4)^2}-\frac{11i\left(-16i+31\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{11i\left(16i+31\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((-4*x*(759 + 670*x^2 + 529*x^4 + 88*x^6))/(3 + 2*x^2 + x^4)^2 - ((11*I)*(-
16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((1
1*I)*(16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]]
)/768
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.29

method	result
risch	$\frac{-\frac{11}{24}x^7-\frac{529}{192}x^5-\frac{335}{96}x^3-\frac{253}{64}x}{(x^4+2x^2+3)^2}+\frac{11\left(\sum_{\_R=\operatorname{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(-8\_R^2+23)\ln(x-\_R)}{\_R^3+\_R}\right)}{768}$
default	$\frac{-\frac{11}{24}x^7-\frac{529}{192}x^5-\frac{335}{96}x^3-\frac{253}{64}x}{(x^4+2x^2+3)^2}+\frac{11\left(-47\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-93\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{9216}+\frac{11\left(92\sqrt{3}+\frac{(-47\sqrt{-2+2\sqrt{}}}{}\right.}{}$

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-11/24*x^7-529/192*x^5-335/96*x^3-253/64*x)/(x^4+2*x^2+3)^2+11/768*sum((-8
*_R^2+23)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.13

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx =$$
$$-\frac{2112\,x^7+12696\,x^5+16080\,x^3-\sqrt{6}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{40777i\,\sqrt{2}+220825}\log\left(\sqrt{6}\sqrt{4\right.}{}$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/4608*(2112*x^7 + 12696*x^5 + 16080*x^3 - sqrt(6)*(x^8 + 4*x^6 + 10*x^4 +
 12*x^2 + 9)*sqrt(40777*I*sqrt(2) + 220825)*log(sqrt(6)*sqrt(40777*I*sqrt(2
) + 220825)*(47*I*sqrt(2) + 46) + 71874*x) + sqrt(6)*(x^8 + 4*x^6 + 10*x^4
+ 12*x^2 + 9)*sqrt(40777*I*sqrt(2) + 220825)*log(sqrt(6)*sqrt(40777*I*sqrt(
2) + 220825)*(-47*I*sqrt(2) - 46) + 71874*x) + sqrt(6)*(x^8 + 4*x^6 + 10*x^
4 + 12*x^2 + 9)*sqrt(-40777*I*sqrt(2) + 220825)*log(sqrt(6)*(47*I*sqrt(2) -
 46)*sqrt(-40777*I*sqrt(2) + 220825) + 71874*x) - sqrt(6)*(x^8 + 4*x^6 + 10
```

```
*x^4 + 12*x^2 + 9)*sqrt(-40777*I*sqrt(2) + 220825)*log(sqrt(6)*(-47*I*sqrt(
2) + 46)*sqrt(-40777*I*sqrt(2) + 220825) + 71874*x) + 18216*x)/(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. 2(207) = 414.

Time = 0.72 (sec) , antiderivative size = 1200, normalized size of antiderivative = 4.88

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-88*x**7 - 529*x**5 - 670*x**3 - 759*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
 2304*x**2 + 1728) - sqrt(220825/7077888 + 14641*sqrt(3)/786432)*log(x**2 +
 x*(-47*sqrt(6)*sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/3
66993 + 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 188*sqrt(1825 + 10
89*sqrt(3))/337) - 24765218375*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)/1346
83862049 - 38128468*sqrt(6)*sqrt(1987425*sqrt(3) + 3444194)/371029923 + 904
13874433403/134683862049 + 144251139148*sqrt(3)/371029923) + sqrt(220825/70
77888 + 14641*sqrt(3)/786432)*log(x**2 + x*(-188*sqrt(1825 + 1089*sqrt(3))/
337 - 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 47*sqrt(6)*sqrt(1825
 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/366993) - 24765218375*sqrt
(2)*sqrt(1987425*sqrt(3) + 3444194)/134683862049 - 38128468*sqrt(6)*sqrt(19
87425*sqrt(3) + 3444194)/371029923 + 90413874433403/134683862049 + 14425113
9148*sqrt(3)/371029923) + 2*sqrt(-121*sqrt(2)*sqrt(1987425*sqrt(3) + 344419
4)/3538944 + 220825/7077888 + 14641*sqrt(3)/262144)*atan(733986*sqrt(3)*x/(
15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)
) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425
*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 204732*sqrt(3)*sqrt(1825 + 10
89*sqrt(3))/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 +
 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)
*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 156048*sqrt(1825
 + 1089*sqrt(3))/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1
825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sq
rt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) + 141*sqrt(2)
*sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/(15502*sqrt(-2*s
qrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*
sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444
194) + 1825 + 3267*sqrt(3)))) + 2*sqrt(-121*sqrt(2)*sqrt(1987425*sqrt(3) +
3444194)/3538944 + 220825/7077888 + 14641*sqrt(3)/262144)*atan(733986*sqrt(
3)*x/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*s
qrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1
```

```
987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 141*sqrt(2)*sqrt(1825 +
 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/(15502*sqrt(-2*sqrt(2)*sqrt(
1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425
*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825
+ 3267*sqrt(3))) + 156048*sqrt(1825 + 1089*sqrt(3))/(15502*sqrt(-2*sqrt(2)*
sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(19
87425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) +
1825 + 3267*sqrt(3))) + 204732*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/(15502*sqr
t(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sq
rt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3)
+ 3444194) + 1825 + 3267*sqrt(3))))
```

Maxima [F]

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^2}{(x^4+2\,x^2+3)^3}\,dx$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9) - 11/192*integrate((8*x^2 - 23)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(177) = 354$.

Time = 0.75 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.35

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$

$$= \frac{11}{124416}\sqrt{2}\left(2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 36\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 36\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}+\right.$$

$$+\frac{11}{124416}\sqrt{2}\left(2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 36\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 36\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}\right.$$

$$+\frac{11}{248832}\sqrt{2}\left(36\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 2\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 36\cdot\right.$$

$$\left. + 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$-\frac{11}{248832}\sqrt{2}\left(36\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18} - 2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}} + 2\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 36\cdot\right.$$

$$\left. - 2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right) - \frac{88\,x^7+529\,x^5+670\,x^3+759\,x}{192\,\left(x^4+2\,x^2+3\right)^2}$$

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 11/124416*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6
*sqrt(3) + 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sq
rt(6*sqrt(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/124416
*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(2)*sqr
t(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sqrt(6*sqrt
(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1
/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/248832*sqrt(2)*
(36*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2*3^(3/4)*sqrt(2)
*(-6*sqrt(3) + 18)^(3/2) + 2*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sq
rt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 207*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18
) + 207*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(
3) + 1/2) + sqrt(3)) - 11/248832*sqrt(2)*(36*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*
sqrt(-6*sqrt(3) + 18) - 2*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2*3^(3/
4)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) +
 207*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 207*3^(1/4)*sqrt(6*sqrt(3) + 1
```

```
8))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/192*(88*x
^7 + 529*x^5 + 670*x^3 + 759*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.71

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -\frac{\frac{11\,x^7}{24}+\frac{529\,x^5}{192}+\frac{335\,x^3}{96}+\frac{253\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} + \frac{\operatorname{atan}\left(\frac{x\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}\,448547\mathrm{i}}{31850496\,\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}-\frac{448547\,\sqrt{2}\,x\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}}{63700992\,\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}\right)\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}\,11\mathrm{i}}{2304} - \frac{\operatorname{atan}\left(\frac{x\,\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}\,448547\mathrm{i}}{31850496\,\left(\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}+\frac{448547\,\sqrt{2}\,x\,\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}}{63700992\,\left(\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}\right)\,\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}\,11\mathrm{i}}{2304}$$

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] (atan((x*(10950 - 2^(1/2)*2022i)^(1/2)*448547i)/(31850496*((2^(1/2)*1031658
1i)/10616832 - 21081709/10616832)) - (448547*2^(1/2)*x*(10950 - 2^(1/2)*202
2i)^(1/2))/(63700992*((2^(1/2)*10316581i)/10616832 - 21081709/10616832)))*(
10950 - 2^(1/2)*2022i)^(1/2)*11i)/2304 - ((253*x)/64 + (335*x^3)/96 + (529*
x^5)/192 + (11*x^7)/24)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - (atan((x*(2^(
1/2)*2022i + 10950)^(1/2)*448547i)/(31850496*((2^(1/2)*10316581i)/10616832
+ 21081709/10616832)) + (448547*2^(1/2)*x*(2^(1/2)*2022i + 10950)^(1/2))/(6
3700992*((2^(1/2)*10316581i)/10616832 + 21081709/10616832)))*(2^(1/2)*2022i
 + 10950)^(1/2)*11i)/2304
```

3.122 $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 248

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx &= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192\,(3+2x^2+x^4)} \\
&\quad -\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) \\
&\quad -\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)
\end{aligned}$$

```
[Out] 25/48*x*(-x^2+1)/(x^4+2*x^2+3)^2+1/192*x*(51*x^2+64)/(x^4+2*x^2+3)-1/768*ar
ctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-3873+3057*3^(1/2))^
(1/2)+1/768*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-3873+3
057*3^(1/2))^(1/2)+1/1536*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(3873+3057
*3^(1/2))^(1/2)-1/1536*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(3873+3057*3^
(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1692, 1192, 1183, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = -\frac{1}{256}\sqrt{\frac{1}{3}\left(1019\sqrt{3}-1291\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{256}\sqrt{\frac{1}{3}\left(1019\sqrt{3}-1291\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) - \frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{25x(1-x^2)}{48\left(x^4+2x^2+3\right)^2} + \frac{x(51x^2+64)}{192\left(x^4+2x^2+3\right)}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + (x*(64 + 51*x^2))/(192*(3 + 2*x
^2 + x^4)) - (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
 - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTa
n[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(1291
+ 1019*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sq
rt[(1291 + 1019*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{1}{96}\int\frac{78+230x^2}{(3+2x^2+x^4)^2}\,dx\\ &= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192\,(3+2x^2+x^4)}+\frac{\int\frac{-288+1224x^2}{3+2x^2+x^4}\,dx}{4608}\end{aligned}$$

$$= \frac{25x(1-x^2)}{48(3+2x^2+x^4)^2} + \frac{x(64+51x^2)}{192(3+2x^2+x^4)} + \frac{\int \frac{-288\sqrt{2(-1+\sqrt{3})}-(-288-1224\sqrt{3})x}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2}\,dx}{9216\sqrt{6(-1+\sqrt{3})}} + \frac{\int \frac{-288\sqrt{2(-1+\sqrt{3})}+(-288-1224\sqrt{3})x}{\sqrt{3}+\sqrt{2(-1+\sqrt{3})}x+x^2}\,dx}{9216\sqrt{6(-1+\sqrt{3})}}$$

$$= \frac{25x(1-x^2)}{48(3+2x^2+x^4)^2} + \frac{x(64+51x^2)}{192(3+2x^2+x^4)} + \frac{1}{768}\left(51-4\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2}\,dx + \frac{1}{768}\left(51-4\sqrt{3}\right)\int \frac{1}{\sqrt{3}+\sqrt{2(-1+\sqrt{3})}x+x^2}\,dx + \frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\int \frac{-\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2}\,dx - \frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\int \frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{3}+\sqrt{2(-1+\sqrt{3})}x+x^2}\,dx$$

$$= \frac{25x(1-x^2)}{48(3+2x^2+x^4)^2} + \frac{x(64+51x^2)}{192(3+2x^2+x^4)} + \frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) - \frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right) + \frac{1}{384}\left(-51+4\sqrt{3}\right)\mathrm{Subst}\left(\int \frac{1}{-2(1+\sqrt{3})-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right) + \frac{1}{384}\left(-51+4\sqrt{3}\right)\mathrm{Subst}\left(\int \frac{1}{-2(1+\sqrt{3})-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)$$

$$= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2} + \frac{x(64+51x^2)}{192\,(3+2x^2+x^4)}$$
$$- \frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right)$$
$$+ \frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right)$$
$$+ \frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$
$$- \frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.52

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \frac{1}{768}\left(\frac{4x(292+181x^2+166x^4+51x^6)}{(3+2x^2+x^4)^2} + \frac{3(34+21i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3(34-21i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
[Out] ((4*x*(292 + 181*x^2 + 166*x^4 + 51*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(34 + (2
1*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(34 -
 (21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(17\_R^2-4\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{256}$
default	$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}+\frac{\left(55\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+63\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{3072}+\frac{\left(-16\sqrt{3}+\frac{\left(55\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+63\sqrt{-\cdots\right.}{2}\right.}{7\cdots}$

[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)

[Out] (17/64*x^7+83/96*x^5+181/192*x^3+73/48*x)/(x^4+2*x^2+3)^2+1/256*sum((17*_R^2-4)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.12

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$$
$$=\frac{408\,x^7+1328\,x^5+1448\,x^3+\sqrt{6}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{851i\,\sqrt{2}+1291}\log\left(\sqrt{6}\sqrt{851i\,\sqrt{2}+1\cdots\right.}{}$$

[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")

[Out] 1/1536*(408*x^7 + 1328*x^5 + 1448*x^3 + sqrt(6)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(851*I*sqrt(2) + 1291)*log(sqrt(6)*sqrt(851*I*sqrt(2) + 1291)*(55*I*sqrt(2) - 8) + 6114*x) - sqrt(6)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(851*I*sqrt(2) + 1291)*log(sqrt(6)*sqrt(851*I*sqrt(2) + 1291)*(-55*I*sqrt(2) + 8) + 6114*x) - sqrt(6)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-851*I*sqrt(2) + 1291)*log(sqrt(6)*(55*I*sqrt(2) + 8)*sqrt(-851*I*sqrt(2) + 1291) + 6114*x) + sqrt(6)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-851*I*sqrt(2) + 1291)*log(sqrt(6)*(-55*I*sqrt(2) - 8)*sqrt(-851*I*sqrt(2) + 1291) + 6114*x) + 2336*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(201) = 402$.

Time = 0.70 (sec) , antiderivative size = 1195, normalized size of antiderivative = 4.82

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{(3 + 2x^2 + x^4)^3}\, dx = \text{Too large to display}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (51*x**7 + 166*x**5 + 181*x**3 + 292*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
2304*x**2 + 1728) - sqrt(1291/786432 + 1019*sqrt(3)/786432)*log(x**2 + x*(-
55*sqrt(6)*sqrt(1291 + 1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867169
 + 49606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 220*sqrt(1291 + 1019*sq
rt(3))/851) - 26628761029*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/751982074
561 - 40176070*sqrt(6)*sqrt(1315529*sqrt(3) + 2390882)/2213882457 + 7609499
4709709/751982074561 + 133967471914*sqrt(3)/2213882457) + sqrt(1291/786432
+ 1019*sqrt(3)/786432)*log(x**2 + x*(-220*sqrt(1291 + 1019*sqrt(3))/851 - 4
9606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 55*sqrt(6)*sqrt(1291 + 1019
*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867169) - 26628761029*sqrt(2)*sqr
t(1315529*sqrt(3) + 2390882)/751982074561 - 40176070*sqrt(6)*sqrt(1315529*s
qrt(3) + 2390882)/2213882457 + 76094994709709/751982074561 + 133967471914*s
qrt(3)/2213882457) + 2*sqrt(-sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/393216
 + 1291/786432 + 1019*sqrt(3)/262144)*atan(1734338*sqrt(3)*x/(-6808*sqrt(-2
*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2
)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 23
90882) + 1291 + 3057*sqrt(3))) - 224180*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/(
-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)
) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529
*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) - 148818*sqrt(1291 + 1019*sqrt(
3))/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sq
rt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(13
15529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) + 165*sqrt(2)*sqrt(1291 +
1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/(-6808*sqrt(-2*sqrt(2)*sqrt(1
315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*
sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 +
 3057*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/393216 +
 1291/786432 + 1019*sqrt(3)/262144)*atan(1734338*sqrt(3)*x/(-6808*sqrt(-2*s
qrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*
sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390
882) + 1291 + 3057*sqrt(3))) - 165*sqrt(2)*sqrt(1291 + 1019*sqrt(3))*sqrt(1
315529*sqrt(3) + 2390882)/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 239
0882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*s
qrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) + 14
8818*sqrt(1291 + 1019*sqrt(3))/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3)
```

```
+ 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 23908
82)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)))
 + 224180*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/(-6808*sqrt(-2*sqrt(2)*sqrt(131
5529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sq
rt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3
057*sqrt(3))))
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^3}\,dx$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
[Out] 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) + 1/64*integrate((17*x^2 - 4)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(177) = 354$.

Time = 0.72 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.33

$$\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3}\,dx =$$

$$-\frac{1}{165888}\sqrt{2}\Big(17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+306\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-306\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}}$$

$$-\frac{1}{165888}\sqrt{2}\Big(17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+306\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-306\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}}$$

$$-\frac{1}{331776}\sqrt{2}\Big(306\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+17\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+$$

$$+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\Big)$$

$$+\frac{1}{331776}\sqrt{2}\Big(306\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+17\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+$$

$$-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\Big)+\frac{51\,x^7+166\,x^5+181\,x^3+292\,x}{192\left(x^4+2\,x^2+3\right)^2}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/165888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3
/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/165
888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sq
rt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt(2)*sqrt
(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x
 - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/331776*sq
rt(2)*(306*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 17*3^(3/4)
*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 17*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 306*
3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 144*3^(1/4)*sqrt(2)*sqrt(-6*sq
rt(3) + 18) + 144*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(
-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/331776*sqrt(2)*(306*3^(3/4)*sqrt(2)*(sqr
t(3) + 3)*sqrt(-6*sqrt(3) + 18) - 17*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2
) + 17*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(6*sqrt(3) + 18)*(s
qrt(3) - 3) + 144*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 144*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
+ 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.70

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \frac{\frac{17\,x^7}{64}+\frac{83\,x^5}{96}+\frac{181\,x^3}{192}+\frac{73\,x}{48}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}\,851\mathrm{i}}{1179648\,\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}+\frac{851\,\sqrt{2}\,x\,\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}}{2359296\,\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}\right)\,\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}\,1\mathrm{i}}{768} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}\,851\mathrm{i}}{1179648\,\left(-\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}-\frac{851\,\sqrt{2}\,x\,\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}}{2359296\,\left(-\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}\right)\,\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}\,1\mathrm{i}}{768}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] ((73*x)/48 + (181*x^3)/192 + (83*x^5)/96 + (17*x^7)/64)/(12*x^2 + 10*x^4 +
4*x^6 + x^8 + 9) + (atan((x*(7746 - 2^(1/2)*5106i)^(1/2)*851i)/(1179648*((2
^(1/2)*851i)/98304 + 46805/393216)) + (851*2^(1/2)*x*(7746 - 2^(1/2)*5106i)
^(1/2))/(2359296*((2^(1/2)*851i)/98304 + 46805/393216)))*(7746 - 2^(1/2)*51
06i)^(1/2)*1i)/768 - (atan((x*(2^(1/2)*5106i + 7746)^(1/2)*851i)/(1179648*(
```

```
(2^(1/2)*851i)/98304 - 46805/393216)) - (851*2^(1/2)*x*(2^(1/2)*5106i + 774
6)^(1/2))/(2359296*((2^(1/2)*851i)/98304 - 46805/393216)))*(2^(1/2)*5106i +
 7746)^(1/2)*1i)/768
```

3.123 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 253

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)} + \frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304} - \frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304} - \frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{4608} + \frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{4608}$$

```
[Out] -4/27/x-25/144*x*(x^2+5)/(x^4+2*x^2+3)^2-1/1728*x*(242*x^2+325)/(x^4+2*x^2+
3)-1/13824*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-179133+165483*3^(1/2))^
(1/2)+1/13824*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-179133+165483*3^(1/2
))^(1/2)+1/6912*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(17
9133+165483*3^(1/2))^(1/2)-1/6912*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^
(1/2))^(1/2))*(179133+165483*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx=\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}-\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}-\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}+\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}-\frac{25x(x^2+5)}{144\left(x^4+2x^2+3\right)^2}-\frac{x(242x^2+325)}{1728\left(x^4+2x^2+3\right)}-\frac{4}{27x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] -4/(27*x) - (25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) - (x*(325 + 242*x^2)
)/(1728*(3 + 2*x^2 + x^4)) + (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[
2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(59711 + 5516
1*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])
/2304 - (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/4608 + (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2
*(-1 + Sqrt[3])]*x + x^2])/4608
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{25x(5+x^2)}{144\,(3+2x^2+x^4)^2}+\frac{1}{96}\int\frac{128+30x^2-\frac{250x^4}{3}}{x^2\,(3+2x^2+x^4)^2}\,dx\\ &= -\frac{25x(5+x^2)}{144\,(3+2x^2+x^4)^2}-\frac{x(325+242x^2)}{1728\,(3+2x^2+x^4)}+\frac{\int\frac{2048-\frac{56x^2}{3}-\frac{1936x^4}{3}}{x^2(3+2x^2+x^4)}\,dx}{4608}\end{aligned}$$

$$=-\frac{25x(5+x^2)}{144(3+2x^2+x^4)^2}-\frac{x(325+242x^2)}{1728(3+2x^2+x^4)}+\frac{\int\left(\frac{2048}{3x^2}-\frac{8(173+166x^2)}{3+2x^2+x^4}\right)dx}{4608}$$

$$=-\frac{4}{27x}-\frac{25x(5+x^2)}{144(3+2x^2+x^4)^2}-\frac{x(325+242x^2)}{1728(3+2x^2+x^4)}-\frac{1}{576}\int\frac{173+166x^2}{3+2x^2+x^4}\,dx$$

$$=-\frac{4}{27x}-\frac{25x(5+x^2)}{144(3+2x^2+x^4)^2}-\frac{x(325+242x^2)}{1728(3+2x^2+x^4)}$$
$$-\frac{\int\frac{173\sqrt{2\left(-1+\sqrt{3}\right)}-\left(173-166\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{1152\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int\frac{173\sqrt{2\left(-1+\sqrt{3}\right)}+\left(173-166\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{1152\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$=-\frac{4}{27x}-\frac{25x(5+x^2)}{144(3+2x^2+x^4)^2}-\frac{x(325+242x^2)}{1728(3+2x^2+x^4)}$$
$$-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\int\frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{4608}$$
$$+\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{4608}$$
$$-\frac{\sqrt{\frac{1}{3}\left(112597+57436\sqrt{3}\right)}\int\frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{2304}$$
$$-\frac{\sqrt{\frac{1}{3}\left(112597+57436\sqrt{3}\right)}\int\frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{2304}$$

$$=-\frac{4}{27x}-\frac{25x(5+x^2)}{144(3+2x^2+x^4)^2}-\frac{x(325+242x^2)}{1728(3+2x^2+x^4)}$$
$$-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{4608}$$
$$+\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{4608}$$
$$+\frac{\sqrt{\frac{1}{3}\left(112597+57436\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx,x,-\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)}{1152}$$
$$+\frac{\sqrt{\frac{1}{3}\left(112597+57436\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx,x,\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)}{1152}$$

$$= -\frac{4}{27x} - \frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2} - \frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)} + \frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304} - \frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304} - \frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{4608} + \frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{4608}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = \frac{-\frac{12\left(768+1849x^2+1412x^4+611x^6+166x^8\right)}{x\left(3+2x^2+x^4\right)^2} + \frac{3i\left(332i+7\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} - \frac{3i\left(-332i+7\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{6912}$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] ((-12*(768 + 1849*x^2 + 1412*x^4 + 611*x^6 + 166*x^8))/(x*(3 + 2*x^2 + x^4)
^2) + ((3*I)*(332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*
Sqrt[2]] - ((3*I)*(-332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[
1 + I*Sqrt[2]])/6912
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

method	result
risch	$\frac{-\frac{83}{288}x^8-\frac{611}{576}x^6-\frac{353}{144}x^4-\frac{1849}{576}x^2-\frac{4}{3}}{x(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(12\_Z^4+238844\_Z^2+3042735921\right)}\_R\ln\left(-1950\_R^3-37653769\_R+2909\right.\right.}{2304}$
default	$-\frac{4}{27x}-\frac{\frac{121}{32}x^7+\frac{809}{64}x^5+\frac{419}{16}x^3+\frac{2475}{64}x}{27(x^4+2x^2+3)^2}-\frac{\left(325\sqrt{-2+2\sqrt{3}}\sqrt{3}-21\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{27648}-\frac{\left(692\sqrt{3}+\frac{(325\sqrt{}\right.}{}$

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-83/288*x^8-611/576*x^6-353/144*x^4-1849/576*x^2-4/3)/x/(x^4+2*x^2+3)^2+1/
2304*sum(_R*ln(-1950*_R^3-37653769*_R+2909135979*x),_R=RootOf(12*_Z^4+23884
4*_Z^2+3042735921))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.15

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx = -\frac{3984\,x^8+14664\,x^6+33888\,x^4+\sqrt{6}(x^9+4\,x^7+10\,x^5+12\,x^3+9\,x)\sqrt{52739i\,\sqrt{2}-59711}\log\left(\sqrt{6}\sqrt{5\right.}{}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/13824*(3984*x^8 + 14664*x^6 + 33888*x^4 + sqrt(6)*(x^9 + 4*x^7 + 10*x^5
+ 12*x^3 + 9*x)*sqrt(52739*I*sqrt(2) - 59711)*log(sqrt(6)*sqrt(52739*I*sqrt
(2) - 59711)*(325*I*sqrt(2) + 346) + 330966*x) - sqrt(6)*(x^9 + 4*x^7 + 10*
x^5 + 12*x^3 + 9*x)*sqrt(52739*I*sqrt(2) - 59711)*log(sqrt(6)*sqrt(52739*I*
sqrt(2) - 59711)*(-325*I*sqrt(2) - 346) + 330966*x) - sqrt(6)*(x^9 + 4*x^7
+ 10*x^5 + 12*x^3 + 9*x)*sqrt(-52739*I*sqrt(2) - 59711)*log(sqrt(6)*(325*I*
sqrt(2) - 346)*sqrt(-52739*I*sqrt(2) - 59711) + 330966*x) + sqrt(6)*(x^9 +
4*x^7 + 10*x^5 + 12*x^3 + 9*x)*sqrt(-52739*I*sqrt(2) - 59711)*log(sqrt(6)*(
-325*I*sqrt(2) + 346)*sqrt(-52739*I*sqrt(2) - 59711) + 330966*x) + 44376*x^
2 + 18432)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.30

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx = \frac{-166x^8-611x^6-1412x^4-1849x^2-768}{576x^9+2304x^7+5760x^5+6912x^3+5184x} + \text{RootSum}\left(4174708211712t^4+15652880384t^2+37564641, \left(t\mapsto t\log\left(-\frac{98146713600t^3}{11971753}-\frac{9639364864}{323237331}\right.\right.\right.$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-166*x**8 - 611*x**6 - 1412*x**4 - 1849*x**2 - 768)/(576*x**9 + 2304*x**7
+ 5760*x**5 + 6912*x**3 + 5184*x) + RootSum(4174708211712*_t**4 + 156528803
84*_t**2 + 37564641, Lambda(_t, _t*log(-98146713600*_t**3/11971753 - 963936
4864*_t/323237331 + x)))
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^3x^2}\,dx$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/576*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768)/(x^9 + 4*x^7 + 10*x^
5 + 12*x^3 + 9*x) - 1/576*integrate((166*x^2 + 173)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(182) = 364.

Time = 0.74 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.30

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx$$

$$=\frac{1}{746496}\sqrt{2}\left(83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1494\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1494\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}\cdots}\right.$$

$$+\frac{1}{746496}\sqrt{2}\left(83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1494\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1494\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\cdots}\right.$$

$$+\frac{1}{1492992}\sqrt{2}\left(1494\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+83\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}\cdots\right.$$
$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$-\frac{1}{1492992}\sqrt{2}\left(1494\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+83\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}\cdots\right.$$
$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)-\frac{242\,x^7+809\,x^5+1676\,x^3+2475\,x}{1728\,\left(x^4+2\,x^2+3\right)^2}-\frac{4}{27\,x}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 1/746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/
746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/14
92992*sqrt(2)*(1494*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 8
3*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 1494*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)
*sqrt(-6*sqrt(3) + 18) - 3114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(
1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/1492992*sqrt(2)*(1494*3^(3/4
```

```
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 83*3^(3/4)*sqrt(2)*(-6*sqrt
(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*sqrt(6*s
qrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 3
114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
 1/2) + sqrt(3)) - 1/1728*(242*x^7 + 809*x^5 + 1676*x^3 + 2475*x)/(x^4 + 2*
x^2 + 3)^2 - 4/27/x
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{\frac{83\,x^8}{288}+\frac{611\,x^6}{576}+\frac{353\,x^4}{144}+\frac{1849\,x^2}{576}+\frac{4}{3}}{x^9+4\,x^7+10\,x^5+12\,x^3+9\,x} + \frac{\operatorname{atan}\left(\frac{x\,\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}\,52739\mathrm{i}}{859963392\,\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}+\frac{52739\,\sqrt{2}\,x\,\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}}{1719926784\,\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}\right)\,\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}\,1\mathrm{i}}{6912} - \frac{\operatorname{atan}\left(\frac{x\,\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}\,52739\mathrm{i}}{859963392\,\left(\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}-\frac{52739\,\sqrt{2}\,x\,\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}}{1719926784\,\left(\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}\right)\,\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}\,1\mathrm{i}}{6912}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^3),x)
```

```
[Out] (atan((x*(- 2^(1/2)*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*91
23847i)/286654464 - 17140175/286654464)) + (52739*2^(1/2)*x*(- 2^(1/2)*3164
34i - 358266)^(1/2))/(1719926784*((2^(1/2)*9123847i)/286654464 - 17140175/2
86654464)))*(- 2^(1/2)*316434i - 358266)^(1/2)*1i)/6912 - (atan((x*(2^(1/2)
*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*9123847i)/286654464 +
 17140175/286654464)) - (52739*2^(1/2)*x*(2^(1/2)*316434i - 358266)^(1/2))/
(1719926784*((2^(1/2)*9123847i)/286654464 + 17140175/286654464)))*(2^(1/2)*
316434i - 358266)^(1/2)*1i)/6912 - ((1849*x^2)/576 + (353*x^4)/144 + (611*x
^6)/576 + (83*x^8)/288 + 4/3)/(9*x + 12*x^3 + 10*x^5 + 4*x^7 + x^9)
```

3.124 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 262

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx &= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432(3+2x^2+x^4)^2}+\frac{x(1474+1025x^2)}{5184(3+2x^2+x^4)} \\
&- \frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736} \\
&+ \frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736} \\
&+ \frac{\sqrt{\frac{1}{3}\left(-10004741+11240451\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{41472} \\
&- \frac{\sqrt{\frac{1}{3}\left(-10004741+11240451\sqrt{3}\right)}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{41472}
\end{aligned}$$

```
[Out] -4/81/x^3+7/27/x+25/432*x*(5*x^2+7)/(x^4+2*x^2+3)^2+1/5184*x*(1025*x^2+1474
)/(x^4+2*x^2+3)+1/124416*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-30014223+
33721353*3^(1/2))^(1/2)-1/124416*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-3
0014223+33721353*3^(1/2))^(1/2)-1/62208*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/
(2+2*3^(1/2))^(1/2))*(30014223+33721353*3^(1/2))^(1/2)+1/62208*arctan((2*x+
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(30014223+33721353*3^(1/2))^(1/2
)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$$
$$=-\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}+\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}-\frac{4}{81x^3}+\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}-\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}+\frac{25x(5x^2+7)}{432\left(x^4+2x^2+3\right)^2}+\frac{x(1025x^2+1474)}{5184\left(x^4+2x^2+3\right)}+\frac{7}{27x}$$

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] -4/(81*x^3) + 7/(27*x) + (25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4)^2) + (x*
(1474 + 1025*x^2))/(5184*(3 + 2*x^2 + x^4)) - (Sqrt[(10004741 + 11240451*Sq
rt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/207
36 + (Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/20736 + (Sqrt[(-10004741 + 11240451*Sqrt[3])
/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472 - (Sqrt[(-10004741
 + 11240451*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4147
2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\text{integral} = \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{128 - \frac{160x^2}{3} + 50x^4 + \frac{1250x^6}{9}}{x^4\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\int \frac{2048-\frac{6656x^2}{3}+\frac{2576x^4}{9}+\frac{8200x^6}{9}}{x^4(3+2x^2+x^4)}\,dx}{4608}$$

$$= \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\int \left(\frac{2048}{3x^4}-\frac{3584}{3x^2}+\frac{8(2242+2369x^2)}{9(3+2x^2+x^4)}\right)dx}{4608}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\int \frac{2242+2369x^2}{3+2x^2+x^4}\,dx}{5184}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)}$$
$$+ \frac{\int \frac{2242\sqrt{2\left(-1+\sqrt{3}\right)}-\left(2242-2369\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{10368\sqrt{6\left(-1+\sqrt{3}\right)}} + \frac{\int \frac{2242\sqrt{2\left(-1+\sqrt{3}\right)}+\left(2242-2369\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{10368\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)}$$
$$+ \frac{\left(2242-2369\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{20736\sqrt{6\left(-1+\sqrt{3}\right)}}$$
$$+ \frac{\left(7107+2242\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{62208}$$
$$+ \frac{\left(7107+2242\sqrt{3}\right)\int \frac{1}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{62208}$$
$$+ \frac{\left(-2242+2369\sqrt{3}\right)\int \frac{-\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{20736\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7+5x^2)}{432\,(3+2x^2+x^4)^2} + \frac{x(1474+1025x^2)}{5184\,(3+2x^2+x^4)}$$
$$+ \frac{\sqrt{-\frac{10004741}{12}+\frac{3746817\sqrt{3}}{4}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{20736}$$
$$- \frac{\sqrt{-\frac{10004741}{12}+\frac{3746817\sqrt{3}}{4}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{20736}$$
$$- \frac{\left(7107+2242\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, -\sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)}{31104}$$
$$- \frac{\left(7107+2242\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{-2\left(1+\sqrt{3}\right)-x^2}\,dx, x, \sqrt{2\left(-1+\sqrt{3}\right)}+2x\right)}{31104}$$
$$= -\frac{4}{81x^3} + \frac{7}{27x} + \frac{25x(7+5x^2)}{432\,(3+2x^2+x^4)^2} + \frac{x(1474+1025x^2)}{5184\,(3+2x^2+x^4)}$$
$$- \frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}$$
$$+ \frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}$$
$$+ \frac{\sqrt{-\frac{10004741}{12}+\frac{3746817\sqrt{3}}{4}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{20736}$$
$$- \frac{\sqrt{-\frac{10004741}{12}+\frac{3746817\sqrt{3}}{4}}\log\left(\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)}{20736}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.53

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\,(3+2x^2+x^4)^3}\,dx$$
$$= \frac{\frac{4(-2304+9024x^2+20090x^4+19939x^6+8644x^8+2369x^{10})}{x^3(3+2x^2+x^4)^2} + \frac{\left(4738+127i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{\left(4738-127i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{20736}$$

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

[Out] ((4*(-2304 + 9024*x^2 + 20090*x^4 + 19939*x^6 + 8644*x^8 + 2369*x^10))/(x^3 *(3 + 2*x^2 + x^4)^2) + ((4738 + (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((4738 - (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*S qrt[2]]])/Sqrt[1 + I*Sqrt[2]])/20736

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\frac{2369}{5184}x^{10}+\frac{2161}{1296}x^{8}+\frac{19939}{5184}x^{6}+\frac{10045}{2592}x^{4}+\frac{47}{27}x^{2}-\frac{4}{9}}{x^{3}(x^{4}+2x^{2}+3)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(12\_Z^{4}+40018964\_Z^{2}+126347738683401)}\_R\ln\left(29190\_R^{3}+10162\right.\right.}{20736}$
default	$-\frac{4}{81x^{3}}+\frac{7}{27x}+\frac{\frac{1025}{192}x^{7}+\frac{881}{48}x^{5}+\frac{7523}{192}x^{3}+\frac{1087}{32}x}{27(x^{4}+2x^{2}+3)^{2}}+\frac{\left(4865\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+381\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{248832}+\frac{\left(896\right.}{}$

[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)

[Out] (2369/5184*x^10+2161/1296*x^8+19939/5184*x^6+10045/2592*x^4+47/27*x^2-4/9)/ x^3/(x^4+2*x^2+3)^2+1/20736*sum(_R*ln(29190*_R^3+101628741761*_R+1327488158 33469*x),_R=RootOf(12*_Z^4+40018964*_Z^2+126347738683401))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.16

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$$
$$=\frac{56856\,x^{10}+207456\,x^{8}+478536\,x^{6}+482160\,x^{4}+\sqrt{6}(x^{11}+4\,x^{9}+10\,x^{7}+12\,x^{5}+9\,x^{3})\sqrt{11809919i\,\sqrt{2}-}}{}$$

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="fricas")

[Out] 1/124416*(56856*x^10 + 207456*x^8 + 478536*x^6 + 482160*x^4 + sqrt(6)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(11809919*I*sqrt(2) - 10004741)*log (sqrt(6)*sqrt(11809919*I*sqrt(2) - 10004741)*(4865*I*sqrt(2) + 4484) + 6744 2706*x) - sqrt(6)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(11809919*I* sqrt(2) - 10004741)*log(sqrt(6)*sqrt(11809919*I*sqrt(2) - 10004741)*(-4865* I*sqrt(2) - 4484) + 67442706*x) - sqrt(6)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(-11809919*I*sqrt(2) - 10004741)*log(sqrt(6)*(4865*I*sqrt(2) -

```
4484)*sqrt(-11809919*I*sqrt(2) - 10004741) + 67442706*x) + sqrt(6)*(x^11 +
4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(-11809919*I*sqrt(2) - 10004741)*log(s
qrt(6)*(-4865*I*sqrt(2) + 4484)*sqrt(-11809919*I*sqrt(2) - 10004741) + 6744
2706*x) + 216576*x^2 - 55296)/(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$$
$$=\operatorname{RootSum}\left(338151365148672t^4+2622682824704t^2+19257390441,\left(t\mapsto t\log\left(\frac{357010935644160t^3}{182097141061}+\cdots\right.\right.\right.$$
$$+\frac{2369x^{10}+8644x^8+19939x^6+20090x^4+9024x^2-2304}{5184x^{11}+20736x^9+51840x^7+62208x^5+46656x^3}$$

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)
```

```
[Out] RootSum(338151365148672*_t**4 + 2622682824704*_t**2 + 19257390441, Lambda(_
t, _t*log(357010935644160*_t**3/182097141061 + 26016957890816*_t/1638874269
549 + x))) + (2369*x**10 + 8644*x**8 + 19939*x**6 + 20090*x**4 + 9024*x**2
- 2304)/(5184*x**11 + 20736*x**9 + 51840*x**7 + 62208*x**5 + 46656*x**3)
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx=\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^3x^4}\,dx$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 1/5184*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^4 + 9024*x^2 - 2304)/(x^
11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) + 1/5184*integrate((2369*x^2 + 2242)/
(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(189) = 378$.

Time = 0.77 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.25

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx =$$

$$-\frac{1}{13436928}\sqrt{2}\left(2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+42642\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-42642\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\right.$$

$$-\frac{1}{13436928}\sqrt{2}\left(2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+42642\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-42642\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\right.$$

$$-\frac{1}{26873856}\sqrt{2}\left(42642\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+2369\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+\right.\right.$$

$$\left.+2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)$$

$$+\frac{1}{26873856}\sqrt{2}\left(42642\cdot 3^{\frac{3}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}-2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}+2369\cdot 3^{\frac{3}{4}}\left(6\sqrt{3}+\right.\right.$$

$$\left.-2\cdot 3^{\frac{1}{4}}x\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}+\sqrt{3}\right)+\frac{1025\,x^7+3524\,x^5+7523\,x^3+6522\,x}{5184\left(x^4+2\,x^2+3\right)^2}+\frac{21\,x^2-4}{81\,x^3}$$

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 42642*3^
(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(sqrt(3) +
 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 80712*3^
(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3) + 18))*a
rctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
 1/2)) - 1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
42642*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(s
qrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) -
80712*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3)
+ 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*s
qrt(3) + 1/2)) - 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sq
rt(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3
^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3)
 - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(6*
sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) +
1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 1
8) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3^(3/4)*(6*sqrt(3)
 + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 80712*3^(
```

```
1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(6*sqrt(3) + 18))*lo
g(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/5184*(1025*x^7
+ 3524*x^5 + 7523*x^3 + 6522*x)/(x^4 + 2*x^2 + 3)^2 + 1/81*(21*x^2 - 4)/x^3
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx = \frac{\frac{2369\,x^{10}}{5184}+\frac{2161\,x^8}{1296}+\frac{19939\,x^6}{5184}+\frac{10045\,x^4}{2592}+\frac{47\,x^2}{27}-\frac{4}{9}}{x^{11}+4\,x^9+10\,x^7+12\,x^5+9\,x^3}$$
$$-\frac{\operatorname{atan}\left(\frac{x\,\sqrt{-60028446-\sqrt{2}\,70859514\mathrm{i}}\,11809919\mathrm{i}}{626913312768\,\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}+\frac{11809919\,\sqrt{2}\,x\,\sqrt{-60028446-\sqrt{2}\,70859514\mathrm{i}}}{1253826625536\,\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}\right)\,\sqrt{-60028446-\ldots}}{62208}$$
$$+\frac{\operatorname{atan}\left(\frac{x\,\sqrt{-60028446+\sqrt{2}\,70859514\mathrm{i}}\,11809919\mathrm{i}}{626913312768\,\left(\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}-\frac{11809919\,\sqrt{2}\,x\,\sqrt{-60028446+\sqrt{2}\,70859514\mathrm{i}}}{1253826625536\,\left(\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}\right)\,\sqrt{-60028446+\sqrt{2}\ldots}}{62208}$$

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^3),x)
```

```
[Out] ((47*x^2)/27 + (10045*x^4)/2592 + (19939*x^6)/5184 + (2161*x^8)/1296 + (236
9*x^10)/5184 - 4/9)/(9*x^3 + 12*x^5 + 10*x^7 + 4*x^9 + x^11) - (atan((x*(-
2^(1/2)*70859514i - 60028446)^(1/2)*11809919i)/(626913312768*((2^(1/2)*1323
8919199i)/104485552128 - 57455255935/208971104256)) + (11809919*2^(1/2)*x*(
- 2^(1/2)*70859514i - 60028446)^(1/2))/(1253826625536*((2^(1/2)*13238919199
i)/104485552128 - 57455255935/208971104256)))*(- 2^(1/2)*70859514i - 600284
46)^(1/2)*1i)/62208 + (atan((x*(2^(1/2)*70859514i - 60028446)^(1/2)*1180991
9i)/(626913312768*((2^(1/2)*13238919199i)/104485552128 + 57455255935/208971
104256)) - (11809919*2^(1/2)*x*(2^(1/2)*70859514i - 60028446)^(1/2))/(12538
26625536*((2^(1/2)*13238919199i)/104485552128 + 57455255935/208971104256)))
*(2^(1/2)*70859514i - 60028446)^(1/2)*1i)/62208
```

3.125 $\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 33, antiderivative size = 149

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4} dx = \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}-\frac{(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(c^2e+b^2g-c(bf+ag))\log(a+bx^2+cx^4)}{4c^3}$$

```
[Out] 1/2*(-b*g+c*f)*x^2/c^2+1/4*g*x^4/c+1/4*(c^2*e+b^2*g-c*(a*g+b*f))*ln(c*x^4+b
*x^2+a)/c^3-1/2*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*c*(3*a*g+b*f))*arctanh((2*
c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1677, 1671, 648, 632, 212, 642}

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4} dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)}{2c^3\sqrt{b^2-4ac}}+\frac{\log(a+bx^2+cx^4)(-c(ag+bf)+b^2g+c^2e)}{4c^3}+\frac{x^2(cf-bg)}{2c^2}+\frac{gx^4}{4c}$$

```
[In] Int[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*f - b*g)*x^2)/(2*c^2) + (g*x^4)/(4*c) - ((2*c^3*d - c^2*(b*e + 2*a*f) -
 b^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
3*Sqrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2+gx^3}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{cf-bg}{c^2}+\frac{gx}{c}+\frac{c^2d-acf+abg+(c^2e+b^2g-c(bf+ag))\,x}{c^2\,(a+bx+cx^2)}\right)\,dx,x,x^2\right)$$

$$=\frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{\text{Subst}\left(\int\frac{c^2d-acf+abg+(c^2e+b^2g-c(bf+ag))x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}$$

$$=\frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{(c^2e+b^2g-c(bf+ag))\,\text{Subst}\left(\int\frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}+\frac{(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\,\text{Subst}\left(\int\frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}$$

$$=\frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{(c^2e+b^2g-c(bf+ag))\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^3}$$

$$=\frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}-\frac{(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(c^2e+b^2g-c(bf+ag))\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int\frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx$$

$$=\frac{2c(cf-bg)x^2+c^2gx^4+\frac{2(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2e+b^2g-c(bf+ag))\log(a+}{4c^3}$$

```
[In] Integrate[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*f - b*g)*x^2 + c^2*g*x^4 + (2*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g
+ b*c*(b*f + 3*a*g))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + (c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

method	result
default	$-\frac{-\frac{1}{2} c g \,x^{4}+b g \,x^{2}-c f \,x^{2}}{2 c^{2}}+\frac{\frac{\left(-a c g +b^{2} g -f b c +e \,c^{2}\right) \ln \left(c \,x^{4}+b \,x^{2}+a \right)}{2 c}+\frac{2 \left(a b g -a c f +c^{2} d -\frac{\left(-a c g +b^{2} g -f b c +e \,c^{2}\right) b}{2 c}\right) \arctan \left(\frac{2 c \,x^{2}+b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{2 c^{2}}$
risch	Expression too large to display

```
[In] int(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2*(-1/2*c*g*x^4+b*g*x^2-c*f*x^2)+1/2/c^2*(1/2*(-a*c*g+b^2*g-b*c*f+c^
2*e)/c*ln(c*x^4+b*x^2+a)+2*(a*b*g-a*c*f+c^2*d-1/2*(-a*c*g+b^2*g-b*c*f+c^2*e
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.26

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx$$
$$=\left[\frac{(b^2c^2-4\,ac^3)gx^4+2\,((b^2c^2-4\,ac^3)f-(b^3c-4\,abc^2)g)x^2+(2\,c^3d-bc^2e+(b^2c-2\,ac^2)f-(b^3-3\,ab\right.$$

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b
*c^2)*g)*x^2 + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c
 - 4*a*b*c^2)*f + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/
(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3
)*f - (b^3*c - 4*a*b*c^2)*g)*x^2 - 2*(2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)
*f - (b^3 - 3*a*b*c)*g)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*c^2)*f +
(b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \frac{cgx^4+2\,cfx^2-2\,bgx^2}{4\,c^2} + \frac{(c^2e-bcf+b^2g-acg)\log(cx^4+bx^2+a)}{4\,c^3} + \frac{(2\,c^3d-bc^2e+b^2cf-2\,ac^2f-b^3g+3\,abcg)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*g*x^4 + 2*c*f*x^2 - 2*b*g*x^2)/c^2 + 1/4*(c^2*e - b*c*f + b^2*g - a*
c*g)*log(c*x^4 + b*x^2 + a)/c^3 + 1/2*(2*c^3*d - b*c^2*e + b^2*c*f - 2*a*c^
2*f - b^3*g + 3*a*b*c*g)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^
2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 1834, normalized size of antiderivative = 12.31

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(f/(2*c) - (b*g)/(2*c^2)) + (g*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 1
0*a*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x^2*
(((((4*c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a*b*c
^4*g)/c^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b
^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*c^3*d - b^
3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(8*c^3*(4*a*c - b^2)^(1/2
)) - (b*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)*(2*b^
4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*
a*b^2*c*g))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((((4*
c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a*b*c^4*g)/c
^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f
+ 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*g + 2*b^2*c^2
*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(2*
(16*a*c^4 - 4*b^2*c^3)) - (b^5*g^2 + b*c^4*e^2 + b^3*c^2*f^2 - c^5*d*e + 2*
a^2*b*c^2*g^2 + a*c^4*d*g + a*c^4*e*f + b*c^4*d*f - 2*b^4*c*f*g - a*b*c^3*f
^2 - 3*a*b^3*c*g^2 - b^2*c^3*d*g - 2*b^2*c^3*e*f - a^2*c^3*f*g + 2*b^3*c^2*
e*g + 4*a*b^2*c^2*f*g - 3*a*b*c^3*e*g)/c^4 + (b*(2*c^3*d - b^3*g - 2*a*c^2*
f - b*c^2*e + b^2*c*f + 3*a*b*c*g)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c -
 b^2)^(1/2))) + ((((8*a^2*c^4*g - 8*a*c^5*e + 8*a*b*c^4*f - 8*a*b^2*c^3*g)/
c^4 - (8*a*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f
 + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*c^3*d - b^3*g -
2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(8*c^3*(4*a*c - b^2)^(1/2)) - (
a*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)*(2*b^4*g +
2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*
c*g))/(c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((((8*a^2*c^4*
g - 8*a*c^5*e + 8*a*b*c^4*f - 8*a*b^2*c^3*g)/c^4 - (8*a*c^2*(2*b^4*g + 2*b^
2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g)
)/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e
- 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*
c^4*e^2 + a*b^4*g^2 + a^3*c^2*g^2 + a*b^2*c^2*f^2 - 2*a^2*b^2*c*g^2 - 2*a^2
*c^3*e*g + 2*a*b^2*c^2*e*g + 2*a^2*b*c^2*f*g - 2*a*b*c^3*e*f - 2*a*b^3*c*f*
g)/c^4 + (a*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)^2
)/(c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(4*c^6*d^2 + b^6*g^2 +
4*a^2*c^4*f^2 + b^2*c^4*e^2 + b^4*c^2*f^2 - 4*a*b^2*c^3*f^2 - 8*a*c^5*d*f -
 4*b*c^5*d*e - 2*b^5*c*f*g + 9*a^2*b^2*c^2*g^2 - 6*a*b^4*c*g^2 + 4*b^2*c^4*
```

```
d*f - 4*b^3*c^3*d*g - 2*b^3*c^3*e*f + 2*b^4*c^2*e*g - 6*a*b^2*c^3*e*g + 10*
a*b^3*c^2*f*g - 12*a^2*b*c^3*f*g + 12*a*b*c^4*d*g + 4*a*b*c^4*e*f))*(2*c^3*
d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(2*c^3*(4*a*c - b^2
)^(1/2))
```

3.126 $\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 594

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}$$
$$+\frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b^3cf+bc^2(cd-3af)-b^4g-b^2c(ce-4ag)+2ac^2(\cdots}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$-\frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)-\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+19af)-5\cdots}{\sqrt{b^2-4\cdots}}\cdots\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)+\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+19af)-5\cdots}{\sqrt{b^2-4\cdots}}\cdots\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] (-2*b*g+c*f)*x/c^3+1/3*g*x^3/c^2+1/2*x*(a*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*
c*(3*a*g+b*f))+(b^3*c*f+b*c^2*(-3*a*f+c*d)-b^4*g-b^2*c*(-4*a*g+c*e)+2*a*c^2
*(-a*g+c*e))*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*f-b*c^2*(13*a*f+c*d)-5*b^4*g-b^
2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e)+(-3*b^4*c*f+4*a*c^3*(-5*a*f+c*d)+b
^2*c^2*(19*a*f+c*d)+5*b^5*g+b^3*c*(-34*a*g+c*e)-4*a*b*c^2*(-13*a*g+2*c*e))/
(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*f-b*
c^2*(13*a*f+c*d)-5*b^4*g-b^2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e)+(3*b^4*
c*f-4*a*c^3*(-5*a*f+c*d)-b^2*c^2*(19*a*f+c*d)-5*b^5*g-b^3*c*(-34*a*g+c*e)+4
*a*b*c^2*(-13*a*g+2*c*e))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*a*c+b^2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 11.11 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1682, 1690, 1180, 211}

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx =$$
$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b^2c(ce-24ag)-\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-b^2c(ce-24ag)+\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x(a(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)+x^2(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$+\frac{x(cf-2bg)}{c^3}+\frac{gx^3}{3c^2}$$

```
[In] Int[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*f - 2*b*g)*x)/c^3 + (g*x^3)/(3*c^2) + (x*(a*(2*c^3*d - c^2*(b*e + 2*a*f
) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f + b*c^2*(c*d - 3*a*f) - b^4*g - b
^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e
- 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) -
b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e
- 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*
c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 1
9*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

integral

$$= \frac{x(a(2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)) + (b^3cf + bc^2(cd - 3af) - b^4g - b^2c(ce - 4ag) + 2ac^2(ce}{2c^3\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)} - \frac{\int \frac{\frac{a^2\left(2c^3d - c^2(be+2af) - b^3g + bc(bf+3ag)\right)}{c^3} + \frac{a\left(b^3cf - bc^2(cd+5af) - b^4g - b^2c(ce-6ag) + 6ac^2(ce-ag)\right)x^2}{c^3} - \frac{2a\left(b^2-4ac\right)(cf-bg)x^4}{c^2} + 2a\left(4a - \frac{b^2}{c}\right)gx^6}{a+bx^2+cx^4}\,dx}{2a\left(b^2 - 4ac\right)}$$

$$= \frac{x(a(2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)) + (b^3cf + bc^2(cd - 3af) - b^4g - b^2c(ce - 4ag) +}{2c^3\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)} - \frac{\int \left(-\frac{2a\left(b^2-4ac\right)(cf-2bg)}{c^3} - \frac{2a\left(b^2-4ac\right)gx^2}{c^2} + \frac{a^2\left(2c^3d - c^2(be+10af) - 5b^3g + bc(3bf+19ag)\right) + a\left(3b^3cf - bc^2(cd+13af) - 5b^4}{c^3\left(a+bx^2+cx^4\right)}\right.}{2a\left(b^2 - 4ac\right)}$$

$$= \frac{(cf - 2bg)x}{c^3} + \frac{gx^3}{3c^2} + \frac{x(a(2c^3d - c^2(be + 2af) - b^3g + bc(bf + 3ag)) + (b^3cf + bc^2(cd - 3af) - b^4g - b^2c(ce - 4ag)}{2c^3\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)} - \frac{\int \frac{a^2\left(2c^3d - c^2(be+10af) - 5b^3g + bc(3bf+19ag)\right) + a\left(3b^3cf - bc^2(cd+13af) - 5b^4g - b^2c(ce-24ag) + 2ac^2(3ce-7ag)\right)x^2}{a+bx^2+cx^4}\,dx}{2ac^3\left(b^2 - 4ac\right)}$$

$$= \frac{(cf-2bg)x}{c^3} + \frac{gx^3}{3c^2} + \frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b^3cf+bc^2(cd-3af)-b^4g-b^2c(ce-4ag)+}{2c^3(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)-\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+}{}\right.}{4c^3(b^2-4ac)} - \frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)+\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+}{}\right.}{4c^3(b^2-4ac)}$$

$$= \frac{(cf-2bg)x}{c^3} + \frac{gx^3}{3c^2} + \frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b^3cf+bc^2(cd-3af)-b^4g-b^2c(ce-4ag)+}{2c^3(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)-\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+}{}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)+\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+}{}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.21

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{12\sqrt{c}(cf-2bg)x+4c^{3/2}gx^3+\frac{6\sqrt{c}x(b(c^3d-bc^2e+b^2cf-b^3g)x^2+a^2c(3bg-2c(f+gx^2))+a(-b^3g+2c^3(d+ex^2)-bc^2(e+3fx^2)+b^2c(f}{(b^2-4ac)(a+bx^2+cx^4)}}{}$$

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*f - 2*b*g)*x + 4*c^(3/2)*g*x^3 + (6*Sqrt[c]*x*(b*(c^3*d - b*
c^2*e + b^2*c*f - b^3*g)*x^2 + a^2*c*(3*b*g - 2*c*(f + g*x^2)) + a*(-(b^3*g
) + 2*c^3*(d + e*x^2) - b*c^2*(e + 3*f*x^2) + b^2*c*(f + 4*g*x^2))))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*g - b^3*c*(c*e + 3*Sqrt[
b^2 - 4*a*c]*f - 34*a*g) + b^4*(3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + 2*a*c^2*(-
2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e + 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*c]*g) - b
^2*c*(c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f + 24*a*Sqrt[b^2 - 4*a*c]*g)
+ b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d + 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*f - 4*a*
g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*g + b^3*c*(c*e - 3
*Sqrt[b^2 - 4*a*c]*f - 34*a*g) + b^4*(-3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + b^2
```

```
*c*(c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f - 24*a*Sqrt[b^2 - 4*a*c]*g) +
2*a*c^2*(2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e - 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*
c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*f
+ 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.59

method	result
risch	$\frac{g\,x^{3}}{3c^{2}}-\frac{2bgx}{c^{3}}+\frac{fx}{c^{2}}+\frac{\frac{\left(2g\,a^{2}c^{2}-4a\,b^{2}cg+3ab\,c^{2}f-2a\,c^{3}e+b^{4}g-b^{3}cf+b^{2}c^{2}e-b\,c^{3}d\right)x^{3}}{8ac-2b^{2}}-\frac{a\left(3abgc-2a\,c^{2}f-b^{3}g+b^{2}cf-b\,c^{2}e+2c^{3}d\right)x}{2\left(4ac-b^{2}\right)}}{c^{3}\left(c\,x^{4}+b\,x^{2}+a\right)}+$
default	$-\frac{-\frac{1}{3}g\,x^{3}c+2bgx-cfx}{c^{3}}+\frac{\frac{\left(2g\,a^{2}c^{2}-4a\,b^{2}cg+3ab\,c^{2}f-2a\,c^{3}e+b^{4}g-b^{3}cf+b^{2}c^{2}e-b\,c^{3}d\right)x^{3}}{8ac-2b^{2}}-\frac{a\left(3abgc-2a\,c^{2}f-b^{3}g+b^{2}cf-b\,c^{2}e+2c^{3}d\right)x}{2\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+$

```
[In] int(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*g*x^3/c^2-2/c^3*b*g*x+f*x/c^2+(1/2*(2*a^2*c^2*g-4*a*b^2*c*g+3*a*b*c^2*f
-2*a*c^3*e+b^4*g-b^3*c*f+b^2*c^2*e-b*c^3*d)/(4*a*c-b^2)*x^3-1/2*a*(3*a*b*c*
g-2*a*c^2*f-b^3*g+b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^2)*x)/c^3/(c*x^4+b*x^2+
a)+1/4/c^3*sum((-(14*a^2*c^2*g-24*a*b^2*c*g+13*a*b*c^2*f-6*a*c^3*e+5*b^4*g-
3*b^3*c*f+b^2*c^2*e+b*c^3*d)/(4*a*c-b^2)*_R^2+a*(19*a*b*c*g-10*a*c^2*f-5*b^
3*g+3*b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=Roo
tOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(gx^6+fx^4+ex^2+d)x^4}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*((b*c^3*d - (b^2*c^2 - 2*a*c^3)*e + (b^3*c - 3*a*b*c^2)*f - (b^4 - 4*a*
b^2*c + 2*a^2*c^2)*g)*x^3 + (2*a*c^3*d - a*b*c^2*e + (a*b^2*c - 2*a^2*c^2)*
f - (a*b^3 - 3*a^2*b*c)*g)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*
x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*integrate(-(2*a*c^3*d - a*b*c^2*e -
(b*c^3*d + (b^2*c^2 - 6*a*c^3)*e - (3*b^3*c - 13*a*b*c^2)*f + (5*b^4 - 24*a
*b^2*c + 14*a^2*c^2)*g)*x^2 + (3*a*b^2*c - 10*a^2*c^2)*f - (5*a*b^3 - 19*a^
2*b*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*c^4) + 1/3*(c*g*x^3 + 3*(c
*f - 2*b*g)*x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10752 vs. 2(550) = 1100.

Time = 2.34 (sec) , antiderivative size = 10752, normalized size of antiderivative = 18.10

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^3*d*x^3 - b^2*c^2*e*x^3 + 2*a*c^3*e*x^3 + b^3*c*f*x^3 - 3*a*b*c^2*
f*x^3 - b^4*g*x^3 + 4*a*b^2*c*g*x^3 - 2*a^2*c^2*g*x^3 + 2*a*c^3*d*x - a*b*c
^2*e*x + a*b^2*c*f*x - 2*a^2*c^2*f*x - a*b^3*g*x + 3*a^2*b*c*g*x)/((b^2*c^3
 - 4*a*c^4)*(c*x^4 + b*x^2 + a)) + 1/16*((2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 -
```

4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*(b^2*c^3 - 4
*a*c^4)^2*d + (2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c
^3 - 4*a*c^4)^2*e - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*
a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*f + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^
2*b^2*c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^5*c - 110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*
c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 +
 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 28*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 24*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4
*a*c)*b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2
*c^3 - 4*a*c^4)^2*g - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 - 2*a*b^4*c^8 + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*c^9 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^9 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^9 + 16*a^2*b^2*c^9
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^10 - 32*a^3*c^10 + 2*(b^2
 - 4*a*c)*a*b^2*c^8 - 8*(b^2 - 4*a*c)*a^2*c^9)*d*abs(b^2*c^3 - 4*a*c^4) + 2
*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^4*c^7 - 2*a*b^5*c^7 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b*c^8 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 + 16*a^2*b^3*c^8 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^9 - 32*a^3*b*c^9 + 2*(b^2 - 4*a*c)*a*b^3*c
^7 - 8*(b^2 - 4*a*c)*a^2*b*c^8)*e*abs(b^2*c^3 - 4*a*c^4) - 2*(3*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4

*a*c)*c)*a^2*b^4*c^6 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 6*a*b^6*c^6 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 + 4 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 + 68*a^2*b^4*c^7 - 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^8 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a ^3*b*c^8 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 - 256*a^3 *b^2*c^8 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^9 + 320*a^4*c^9 + 6*(b^2 - 4*a*c)*a*b^4*c^6 - 44*(b^2 - 4*a*c)*a^2*b^2*c^7 + 80*(b^2 - 4*a *c)*a^3*c^8)*f*abs(b^2*c^3 - 4*a*c^4) + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^ 5 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 10*a*b^7*c^5 + 2 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^6 + 78*sqrt(2)*sqrt(b* c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a* c)*c)*a*b^5*c^6 + 118*a^2*b^5*c^6 - 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 464*a^3*b^3*c^7 + 76*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 608*a^4*b*c^8 + 10*(b^2 - 4*a*c)*a*b^5*c^5 - 78*(b^2 - 4*a*c)*a^2*b^3*c^6 + 152*(b^2 - 4*a*c)*a ^3*b*c^7)*g*abs(b^2*c^3 - 4*a*c^4) - (2*b^7*c^11 - 8*a*b^5*c^12 - 32*a^2*b^ 3*c^13 + 128*a^3*b*c^14 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4 *a*c)*c)*b^7*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c) *c)*a*b^5*c^10 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a ^2*b^3*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 *c^11 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b* c^12 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2 *c^12 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b* c^13 - 2*(b^2 - 4*a*c)*b^5*c^11 + 32*(b^2 - 4*a*c)*a^2*b*c^13)*d - (2*b^8*c ^10 - 32*a*b^6*c^11 + 160*a^2*b^4*c^12 - 256*a^3*b^2*c^13 - sqrt(2)*sqrt(b^ 2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 + 2*sqrt(2)*sqrt(b^2 - 4*a *c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)* sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 - 24*sqrt(2)*sqrt(b^2 - 4*a*c) *sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr t(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt (b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^11 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt (b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^12 - 2*(b^2 - 4*a*c)*b^6*c^10 + 24*(b ^2 - 4*a*c)*a*b^4*c^11 - 64*(b^2 - 4*a*c)*a^2*b^2*c^12)*e + (6*b^9*c^9 - 86 *a*b^7*c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 + 640*a^4*b*c^13 - 3*sqrt (2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^7 + 43*sqrt(2)* sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 + 6*sqrt(2)*sqr t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 - 220*sqrt(2)*sqrt(b ^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 - 62*sqrt(2)*sqrt(b

^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^11 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^12 - 6*(b^2 - 4*a*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^2*b^3*c^11 + 160*(b^2 - 4*a*c)*a^3*b*c^12)*f - (10*b^10*c^8 - 148*a*b^8*c^9 + 808*a^2*b^6*c^10 - 1920*a^3*b^4*c^11 + 1664*a^4*b^2*c^12 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^10*c^6 + 74*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^7 - 404*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^8 - 108*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^9 + 376*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 + 54*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 832*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^10 - 416*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 - 188*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 10*(b^2 - 4*a*c)*b^8*c^8 + 108*(b^2 - 4*a*c)*a*b^6*c^9 - 376*(b^2 - 4*a*c)*a^2*b^4*c^10 + 416*(b^2 - 4*a*c)*a^3*b^2*c^11)*g)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*a*b*c^4 + sqrt((b^3*c^3 - 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4 - 4*a*c^5)))/(b^2*c^4 - 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*c^8 + 48*a^3*b^2*c^9 + 16*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*c^10 - 8*a^2*b^2*c^10 + 16*a^3*c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) + 1/16*((2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*(b^2*c^3 - 4*a*c^4)^2*d + (2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c^3 - 4*a*c^4)^2*e - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt

(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 2
6*(b^2 - 4*a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*f + (10*b^6*c^2 - 88*a*b^4*c
^3 + 220*a^2*b^2*c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^5*c - 110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
3 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 -
10*(b^2 - 4*a*c)*b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^
2*c^4)*(b^2*c^3 - 4*a*c^4)^2*g - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 - 2*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 + 2*a*b^4*c^8 + 16*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^9 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^9 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^9 - 16*a
^2*b^2*c^9 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^10 + 32*a^3*c^
10 - 2*(b^2 - 4*a*c)*a*b^2*c^8 + 8*(b^2 - 4*a*c)*a^2*c^9)*d*abs(b^2*c^3 - 4
*a*c^4) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^4*c^7 + 2*a*b^5*c^7 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^8 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 +
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 - 16*a^2*b^3*c^8 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^9 + 32*a^3*b*c^9 - 2*(b^2 - 4*a
*c)*a*b^3*c^7 + 8*(b^2 - 4*a*c)*a^2*b*c^8)*e*abs(b^2*c^3 - 4*a*c^4) - 2*(3*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^5*c^6 + 6*a*b^6*c^6 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^2*c^7 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 3*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 - 68*a^2*b^4*c^7 - 160*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^8 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^
8 + 256*a^3*b^2*c^8 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^9 -
320*a^4*c^9 - 6*(b^2 - 4*a*c)*a*b^4*c^6 + 44*(b^2 - 4*a*c)*a^2*b^2*c^7 - 80
*(b^2 - 4*a*c)*a^3*c^8)*f*abs(b^2*c^3 - 4*a*c^4) + 2*(5*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 + 10*a*
b^7*c^5 + 232*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^6 + 78*sqrt

(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + 5*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^5*c^6 - 118*a^2*b^5*c^6 - 304*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^2*c^7 - 39*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 464*a^3
*b^3*c^7 + 76*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 608*a^4*b
c^8 - 10(b^2 - 4*a*c)*a*b^5*c^5 + 78*(b^2 - 4*a*c)*a^2*b^3*c^6 - 152*(b^2
 - 4*a*c)*a^3*b*c^7)*g*abs(b^2*c^3 - 4*a*c^4) - (2*b^7*c^11 - 8*a*b^5*c^12
 - 32*a^2*b^3*c^13 + 128*a^3*b*c^14 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^7*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^5*c^10 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6*c^10 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^5*c^11 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b*c^12 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^12 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^13 - 2*(b^2 - 4*a*c)*b^5*c^11 + 32*(b^2 - 4*a*c)*a^2*b*c^13)*d
 - (2*b^8*c^10 - 32*a*b^6*c^11 + 160*a^2*b^4*c^12 - 256*a^3*b^2*c^13 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^9 - 80*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 - 24*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 128*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 + 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 12*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^11 - 32*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^12 - 2*(b^2 - 4*a*c)*b^6*c
^10 + 24*(b^2 - 4*a*c)*a*b^4*c^11 - 64*(b^2 - 4*a*c)*a^2*b^2*c^12)*e + (6*b
^9*c^9 - 86*a*b^7*c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 + 640*a^4*b*c^
13 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^7 +
43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 + 6*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8 - 220*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 - 62*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 3*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^9 + 464*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 + 192*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 31*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - 320*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^11 - 160*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 96*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 80*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^12 - 6*(b^2 -
 4*a*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^2*b^3*c
^11 + 160*(b^2 - 4*a*c)*a^3*b*c^12)*f - (10*b^10*c^8 - 148*a*b^8*c^9 + 808*
a^2*b^6*c^10 - 1920*a^3*b^4*c^11 + 1664*a^4*b^2*c^12 - 5*sqrt(2)*sqrt(b^2 -

```
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^10*c^6 + 74*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^7 - 404*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^8 - 108*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^9 + 376*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 + 54*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 832*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^10 - 416*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 - 188*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 10*(b^2 - 4*a*c)*b^8*c^8 +
108*(b^2 - 4*a*c)*a*b^6*c^9 - 376*(b^2 - 4*a*c)*a^2*b^4*c^10 + 416*(b^2 - 4
*a*c)*a^3*b^2*c^11)*g)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*a*b*c^4 - sqr
t((b^3*c^3 - 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4 - 4*a*c^5)))
/(b^2*c^4 - 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*c^8 + 48*a^3*
b^2*c^9 + 16*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*c^10 - 8*a^2*
b^2*c^10 + 16*a^3*c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) + 1/3*(c^4*g*x^3 + 3
*c^4*f*x - 6*b*c^3*g*x)/c^6
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 47339, normalized size of antiderivative = 79.70

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^3*(b^4*g + b^2*c^2*e + 2*a^2*c^2*g - 2*a*c^3*e - b*c^3*d - b^3*c*f + 3*
a*b*c^2*f - 4*a*b^2*c*g))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2*f - 2*a*c^3*d +
 a*b^3*g + a*b*c^2*e - a*b^2*c*f - 3*a^2*b*c*g))/(2*(4*a*c - b^2)))/(a*c^3
+ c^4*x^4 + b*c^3*x^2) + x*(f/c^2 - (2*b*g)/c^3) + atan(((((2048*a^4*c^10*d
 - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c
^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a
^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*
g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 8
0*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6
 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b
^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9
)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*
c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2
 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c
^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2
```

```
+ 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44
800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 +
 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 -
 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)
^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c
^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*
e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5
*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5
*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 2
58*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g
*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) +
192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^
4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d
*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 26
88*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^
4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*
f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g +
 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(
4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119
616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*
b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^
(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c -
 b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*
f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/
(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*
b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192
*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 -
 8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(
1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640
*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 +
 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a
^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*
b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*
c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*
a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*
c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) -
 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g -
 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35
840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1
536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(
4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^1
```

1*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11

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*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c
 - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b
^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(
-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*
a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^
6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7
*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*
c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^
4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(
1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b
^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b
^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e
*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 1
2*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^
(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*
a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10
 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i - (((2048*a^4*c^10*d -
10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*
e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*
b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g -
 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a
*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 4
8*a^2*b^2*c^7)) + (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)
^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(
1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5
*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 -
80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*
d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2
077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800
*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 11
6928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49
*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*
d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f
 + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*
f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*
e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*
a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-
(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192
*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c
*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f
 - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*
a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b
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^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a

^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 +
3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*g^2 + 8*a^2*c^
8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*
a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^
6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a
^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2
*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f
 + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a
*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a
*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g +
394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5
*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1
804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a
*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2)
 + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 76
8*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7
*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288
*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b
^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*
c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c -
b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*
b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*
g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615
*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 153
60*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*
a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*
a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^
3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c -
b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*
c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4
*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c
^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*
f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c -
 b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b
^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2
) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)
^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*
c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*
(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a
*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/
2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c
 - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^

6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3
840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i)/((((2048*a^4*c^10*d - 102
40*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*e +
 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*b^2
*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g - 32
*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a*b^
9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a
^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)
^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2
) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^
2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 806
40*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2
 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077
*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^
5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 11692
8*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^
3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
 - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g
 - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f +
35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f -
 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(
-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b
^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*
a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^
2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^
2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 5
12*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3
*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*
c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4
*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352
*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9
)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c
 - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^
4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^
2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2)
 + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^
9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-
(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4
096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^
10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5
*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) +
 b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*
a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a

*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b
*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a
^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9
*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^
7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^
7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a
*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360
*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^
7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^
5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c
- b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*
e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^
2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^
7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a
*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b
^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6
*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f
+ 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b
^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5
*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2)
+ 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9
)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^
5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-
(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b
*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2)
- 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c -
b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*
c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 384
0*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*g^2 + 8*a^2*c^8*d
^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*a^5
*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^6*c
^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a^3*
b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2*c^
4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f +
336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a*b^
3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a*b^
5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + 394
*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5*d*
g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1804
*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^
2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) +
b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*

c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) + (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*

b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*
g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615
*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 153
60*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*
a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*
a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^
3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c -
b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*
c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4
*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c
^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*
f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c -
b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b
^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2
) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)
^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*
c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*
(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a
*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/
2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c
- b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3
840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 -
1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6
)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*
c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*
c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g
^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7
*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f
^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2
+ 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*
g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4
*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*
c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c
^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*
f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^
9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)
^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g +
43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^
(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e
- 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b

^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8
*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g +
8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144
*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e
*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b
^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2
) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g
+ 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c
- b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d
g(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*
a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^
9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 +
b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*
b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*g^2 + 8*a^2*c^8*d^2 - 7
2*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*a^5*c^5*g
^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^6*c^3*f^2
- 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a^3*b^2*c^
5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2*c^4*g^2
- 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^
4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*
d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*
e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + 394*a*b^7
*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 37
4*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1804*a^2*b
^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c
^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c
^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^
2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*
c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^
2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 +
25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(
1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g
^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*
a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^
9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f
*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9
*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^
9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 4
3520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(
1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e -
128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^

2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (2744*a^7*c^3*g^3 - 225*a^4*b^6*g^3 - 216*a^4*c^6*e^3 + 3*a*b^3*c^6*d^3 + 4*a^2*b*c^7*d^3 + 1300*a^5*b*c^4*f^3 - 24*a^3*c^7*d^2*e + 2060*a^5*b^4*c*g^3 - 125*a^2*b^8*e*g^2 + 56*a^4*c^6*d^2*g - 600*a^5*c^5*e*f^2 + 175*a^3*b^7*f*g^2 + 1512*a^5*c^5*e^2*g - 3528*a^6*c^4*e*g^2 + 1400*a^6*c^4*f^2*g - 5*a^2*b^4*c^4*e^3 + 66*a^3*b^2*c^5*e^3 + 63*a^3*b^5*c^2*f^3 - 573*a^4*b^3*c^3*f^3 - 5334*a^6*b^2*c^2*g^3 + 75*a*b^9*d*g^2 + 240*a^4*c^6*d*e*f - 560*a^5*c^5*d*f*g + 6*a*b^4*c^5*d^2*e + 3*a*b^5*c^4*d*e^2 + 204*a^3*b*c^6*d*e^2 - 18*a*b^5*c^4*d^2*f + 27*a*b^7*c^2*d*f^2 + 12*a^3*b*c^6*d^2*f - 420*a^4*b*c^5*d*f^2 + 30*a*b^6*c^3*d^2*g - 845*a^2*b^7*c*d*g^2 + 924*a^4*b*c^5*e^2*f + 2044*a^5*b*c^4*d*g^2 + 1350*a^3*b^6*c*e*g^2 - 210*a^3*b^6*c*f^2*g - 1485*a^4*b^5*c*f*g^2 + 364*a^6*b*c^3*f*g^2 - 42*a^2*b^2*c^6*d^2*e - 51*a^2*b^3*c^5*d*e^2 + 81*a^2*b^3*c^5*d^2*f - 279*a^2*b^5*c^3*d*f^2 + 801*a^3*b^3*c^4*d*f^2 - 149*a^2*b^4*c^4*d^2*g + 30*a^2*b^5*c^3*e^2*f - 45*a^2*b^6*c^2*e*f^2 + 78*a^3*b^2*c^5*d^2*g - 339*a^3*b^3*c^4*e^2*f + 402*a^3*b^4*c^3*e*f^2 + 3198*a^3*b^5*c^2*d*g^2 - 762*a^4*b^2*c^4*e*f^2 - 4571*a^4*b^3*c^3*d*g^2 - 50*a^2*b^6*c^2*e^2*g + 600*a^3*b^4*c^3*e^2*g - 2002*a^4*b^2*c^4*e^2*g - 4835*a^4*b^4*c^2*e*g^2 + 6598*a^5*b^2*c^3*e*g^2 + 1927*a^4*b^4*c^2*f^2*g - 4722*a^5*b^2*c^3*f^2*g + 3061*a^5*b^3*c^2*f*g^2 - 90*a*b^8*c*d*f*g - 18*a*b^6*c^3*d*e*f + 30*a*b^7*c^2*d*e*g - 1352*a^4*b*c^5*d*e*g + 150*a^2*b^7*c*e*f*g - 2312*a^5*b*c^4*e*f*g + 246*a^2*b^4*c^4*d*e*f - 804*a^3*b^2*c^5*d*e*f - 424*a^2*b^5*c^3*d*e*g + 1578*a^3*b^3*c^4*d*e*g + 972*a^2*b^6*c^2*d*f*g - 3244*a^3*b^4*c^3*d*f*g + 3276*a^4*b^2*c^4*d*f*g - 1480*a^3*b^5*c^2*e*f*g + 4122*a^4*b^3*c^3*e*f*g)/(4*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2)

+ b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*
b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*
b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e
 - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g
 - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e +
 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1
/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^1
0*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g -
 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d
*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*
a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8
*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d
*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f -
 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*
b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*
g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^
3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*
c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*
c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a
*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e
f(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^
9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*2i +
 atan(((((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*
b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 422
4*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c
^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c
^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8
 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*((c^6*d^2*(-(4*a*c - b^2)^
9)^(1/2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b
^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3
840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f
^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5
*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2
 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30
240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2
)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 +
 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 -
215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^
2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b
^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^
12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^

8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^

3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*
e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*
b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^
8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^
2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4
*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4
*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) +
 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c -
b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*
c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) -
(x*(25*b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^
6*f^2 + b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 1
6*a*b^4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 4
81*a^2*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*
b^4*c^3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b
^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e
*f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f
 - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*
e*f - 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^
2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g
- 1132*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(
16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) -
b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4
*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c
^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*
a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 -
512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4
*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5
*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*
b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*
b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*
a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e
 + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g
+ 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e -
 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1
/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^1
0*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g -
 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d
*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*
a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8
*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d
*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f -
 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*

b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*
g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^
3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*
c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*
c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a
*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e
f(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^
9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i -
(((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^
9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*
b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g -
19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e -
48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*
c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) + (x*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/
2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2
*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^
5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 2
6880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d
^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 384
0*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^
4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(
1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767
*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040
*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*
(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^
5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3
*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*
d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^
9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*
a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*
f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c
- b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*
c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f -
1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^
2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*
c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*
e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784
*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c
^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) -
6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3
*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6
*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(
-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*

c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2
) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b
^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)
*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*
c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c
 - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e
^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*
b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*
a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3
*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6
*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^
2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*
c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*
c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7
168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*
b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*
a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2)
+ 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^
4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*
b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)
^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e -
 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*
b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5
*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g +
 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b
^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*
c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-
(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^
3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*
f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*
f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c -
 b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(
-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a
*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 -
1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25
*b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2
+ b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^
4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2
*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^
3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5
*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 1

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0*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*
a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f -
148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*
d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132
*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^
6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c -
 b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2
 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*
c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^
3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c
^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f
^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*
c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^
3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^
6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c -
b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 716
8*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^
12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*
b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) +
2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*
e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^
5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(
1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 1
65*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^
5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e
*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 2
2400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2
*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^
5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4
*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*
g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*
g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b
^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(
4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b
^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 12
80*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i)/((((20
48*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d -
192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^
7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712
*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*
b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 +
12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b
```

^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)

^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9

)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*
c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3
*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^
5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^
5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f +
258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*
g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) -
 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b
^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*
d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2
688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a
^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d
f(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g
- 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c -
 b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-
(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 11
9616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10
*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)
^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c
- b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2
*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3
*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (((2048*a^4*c^
10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b
^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 107
52*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*
c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f
 + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*
c^6 - 48*a^2*b^2*c^7)) + (x*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^
2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2
)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9
*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*
f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^
3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e
^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 +
 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^
2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^
2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)
^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^
6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c
^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*
c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*
c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f
+ 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*

f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2)
 - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a
*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^
7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f +
 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400
*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4
*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*
g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c
 - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*
(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g +
119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g +
10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*
c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c
^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a
^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 -
192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2 - 25*b^
15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 806
40*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2
 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077
*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^
5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 11692
8*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^
3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
 + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g
 + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f -
35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f +
 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(
-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b
^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*
a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^
2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 5
12*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3
*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*
c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4
*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352
*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9
)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c
 - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^

4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^
2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^
9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-
(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4
096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^
10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*g^2 + 8*a
^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 -
392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114
*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 -
718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^
4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^
4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g -
14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g +
86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e
*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^
3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*
g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2 - 25*b^15
*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2)
+ 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 80640
*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 -
288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077*a
^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^5*
b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*
c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 116928*
a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^3*
c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) +
615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g +
15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f - 35
840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f + 1
536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(
4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b^1
1*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*
c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*
b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*
(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 512
*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3*b
^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*c^
7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a
*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352*a
^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^
(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c -
b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^4*

b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*
e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) +
12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)
^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4
*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10
 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (2744*a^7*c^3*g^3 - 225
*a^4*b^6*g^3 - 216*a^4*c^6*e^3 + 3*a*b^3*c^6*d^3 + 4*a^2*b*c^7*d^3 + 1300*a
^5*b*c^4*f^3 - 24*a^3*c^7*d^2*e + 2060*a^5*b^4*c*g^3 - 125*a^2*b^8*e*g^2 +
56*a^4*c^6*d^2*g - 600*a^5*c^5*e*f^2 + 175*a^3*b^7*f*g^2 + 1512*a^5*c^5*e^2
*g - 3528*a^6*c^4*e*g^2 + 1400*a^6*c^4*f^2*g - 5*a^2*b^4*c^4*e^3 + 66*a^3*b
^2*c^5*e^3 + 63*a^3*b^5*c^2*f^3 - 573*a^4*b^3*c^3*f^3 - 5334*a^6*b^2*c^2*g^
3 + 75*a*b^9*d*g^2 + 240*a^4*c^6*d*e*f - 560*a^5*c^5*d*f*g + 6*a*b^4*c^5*d^
2*e + 3*a*b^5*c^4*d*e^2 + 204*a^3*b*c^6*d*e^2 - 18*a*b^5*c^4*d^2*f + 27*a*b
^7*c^2*d*f^2 + 12*a^3*b*c^6*d^2*f - 420*a^4*b*c^5*d*f^2 + 30*a*b^6*c^3*d^2*
g - 845*a^2*b^7*c*d*g^2 + 924*a^4*b*c^5*e^2*f + 2044*a^5*b*c^4*d*g^2 + 1350
*a^3*b^6*c*e*g^2 - 210*a^3*b^6*c*f^2*g - 1485*a^4*b^5*c*f*g^2 + 364*a^6*b*c
^3*f*g^2 - 42*a^2*b^2*c^6*d^2*e - 51*a^2*b^3*c^5*d*e^2 + 81*a^2*b^3*c^5*d^2
*f - 279*a^2*b^5*c^3*d*f^2 + 801*a^3*b^3*c^4*d*f^2 - 149*a^2*b^4*c^4*d^2*g
+ 30*a^2*b^5*c^3*e^2*f - 45*a^2*b^6*c^2*e*f^2 + 78*a^3*b^2*c^5*d^2*g - 339*
a^3*b^3*c^4*e^2*f + 402*a^3*b^4*c^3*e*f^2 + 3198*a^3*b^5*c^2*d*g^2 - 762*a^
4*b^2*c^4*e*f^2 - 4571*a^4*b^3*c^3*d*g^2 - 50*a^2*b^6*c^2*e^2*g + 600*a^3*b
^4*c^3*e^2*g - 2002*a^4*b^2*c^4*e^2*g - 4835*a^4*b^4*c^2*e*g^2 + 6598*a^5*b
^2*c^3*e*g^2 + 1927*a^4*b^4*c^2*f^2*g - 4722*a^5*b^2*c^3*f^2*g + 3061*a^5*b
^3*c^2*f*g^2 - 90*a*b^8*c*d*f*g - 18*a*b^6*c^3*d*e*f + 30*a*b^7*c^2*d*e*g -
 1352*a^4*b*c^5*d*e*g + 150*a^2*b^7*c*e*f*g - 2312*a^5*b*c^4*e*f*g + 246*a^
2*b^4*c^4*d*e*f - 804*a^3*b^2*c^5*d*e*f - 424*a^2*b^5*c^3*d*e*g + 1578*a^3*
b^3*c^4*d*e*g + 972*a^2*b^6*c^2*d*f*g - 3244*a^3*b^4*c^3*d*f*g + 3276*a^4*b
^2*c^4*d*f*g - 1480*a^3*b^5*c^2*e*f*g + 4122*a^4*b^3*c^3*e*f*g)/(4*(64*a^3*
c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))))*((c^6*d^2*(-(4*a*c - b^2)
^9)^(1/2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*
b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 +
3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*
f^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^
5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^
2 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 3
0240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^
2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2
+ 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 -
 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c
^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*
b^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b
^12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b
^8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g

```
 - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^
12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-
(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a
^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6
*d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g +
1548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a
^4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b
^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g
+ 14784*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 4
2*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^
(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39
132*a^3*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161
280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^
3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) +
44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^
9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(
-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 24
0*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12))
)^(1/2)*2i + (g*x^3)/(3*c^2)
```

3.127 $\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 471

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \frac{gx}{c^2} - \frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)+\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\right.}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)-\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\right.}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] g*x/c^2-1/2*x*(b*c*(a*f+c*d)-a*b^2*g-2*a*c*(-a*g+c*e)+(2*c^3*d-c^2*(2*a*f+b
*e)-b^3*g+b*c*(3*a*g+b*f))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan
(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^3*d-c^2*(-6*a*f+b*e)+
3*b^3*g-b*c*(13*a*g+b*f)+(b^3*c*f-4*b*c^2*(2*a*f+c*d)-3*b^4*g+4*a*c^2*(-5*a
*g+c*e)+b^2*c*(19*a*g+c*e))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*(2*c^3*d-c^2*(-6*a*f+b*e)+3*b^3*g-b*c*(13*a*g+b*f)+(-b^3*c*f
+4*b*c^2*(2*a*f+c*d)+3*b^4*g-4*a*c^2*(-5*a*g+c*e)-b^2*c*(19*a*g+c*e))/(-4*a
*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 4.20 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1682, 1690, 1180, 211}

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx =$$

$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3\right.}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3\right.}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

$$-\frac{x(x^2(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)-ab^2g+bc(af+cd)-2ac(ce-ag))}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$+\frac{gx}{c^2}$$

```
[In] Int[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (g*x)/c^2 - (x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (2*c^3*d -
c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(2*c^2*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f +
13*a*g) + (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g
) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g
) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^
2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\text{integral} = -\frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))x^2)}{2c^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{-\frac{a\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)}{c^2}+\frac{a\left(2c^3d-c^2(be-6af)+b^3g-bc(bf+5ag)\right)x^2}{c^2}+2a\left(4a-\frac{b^2}{c}\right)gx^4}{a+bx^2+cx^4}\,dx}{2a(b^2-4ac)}$$

$$= -\frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))x^2)}{2c^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \left(-\frac{2a(b^2-4ac)g}{c^2}-\frac{a(bc(cd+af)-3ab^2g-2ac(ce-5ag))-a(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag))x^2}{c^2(a+bx^2+cx^4)}\right)dx}{2a(b^2-4ac)}$$

$$= \frac{gx}{c^2} - \frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))x^2)}{2c^2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{a(bc(cd+af)-3ab^2g-2ac(ce-5ag))-a(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag))x^2}{a+bx^2+cx^4}\,dx}{2ac^2(b^2-4ac)}$$

$$
\begin{aligned}
&= \frac{gx}{c^2} \\
&- \frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&- \frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)-\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4}}}{4c^2\,(b^2-4ac)} \\
&- \frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)+\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4}}}{4c^2\,(b^2-4ac)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{gx}{c^2} \\
&- \frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&- \frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)+\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\
&- \frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)-\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b+}}\right.}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.22

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{4\sqrt{c}gx - \frac{2\sqrt{c}x(-b^3gx^2+b^2(-ag+cfx^2)+2c(a^2g+c^2dx^2-ac(e+fx^2))+bc(c(d-ex^2)+a(f+3gx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4g+b^2c\left(ce-\sqrt{b^2-4ac}f+19\right.\right.}{}}{}$$

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*g*x - (2*Sqrt[c]*x*(-(b^3*g*x^2) + b^2*(-(a*g) + c*f*x^2) + 2*c*
(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2)) + b*c*(c*(d - e*x^2) + a*(f + 3*g*x^2
))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*g + b^2*c*(c*e
 - Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e +
3*a*Sqrt[b^2 - 4*a*c]*f - 10*a^2*g) + b^3*(c*f + 3*Sqrt[b^2 - 4*a*c]*g) - b
*c*(4*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f + 13*a*Sqrt[b^2 - 4*a*c]*g))*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*g - b^2*c*(c*e + Sqrt[b^2
 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + 3*a*Sqrt[b
^2 - 4*a*c]*f + 10*a^2*g) + b^3*(-(c*f) + 3*Sqrt[b^2 - 4*a*c]*g) + b*c*(4*c
^2*d - c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f - 13*a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.58

method	result
risch	$\frac{gx}{c^2}+\frac{\frac{\left(3abgc-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x^3}{8ac-2b^2}+\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x}{8ac-2b^2}}{c^2(c\,x^4+b\,x^2+a)}+\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\left(13abgc-6a\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{gx}{c^2}-\frac{-\frac{\left(3abgc-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x^3}{2\left(4ac-b^2\right)}-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(13\sqrt{-4ac+b^2}\,abcg-6a\,c^2f\sqrt{-4ac+b^2}-3b^3\cdots\right.}{\cdots}\right.}{\cdots}$

[In] int(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] g*x/c^2+(1/2*(3*a*b*c*g-2*a*c^2*f-b^3*g+b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^2
)*x^3+1/2*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)*x)/c^2/
(c*x^4+b*x^2+a)+1/4/c^2*sum((-(13*a*b*c*g-6*a*c^2*f-3*b^3*g+b^2*c*f+b*c^2*e
-2*c^3*d)/(4*a*c-b^2)*_R^2-(10*a^2*c*g-3*a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)
/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23774 vs. 2(430) = 860.

Time = 181.70 (sec) , antiderivative size = 23774, normalized size of antiderivative = 50.48

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(gx^6+fx^4+ex^2+d)x^2}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g)*x^3 + (
b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x)/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + g*x/c^2 + 1/2*
integrate((b*c^2*d - 2*a*c^2*e + a*b*c*f - (2*c^3*d - b*c^2*e - (b^2*c - 6*
a*c^2)*f + (3*b^3 - 13*a*b*c)*g)*x^2 - (3*a*b^2 - 10*a^2*c)*g)/(c*x^4 + b*x
^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9152 vs. $2(430) = 860$.

Time = 2.04 (sec) , antiderivative size = 9152, normalized size of antiderivative = 19.43

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] g*x/c^2 - 1/2*(2*c^3*d*x^3 - b*c^2*e*x^3 + b^2*c*f*x^3 - 2*a*c^2*f*x^3 - b^
3*g*x^3 + 3*a*b*c*g*x^3 + b*c^2*d*x - 2*a*c^2*e*x + a*b*c*f*x - a*b^2*g*x +
 2*a^2*c*g*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/16*(2*(2*b^2*c^
5 - 8*a*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2
*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^5 - 2*(b^2 - 4*a*c)*c^
5)*(b^2*c^2 - 4*a*c^3)^2*d - (2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*
```

a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4*a*c^3)^2*e
- (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4*a*c^3)^
2*f + (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c
^2 - 4*a*c^3)^2*g - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^6 - 2*b^5*c^6 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^7 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^7 + 16*a*b^3*c^7 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^8 - 32*a^2*b*c^8 + 2*(b^2 - 4*a*c)*b^
3*c^6 - 8*(b^2 - 4*a*c)*a*b*c^7)*d*abs(b^2*c^2 - 4*a*c^3) + 4*(sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^6 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 -
2*a*b^4*c^6 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^7 + 16*a^2*b^2*c^7 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^8 - 32*a^3*c^8 + 2*(b^2 - 4*a*c)*a*b^2*c^6 - 8*(b^2 - 4*a*c)*a
^2*c^7)*e*abs(b^2*c^2 - 4*a*c^3) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*a*b^5*c^5 + 16*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6
+ 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 32
*a^3*b*c^7 + 2*(b^2 - 4*a*c)*a*b^3*c^5 - 8*(b^2 - 4*a*c)*a^2*b*c^6)*f*abs(b
^2*c^2 - 4*a*c^3) + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3
- 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a

^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^6 - 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*c^7 + 320*a^4*c^7 + 6*(b^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2
- 4*a*c)*a^2*b^2*c^5 + 80*(b^2 - 4*a*c)*a^3*c^6)*g*abs(b^2*c^2 - 4*a*c^3) -
4*(2*b^6*c^9 - 16*a*b^4*c^10 + 32*a^2*b^2*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c^8 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^4*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^10 - 2*(b^2 - 4*a*c)*b^4*c^9 + 8*(b^2 - 4*a*c)*a*b^2*c^10
)*d + (2*b^7*c^8 - 8*a*b^5*c^9 - 32*a^2*b^3*c^10 + 128*a^3*b*c^11 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^8 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 32*(b^
2 - 4*a*c)*a^2*b*c^10)*e + (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 25
6*a^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6
- 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7
- 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2
- 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^
9)*f - (6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*
a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*
c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 -
220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6
- 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sq

rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2
- 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c
^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9)*g)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4
*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^
3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5
*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*
c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(b^2*c^2 - 4*a*c^3)*abs(c)) - 1/16*(2*
(2*b^2*c^5 - 8*a*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^5 - 2*(b^2 -
4*a*c)*c^5)*(b^2*c^2 - 4*a*c^3)^2*d - (2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4*a*
c^3)^2*e - (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 -
4*a*c^3)^2*f + (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^
3)*(b^2*c^2 - 4*a*c^3)^2*g - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5
*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^6 + 2*b^5*c^6 + 16*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^7 - 16*a*b^3*c^7 - 4
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^8 + 32*a^2*b*c^8 - 2*(b^2 -
4*a*c)*b^3*c^6 + 8*(b^2 - 4*a*c)*a*b*c^7)*d*abs(b^2*c^2 - 4*a*c^3) + 4*(sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^6 + 2*a*b^4*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c - s

qrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 16*a^2*b^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*c^8 + 32*a^3*c^8 - 2*(b^2 - 4*a*c)*a*b^2*c^6 + 8*(b^2 -
 4*a*c)*a^2*c^7)*e*abs(b^2*c^2 - 4*a*c^3) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*a*b^5*c^5 + 16
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^6 - 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^7 + 32*a^3*b*c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2 - 4*a*c)*a^2*b*c^6
)*f*abs(b^2*c^2 - 4*a*c^3) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^6*c^3 - 34*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^5 - 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 -
 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 +
 44*(b^2 - 4*a*c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)*g*abs(b^2*c^2 - 4
*a*c^3) - 4*(2*b^6*c^9 - 16*a*b^4*c^10 + 32*a^2*b^2*c^11 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^4*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^10 - 2*(b^2 - 4*a*c)*b^4*c^9 + 8*(b^2 - 4*a*c)*a
*b^2*c^10)*d + (2*b^7*c^8 - 8*a*b^5*c^9 - 32*a^2*b^3*c^10 + 128*a^3*b*c^11
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 16*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 - 64*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 32*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8
 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*e + (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4
*c^9 - 256*a^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^7*c^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^4*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^5*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7
 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^
8 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^

```
8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8
- 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9
- 2*(b^2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a
^2*b^2*c^9)*f - (6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c
^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^
6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c
^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*
c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^
7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8
 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^
8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^
8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9
- 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*
a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9)*g)*arctan(2*sqrt(1/2)*x/sqrt((b^
3*c^2 - 4*a*b*c^3 - sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3
)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6
- 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 -
32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(b^2*c^2 - 4*a*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 36589, normalized size of antiderivative = 77.68

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^3*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(2*(4*
a*c - b^2)) + (x*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(2*
(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7*g -
16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 153
6*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g
 + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*
c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g
)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((c^5*d^2
*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2
- a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2)
+ 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b
```

^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2
- 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2
*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^
3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e
*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^
8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10
*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 38
4*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^
3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g
 - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752
*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*
c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*
a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^
4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*
f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c -
b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^
2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280
*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^
13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*
a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^
7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3
840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9
*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^
5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a
^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*
b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1
8*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g
 - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^
3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4
*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g +
128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2
*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6
*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 154
8*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^
6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 5
1*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9
)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c -
 b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*

g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2

+ 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*
b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(
-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30
240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360
*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*
a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f +
6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*
c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f
+ 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3
*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c
^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g -
1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)
^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*
f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^
2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1
/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^
2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4
*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c
^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*
c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
c^4)))((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 7
68*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c -
b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c
- b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^
7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27
*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^
3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 +
10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25
*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g
- 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*
f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c
^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2
*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*
d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 224
0*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^
4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e
*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*
a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*
c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*
f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*
c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) -
2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(
1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c

- b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*1i)/((((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*

b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^
2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^
5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2)
 - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f
*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*
(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*
c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e -
128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*
b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e
*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1
536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b
^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2)
- 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 2240
0*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*
(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a
*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2
)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11
+ a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*
a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024
*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((
c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c
^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(
1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)
^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 51
2*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c
^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2
+ 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4
*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g
^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^
6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a
^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2
*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d
*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072
*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*
c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f
- 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a
^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*
g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g -
8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*
b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a
*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)

^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (x*(9 *b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^ 3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^ 2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d* f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a* b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8* a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4 *a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b ^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^ 4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a* b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4* b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f ^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a ^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6* b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/ 2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c ^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b ^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6* d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b ^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30 720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/ 2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4 *a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b* c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2* b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6 *b^2*c^10)))^(1/2) + (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 7 68*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5 *f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4 *b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7 *c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a *b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c ^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e ^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f ^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7* b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 -

512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(

4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (8*a*c^7*d^3 + 9*b^8*d*g^2 + 6*b^2*c^6*d^3 - 63*a^3*b^5*g^3 + 216*a^4*c^4*f^3 - 3*a*b^3*c^4*e^3 - 4*a^2*b*c^5*e^3 + 8*a^2*c^6*d*e^2 + 573*a^4*b^3*c*g^3 - 1300*a^5*b*c^2*g^3 + 72*a^2*c^6*d^2*f + 216*a^3*c^5*d*f^2 - 5*b^3*c^5*d^2*e + b^4*c^4*d*e^2 + 24*a^3*c^5*e^2*f + 200*a^4*c^4*d*g^2 - 5*b^4*c^4*d^2*f + b^6*c^2*d*f^2 + 45*a^2*b^6*f*g^2 + 15*b^5*c^3*d^2*g + 600*a^5*c^3*f*g^2 + 5*a^2*b^4*c^2*f^3 - 66*a^3*b^2*c^3*f^3 - 27*a*b^7*e*g^2 - 28*a*b*c^6*d^2*e - 78*a*b^6*c*d*g^2 - 80*a^3*c^5*d*e*g + 2*b^5*c^3*d*e*f - 6*b^6*c^2*d*e*g - 240*a^4*c^4*e*f*g + 18*a*b^2*c^5*d*e^2 + 26*a*b^2*c^5*d^2*f - 12*a*b^4*c^3*d*f^2 - 53*a*b^3*c^4*d^2*g - 6*a*b^4*c^3*e^2*f - 3*a*b^5*c^2*e*f^2 - 76*a^2*b*c^5*d^2*g - 204*a^3*b*c^4*e*f^2 + 18*a*b^5*c^2*e^2*g + 279* a^2*b^5*c*e*g^2 - 12*a^3*b*c^4*e^2*g + 420*a^4*b*c^3*e*g^2 - 30*a^2*b^5*c*f^2*g - 402*a^3*b^4*c*f*g^2 - 924*a^4*b*c^3*f^2*g - 6*b^7*c*d*f*g + 2*a^2*b^2*c^4*d*f^2 + 42*a^2*b^2*c^4*e^2*f + 51*a^2*b^3*c^3*e*f^2 + 133*a^2*b^4*c^2*d*g^2 + 114*a^3*b^2*c^3*d*g^2 - 81*a^2*b^3*c^3*e^2*g - 801*a^3*b^3*c^2*e*g^2 + 339*a^3*b^3*c^2*f^2*g + 762*a^4*b^2*c^2*f*g^2 + 18*a*b^6*c*e*f*g + 6*a*b^3*c^4*d*e*f - 152*a^2*b*c^5*d*e*f - 28*a*b^4*c^3*d*e*g + 62*a*b^5*c^2*d*f*g - 536*a^3*b*c^4*d*f*g + 276*a^2*b^2*c^4*d*e*g - 42*a^2*b^3*c^3*d*f*g - 246*a^2*b^4*c^2*e*f*g + 804*a^3*b^2*c^3*e*f*g)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*2i - atan(((((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a

```
*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a
^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^
(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*
c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4
*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*
b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d
*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324
*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*
c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*
g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*
a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(
1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g +
 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2
*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) +
 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)
^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c
- b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*
c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 +
3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 -
 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9
*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^
2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2
 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*
b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6
*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 1065
6*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*
c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 30
72*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3
584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*
g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*
c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f -
 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3
*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6
*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g +
576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c
^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f
*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g +
 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2
*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b
*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2)
- 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^
8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) -
```

(x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f
^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b
^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a
^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c
^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g +
52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g
 - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*
g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^
5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-
(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 +
 a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 +
 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^
6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512
*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*
c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2
077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800
*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d
*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5
*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9
)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^
6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d
*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*
a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4
*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e
*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*
a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^
2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g
 - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9
)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a
*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g
*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a
^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24
*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 614
4*a^6*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c
^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2
*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 1
0752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e -
16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^
3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2
- c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5
*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6
*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26
880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c
^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1
504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^

9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4
*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 10
24*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g +
12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*
a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e
*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a
^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c
^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f +
7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a
^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^
5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152
*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5
*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^
3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6
*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^
(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b
^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b
^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*
b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a
^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^
2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2
) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3
*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^
2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a
^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*
c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-
(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8
*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*
c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^
10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e +
384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*
b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d
*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 107
52*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^
7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(
4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*
a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*
c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c
- b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(
-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*
c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 12
80*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (x*(9*b^8*

g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c
^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2
 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3
*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 8
0*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c
^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b
*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*
b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g
^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96
*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c
^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 -
3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^
9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c
^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*
a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a
*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) -
18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*
g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a
^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^
4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g +
 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^
2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^
6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 15
48*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a
^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) -
51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^
9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c
- b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f
g(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*
c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*
c^10)))^(1/2)*1i)/((((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768
*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f
 - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b
^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c
^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b
^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*
(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2
 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2
 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*
c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 5
12*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^
5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) -

2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 448
00*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9
*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c
^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)
^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*
a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5
*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 3
6*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b
^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7
*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 51
2*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*
c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f
*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4
*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f
g(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44
*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 -
24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6
144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 76
8*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d
^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*
c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c
^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^
2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d
^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^
3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2
*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 -
30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b
^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 153
60*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f -
6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f
 + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^
4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*
f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a
^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2
*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g
- 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^
2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^
4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*
b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^
(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c -
b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-
(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(40
96*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6
*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (x*(9*b^8*g^2 - 8*a*

c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 2
00*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c
*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114
*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*
e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 1
26*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f +
 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*
g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^
2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*
c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c
^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 2
7*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b
^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2
+ 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 2
5*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*
g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d
*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*
c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^
2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6
*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 22
40*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b
^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*
e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10
*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8
*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6
*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2
*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) +
 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^
(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*
a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1
/2) + (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*
d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^
3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 19
2*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a
^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a
^2*b^2*c^5)) + (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*
a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^
7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3
840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9

*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^
5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a
^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*
b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1
8*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g
 - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^
3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4
*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g +
128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2
*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6
*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 154
8*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^
6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 5
1*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9
)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c -
 b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*
g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c
^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c
^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7
))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*
d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768
*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840
*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2
- 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b
^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2
 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b
^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5
*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*
g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(
-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c
^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 1
28*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b
^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*
f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 15
36*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^
5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) -
 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400
*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(
-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*
b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2)
 - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)
^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 +
 a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a
^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a

^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (8*a*c^7*d^3 + 9*b^8*d*g^2 + 6*b^2*c^6*d^3 - 63*a^3*b^5*g^3 + 216*a^4*c^4*f^3 - 3*a*b^3*c^4*e^3 - 4*a^2*b*c^5*e^3 + 8*a^2*c^6*d*e^2 + 573*a^4*b^3*c*g^3 - 1300*a^5*b*c^2*g^3 + 72*a^2*c^6*d^2*f + 216*a^3*c^5*d*f^2 - 5*b^3*c^5*d^2*e + b^4*c^4*d*e^2 + 24*a^3*c^5*e^2*f + 200*a^4*c^4*d*g^2 - 5*b^4*c^4*d^2*f + b^6*c^2*d*f^2 + 45*a^2*b^6*f*g^2 + 15*b^5*c^3*d^2*g + 600*a^5*c^3*f*g^2 + 5*a^2*b^4*c^2*f^3 - 66*a^3*b^2*c^3*f^3 - 27*a*b^7*e*g^2 - 28*a*b*c^6*d^2*e - 78*a*b^6*c*d*g^2 - 80*a^3*c^5*d*e*g + 2*b^5*c^3*d*e*f - 6*b^6*c^2*d*e*g - 240*a^4*c^4*e*f*g + 18*a*b^2*c^5*d*e^2 + 26*a*b^2*c^5*d^2*f - 12*a*b^4*c^3*d*f^2 - 53*a*b^3*c^4*d^2*g - 6*a*b^4*c^3*e^2*f - 3*a*b^5*c^2*e*f^2 - 76*a^2*b*c^5*d^2*g - 204*a^3*b*c^4*e*f^2 + 18*a*b^5*c^2*e^2*g + 279*a^2*b^5*c*e*g^2 - 12*a^3*b*c^4*e^2*g + 420*a^4*b*c^3*e*g^2 - 30*a^2*b^5*c*f^2*g - 402*a^3*b^4*c*f*g^2 - 924*a^4*b*c^3*f^2*g - 6*b^7*c*d*f*g + 2*a^2*b^2*c^4*d*f^2 + 4

2*a^2*b^2*c^4*e^2*f + 51*a^2*b^3*c^3*e*f^2 + 133*a^2*b^4*c^2*d*g^2 + 114*a^3*b^2*c^3*d*g^2 - 81*a^2*b^3*c^3*e^2*g - 801*a^3*b^3*c^2*e*g^2 + 339*a^3*b^3*c^2*f^2*g + 762*a^4*b^2*c^2*f*g^2 + 18*a*b^6*c*e*f*g + 6*a*b^3*c^4*d*e*f - 152*a^2*b*c^5*d*e*f - 28*a*b^4*c^3*d*e*g + 62*a*b^5*c^2*d*f*g - 536*a^3*b*c^4*d*f*g + 276*a^2*b^2*c^4*d*e*g - 42*a^2*b^3*c^3*d*f*g - 246*a^2*b^4*c^2*e*f*g + 804*a^3*b^2*c^3*e*f*g)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*2i + (g*x)/c^2

3.128 $\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 449

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)+\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)-\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(c*(b^2*d-2*a*(-a*f+c*d)-a*b*(a*g+c*e)/c)+(b*c*(a*f+c*d)-a*b^2*g-2*a*
c*(-a*g+c*e))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*f+c*d)+a*b^2*g/c-2*a*(3*a*g+c*e)+
(b^2*c*(-a*f+c*d)-4*a*c^2*(a*f+3*c*d)-a*b^3*g+4*a*b*c*(2*a*g+c*e))/c/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1
/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*f+c*d)+a*b^
2*g/c-2*a*(3*a*g+c*e)+(-b^2*c*(-a*f+c*d)+4*a*c^2*(a*f+3*c*d)+a*b^3*g-4*a*b*
c*(2*a*g+c*e))/c/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1692, 1180, 211}

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2g}{c}+\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2g}{c}-\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x\left(x^2(-ab^2g+bc(af+cd)-2ac(ce-ag))+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d + a*f) -
a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) + (b^2*c*(c*d - a*f)
- 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*
a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a
*b^2*g)/c - 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f)
- a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d+2a(3cd+af)-\frac{ab(ce+ag)}{c}+\left(-b(cd+af)-\frac{ab^2g}{c}+2a(ce+3ag)\right)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)-\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)} + \frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)+\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)+\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)-\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.14

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}x\left(b\left(-ace-a^2g+c^2dx^2+acfx^2\right)+b^2\left(cd-agx^2\right)+2ac\left(-c\left(d+ex^2\right)+a\left(f+gx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(-ab^3g+bc\left(c\sqrt{b^2-4ac}d+4ace+a\sqrt{b^2-4ac}f+8a^2g\right)+\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b*(-(a*c*e) - a^2*g + c^2*d*x^2 + a*c*f*x^2) + b^2*(c*d - a*
g*x^2) + 2*a*c*(-(c*(d + e*x^2)) + a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*g) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e
+ a*Sqrt[b^2 - 4*a*c]*f + 8*a^2*g) + b^2*(c^2*d - a*c*f + a*Sqrt[b^2 - 4*a*
c]*g) - 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f + 3*a*Sqrt[b^2 - 4
*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*g + b*c*(c*Sqrt
[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f - 8*a^2*g) + 2*a*c*(6*c^2
*d - c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f - 3*a*Sqrt[b^2 - 4*a*c]*g) + b^2*(-(c^
2*d) + a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
a*c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{\left(2a^{2}cg-a\,b^{2}g+abcf-2a\,c^{2}e+b\,c^{2}d\right)x^{3}}{2a\left(4ac-b^{2}\right)c}+\frac{\left(a^{2}bg-2a^{2}cf+abce+2a\,c^{2}d-b^{2}cd\right)x}{2ac\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\left(\frac{\left(6a^{2}cg-a\,b^{2}g-abcf+2a\right.}{4ac-b^{2}}\right.}{}$
default	$\frac{-\frac{\left(2a^{2}cg-a\,b^{2}g+abcf-2a\,c^{2}e+b\,c^{2}d\right)x^{3}}{2a\left(4ac-b^{2}\right)c}+\frac{\left(a^{2}bg-2a^{2}cf+abce+2a\,c^{2}d-b^{2}cd\right)x}{2ac\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\frac{\left(6a^{2}cg\sqrt{-4ac+b^{2}}-a\,b^{2}g\sqrt{-4ac+b^{2}}-\sqrt{-4ac+b^{2}}\,abcf+2\sqrt{\right.}}{}}{}$

```
[In] int((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/2
*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x)/(c*x^4+b*
x^2+a)+1/4/a/c*sum(((6*a^2*c*g-a*b^2*g-a*b*c*f+2*a*c^2*e-b*c^2*d)/(4*a*c-b^
2)*_R^2-(a^2*b*g-2*a^2*c*f+a*b*c*e-6*a*c^2*d+b^2*c*d)/(4*a*c-b^2))/(2*_R^3*
c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19375 vs. 2(408) = 816.

Time = 151.26 (sec) , antiderivative size = 19375, normalized size of antiderivative = 43.15

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d + ex^2 + fx^4 + gx^6}{(a + bx^2 + cx^4)^2} \, dx = \int \frac{gx^6 + fx^4 + ex^2 + d}{(cx^4 + bx^2 + a)^2} \, dx$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*((b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x^3 - (a*b*c*e -
 2*a^2*c*f + a^2*b*g - (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*
b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) - 1/2*integrate(-(a
*b*c*e - 2*a^2*c*f + a^2*b*g + (b*c^2*d - 2*a*c^2*e + a*b*c*f + (a*b^2 - 6*
a^2*c)*g)*x^2 + (b^2*c - 6*a*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a
^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8905 vs. 2(408) = 816.

Time = 1.89 (sec) , antiderivative size = 8905, normalized size of antiderivative = 19.83

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*(b*c^2*d*x^3 - 2*a*c^2*e*x^3 + a*b*c*f*x^3 - a*b^2*g*x^3 + 2*a^2*c*g*x^
3 + b^2*c*d*x - 2*a*c^2*d*x - a*b*c*e*x + 2*a^2*c*f*x - a^2*b*g*x)/((c*x^4
+ b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c
- 4*a^2*c^2)^2*d - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*e +
(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*f + (2*a*b^4*c^2
- 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 -
2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c^2)
^2*g + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5*c^4 - 2*a*b^6*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^2*c^5 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 28*a^2*b^4*c^5 - 96*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^6 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^6 - 128*a^3*b^2*c^6 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
c^7 + 192*a^4*c^7 + 2*(b^2 - 4*a*c)*a*b^4*c^4 - 20*(b^2 - 4*a*c)*a^2*b^2*c^
5 + 48*(b^2 - 4*a*c)*a^3*c^6)*d*abs(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(
```

b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*a^2*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*a^4*b*c^6 + 2*(b^2 - 4*a*c)*a^2*b^3*c^4 - 8*(b^2 - 4*a*c)*a^3*b*c^5)*e*abs(a*b^2*c - 4*a^2*c^2) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*a^3*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 32*a^5*c^6 + 2*(b^2 - 4*a*c)*a^3*b^2*c^4 - 8*(b^2 - 4*a*c)*a^4*c^5)*f*abs(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*a^3*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 32*a^5*b*c^5 + 2*(b^2 - 4*a*c)*a^3*b^3*c^3 - 8*(b^2 - 4*a*c)*a^4*b*c^4)*g*abs(a*b^2*c - 4*a^2*c^2) + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*a^4*b^3*c^8 - 384*a^5*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3*c^7 - 96*(b^2 - 4*a*c)*a^4*b*c^8)*d + 4*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 8*(b^2 - 4*a*c)*a^4*b^2*c^7)*e - (2*a^3*b^7*c^5 - 8*a^4*b^5*c^6 - 32*a^5*b^3*c^7 + 128*a^6*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 16*sqrt(2)*sqrt(

b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 - 2*(b^2 - 4*a*c)*a^3*b^5*c^5 + 32*(b^2 - 4*a*c)*a^5*b*c^7)*f - (2*a^3*b^8*c^4 - 32*a^4*b^6*c^5 + 160*a^5*b^4*c^6 - 256*a^6*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 2*(b^2 - 4*a*c)*a^3*b^6*c^4 + 24*(b^2 - 4*a*c)*a^4*b^4*c^5 - 64*(b^2 - 4*a*c)*a^5*b^2*c^6)*g)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c - 4*a^2*b*c^2 + sqrt((a*b^3*c - 4*a^2*b*c^2)^2 - 4*(a^2*b^2*c - 4*a^3*c^2)*(a*b^2*c^2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^2*c^3)))/((a^3*b^6*c^3 - 12*a^4*b^4*c^4 - 2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 + 16*a^4*b^3*c^5 + a^3*b^4*c^5 - 64*a^6*c^6 - 32*a^5*b*c^6 - 8*a^4*b^2*c^6 + 16*a^5*c^7)*abs(a*b^2*c - 4*a^2*c^2)*abs(c)) - 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*e + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*f + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c^2)^2*g - 2*(sqrt(2

)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5
*c^4 + 2*a*b^6*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5
 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 28*a^2*b^4*c^5 - 96*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b*c^6 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 128*a^
3*b^2*c^6 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 192*a^4*c^
7 - 2*(b^2 - 4*a*c)*a*b^4*c^4 + 20*(b^2 - 4*a*c)*a^2*b^2*c^5 - 48*(b^2 - 4*
a*c)*a^3*c^6)*d*abs(a*b^2*c - 4*a^2*c^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*
c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 2*a^2*b^5*c^4
 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^5 - 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^6 + 32*a^4*b*c^6 - 2*(b^2 - 4*a*c)*a^2*b^3*c^4 + 8*(b^2 - 4*a*c)*
a^3*b*c^5)*e*abs(a*b^2*c - 4*a^2*c^2) + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4
 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 2*a^3*b^4*c^4 +
16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^2*c^5 - 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c
^6 + 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(b^2 - 4*a*c)*a^4*c^5)*f*
abs(a*b^2*c - 4*a^2*c^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*a^3*b^5*c^3 + 16*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4
 - 16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 32
*a^5*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^3 + 8*(b^2 - 4*a*c)*a^4*b*c^4)*g*abs
(a*b^2*c - 4*a^2*c^2) + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*a^4*b^3*c^8 -
 384*a^5*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^7*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^5*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^6*c^5 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^6 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^3*c^7 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3*c^7 - 96*(b^
2 - 4*a*c)*a^4*b*c^8)*d + 4*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^2*c^
8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 +

```
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2
 - 4*a*c)*a^3*b^4*c^6 + 8*(b^2 - 4*a*c)*a^4*b^2*c^7)*e - (2*a^3*b^7*c^5 - 8
*a^4*b^5*c^6 - 32*a^5*b^3*c^7 + 128*a^6*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 - 2*(b^2 - 4*a*c)*a^3*b^5*c^5 + 32*(b^2 -
 4*a*c)*a^5*b*c^7)*f - (2*a^3*b^8*c^4 - 32*a^4*b^6*c^5 + 160*a^5*b^4*c^6 -
256*a^6*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^5*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
6*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^2*c^6 - 2*(b^2 - 4*a*c)*a^3*b^6*c^4 + 24*(b^2 - 4*a*c)*a^4*b^4*c^5 - 64
*(b^2 - 4*a*c)*a^5*b^2*c^6)*g)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c - 4*a^2*b
*c^2 - sqrt((a*b^3*c - 4*a^2*b*c^2)^2 - 4*(a^2*b^2*c - 4*a^3*c^2)*(a*b^2*c^
2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^2*c^3)))/((a^3*b^6*c^3 - 12*a^4*b^4*c^4 -
 2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 + 16*a^4*b^3*c^5 + a^3*b^4*c^5 - 64*a^6*c^6
 - 32*a^5*b*c^6 - 8*a^4*b^2*c^6 + 16*a^5*c^7)*abs(a*b^2*c - 4*a^2*c^2)*abs(
c))
```

Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 32587, normalized size of antiderivative = 72.58

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(2*a*c^2*d - b^2*c*d + a^2*b*g - 2*a^2*c*f + a*b*c*e))/(2*a*c*(4*a*c -
b^2)) - (x^3*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(2*a*c*
```

(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 9

60*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9
)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g +
1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f
g(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10
*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2
c^8)))^(1/2) + (x(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*
g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2
+ 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2
*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e
 - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g +
 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2
*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c
 - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840
*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 +
 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-
(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*
a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^
2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(
1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2
g^2(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*
a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^
8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c
^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^
2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^
7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 307
2*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3
*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 268
8*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*
c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g
+ 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6
*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9
*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6
+ 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i - (((6144*a^5*c^7*d + 204
8*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d +
 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*
f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5
*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b
*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x
*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*
a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2
+ 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a
^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(
4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*

f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1
504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^
7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b
*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192
*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c
^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f -
 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c
^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g +
 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b
^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(
4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*
b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2)
 - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7
- 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*
c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*
a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a
^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) -
288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9
*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c
- b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 +
96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(
1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^
6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g
- 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d
*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b
^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f
 - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2
*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f
 + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a
^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3
*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c
- b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^
4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a
^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*
a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144
*a^8*b^2*c^8)))^(1/2) - (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 +
a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^
4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4
*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b
*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c
^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2
*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a

^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^
2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c
^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*
c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2
 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^
3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b
^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 -
 a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 307
2*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*
a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g
 + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^
5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d
*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3
*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e
*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*
a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*
c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192
*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d
e(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(
4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*
b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i)/((((6144*a^5*c^7
*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2
*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*
b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192
*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 10
24*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^
3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*
d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*
c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^
2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7
*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3
e^2(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*
b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a
*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2
*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 921
6*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 35
84*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d
*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a
^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c
^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a
^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c
^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 1
28*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2
*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g +

128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^
9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*
b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*
b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*
a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)
))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 -
9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^
2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^
(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 -
a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(
-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^
2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b
^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 +
1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*
c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6
*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 1
92*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9
*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f
 - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8
*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g
 + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4
*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(
-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^
6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/
2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c
^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^
7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^
4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 1
6*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2
+ 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e -
40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8
*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f
 - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4
*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^1
1*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768
*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2
+ 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5
*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 51
2*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4
*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c
^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g
^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*
f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^
10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e +

1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b
^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2)
- 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b
^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*
d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*
a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f
*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a
*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1
280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (((6144*a^
5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^
4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32
*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g
- 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e
- 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b
^2*c^3)) + (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b
*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a
^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c
- b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^
3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^
2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) -
a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(
-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^
7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e
- 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e
+ 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*
c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e
+ 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*
b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g +
12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*
b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*
g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^
3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f
*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 +
a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840
*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 +
192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2
c^2)))((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d
^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c
^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2
)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*
d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*
e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b
^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*

c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*
g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216
*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 358
4*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*
e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^
2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^
6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^
3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^
5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 12
8*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*
e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 1
28*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9
)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b
^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b
^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b
^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^
2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3
*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d
*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*
g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^
3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(1
6*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2
- b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5
*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^
3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2
 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2
*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*
b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c -
 b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*
c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8
*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a
^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*
d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*
a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*
a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2
*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g -
3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*
c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g
+ 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^
5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (8*a^
3*c^5*e^3 + 5*b^3*c^5*d^3 + 5*a^4*b^4*g^3 + 216*a^6*c^2*g^3 - 4*a^4*b*c^3*f
^3 + 72*a^2*c^6*d^2*e - 66*a^5*b^2*c*g^3 - 3*b^4*c^4*d^2*e + a^2*b^6*e*g^2

+ 216*a^3*c^5*d^2*g + 8*a^4*c^4*e*f^2 + b^5*c^3*d^2*f - 3*a^3*b^5*f*g^2 + 7
2*a^4*c^4*e^2*g + 216*a^5*c^3*e*g^2 + b^6*c^2*d^2*g + 24*a^5*c^3*f^2*g + 6*
a^2*b^2*c^4*e^3 - 3*a^3*b^3*c^2*f^3 - 36*a*b*c^6*d^3 + a*b^7*d*g^2 + 48*a^3
*c^5*d*e*f + 144*a^4*c^4*d*f*g + 18*a*b^2*c^5*d^2*e + 3*a*b^3*c^4*d*e^2 - 6
0*a^2*b*c^5*d*e^2 - a*b^3*c^4*d^2*f + a*b^5*c^2*d*f^2 - 60*a^2*b*c^5*d^2*f
- 28*a^3*b*c^4*d*f^2 - 10*a*b^4*c^3*d^2*g - 21*a^2*b^5*c*d*g^2 - 28*a^3*b*c
^4*e^2*f - 396*a^4*b*c^3*d*g^2 - 12*a^3*b^4*c*e*g^2 - 6*a^3*b^4*c*f^2*g + 5
1*a^4*b^3*c*f*g^2 - 204*a^5*b*c^2*f*g^2 - 9*a^2*b^3*c^3*d*f^2 - 6*a^2*b^2*c
^4*d^2*g - 5*a^2*b^3*c^3*e^2*f + a^2*b^4*c^2*e*f^2 + 18*a^3*b^2*c^3*e*f^2 +
155*a^3*b^3*c^2*d*g^2 - 5*a^2*b^4*c^2*e^2*g + 26*a^3*b^2*c^3*e^2*g + 2*a^4
*b^2*c^2*e*g^2 + 42*a^4*b^2*c^2*f^2*g + 2*a*b^6*c*d*f*g - 4*a*b^4*c^3*d*e*f
- 4*a*b^5*c^2*d*e*g - 312*a^3*b*c^4*d*e*g + 2*a^2*b^5*c*e*f*g - 152*a^4*b*
c^3*e*f*g + 52*a^2*b^2*c^4*d*e*f + 70*a^2*b^3*c^3*d*e*g - 30*a^2*b^4*c^2*d*
f*g + 100*a^3*b^2*c^3*d*f*g + 6*a^3*b^3*c^2*e*f*g)/(4*(64*a^5*c^4 - a^2*b^6
*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3))))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2
- b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5
*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^
3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2
- 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2
*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*
b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*
c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8
*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a
^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*
d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*
a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*
a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2
*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g -
3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*
c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g
+ 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^
5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*2i - at
an(((((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c
^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*
b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*
a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 10
24*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*
c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^
2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c
^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*
c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2

- 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f +

2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c ^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5 *d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a ^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4* f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2* f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7* e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b *c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 12 8*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7* c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a *c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6* c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5* b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^ 7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4 *a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8))) ^(1/2)*1i - (((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a ^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5 *f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^ 3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12* a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^1 1*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768 *a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5 *c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 51 2*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4 *a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c ^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2) ^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g ^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6* f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^ 10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b ^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b ^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*

d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*
a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f
*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a
*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1
280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*
c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 +
a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*
d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b
*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^
4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d
^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*
b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c -
b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2
+ a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3
072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g -
2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f
*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*
a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5
*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a
^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4
*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g +
6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^
3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 1
92*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3
*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32
*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^
6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^2*c^6*d
^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3
*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c
^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c
^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^
4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*
c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a
^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c
^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(
4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*
c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2
*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^
2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^
9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*
b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) +
a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c
^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*

a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 35
84*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*
d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*
a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*
f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*
a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*
c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g +
960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^
9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g +
 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*
f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^1
0*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^
2*c^8)))^(1/2)*1i)/((((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d
+ 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*
c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5
*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*
a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6
*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g
^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*
c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504
*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*
e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*
d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a
^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b
^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*
a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g -
2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c
^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 9
60*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9
)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 1
28*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*
b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g
 - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b
^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f
*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)
^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8
*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(102
4*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a
^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b
^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7
68*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^
2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b
^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 -

512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 614

4*a^8*b^2*c^8)))^(1/2) + (((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g

- 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^
8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*
g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^
9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*
c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*
(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2
 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^
2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d
*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g
 - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f
+ 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c
^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))
*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*
a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2
+ 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a
^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(
4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*
f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1
504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^
7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b
*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192
*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c
^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f +
 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c
^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g +
 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b
^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(
4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*
b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2)
 + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7
- 6144*a^8*b^2*c^8)))^(1/2) + (8*a^3*c^5*e^3 + 5*b^3*c^5*d^3 + 5*a^4*b^4*g^
3 + 216*a^6*c^2*g^3 - 4*a^4*b*c^3*f^3 + 72*a^2*c^6*d^2*e - 66*a^5*b^2*c*g^3
 - 3*b^4*c^4*d^2*e + a^2*b^6*e*g^2 + 216*a^3*c^5*d^2*g + 8*a^4*c^4*e*f^2 +
b^5*c^3*d^2*f - 3*a^3*b^5*f*g^2 + 72*a^4*c^4*e^2*g + 216*a^5*c^3*e*g^2 + b^
6*c^2*d^2*g + 24*a^5*c^3*f^2*g + 6*a^2*b^2*c^4*e^3 - 3*a^3*b^3*c^2*f^3 - 36
*a*b*c^6*d^3 + a*b^7*d*g^2 + 48*a^3*c^5*d*e*f + 144*a^4*c^4*d*f*g + 18*a*b^
2*c^5*d^2*e + 3*a*b^3*c^4*d*e^2 - 60*a^2*b*c^5*d*e^2 - a*b^3*c^4*d^2*f + a*
b^5*c^2*d*f^2 - 60*a^2*b*c^5*d^2*f - 28*a^3*b*c^4*d*f^2 - 10*a*b^4*c^3*d^2*
g - 21*a^2*b^5*c*d*g^2 - 28*a^3*b*c^4*e^2*f - 396*a^4*b*c^3*d*g^2 - 12*a^3*
b^4*c*e*g^2 - 6*a^3*b^4*c*f^2*g + 51*a^4*b^3*c*f*g^2 - 204*a^5*b*c^2*f*g^2
- 9*a^2*b^3*c^3*d*f^2 - 6*a^2*b^2*c^4*d^2*g - 5*a^2*b^3*c^3*e^2*f + a^2*b^4
*c^2*e*f^2 + 18*a^3*b^2*c^3*e*f^2 + 155*a^3*b^3*c^2*d*g^2 - 5*a^2*b^4*c^2*e

^2*g + 26*a^3*b^2*c^3*e^2*g + 2*a^4*b^2*c^2*e*g^2 + 42*a^4*b^2*c^2*f^2*g + 2*a*b^6*c*d*f*g - 4*a*b^4*c^3*d*e*f - 4*a*b^5*c^2*d*e*g - 312*a^3*b*c^4*d*e*g + 2*a^2*b^5*c*e*f*g - 152*a^4*b*c^3*e*f*g + 52*a^2*b^2*c^4*d*e*f + 70*a^2*b^3*c^3*d*e*g - 30*a^2*b^4*c^2*d*f*g + 100*a^3*b^2*c^3*d*f*g + 6*a^3*b^3*c^2*e*f*g)/(4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3))))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*2i

3.129 $\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 460

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx = -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^2cd-2ac(5cd-af)-ab(ce+ag)+\frac{3b^3cd-4abc(4cd+af)-ab^2(ce-ag)+4a^2c(3ce+ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^2cd-2ac(5cd-af)-ab(ce+ag)-\frac{3b^3cd-4abc(4cd+af)-ab^2(ce-ag)+4a^2c(3ce+ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -d/a^2/x-1/2*x*(a*(b^3*d/a-b*(b*e+3*c*d)+a*(b*f+2*c*e)-2*a^2*g)+(b^2*c*d-2*
a*c*(-a*f+c*d)-a*b*(a*g+c*e))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arc
tan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^2*c*d-2*a*c*(-a*f+
5*c*d)-a*b*(a*g+c*e)+(3*b^3*c*d-4*a*b*c*(a*f+4*c*d)-a*b^2*(-a*g+c*e)+4*a^2*
c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*(3*b^2*c*d-2*a*c*(-a*f+5*c*d)-a*b*(a*g+c*e)+(-3*b^3*c*d+4*a*b*c*(a*f+
4*c*d)+a*b^2*(-a*g+c*e)-4*a^2*c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1683, 1678, 1180, 211}

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{2\sqrt{2}a^2\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{2\sqrt{2}a^2\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2(-ab(ag+ce)-2ac(cd-af)+b^2cd)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{d}{a^2x}$$

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f) - 2*a^2
*g) + (b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2))/(2*a^2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e +
 a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3
*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*c*d - 4
*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\text{integral} = -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-2\left(b^2-4ac\right)d+\left(\frac{b^3d}{a}-b(5cd+be)+a(6ce-bf)+2a^2g\right)x^2+\left(\frac{b^2cd}{a}-c(2cd+be)+a(2cf-bg)\right)x^4}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \left(\frac{2\left(-b^2+4ac\right)d}{ax^2}+\frac{3b^3d-ab^2e-ab(13cd+af)+2a^2(3ce+ag)+\left(3b^2cd-2ac(5cd-af)-ab(ce+ag)\right)x^2}{a\left(a+bx^2+cx^4\right)}\right)dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{3b^3d-ab^2e-ab(13cd+af)+2a^2(3ce+ag)+\left(3b^2cd-2ac(5cd-af)-ab(ce+ag)\right)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2g\right) + (b^2cd - 2ac(cd - af) - ab(ce + ag))\,x^2\right)}{2a^2\,(b^2 - 4ac)\,(a + bx^2 + cx^4)} - \frac{\left(3b^2cd - 2ac(5cd - af) - ab(ce + ag) - \frac{3b^3cd - 4abc(4cd + af) - ab^2(ce - ag) + 4a^2c(3ce + ag)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} +}}{4a^2\,(b^2 - 4ac)} - \frac{\left(3b^2cd - 2ac(5cd - af) - ab(ce + ag) + \frac{3b^3cd - 4abc(4cd + af) - ab^2(ce - ag) + 4a^2c(3ce + ag)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} +}}{4a^2\,(b^2 - 4ac)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2g\right) + (b^2cd - 2ac(cd - af) - ab(ce + ag))\,x^2\right)}{2a^2\,(b^2 - 4ac)\,(a + bx^2 + cx^4)} - \frac{\left(3b^2cd - 2ac(5cd - af) - ab(ce + ag) + \frac{3b^3cd - 4abc(4cd + af) - ab^2(ce - ag) + 4a^2c(3ce + ag)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b - \sqrt{b^2}}}\right.}{2\sqrt{2}a^2\sqrt{c}\,(b^2 - 4ac)\,\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(3b^2cd - 2ac(5cd - af) - ab(ce + ag) - \frac{3b^3cd - 4abc(4cd + af) - ab^2(ce - ag) + 4a^2c(3ce + ag)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b + \sqrt{b^2}}}\right.}{2\sqrt{2}a^2\sqrt{c}\,(b^2 - 4ac)\,\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.15

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2\,(a + bx^2 + cx^4)^2}\,dx =$$

$$-\frac{\frac{4d}{x} - \frac{2x(-b^3d + b^2(ae - cdx^2) + ab(3cd - af + cex^2 + agx^2) + 2a(a^2g + c^2dx^2 - ac(e + fx^2)))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\left(3b^3cd + b^2\left(3c\sqrt{b^2 - 4ac}d - ace + a^2g\right) + 2ac\right.}{}}{}$$

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((4*d)/x - (2*x*(-(b^3*d) + b^2*(a*e - c*d*x^2) + a*b*(3*c*d - a*f + c
*e*x^2 + a*g*x^2) + 2*a*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2))))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]
*d - a*c*e + a^2*g) + 2*a*c*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqrt[b^
2 - 4*a*c]*f + 2*a^2*g) - a*b*(16*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f +
 a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*(-3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d + a*c*e - a^2*g) - 2*a*c*(5*c*S
qrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 2*a^2*g) + a*b*(16*c
^2*d - c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f - a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d}{a^2 x}+\frac{-\frac{\left(a^2 b g-2 a^2 c f+a b c e+2 a c^2 d-b^2 c d\right) x^3}{2\left(4 a c-b^2\right)}-\frac{\left(2 a^3 g-a^2 b f-2 a^2 c e+a b^2 e+3 a b c d-b^3 d\right) x}{2\left(4 a c-b^2\right)}}{c x^4+b x^2+a}+ 2c\left(\frac{\left(-a^2 b g\sqrt{-4ac+b^2}+2a^2 c f\sqrt{-4ac+b^2}-abce\right.}{}\right.$
risch	Expression too large to display

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d/a^2/x+1/a^2*((-1/2*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/(4*a*c-
b^2)*x^3-1/2*(2*a^3*g-a^2*b*f-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(4*a*c-b^2
)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(-a^2*b*g*(-4*a*c+b^2)^(1/2)+2*a^
2*c*f*(-4*a*c+b^2)^(1/2)-a*b*c*e*(-4*a*c+b^2)^(1/2)-10*(-4*a*c+b^2)^(1/2)*a
*c^2*d+3*(-4*a*c+b^2)^(1/2)*b^2*c*d-4*a^3*g*c-a^2*b^2*g+4*a^2*b*c*f-12*a^2*
c^2*e+a*b^2*c*e+16*a*b*c^2*d-3*b^3*c*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))-1/8*(-a^2*b*g*(-4*a*c+b^2)^(1/2)+2*a^2*c*f*(-4*a*c+b^2)^(1/2)-a*b*c*e*
(-4*a*c+b^2)^(1/2)-10*(-4*a*c+b^2)^(1/2)*a*c^2*d+3*(-4*a*c+b^2)^(1/2)*b^2*c
*d+4*a^3*g*c+a^2*b^2*g-4*a^2*b*c*f+12*a^2*c^2*e-a*b^2*c*e-16*a*b*c^2*d+3*b^
3*c*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23991 vs. 2(418) = 836.

Time = 144.80 (sec) , antiderivative size = 23991, normalized size of antiderivative = 52.15

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{gx^6+fx^4+ex^2+d}{\left(cx^4+bx^2+a\right)^2x^2}\,dx$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*((a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*
f - 2*a^3*g + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 -
4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3
*b^2 - 4*a^4*c)*x) + 1/2*integrate((a^2*b*f - 2*a^3*g + (a*b*c*e - 2*a^2*c*
f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 -
 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9167 vs. $2(418) = 836$.

Time = 1.75 (sec) , antiderivative size = 9167, normalized size of antiderivative = 19.93

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c*f*x^4 - a^2*b*
g*x^4 + 3*b^3*d*x^2 - 11*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*b*
f*x^2 - 2*a^3*g*x^2 + 2*a*b^2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2*b^
2 - 4*a^3*c)) + 1/16*((6*b^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 22*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
```

qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*c^4 - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2 - 4*a*c)*a*
c^4)*(a^2*b^2 - 4*a^3*c)^2*d - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a^2*b^2 -
 4*a^3*c)^2*e + 2*(2*a^2*b^2*c^3 - 8*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a^2*c^3)*(a^2*b^2 - 4*a^3*c)^2*f
 - (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*g - 2*(3*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 37*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^6*c^2 - 6*a^2*b^7*c^2 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^3*c^3 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 3*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 74*a^3*b^5*c^3 - 208*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 104*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^3*c^4 - 304*a^4*b^3*c^4 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^5 + 416*a^5*b*c^5 + 6*(b^2 - 4*a*c)*a^2*b^5*c^2 - 50*(b^2 - 4*a*c)*a^
3*b^3*c^3 + 104*(b^2 - 4*a*c)*a^4*b*c^4)*d*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 14*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^5*c^2 - 2*a^3*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 28*a^4*b^4*c^3 - 96*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b*c^4 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4
- 128*a^5*b^2*c^4 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 19
2*a^6*c^5 + 2*(b^2 - 4*a*c)*a^3*b^4*c^2 - 20*(b^2 - 4*a*c)*a^4*b^2*c^3 + 48
*(b^2 - 4*a*c)*a^5*c^4)*e*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
5*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 - 2*a^4*b
^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 + 8*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^3*c^3 + 16*a^5*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b*c^4 - 32*a^6*b*c^4 + 2*(b^2 - 4*a*c)*a^4*b^3*c^2 - 8*(b^2 - 4
*a*c)*a^5*b*c^3)*f*abs(a^2*b^2 - 4*a^3*c) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2

- 4*a*c)*c)*a^5*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c
^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - 2*a^5*b^4*c^2
+ 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^3 + 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^2*c^3 + 16*a^6*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6
*c^4 - 32*a^7*c^4 + 2*(b^2 - 4*a*c)*a^5*b^2*c^2 - 8*(b^2 - 4*a*c)*a^6*c^3)*
g*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^3 - 80*a^5*b^6*c^4 + 352*a^6*b^4*c^
5 - 512*a^7*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b^8*c + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^6*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^7*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^6*b^4*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^5*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^6*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^7*b^2*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*b^3*c^4 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^5*b^4*c^4 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*b^2*c^5 - 6*(b^2 - 4*a*c)*a^4*b^6*c^3 + 56*(b^2 - 4*a*c)*a^5*b^4*
c^4 - 128*(b^2 - 4*a*c)*a^6*b^2*c^5)*d - (2*a^5*b^7*c^3 - 40*a^6*b^5*c^4 +
224*a^7*b^3*c^5 - 384*a^8*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^5*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^6*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^5*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^7*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^6*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^5*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^8*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^7*b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^6*b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^7*b*c^5 - 2*(b^2 - 4*a*c)*a^5*b^5*c^3 + 32*(b^2 - 4*a*c)*a^6*b
^3*c^4 - 96*(b^2 - 4*a*c)*a^7*b*c^5)*e - 4*(2*a^6*b^6*c^3 - 16*a^7*b^4*c^4
+ 32*a^8*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^7*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
6*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
8*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7
*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^
4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2
c^4 - 2(b^2 - 4*a*c)*a^6*b^4*c^3 + 8*(b^2 - 4*a*c)*a^7*b^2*c^4)*f + (2*a^
6*b^7*c^2 - 8*a^7*b^5*c^3 - 32*a^8*b^3*c^4 + 128*a^9*b*c^5 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*s

qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^4 - 2*(b^2 - 4*a*c)*a^6*b^5*c^2 + 32*(
b^2 - 4*a*c)*a^8*b*c^4)*g)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c +
 sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^
2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6*c - 12*a^6*b^4*c^2 - 2*a^5*b^5*c^2
 + 48*a^7*b^2*c^3 + 16*a^6*b^3*c^3 + a^5*b^4*c^3 - 64*a^8*c^4 - 32*a^7*b*c^
4 - 8*a^6*b^2*c^4 + 16*a^7*c^5)*abs(a^2*b^2 - 4*a^3*c)*abs(c)) + 1/16*((6*b
^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^2*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^
4 - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2 - 4*a*c)*a*c^4)*(a^2*b^2 - 4*a^3*c)^2
*d - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(2*a^2*b
^2*c^3 - 8*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 -
 2*(b^2 - 4*a*c)*a^2*c^3)*(a^2*b^2 - 4*a^3*c)^2*f - (2*a^2*b^3*c^2 - 8*a^3*
b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a
*c)*a^2*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*g - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^7*c - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c
^2 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 6*a^2*b^7*c^2
+ 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 50*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^5*c^3 - 74*a^3*b^5*c^3 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^4 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^
4 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 304*a^4*b^3*c^
4 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 416*a^5*b*c^5 -
6*(b^2 - 4*a*c)*a^2*b^5*c^2 + 50*(b^2 - 4*a*c)*a^3*b^3*c^3 - 104*(b^2 - 4*a
*c)*a^4*b*c^4)*d*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^6*c - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^
2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*a^3*b^6*c^2 +
 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 20*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c

)*c)*a^3*b^4*c^3 - 28*a^4*b^4*c^3 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 128*a^5*b^2*c^4 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 192*a^6*c^5 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 20*(b^2 - 4*a*c)*a^4*b^2*c^3 - 48*(b^2 - 4*a*c)*a^5*c^4)*e*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 2*a^4*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 16*a^5*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 32*a^6*b*c^4 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 8*(b^2 - 4*a*c)*a^5*b*c^3)*f*abs(a^2*b^2 - 4*a^3*c) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 + 2*a^5*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 16*a^6*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^4 + 32*a^7*c^4 - 2*(b^2 - 4*a*c)*a^5*b^2*c^2 + 8*(b^2 - 4*a*c)*a^6*c^3)*g*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^3 - 80*a^5*b^6*c^4 + 352*a^6*b^4*c^5 - 512*a^7*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 - 6*(b^2 - 4*a*c)*a^4*b^6*c^3 + 56*(b^2 - 4*a*c)*a^5*b^4*c^4 - 128*(b^2 - 4*a*c)*a^6*b^2*c^5)*d - (2*a^5*b^7*c^3 - 40*a^6*b^5*c^4 + 224*a^7*b^3*c^5 - 384*a^8*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 - 2*(b^2 - 4*a*c)*a^5*b^5*c^3 + 32*(b^2 - 4*a*c)*a^6*b^3*c^4 - 96*(b^2 - 4*a*c)*a^7*b*c^5)*e - 4*(2*a^6*b^6*c^3 - 16*a^7*b^4*c^4 + 32*a^8*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c + 8*sqrt(2)*sq

```
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^2 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^3 - 8*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 2*(b^2 - 4*a*c)*a^6
*b^4*c^3 + 8*(b^2 - 4*a*c)*a^7*b^2*c^4)*f + (2*a^6*b^7*c^2 - 8*a^7*b^5*c^3
- 32*a^8*b^3*c^4 + 128*a^9*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^6*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^7*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^6*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^8*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^6*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^9*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^8*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^8*b*c^4 - 2*(b^2 - 4*a*c)*a^6*b^5*c^2 + 32*(b^2 - 4*a*c)*a^8*b*c^4)*g)
*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a^3*b*c
)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^
2)))/((a^5*b^6*c - 12*a^6*b^4*c^2 - 2*a^5*b^5*c^2 + 48*a^7*b^2*c^3 + 16*a^6
*b^3*c^3 + a^5*b^4*c^3 - 64*a^8*c^4 - 32*a^7*b*c^4 - 8*a^6*b^2*c^4 + 16*a^7
*c^5)*abs(a^2*b^2 - 4*a^3*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 12.15 (sec) , antiderivative size = 40860, normalized size of antiderivative = 88.83

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2
) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^
2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^
2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c
^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*
d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^
4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a
^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g
 - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*
d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c
^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*
b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*
c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f
```

- 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(x*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 131072*a^16*c^7*g - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g - 2560*a^12*b^8*c^3*g + 20480*a^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 1638

40*a^15*b^2*c^6*g + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*1i + (((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c

- b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(393216*a^15*c^8*e + 131072*a^16*c^7*g + x*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*

c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c
*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^1
0*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b
^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^
3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 157286
4*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9
*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c
^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 10
2400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^
10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5
*c^5*f + 81920*a^14*b^3*c^6*f - 128*a^11*b^10*c^2*g + 2560*a^12*b^8*c^3*g -
 20480*a^13*b^6*c^4*g + 81920*a^14*b^4*c^5*g - 163840*a^15*b^2*c^6*g - 8519
68*a^14*b*c^8*d - 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13
*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144
*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*
a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^
8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b
^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*
c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2
*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2
 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496
*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*
c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^
6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*
f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f
+ 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a
^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^
12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b
^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c
^3*f*g - 3072*a^12*b^5*c^4*f*g))*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g
^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11
*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^
9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*
b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^
5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2
7*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b
^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 -
512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*
c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^
11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^
4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 1
2*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^
6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*
d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10
*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^

2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g +
128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a
^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c
^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c -
b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(
4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c
*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) -
6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c
- 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 -
6144*a^10*b^2*c^6)))^(1/2)*1i)/((((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*
g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^1
1*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b
^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9
*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c
^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) +
27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*
b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 -
512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7
*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b
^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a
^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) +
12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b
^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2
*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 1
0*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c
^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g
+ 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*
a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*
c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c
- b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-
(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*
c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2)
- 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c
- 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5
- 6144*a^10*b^2*c^6)))^(1/2)*(393216*a^15*c^8*e + 131072*a^16*c^7*g + x*((2
13*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13
*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*
c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*
f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 1
0656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a
^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*
e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2
*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^
8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a

^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 128*a^11*b^10*c^2*g + 2560*a^12*b^8*c^3*g - 20480*a^13*b^6*c^4*g + 81920*a^14*b^4*c^5*g - 163840*a^15*b^2*c^6*g - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9

*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^
6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*
a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e
^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96
*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d
*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^
6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b
*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^
2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^
4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*
f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c -
b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3
*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 1
0752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6
*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3
*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2
*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) +
18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^
(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*
a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1
/2) - (((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2
) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^
2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^
2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c
^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*
d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^
4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a
^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g
 - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*
d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c
^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*
b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*
c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f
- 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d
*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 107
52*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b
^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f
*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c
e^2(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18
*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1
/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^

7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2
)*(x*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2
+ 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2
+ a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3
*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^
2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*
b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7
*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g -
3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*
f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5
*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^
10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^
5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f -
1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)
^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g
- 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752
*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5
*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g
+ 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a
^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c -
b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*
b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*
(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b
^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)
- 131072*a^16*c^7*g - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^1
1*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^
6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 1
7920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 55705
6*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^
7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g
- 2560*a^12*b^8*c^3*g + 20480*a^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 1638
40*a^15*b^2*c^6*g + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a
^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 +
16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a
^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*
a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^
6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*
c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^
6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2
+ 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568
*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*

b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*
d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d
*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f +
 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a
^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^1
1*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b
^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c
^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((213*a*b^11*c^2*d
^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*
a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c
 - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c -
b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^
4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4
*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*
b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*
b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 102
4*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g -
 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4
*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b
^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c
^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f +
512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8
*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f
+ 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536
*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3
*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*
a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(
1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b
^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-
(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*
c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2) - 128000*a^10*c^9*d^3 +
 1024*a^13*c^6*f^3 - 4608*a^11*b*c^7*e^3 - 24*a^11*b^7*c*g^3 - 46080*a^11*c
^8*d*e^2 - 512*a^14*b*c^4*g^3 + 76800*a^11*c^8*d^2*f - 15360*a^12*c^7*d*f^2
 + 9216*a^12*c^7*e^2*f - 5120*a^13*c^6*d*g^2 + 1024*a^14*c^5*f*g^2 - 504*a^
6*b^8*c^5*d^3 + 8112*a^7*b^6*c^6*d^3 - 48704*a^8*b^4*c^7*d^3 + 129280*a^9*b
^2*c^8*d^3 + 40*a^8*b^7*c^4*e^3 - 608*a^9*b^5*c^5*e^3 + 2944*a^10*b^3*c^6*e
^3 + 48*a^10*b^6*c^3*f^3 - 320*a^11*b^4*c^4*f^3 + 256*a^12*b^2*c^5*f^3 + 16
0*a^12*b^5*c^2*g^3 - 128*a^13*b^3*c^3*g^3 - 30720*a^12*c^7*d*e*g + 6144*a^1
3*c^6*e*f*g + 84480*a^10*b*c^8*d^2*e - 24*a^8*b^10*c*d*g^2 + 2560*a^11*b*c^
7*d^2*g - 7680*a^12*b*c^6*e*f^2 + 8*a^9*b^9*c*e*g^2 - 7680*a^12*b*c^6*e^2*g
 - 3584*a^13*b*c^5*e*g^2 + 8*a^10*b^8*c*f*g^2 - 3584*a^13*b*c^5*f^2*g + 360
*a^6*b^9*c^4*d^2*e - 5736*a^7*b^7*c^5*d^2*e - 240*a^7*b^8*c^4*d*e^2 + 33888
*a^8*b^5*c^6*d^2*e + 3792*a^8*b^6*c^5*d*e^2 - 87936*a^9*b^3*c^7*d^2*e - 216

96*a^9*b^4*c^6*d*e^2 + 52992*a^10*b^2*c^7*d*e^2 - 216*a^6*b^10*c^3*d^2*f + 3744*a^7*b^8*c^4*d^2*f - 25200*a^8*b^6*c^5*d^2*f - 72*a^8*b^8*c^3*d*f^2 + 81984*a^9*b^4*c^6*d^2*f + 1296*a^9*b^6*c^4*d*f^2 - 128256*a^10*b^2*c^7*d^2*f - 7872*a^10*b^4*c^5*d*f^2 + 19200*a^11*b^2*c^6*d*f^2 + 72*a^6*b^11*c^2*d^2*g - 1128*a^7*b^9*c^3*d^2*g + 6488*a^8*b^7*c^4*d^2*g - 24*a^8*b^8*c^3*e^2*f - 16032*a^9*b^5*c^5*d^2*g + 336*a^9*b^6*c^4*e^2*f + 24*a^9*b^7*c^3*e*f^2 + 368*a^9*b^8*c^2*d*g^2 + 13440*a^10*b^3*c^6*d^2*g - 960*a^10*b^4*c^5*e^2*f - 672*a^10*b^5*c^4*e*f^2 - 1840*a^10*b^6*c^3*d*g^2 - 2304*a^11*b^2*c^6*e^2*f + 4224*a^11*b^3*c^5*e*f^2 + 2880*a^11*b^4*c^4*d*g^2 + 1792*a^12*b^2*c^5*d*g^2 + 8*a^8*b^9*c^2*e^2*g - 72*a^9*b^7*c^3*e^2*g - 288*a^10*b^5*c^4*e^2*g - 136*a^10*b^7*c^2*e*g^2 + 3712*a^11*b^3*c^5*e^2*g + 480*a^11*b^5*c^3*e*g^2 + 640*a^12*b^3*c^4*e*g^2 - 40*a^10*b^7*c^2*f^2*g + 96*a^11*b^5*c^3*f^2*g + 80*a^11*b^6*c^2*f*g^2 + 1152*a^12*b^3*c^4*f^2*g - 960*a^12*b^4*c^3*f*g^2 + 1792*a^13*b^2*c^4*f*g^2 + 21504*a^11*b*c^7*d*e*f + 17408*a^12*b*c^6*d*f*g + 144*a^7*b^9*c^3*d*e*f - 2256*a^8*b^7*c^4*d*e*f + 12480*a^9*b^5*c^5*d*e*f - 28416*a^10*b^3*c^6*d*e*f - 48*a^7*b^10*c^2*d*e*g + 592*a^8*b^8*c^3*d*e*g - 1632*a^9*b^6*c^4*d*e*g - 4992*a^10*b^4*c^5*d*e*g + 28160*a^11*b^2*c^6*d*e*g + 96*a^8*b^9*c^2*d*f*g - 1616*a^9*b^7*c^3*d*f*g + 9408*a^10*b^5*c^4*d*f*g - 22272*a^11*b^3*c^5*d*f*g - 32*a^9*b^8*c^2*e*f*g + 672*a^10*b^6*c^3*e*f*g - 3456*a^11*b^4*c^4*e*f*g + 3584*a^12*b^2*c^5*e*f*g))*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*2i - (d/a - (x^2*(3*b^3* d - 2*a^3*g - a*b^2*e + a^2*b*f + 2*a^2*c*e - 11*a*b*c*d))/(2*a^2*(4*a*c - b^2)) + (x^4*(10*a*c^2*d - 3*b^2*c*d + a^2*b*g - 2*a^2*c*f + a*b*c*e))/(2*a^2*(4*a*c - b^2)))/(a*x + b*x^3 + c*x^5) + atan(((((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(x*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*

c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g
- 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2
d^2(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*
a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1
/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c -
b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^
7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 384
0*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b
^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 9
83040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 131072*a^16*c^7*g - 393216*a^1
5*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d -
256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 6
4*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^1
2*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*
c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f -
81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g - 2560*a^12*b^8*c^3*g + 20480*a
^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 163840*a^15*b^2*c^6*g + 851968*a^14*
b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2
+ 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^1
2*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*
c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c
^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e
^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2
+ 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 5
12*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920
*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*
c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e
+ 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e -
253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168
*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^
9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*
c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c
^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e
*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g -
3072*a^12*b^5*c^4*f*g))*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 -
9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 +
3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2
+ 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^
2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 +
44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^
9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e
^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*
b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e
+ 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f
+ 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*

e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^
8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*
e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 57
6*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2
*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f -
2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6
*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c
^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g +
384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b
^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6
*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^
10*b^2*c^6)))^(1/2)*1i + (((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2
- 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2
+ 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2
+ 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g
^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 +
44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b
^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*
e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7
*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e
+ 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*
f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c
*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b
^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d
*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 5
76*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^
2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f -
2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^
6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*
c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g
+ 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9
)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-
(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*
b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^
6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a
^10*b^2*c^6)))^(1/2)*(393216*a^15*c^8*e + 131072*a^16*c^7*g + x*((a^5*g^2*(
-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 -
26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-
(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4
*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3
*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d

^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 15
04*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 5
12*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*
f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*
c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e
g(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 154
8*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^
6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4
*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*
a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c
^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g
 + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*
a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*
e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*
a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2
*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(
32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a
^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*
c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*
a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c
^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d -
 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*
a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^1
3*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^
3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f -
128*a^11*b^10*c^2*g + 2560*a^12*b^8*c^3*g - 20480*a^13*b^6*c^4*g + 81920*a^
14*b^4*c^5*g - 163840*a^15*b^2*c^6*g - 851968*a^14*b*c^8*d - 65536*a^15*b*c
^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8
192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^1
0*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^
4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4
*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*
e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2
- 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 10
24*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^
14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c
^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e -
 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e
- 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144
*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b
^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5
*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*
c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*

e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))
*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^
11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*
b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^
4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2
 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 -
25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*
c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5
*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 307
2*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f -
18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g
 + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c
^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*
e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344
*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^
(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 1
92*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7
*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3
*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6
*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b
*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) -
44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)
^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*
c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*1i)/
((((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b
^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9
*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a
^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^
2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 -
 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7
*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^
5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 30
72*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f -
 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*
g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*
c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d
*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 134
4*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g -
192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^
7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^
3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g +

```
6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*
b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) -
 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8
*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(39
3216*a^15*c^8*e + 131072*a^16*c^7*g + x*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2)
- a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a
^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) -
 a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 7
68*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4
*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1
/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 384
0*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3
*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 153
60*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6
*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f
 + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1
/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064
*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b
^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d
*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4
*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^
4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f
- 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^
6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(
4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*
d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*
a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^
(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*
b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^
4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2
- 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^
14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3
*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d +
 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*
a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^1
4*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4
*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 128*a^11*b^10*c^2*g + 25
60*a^12*b^8*c^3*g - 20480*a^13*b^6*c^4*g + 81920*a^14*b^4*c^5*g - 163840*a^
15*b^2*c^6*g - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f) + x*(204800*a^12*c
^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^
10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^
8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*
b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5
```

*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f
^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2
 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 40
96*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12
*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6
*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e +
 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f +
1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 4096
0*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b
^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5
*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^
3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*
g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((a^5*g^2*(-(4*a*c - b^
2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b
*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2
)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2
 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c
- b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c
^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96
*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c
^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9
*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^
3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c
- b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^
3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*
e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a
^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*
d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 729
6*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*
b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*
e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2
*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9
)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*
c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^1
1*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 +
 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2) - (((a^5*g^2*(-(4*a*c - b^2)
^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c
^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^
9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)
^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 -
 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4
*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a

^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3
*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c
^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*
b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*
d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e
- 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6
*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*
g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*
a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^
2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*
g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c
^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) +
6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^
(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c
- b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*
c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3
840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(x*((a^5*g^2*(-(4*a*c - b^2)^9
)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7
*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)
^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(
1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 3
0240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^
2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e
^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6
*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g
^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5
*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^
10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^
2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*
e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e -
98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b
^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g
+ 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^
6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*
c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g
- 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2
d^2(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*
a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1
/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c -
b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^
7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 384
0*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b
^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 9
83040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 131072*a^16*c^7*g - 393216*a^1

5*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g - 2560*a^12*b^8*c^3*g + 20480*a^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 163840*a^15*b^2*c^6*g + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c -

b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b
^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6
*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^
10*b^2*c^6)))^(1/2) - 128000*a^10*c^9*d^3 + 1024*a^13*c^6*f^3 - 4608*a^11*b
*c^7*e^3 - 24*a^11*b^7*c*g^3 - 46080*a^11*c^8*d*e^2 - 512*a^14*b*c^4*g^3 +
76800*a^11*c^8*d^2*f - 15360*a^12*c^7*d*f^2 + 9216*a^12*c^7*e^2*f - 5120*a^
13*c^6*d*g^2 + 1024*a^14*c^5*f*g^2 - 504*a^6*b^8*c^5*d^3 + 8112*a^7*b^6*c^6
*d^3 - 48704*a^8*b^4*c^7*d^3 + 129280*a^9*b^2*c^8*d^3 + 40*a^8*b^7*c^4*e^3
- 608*a^9*b^5*c^5*e^3 + 2944*a^10*b^3*c^6*e^3 + 48*a^10*b^6*c^3*f^3 - 320*a
^11*b^4*c^4*f^3 + 256*a^12*b^2*c^5*f^3 + 160*a^12*b^5*c^2*g^3 - 128*a^13*b^
3*c^3*g^3 - 30720*a^12*c^7*d*e*g + 6144*a^13*c^6*e*f*g + 84480*a^10*b*c^8*d
^2*e - 24*a^8*b^10*c*d*g^2 + 2560*a^11*b*c^7*d^2*g - 7680*a^12*b*c^6*e*f^2
+ 8*a^9*b^9*c*e*g^2 - 7680*a^12*b*c^6*e^2*g - 3584*a^13*b*c^5*e*g^2 + 8*a^1
0*b^8*c*f*g^2 - 3584*a^13*b*c^5*f^2*g + 360*a^6*b^9*c^4*d^2*e - 5736*a^7*b^
7*c^5*d^2*e - 240*a^7*b^8*c^4*d*e^2 + 33888*a^8*b^5*c^6*d^2*e + 3792*a^8*b^
6*c^5*d*e^2 - 87936*a^9*b^3*c^7*d^2*e - 21696*a^9*b^4*c^6*d*e^2 + 52992*a^1
0*b^2*c^7*d*e^2 - 216*a^6*b^10*c^3*d^2*f + 3744*a^7*b^8*c^4*d^2*f - 25200*a
^8*b^6*c^5*d^2*f - 72*a^8*b^8*c^3*d*f^2 + 81984*a^9*b^4*c^6*d^2*f + 1296*a^
9*b^6*c^4*d*f^2 - 128256*a^10*b^2*c^7*d^2*f - 7872*a^10*b^4*c^5*d*f^2 + 192
00*a^11*b^2*c^6*d*f^2 + 72*a^6*b^11*c^2*d^2*g - 1128*a^7*b^9*c^3*d^2*g + 64
88*a^8*b^7*c^4*d^2*g - 24*a^8*b^8*c^3*e^2*f - 16032*a^9*b^5*c^5*d^2*g + 336
*a^9*b^6*c^4*e^2*f + 24*a^9*b^7*c^3*e*f^2 + 368*a^9*b^8*c^2*d*g^2 + 13440*a
^10*b^3*c^6*d^2*g - 960*a^10*b^4*c^5*e^2*f - 672*a^10*b^5*c^4*e*f^2 - 1840*
a^10*b^6*c^3*d*g^2 - 2304*a^11*b^2*c^6*e^2*f + 4224*a^11*b^3*c^5*e*f^2 + 28
80*a^11*b^4*c^4*d*g^2 + 1792*a^12*b^2*c^5*d*g^2 + 8*a^8*b^9*c^2*e^2*g - 72*
a^9*b^7*c^3*e^2*g - 288*a^10*b^5*c^4*e^2*g - 136*a^10*b^7*c^2*e*g^2 + 3712*
a^11*b^3*c^5*e^2*g + 480*a^11*b^5*c^3*e*g^2 + 640*a^12*b^3*c^4*e*g^2 - 40*a
^10*b^7*c^2*f^2*g + 96*a^11*b^5*c^3*f^2*g + 80*a^11*b^6*c^2*f*g^2 + 1152*a^
12*b^3*c^4*f^2*g - 960*a^12*b^4*c^3*f*g^2 + 1792*a^13*b^2*c^4*f*g^2 + 21504
*a^11*b*c^7*d*e*f + 17408*a^12*b*c^6*d*f*g + 144*a^7*b^9*c^3*d*e*f - 2256*a
^8*b^7*c^4*d*e*f + 12480*a^9*b^5*c^5*d*e*f - 28416*a^10*b^3*c^6*d*e*f - 48*
a^7*b^10*c^2*d*e*g + 592*a^8*b^8*c^3*d*e*g - 1632*a^9*b^6*c^4*d*e*g - 4992*
a^10*b^4*c^5*d*e*g + 28160*a^11*b^2*c^6*d*e*g + 96*a^8*b^9*c^2*d*f*g - 1616
*a^9*b^7*c^3*d*f*g + 9408*a^10*b^5*c^4*d*f*g - 22272*a^11*b^3*c^5*d*f*g - 3
2*a^9*b^8*c^2*e*f*g + 672*a^10*b^6*c^3*e*f*g - 3456*a^11*b^4*c^4*e*f*g + 35
84*a^12*b^2*c^5*e*f*g))*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 -
9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 +
3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 +
 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2
 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 4
4800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9
*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^
2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b
^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e +

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 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f
+ 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e
*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8
*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e
 + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576
*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2
240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*
b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^
2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g +
384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^
(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b
^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4
*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^
2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*
b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^1
0*b^2*c^6)))^(1/2)*2i
```

3.130 $\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 542

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx = -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}$$
$$+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c(b^3d-ab^2e-ab(3cd-af)+2a^2(ce-ag\cdots\right.}{2a^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{c}\left(5b^3d-3ab^2e-ab(19cd-af)+2a^2(5ce-ag)+\frac{5b^4d-3ab^3e+4a^2c(7cd-3af)-ab^2(29cd-af)+4a^2b(4ce+ag)}{\sqrt{b^2-4ac}}\right)\,\mathrm{arc}\cdots}{2\sqrt{2}a^3(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(5b^3d-3ab^2e-ab(19cd-af)+2a^2(5ce-ag)-\frac{5b^4d-3ab^3e+4a^2c(7cd-3af)-ab^2(29cd-af)+4a^2b(4ce+ag)}{\sqrt{b^2-4ac}}\right)\,\mathrm{arc}\cdots}{2\sqrt{2}a^3(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*(a^2*(b^4*d/a^2+2*c^2*d+3*b*c*e-b^2
*(b*e+4*c*d)/a+b^2*f-a*(b*g+2*c*f))+c*(b^3*d-a*b^2*e-a*b*(-a*f+3*c*d)+2*a^2
*(-a*g+c*e))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f+19*
c*d)+2*a^2*(-a*g+5*c*e)+(5*b^4*d-3*a*b^3*e+4*a^2*c*(-3*a*f+7*c*d)-a*b^2*(-a
*f+29*c*d)+4*a^2*b*(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f+19*c*d)+2*a^2*(-a*g+5*c
*e)+(-5*b^4*d+3*a*b^3*e-4*a^2*c*(-3*a*f+7*c*d)+a*b^2*(-a*f+29*c*d)-4*a^2*b*
(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2)
```

Rubi [A] (verified)

Time = 4.72 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1683, 1678, 1180, 211}

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}$$
$$+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-a\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2(2a^2(ce-ag)-ab^2e-ab(3cd-af)+\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/3*d/(a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d +
 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - a*(2*c*f + b*g)) + c*(b^3*d - a*
b^2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e - a*g))*x^2))/(2*a^3*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f) +
2*a^2*(5*c*e - a*g) + (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^
2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a
*f) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f)
- a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*
c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

integral

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c(b^3d-ab^2e-ab(3cd-af)+2a^2(ce-ag))\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-2(b^2-4ac)d+\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}-\frac{\left(b^4d-ab^3e+6a^2c(cd-af)-ab^2(6cd-af)+a^2b(5ce+ag)\right)x^4}{a^2}-c\left(\frac{b^3d}{a^2}+2ce-\frac{b(3cd+be)}{a}+bf-2ag\right)x^6}{x^4(a+bx^2+cx^4)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c(b^3d-ab^2e-ab(3cd-af)+2a^2(ce\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \left(\frac{2(-b^2+4ac)d}{ax^4}+\frac{2(-b^2+4ac)(-2bd+ae)}{a^2x^2}+\frac{-5b^4d+3ab^3e-2a^2c(7cd-3af)+ab^2(24cd-af)-a^2b(13ce+ag)-c(5b^3d-3ab^2e-}{a^2(a+bx^2+cx^4)}\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x} + \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c(b^3d-ab^2e-ab(3cd-af)+2a^2(\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-5b^4d+3ab^3e-2a^2c(7cd-3af)+ab^2(24cd-af)-a^2b(13ce+ag)-c\left(5b^3d-3ab^2e-ab(19cd-af)+2a^2(5ce-ag)\right)x^2}{a+bx^2+cx^4}\,dx}{2a^3\left(b^2-4ac\right)}$$

$$= -\frac{d}{3a^2x^3} + \frac{2bd - ae}{a^3x} + \frac{x\left(a^2\left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - a(2cf + bg)\right) + c(b^3d - ab^2e - ab(3cd - af) + 2a^2\right.}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(c\left(5b^3d - 3ab^2e - ab(19cd - af) + 2a^2(5ce - ag) - \frac{5b^4d - 3ab^3e + 4a^2c(7cd - 3af) - ab^2(29cd - af) + 4a^2b(4ce-}{\sqrt{b^2-4ac}}\right.\right.}{4a^3(b^2 - 4ac)} + \frac{\left(c\left(5b^3d - 3ab^2e - ab(19cd - af) + 2a^2(5ce - ag) + \frac{5b^4d - 3ab^3e + 4a^2c(7cd - 3af) - ab^2(29cd - af) + 4a^2b(4ce-}{\sqrt{b^2-4ac}}\right.\right.}{4a^3(b^2 - 4ac)}$$

$$= -\frac{d}{3a^2x^3} + \frac{2bd - ae}{a^3x} + \frac{x\left(a^2\left(\frac{b^4d}{a^2} + 2c^2d + 3bce - \frac{b^2(4cd+be)}{a} + b^2f - a(2cf + bg)\right) + c(b^3d - ab^2e - ab(3cd - af) + 2a^2\right.}{2a^3(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}\left(5b^3d - 3ab^2e - ab(19cd - af) + 2a^2(5ce - ag) + \frac{5b^4d - 3ab^3e + 4a^2c(7cd - 3af) - ab^2(29cd - af) + 4a^2b(4ce}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}a^3(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c}\left(5b^3d - 3ab^2e - ab(19cd - af) + 2a^2(5ce - ag) - \frac{5b^4d - 3ab^3e + 4a^2c(7cd - 3af) - ab^2(29cd - af) + 4a^2b(4ce}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}a^3(b^2 - 4ac)\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.13

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^4(a + bx^2 + cx^4)^2}\,dx$$

$$= \frac{-\frac{4ad}{x^3} + \frac{24bd-12ae}{x} + \frac{6x\left(b^4d + b^3(-ae+cdx^2) + ab^2(af - c(4d+ex^2)) + ab(-a^2g - 3c^2dx^2 + ac(3e+fx^2)) + 2a^2c(c(d+ex^2) - a(f+gx^2))\right)}{(b^2-4ac)(a+bx^2+cx^4)} + \cdots}{}$$

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
+ a*b^2*(a*f - c*(4*d + e*x^2)) + a*b*(-(a^2*g) - 3*c^2*d*x^2 + a*c*(3*e +
f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3
*a*e) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-19*c*Sqrt[b^2
 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) - 2*a^2*(-14*c^2*
d - 5*c*Sqrt[b^2 - 4*a*c]*e + 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*
c]*d + 3*a*e) + a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(19*c*S
```

```
qrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) + 2*a^2*(1
4*c^2*d - 5*c*Sqrt[b^2 - 4*a*c]*e - 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.16

method	result
default	$-\frac{d}{3a^2x^3}-\frac{ae-2bd}{a^3x}+\frac{\frac{c\left(2a^3g-a^2bf-2a^2ce+a\,b^2e+3abcd-b^3d\right)x^3}{8ac-2b^2}+\frac{\left(a^3bg+2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+a\,b^3e+4a\,b^2cd-d\,b^4\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{(2}{}\right.}{}$
risch	Expression too large to display

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d/a^2/x^3-(a*e-2*b*d)/a^3/x+1/a^3*((1/2*c*(2*a^3*g-a^2*b*f-2*a^2*c*e+a
*b^2*e+3*a*b*c*d-b^3*d)/(4*a*c-b^2)*x^3+1/2*(a^3*b*g+2*a^3*c*f-a^2*b^2*f-3*
a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^
2+a)+2/(4*a*c-b^2)*c*(1/8*(2*a^3*g*(-4*a*c+b^2)^(1/2)-a^2*b*f*(-4*a*c+b^2)^
(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*a*b*c*d
*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)+4*a^3*b*g-12*a^3*c*f+a^2*b^2
*f+16*a^2*b*c*e+28*a^2*c^2*d-3*a*b^3*e-29*a*b^2*c*d+5*d*b^4)/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))-1/8*(2*a^3*g*(-4*a*c+b^2)^(1/2)-a^2*b*f*(-4*a*c+b^2
)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*a*b*c
*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)-4*a^3*b*g+12*a^3*c*f-a^2*b
^2*f-16*a^2*b*c*e-28*a^2*c^2*d+3*a*b^3*e+29*a*b^2*c*d-5*d*b^4)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33432 vs. $2(498) = 996$.

Time = 297.37 (sec) , antiderivative size = 33432, normalized size of antiderivative = 61.68

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{gx^6+fx^4+ex^2+d}{\left(cx^4+bx^2+a\right)^2x^4}\,dx$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/6*(3*(a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*
a^2*c^2)*e)*x^6 - (3*a^3*b*g - (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d + 3*(3*
a*b^3 - 11*a^2*b*c)*e - 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*
b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*
c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) -
 1/2*integrate(-(a^3*b*g + (a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*
d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*d -
 (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 + b*x^2 + a), x)/
(a^3*b^2 - 4*a^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10411 vs. $2(498) = 996$.

Time = 1.80 (sec) , antiderivative size = 10411, normalized size of antiderivative = 19.21

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 - a*b^2*c*e*x^3 + 2*a^2*c^2*e*x^3 + a^2*
b*c*f*x^3 - 2*a^3*c*g*x^3 + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x - a*b^3
*e*x + 3*a^2*b*c*e*x + a^2*b^2*f*x - 2*a^3*c*f*x - a^3*b*g*x)/((a^3*b^2 - 4
```

*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b
*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2
+ 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d - (6*a*b^4*c^2 - 44*a^2
*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^
2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(
b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e
+ (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - 2*(2*a^3*
b^2*c^2 - 8*a^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 2*(b
^2 - 4*a*c)*a^3*c^2)*(a^3*b^2 - 4*a^4*c)^2*g + 2*(5*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c
+ 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3
*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4
*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4
*a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c
^4)*d*abs(a^3*b^2 - 4*a^4*c) - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 6*a^4*b^7*c + 152*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c
^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3

- 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 416*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c^2 + 104*(b^2 - 4*a*c)*a^6*b*c^3)*e*abs(a^3*b^2 - 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 2*a^5*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 28*a^6*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 192*a^8*c^4 + 2*(b^2 - 4*a*c)*a^5*b^4*c - 20*(b^2 - 4*a*c)*a^6*b^2*c^2 + 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*a^6*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 16*a^7*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 32*a^8*b*c^3 + 2*(b^2 - 4*a*c)*a^6*b^3*c - 8*(b^2 - 4*a*c)*a^7*b*c^2)*g*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^

2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*c)*
a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e + (2*a^8*b^7*c^2 - 40*a^9*b^
5*c^3 + 224*a^10*b^3*c^4 - 384*a^11*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^8*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^10*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^8*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^11*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^9*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(b^2 - 4*a*
c)*a^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f + 4*(2*a^9*b^6*c^2 - 16*a^1
0*b^4*c^3 + 32*a^11*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^9*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^10*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^9*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^11*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^10*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^9*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^10*b^2*c^3 - 2*(b^2 - 4*a*c)*a^9*b^4*c^2 + 8*(b^2 - 4*a*c)*a^10*b^2*c^3)
*g)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c + sqrt((a^3*b^3 - 4*a^4*
b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4
*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^
3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c
^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152
*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^
3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d - (6*a*b^4*c^2 -
44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3
 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c

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)^2*e + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - 2*(
2*a^3*b^2*c^2 - 8*a^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2
- 2*(b^2 - 4*a*c)*a^3*c^2)*(a^3*b^2 - 4*a^4*c)^2*g - 2*(5*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*a^3*
b^8*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 572*a
^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 88*(b
^2 - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*a*c)
*a^6*c^4)*d*abs(a^3*b^2 - 4*a^4*c) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c + 6*a^4*b^7*c + 152*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*b^5*c^2 - 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7
*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2 - 4*a
*c)*a^4*b^5*c + 50*(b^2 - 4*a*c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*b*c^3)
*e*abs(a^3*b^2 - 4*a^4*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*
b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c + 2*a^5*b^6*c + 64*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 - 28*
a^6*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^6*b^2*c^3 + 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^7*c^4 - 192*a^8*c^4 - 2*(b^2 - 4*a*c)*a^5*b^4*c + 20*(b^2
- 4*a*c)*a^6*b^2*c^2 - 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^4*c) -
 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5 - 8*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^6*b^4*c + 2*a^6*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*
```

```
c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 - 16*a^7*b^3*c^2 - 4*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 + 32*a^8*b*c^3 - 2*(b^2 - 4*a*c)*a^6*b^3
*c + 8*(b^2 - 4*a*c)*a^7*b*c^2)*g*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2
- 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2
 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^
8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^
3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4
*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e + (2*a^8*b^7*c^2 - 40*
a^9*b^5*c^3 + 224*a^10*b^3*c^4 - 384*a^11*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^10*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^11*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(b^2
- 4*a*c)*a^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f + 4*(2*a^9*b^6*c^2 -
16*a^10*b^4*c^3 + 32*a^11*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
```

```
rt(b^2 - 4*a*c)*c)*a^9*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^10*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^9*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^11*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^10*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^9*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^10*b^2*c^3 - 2*(b^2 - 4*a*c)*a^9*b^4*c^2 + 8*(b^2 - 4*a*c)*a^10*b^
2*c^3)*g)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c - sqrt((a^3*b^3 -
4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c -
 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*
a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16
*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) + 1/3*(6*b*d*x^2 - 3*a*e*x^2 - a*d
)/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 12.57 (sec) , antiderivative size = 51386, normalized size of antiderivative = 94.81

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan((((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2
) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a
^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2
- 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^
4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116
928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a
^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 -
 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*
e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 +
 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^
13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^
12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10
*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*
b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g
+ 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*
(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b
^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 1
19616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 1
0*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b
^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g
```

*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4
*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2
*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g -
 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a
^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^
3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1
536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*
c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) +
184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b
^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4
096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a
^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c
^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1
/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640
*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^
2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*
c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 1
16928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9
*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(
1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2
 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^
2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2
 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*
b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*
b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^
10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^
2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*
g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*
g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6
*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2
*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e -
 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e +
 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5
*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d
g(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a
^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c
^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g
 - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6
*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*
c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g -
 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*
b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
+ 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c -
 b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 +
 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840

*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b
^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 9
83040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^1
3*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16
*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^1
3*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5
*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f -
1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 32768
0*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b
^9*c^3*g + 10240*a^17*b^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6
*g - 851968*a^19*b*c^8*e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 4
01408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14
*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b
^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*
a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a
^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 45875
2*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^1
5*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*
b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^
2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d
*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g -
480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e +
530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*
d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d
*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^
8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*
g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f
- 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g
- 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g -
7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 3
2*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*
a^17*b^3*c^6*f*g))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c -
b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/
2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^
5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 7
68*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c
^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c
^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b
^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) +
25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b
^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^
2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*
g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f
+ 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2

) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4
*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g +
2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2
) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a
*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8
*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2
*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f -
14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10
*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1
/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548
*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b
^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c
^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 134
4*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4
*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)
^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + ((-(25*b^15*d^2 +
9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*
b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*
b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 -
9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e +
6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 2
19744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e
^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2
+ a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 -
96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*
d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g
- 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*
g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^
11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f +
98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2
) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*
c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2)
- 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e +
201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*
b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/
2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2
)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3
*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f

+ 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b
^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4
*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 5
1*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9
)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b
^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b
^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12
*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(25*b^15*d^2
 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^
6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^
3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2
- 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e
 + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 -
 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c -
 b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2
*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^
2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2
 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^1
3*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e
*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*
f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*
b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f
+ 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1
/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^
2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e
 + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*
c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^
6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(
1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b
^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c
^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*
f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c -
 b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c
^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g -
 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)
^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(
4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3
*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8
*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^
12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^1
1*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 15
72864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^

14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^
17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^1
1*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^
6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f -
17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 5570
56*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^16*b^9*c^3*g - 10240*a^17*b
^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3*c^6*g + 851968*a^19*b*c^8*
e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 7
3728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^
12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^
13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*
a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 14336
0*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16
*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a
^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16
*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17
*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d
*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e +
11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*
e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3
*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^
6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c
^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e
*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d
*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*
e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g
+ 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 10
24*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*(-(2
5*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^1
1*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^
2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*
b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*
a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7
*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a
^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*
b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*
c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b
^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 +
10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*
a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072
*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e
- 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b
^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c -
b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g +

246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*

c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*
f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 1
86*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2
 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(3932
16*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*
b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(
4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^
8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c
 - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^
2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2
 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3
*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^
3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*
c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c
^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 +
 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d
*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g -
 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4
*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*
b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 15
36*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4
*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*
d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d
*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 270
6*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a
^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-
(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5
*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c
^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*
g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^
6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^
2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) -
 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c
^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(10
48576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*
c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 3
20*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*
a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*
a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^
9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*

c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b^9*c^3*g + 10240*a^17*b^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6*g - 851968*a^19*b*c^8*e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(

1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f
 - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*
b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2)
+ 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064
*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*
b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2
) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*
a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c
e^2(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 4
4*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^
9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240
*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))
^(1/2) - ((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(
1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 8064
0*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f
^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b
*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 +
116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 +
9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^
(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^
2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^
2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a
*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3
*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b
^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a
^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d
*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f
g(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^
6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^
2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e
- 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e
+ 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^
5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*
d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*
a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*
c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*
g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f -
6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5
*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g
- 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4
*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
 + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c
- b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12

+ 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 384
0*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^2
0*c^8*f + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)
^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80
640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c
*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10
*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2
+ 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2
+ 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9
)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*
e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4
*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*
f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615
*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a
^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5
*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724
*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c
*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b
*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*
a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*
d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*
e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*
e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*
a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^
3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 4
2*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^
8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*
d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f
- 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b
^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*
g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a
^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/
2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^1
2 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3
840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^1
5*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5
+ 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*
a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a
^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*
b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*
c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f
+ 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 32
7680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^1
6*b^9*c^3*g - 10240*a^17*b^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3*

c^6*g + 851968*a^19*b*c^8*e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2
- 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b
^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^1
2*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 18718
72*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 3011
2*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 45
8752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*
a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^
15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18
*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^
9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g
 - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*
e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c
^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^
5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2
*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4
*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*
e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*
d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*
g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g
+ 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 327
68*a^17*b^3*c^6*f*g))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c
 - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^
(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27
*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
- 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^
9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^
3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*
c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^
6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2
) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^
7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3
*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12
*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e
*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(
1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*
a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g
 + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(
1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 16
5*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*
b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*
b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*
f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f +
 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)
^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1

548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - 128000*a^15*c^9*e^3 + 1024*a^18*c^6*g^3 + 476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f^3 - 250880*a^14*c^10*d^2*e + 50176*a^15*c^9*d^2*g - 46080*a^16*c^8*e*f^2 + 76800*a^16*c^8*e^2*g - 15360*a^17*c^7*e*g^2 + 9216*a^17*c^7*f^2*g + 1800*a^9*b^9*c^6*d^3 - 29080*a^10*b^7*c^7*d^3 + 176032*a^11*b^5*c^8*d^3 - 473216*a^12*b^3*c^9*d^3 - 504*a^11*b^8*c^5*e^3 + 8112*a^12*b^6*c^6*e^3 - 48704*a^13*b^4*c^7*e^3 + 129280*a^14*b^2*c^8*e^3 + 40*a^13*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^3 + 2944*a^15*b^3*c^6*f^3 + 48*a^15*b^6*c^3*g^3 - 320*a^16*b^4*c^4*g^3 + 256*a^17*b^2*c^5*g^3 + 215040*a^15*c^9*d*e*f - 43008*a^16*c^8*d*f*g + 442880*a^14*b*c^9*d*e^2 - 433664*a^14*b*c^9*d^2*f + 109056*a^15*b*c^8*d*f^2 + 84480*a^15*b*c^8*e^2*f + 43520*a^16*b*c^7*d*g^2 - 7680*a^17*b*c^6*f*g^2 - 1400*a^9*b^10*c^5*d^2*e + 21680*a^10*b^8*c^6*d^2*e + 1680*a^10*b^9*c^5*d*e^2 - 121648*a^11*b^6*c^7*d^2*e - 27176*a^11*b^7*c^6*d*e^2 + 275264*a^12*b^4*c^8*d^2*e + 164448*a^12*b^5*c^7*d*e^2 - 121088*a^13*b^2*c^9*d^2*e - 441216*a^13*b^3*c^8*d*e^2 + 1000*a^9*b^11*c^4*d^2*f - 17800*a^10*b^9*c^5*d^2*f + 124280*a^11*b^7*c^6*d^2*f + 400*a^11*b^9*c^4*d*f^2 - 422944*a^12*b^5*c^7*d^2*f - 6600*a^12*b^7*c^5*d*f^2 + 694912*a^13*b^3*c^8*d^2*f + 40416*a^13*b^5*c^6*d*f^2 - 108928*a^14*b^3*c^7*d*f^2 - 600*a^9*b^12*c^3*d^2*g + 10960*a^10*b^10*c^4*d^2*g - 78904*a^11*b^8*c^5*d^2*g + 360*a^11*b^9*c^4*e^2*f + 278096*a^12*b^6*c^6*d^2*g - 5736*a^12*b^7*c^5*e^2*f - 240*a^12*b^8*c^4*e*f^2 + 120*a^12*b^9*c^3*d*g^2 - 472000*a^13*b^4*c^7*d^2*g + 33888*a^13*b^5*c^6*e^2*f + 3792*a^13*b^6*c^5*e*f^2 - 2216*a^13*b^7*c^4*d*g^2 + 284416*a^14*b^2*c^8*d^2*g - 87936*a^14*b^3*c^7*e^2*f - 21696*a^14*b^4*c^6*e*f^2 + 14688*a^14*b^5*c^5*d*g^2 + 52992*a^15*b^2*c^7*e*f^2 - 41856*a^15*b^3*c^6*d*g^2 - 216*a^11*b^10*c^3*e^2*g + 3744*a^12*b^8*c^4*e^2*g - 25200*a^13*b^6*c^5*e^2*g - 72*a^13*b^8*c^3*e*g^2 + 81984*a^14*b^4*c^6*e^2*g + 1296*a^14*b^6*c^4*e*g^2 - 128256*a^15*b^2*c^7*e^2*g - 7872*a^15*b^4*c^5*e*g^2 + 19200*a^16*b^2*c^6*e*g^2 - 24*a^13*b^8*c^3*f^2*g + 336*a^14*b^6*c^4*f^2*g + 24*a^14*b^7*c^3*f*g^2 - 960*a^15*b^4*c^5*f^2*g - 672*a^15*b^5*c^4*f*g^2 - 2304*a^16*b^2*c^6*f^2*g + 4224*a^16*b^3*c^5*f*g^2 - 306176*a^15*b*c^8*d*e*g + 21504*a^16*b*c^7*e*f*g - 1200*a^10*b^10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f - 130656*a^12*b^6*c^6*d*e*f + 394368*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8*d*e*f + 720*a^10*b^11*c^3*d*e*g - 12816*a^11*b^9*c^4*d*e*g + 89264*a^12*b^7*c^5*d*e*g - 302400*a^13*b^5*c^6*d*e*g + 493824*a^14*b^3*c^7*d*e*g - 240*a^11*b^10*c^3*d*f*g + 3872*a^12*b^8*c^4*d*f*g - 22368*a^13*b^6*c^5*d*f*g + 51840*a^14*b^4*c^6*d*f*g - 25088*a^15*b^2*c^7*d*f*g + 144*a^12*b^9*c^3*e*f*g - 2256*a^13*b^7*c^4 *e*f*g + 12480*a^14*b^5*c^5*e*f*g - 28416*a^15*b^3*c^6*e*f*g))*(-(25*b^15*d ^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213* a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^ 2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d *e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c ^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5* e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2) ^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f ^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b ^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11 *e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^ 5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^ 3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e* f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^ (1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2* b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d *e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4* a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352* a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9) ^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6 *c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4* e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3 *c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^ 2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a ^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a ^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144* a^12*b^2*c^5)))^(1/2)*2i - (d/(3*a) + (x^2*(3*a*e - 5*b*d))/(3*a^2) + (x^4* (15*b^4*d + 14*a^2*c^2*d + 3*a^2*b^2*f - 9*a*b^3*e - 3*a^3*b*g - 6*a^3*c*f - 62*a*b^2*c*d + 33*a^2*b*c*e))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*b^3*d - 2 *a^3*g - 3*a*b^2*e + a^2*b*f + 10*a^2*c*e - 19*a*b*c*d))/(2*a^3*(4*a*c - b^ 2)))/(a*x^3 + b*x^5 + c*x^7) + atan(((((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4 *a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8 *b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*

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c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3
*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c
 - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^
2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 -
512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*
e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g +
15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b
*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 153
6*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*
a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2
)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d
*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*
e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706
*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^
7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*
b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^
3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g
 - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6
*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2
*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a
*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e
*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) -
186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^
2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393
216*a^20*c^8*f - 917504*a^19*c^9*d + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2
) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(
4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^
8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c
 - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^
2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2
 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3
*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^
3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*
c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c
^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 -
 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d
*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g +
 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4
*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*
b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 15
36*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4
*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^
```

2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b^9*c^3*g + 10240*a^17*b^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6*g - 851968*a^19*b*c^8*e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g

+ 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9
216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*((25*b^6*d^2*(-(4*a*c - b^2
)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^
6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 -
26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*
(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^
11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^
5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a
^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^
2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*
a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*
c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*
a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*
c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5
*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 4
3520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c
*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c
*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4
*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*
b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*
b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1
/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f
+ 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b
^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*
a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b
^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2)
+ 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a
^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*
e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44
*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9
)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*
a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^
(1/2)*1i + (((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^1
5*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 806
40*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*
f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*
b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 -
116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 +
9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e
^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*
c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f

^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*
a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^
3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*
b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*
a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*
d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*
f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a
^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d
^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e
 + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e
 + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a
^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3
*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42
*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8
*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d
*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f -
 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^
5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g
 + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^
4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c
 - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12
 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 38
40*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^
20*c^8*f + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^
15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80
640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c
*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10
*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2
- 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2
+ 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9
)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*
e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4
*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*
f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615
*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a
^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5
*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724
*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c
*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b
*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*
a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*
e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*
e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*

a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^16*b^9*c^3*g - 10240*a^17*b^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3*c^6*g + 851968*a^19*b*c^8*e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(

1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*
a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) +
 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9
*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3
*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c
 - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6
*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
 + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7
*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*
g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*
d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*
f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1
/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a
^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g
+ 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1
/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165
*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b
^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b
^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f
 + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f +
10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^
(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 15
48*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7
*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2
*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1
344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b
^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/
2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^
2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*
e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^
6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/((((25*b^6*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6
*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3
*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 -
 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e
- 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 +
219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c -
b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*
e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2
 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)
^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2
+ 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13
*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*
g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f

*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b
^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f -
 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/
2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2
*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e
- 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c
 - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6
*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1
/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^
3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f
 - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^
4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g -
51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^
9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4
*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*
b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*
b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^1
2*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*((25*b^6*d^2*
(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^
6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^
3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2
- 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e
 - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 +
 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c -
 b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2
*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^
2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2
 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^1
3*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e
*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*
f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*
b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f
- 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1
/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^
2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e
 - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*
c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^
6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(
1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b
^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c

^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*
f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c -
 b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c
^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g -
 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)
^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(
4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3
*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8
*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^
12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^1
1*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 15
72864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^
14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^
17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^1
1*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^
6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f +
17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 5570
56*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b^9*c^3*g + 10240*a^17*b
^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6*g - 851968*a^19*b*c^8*
e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 7
3728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^
12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^
13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*
a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 14336
0*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16
*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a
^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16
*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17
*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d
*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e +
 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*
e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3
*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^
6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c
^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e
*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d
*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*
e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g
+ 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 10
24*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*((25
*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11
*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2
 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b
*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a
*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*

c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - (((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c -

b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(
1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 3
9132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 16
1280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b
^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*
c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c
- b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^
2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f
- 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 307
20*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c
^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g +
128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b
^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^
3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 12
80*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^
19*c^9*d - 393216*a^20*c^8*f + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*
a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c
- b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^
6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)
^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35
767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215
040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d
^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2
- 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^
2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2
+ 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a
^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 1
0*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360
*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*
d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9
*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^
(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e -
39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 1
61280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*
b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3
*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c
- b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c
^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f
- 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30
720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*
c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g

+ 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^16*b^9*c^3*g - 10240*a^17*b^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3*c^6*g + 851968*a^19*b*c^8*e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4 *a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - 128000*a^15*c^9*e^3 + 1024*a^18*c^6*g^3 + 476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f^3 - 250880*a^14*c^10*d^2*e + 50176*a^15*c^9*d^2*g - 46080*a^16*c^8*e*f^2 + 76800*a^16*c^8*e^2*g - 15360*a^17*c^7*e*g^2 + 9216*a^17*c^7*f^2*g + 1800*a^9*b^9*c^6*d^3 - 29080*a^10*b^7*c^7*d^3 + 176032*a^11*b^5*c^8*d^3 - 473216*a^12*b^3*c^9*d^3 - 504*a^11*b^8*c^5*e^3 + 8112*a^12*b^6*c^6*e^3 - 48704*a^13*b^4*c^7*e^3 + 129280*a^14*b^2*c^8*e^3 + 40*a^13*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^3 + 2944*a^15*b^3*c^6*f^3 + 48*a^15*b^6*c^3*g^3 - 320*a^16*b^4*c^4*g^3 + 256*a^17*b^2*c^5*g^3 + 215040*a^15*c^9*d*e*f - 43008*a^16*c^8*d*f*g + 442880*a^14*b*c^9*d*e^2 - 433664*a^14*b*c^9*d^2*f + 109056*a^15*b*c^8*d*f^2 + 84480*a^15*b*c^8*e^2*f + 43520*a^16*b*c^7*d*g^2 - 7680*a^17*b*c^6*f*g^2 - 1400*a^9*b^10*c^5*d^2*e + 21680*a^10*b^8*c^6*d^2*e + 1680*a^10*b^9*c^5*d*e^2 - 121648*a^11*b^6*c^7*d^2*e - 27176*a^11*b^7*c^6*d*e^2 + 275264*a^12*b^4*c^8*d^2*e + 164448*a^12*b^5*c^7*d*e^2 - 121088*a^13*b^2*c^9*d^2*e - 441216*a^13*b^3*c^8*d*e^2 + 1000*a^9*b^11*c^4*d^2*f - 17800*a^10*b^9*c^5*d^2*f + 124280*a^11*b^7*c^6*d^2*f + 400*a^11*b^9*c^4*d*f^2 - 422944*a^12*b^5*c^7*d^2*f - 6600*a^12*b^7*c^5*d*f^2 + 694912*a^13*b^3*c^8*d^2*f + 40416*a^13*b^5*c^6*d*f^2 - 108928*a^14*b^3*c^7*d*f^2 - 600*a^9*b^12*c^3*d^2*g + 10960*a^10*b^10*c^4*d^2*g - 78904*a^11*b^8*c^5*d^2*g + 360*a^11*b^9*c^4*e^2*f + 278096*a^12*b^6*c^6*d^2*g - 5736*a^12*b^7*c^5*e^2*f - 240*a^12*b^8*c^4*e*f^2 + 120*a^12*b^9*c^3*d*g^2 - 472000*a^13*b^4*c^7*d^2*g + 33888*a^13

*b^5*c^6*e^2*f + 3792*a^13*b^6*c^5*e*f^2 - 2216*a^13*b^7*c^4*d*g^2 + 284416
*a^14*b^2*c^8*d^2*g - 87936*a^14*b^3*c^7*e^2*f - 21696*a^14*b^4*c^6*e*f^2 +
14688*a^14*b^5*c^5*d*g^2 + 52992*a^15*b^2*c^7*e*f^2 - 41856*a^15*b^3*c^6*d
*g^2 - 216*a^11*b^10*c^3*e^2*g + 3744*a^12*b^8*c^4*e^2*g - 25200*a^13*b^6*c
^5*e^2*g - 72*a^13*b^8*c^3*e*g^2 + 81984*a^14*b^4*c^6*e^2*g + 1296*a^14*b^6
*c^4*e*g^2 - 128256*a^15*b^2*c^7*e^2*g - 7872*a^15*b^4*c^5*e*g^2 + 19200*a^
16*b^2*c^6*e*g^2 - 24*a^13*b^8*c^3*f^2*g + 336*a^14*b^6*c^4*f^2*g + 24*a^14
*b^7*c^3*f*g^2 - 960*a^15*b^4*c^5*f^2*g - 672*a^15*b^5*c^4*f*g^2 - 2304*a^1
6*b^2*c^6*f^2*g + 4224*a^16*b^3*c^5*f*g^2 - 306176*a^15*b*c^8*d*e*g + 21504
*a^16*b*c^7*e*f*g - 1200*a^10*b^10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f - 1
30656*a^12*b^6*c^6*d*e*f + 394368*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8*
d*e*f + 720*a^10*b^11*c^3*d*e*g - 12816*a^11*b^9*c^4*d*e*g + 89264*a^12*b^7
*c^5*d*e*g - 302400*a^13*b^5*c^6*d*e*g + 493824*a^14*b^3*c^7*d*e*g - 240*a^
11*b^10*c^3*d*f*g + 3872*a^12*b^8*c^4*d*f*g - 22368*a^13*b^6*c^5*d*f*g + 51
840*a^14*b^4*c^6*d*f*g - 25088*a^15*b^2*c^7*d*f*g + 144*a^12*b^9*c^3*e*f*g
- 2256*a^13*b^7*c^4*e*f*g + 12480*a^14*b^5*c^5*e*f*g - 28416*a^15*b^3*c^6*e
*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2
- a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^
7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 +
3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4
*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 1169
28*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^
2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2
) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 +
44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 -
3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^1
3*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^1
2*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*
f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b
^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g -
152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(
-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^
8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-
(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 11
9616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10
*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^
7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*
(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*
c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*
e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g +
22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^
4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3
*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 15
36*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c

```
*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^
2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 40
96*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^
11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i
```

3.131 $\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x\right.$

Optimal result

Integrand size = 42, antiderivative size = 20

$$\int x^2\left(a+bx^2+cx^4\right)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx=x^3\left(a+bx^2+cx^4\right)^{1+p}$$

```
[Out] x^3*(c*x^4+b*x^2+a)^(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {1602}

$$\int x^2\left(a+bx^2+cx^4\right)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx=x^3\left(a+bx^2+cx^4\right)^{p+1}$$

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]
```

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=x^3\left(a+bx^2+cx^4\right)^{1+p}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx=x^3\left(a+bx^2+cx^4\right)^{1+p}$$

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^
4),x]
```

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$x^3(c\,x^4+b\,x^2+a)^{1+p}$	21
risch	$(c\,x^4+b\,x^2+a)^p\,x^3(c\,x^4+b\,x^2+a)$	31
norman	$a\,x^3\mathrm{e}^{p\ln(c\,x^4+b\,x^2+a)}+b\,x^5\mathrm{e}^{p\ln(c\,x^4+b\,x^2+a)}+c\,x^7\mathrm{e}^{p\ln(c\,x^4+b\,x^2+a)}$	65
parallelrisch	$\frac{x^7(c\,x^4+b\,x^2+a)^p ac+ab(c\,x^4+b\,x^2+a)^p x^5+a^2(c\,x^4+b\,x^2+a)^p x^3}{a}$	67

```
[In] int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x,method=_RETUR
NVERBOSE)
```

```
[Out] x^3*(c*x^4+b*x^2+a)^(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx$$
$$=\left(cx^7+bx^5+ax^3\right)\left(cx^4+bx^2+a\right)^p$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="fricas")
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.

Time = 158.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int x^2 (a + bx^2 + cx^4)^p (3a + b(5 + 2p)x^2 + c(7 + 4p)x^4) \, dx$$
$$= ax^3 (a + bx^2 + cx^4)^p + bx^5 (a + bx^2 + cx^4)^p + cx^7 (a + bx^2 + cx^4)^p$$

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x)
[Out] a*x**3*(a + b*x**2 + c*x**4)**p + b*x**5*(a + b*x**2 + c*x**4)**p + c*x**7*
(a + b*x**2 + c*x**4)**p
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2 (a + bx^2 + cx^4)^p (3a + b(5 + 2p)x^2 + c(7 + 4p)x^4) \, dx$$
$$= (cx^7 + bx^5 + ax^3) (cx^4 + bx^2 + a)^p$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="maxima")
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int x^2 (a + bx^2 + cx^4)^p (3a + b(5 + 2p)x^2 + c(7 + 4p)x^4) \, dx$$
$$= (cx^4 + bx^2 + a)^p cx^7 + (cx^4 + bx^2 + a)^p bx^5 + (cx^4 + bx^2 + a)^p ax^3$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="giac")
[Out] (c*x^4 + b*x^2 + a)^p*c*x^7 + (c*x^4 + b*x^2 + a)^p*b*x^5 + (c*x^4 + b*x^2
+ a)^p*a*x^3
```

Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2 \left(a+b x^2+c x^4\right)^p \left(3 a+b (5+2 p) x^2+c (7+4 p) x^4\right) \, dx$$
$$=\left(c\,x^7+b\,x^5+a\,x^3\right)\,\left(c\,x^4+b\,x^2+a\right)^p$$

```
[In] int(x^2*(3*a + b*x^2*(2*p + 5) + c*x^4*(4*p + 7))*(a + b*x^2 + c*x^4)^p,x)
[Out] (a*x^3 + b*x^5 + c*x^7)*(a + b*x^2 + c*x^4)^p
```

3.132 $\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 210

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{d^4(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^{10}} + \frac{d^2(4cd^4+3bd^2e^2+2ae^4)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^{10}} - \frac{(6cd^4+3bd^2e^2+ae^4)(d-ex)^{5/2}(d+ex)^{5/2}}{5e^{10}} + \frac{(4cd^2+be^2)(d-ex)^{7/2}(d+ex)^{7/2}}{7e^{10}} - \frac{c(d-ex)^{9/2}(d+ex)^{9/2}}{9e^{10}}$$

```
[Out] 1/3*d^2*(2*a*e^4+3*b*d^2*e^2+4*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^10-1/5
*(a*e^4+3*b*d^2*e^2+6*c*d^4)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^10+1/7*(b*e^2+4
*c*d^2)*(-e*x+d)^(7/2)*(e*x+d)^(7/2)/e^10-1/9*c*(-e*x+d)^(9/2)*(e*x+d)^(9/2
)/e^10-d^4*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^10
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used

$= \{534, 1265, 911, 1167\}$

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(d^2-e^2x^2)^3(ae^4+3bd^2e^2+6cd^4)}{5e^{10}\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^2(d^2-e^2x^2)^2(2ae^4+3bd^2e^2+4cd^4)}{3e^{10}\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^4(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^{10}\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^4(be^2+4cd^2)}{7e^{10}\sqrt{d-ex}\sqrt{d+ex}} - \frac{c(d^2-e^2x^2)^5}{9e^{10}\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^10*Sqrt[d - e*x]*Sqr
t[d + e*x])) + (d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d^2 - e^2*x^2)^2)/(3
*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d^2
- e^2*x^2)^3)/(5*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((4*c*d^2 + b*e^2)*(d^
2 - e^2*x^2)^4)/(7*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*(d^2 - e^2*x^2)^5
)/(9*e^10*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int \frac{x^2(a+bx+cx^2)}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int \left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2\left(\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}\right)\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int \left(\frac{cd^8+bd^6e^2+ad^4e^4}{e^8}-\frac{d^2(4cd^4+3bd^2e^2+2ae^4)x^2}{e^8}+\frac{(6cd^4+3bd^2e^2+ae^4)x^4}{e^8}-\frac{(4cd^2+be^2)x^6}{e^8}+\right.\right.}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{d^4(cd^4+bd^2e^2+ae^4)(d^2-e^2x^2)}{e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{d^2(4cd^4+3bd^2e^2+2ae^4)(d^2-e^2x^2)^2}{3e^{10}\sqrt{d-ex}\sqrt{d+ex}}$$
$$-\frac{(6cd^4+3bd^2e^2+ae^4)(d^2-e^2x^2)^3}{5e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{(4cd^2+be^2)(d^2-e^2x^2)^4}{7e^{10}\sqrt{d-ex}\sqrt{d+ex}}$$
$$-\frac{c(d^2-e^2x^2)^5}{9e^{10}\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{\sqrt{d-ex}\sqrt{d+ex}(21ae^4(8d^4+4d^2e^2x^2+3e^4x^4)+9b(16d^6e^2+8d^4e^4x^2+6d^2e^6x^4+5e^8x^6)+c(128d^8}{315e^{10}}$$

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*a*e^4*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^
4*x^4) + 9*b*(16*d^6*e^2 + 8*d^4*e^4*x^2 + 6*d^2*e^6*x^4 + 5*e^8*x^6) + c*(
128*d^8 + 64*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 40*d^2*e^6*x^6 + 35*e^8*x^8)))/
e^10
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6e^2x^2+168a}{315e^{10}}$
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6e^2x^2+168a}{315e^{10}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6e^2x^2+168a}{315e^{10}}$

```
[In] int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(35*c*e^8*x^8+45*b*e^8*x^6+40*c*d^2*e^6
*x^6+63*a*e^8*x^4+54*b*d^2*e^6*x^4+48*c*d^4*e^4*x^4+84*a*d^2*e^6*x^2+72*b*d
^4*e^4*x^2+64*c*d^6*e^2*x^2+168*a*d^4*e^4+144*b*d^6*e^2+128*c*d^8)/e^10
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.66

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(35\,ce^8x^8+128\,cd^8+144\,bd^6e^2+168\,ad^4e^4+5\,(8\,cd^2e^6+9\,be^8)x^6+3\,(16\,cd^4e^4+18\,bd^2e^6+21\,ae^8)x^4}{315\,e^{10}}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/315*(35*c*e^8*x^8 + 128*c*d^8 + 144*b*d^6*e^2 + 168*a*d^4*e^4 + 5*(8*c*d
^2*e^6 + 9*b*e^8)*x^6 + 3*(16*c*d^4*e^4 + 18*b*d^2*e^6 + 21*a*e^8)*x^4 + 4*
(16*c*d^6*e^2 + 18*b*d^4*e^4 + 21*a*d^2*e^6)*x^2)*sqrt(e*x + d)*sqrt(-e*x +
 d)/e^10
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.88 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.75

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iad^5 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{ad^5 G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{ibd^7 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{13}{4},-\frac{11}{4} & -3,-3,-\frac{5}{2},1 \\ -\frac{7}{2},-\frac{13}{4},-3,-\frac{11}{4},-\frac{5}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{bd^7 G_{6,6}^{2,6}\left(\begin{matrix} -4,-\frac{15}{4},-\frac{7}{2},-\frac{13}{4},-3,1 & \\ -\frac{15}{4},-\frac{13}{4} & -4,-\frac{7}{2},-\frac{7}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{icd^9 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{17}{4},-\frac{15}{4} & -4,-4,-\frac{7}{2},1 \\ -\frac{9}{2},-\frac{17}{4},-4,-\frac{15}{4},-\frac{7}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^{10}} - \frac{cd^9 G_{6,6}^{2,6}\left(\begin{matrix} -5,-\frac{19}{4},-\frac{9}{2},-\frac{17}{4},-4,1 & \\ -\frac{19}{4},-\frac{17}{4} & -5,-\frac{9}{2},-\frac{9}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^{10}}$$

```
[In] integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4
, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - a*d**5*meijerg(((-3
, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), d**
2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6) - I*b*d**7*meijerg(((-
13/4, -11/4), (-3, -3, -5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0), ()),
d**2/(e**2*x**2))/(4*pi**(3/2)*e**8) - b*d**7*meijerg(((-4, -15/4, -7/2, -1
3/4, -3, 1), ()), ((-15/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_polar(-2*
I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**8) - I*c*d**9*meijerg(((-17/4, -15/4), (
-4, -4, -7/2, 1)), ((-9/2, -17/4, -4, -15/4, -7/2, 0), ()), d**2/(e**2*x**2
))/(4*pi**(3/2)*e**10) - c*d**9*meijerg(((-5, -19/4, -9/2, -17/4, -4, 1), (
)), ((-19/4, -17/4), (-5, -9/2, -9/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x*
*2))/(4*pi**(3/2)*e**10)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.40

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^8}{9\,e^2} - \frac{8\,\sqrt{-e^2x^2+d^2}cd^2x^6}{63\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^6}{7\,e^2} - \frac{16\,\sqrt{-e^2x^2+d^2}cd^4x^4}{105\,e^6} - \frac{6\,\sqrt{-e^2x^2+d^2}bd^2x^4}{35\,e^4} - \frac{\sqrt{-e^2x^2+d^2}ax^4}{5\,e^2} - \frac{64\,\sqrt{-e^2x^2+d^2}cd^6x^2}{315\,e^8} - \frac{8\,\sqrt{-e^2x^2+d^2}bd^4x^2}{35\,e^6} - \frac{4\,\sqrt{-e^2x^2+d^2}ad^2x^2}{15\,e^4} - \frac{128\,\sqrt{-e^2x^2+d^2}cd^8}{315\,e^{10}} - \frac{16\,\sqrt{-e^2x^2+d^2}bd^6}{35\,e^8} - \frac{8\,\sqrt{-e^2x^2+d^2}ad^4}{15\,e^6}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/9*sqrt(-e^2*x^2 + d^2)*c*x^8/e^2 - 8/63*sqrt(-e^2*x^2 + d^2)*c*d^2*x^6/e
^4 - 1/7*sqrt(-e^2*x^2 + d^2)*b*x^6/e^2 - 16/105*sqrt(-e^2*x^2 + d^2)*c*d^4
*x^4/e^6 - 6/35*sqrt(-e^2*x^2 + d^2)*b*d^2*x^4/e^4 - 1/5*sqrt(-e^2*x^2 + d^
2)*a*x^4/e^2 - 64/315*sqrt(-e^2*x^2 + d^2)*c*d^6*x^2/e^8 - 8/35*sqrt(-e^2*x
^2 + d^2)*b*d^4*x^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*a*d^2*x^2/e^4 - 128/315
*sqrt(-e^2*x^2 + d^2)*c*d^8/e^10 - 16/35*sqrt(-e^2*x^2 + d^2)*b*d^6/e^8 - 8
/15*sqrt(-e^2*x^2 + d^2)*a*d^4/e^6
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(315\,cd^8+315\,bd^6e^2+315\,ad^4e^4-(840\,cd^7+630\,bd^5e^2+420\,ad^3e^4-(1932\,cd^6+1071\,bd^4e^2+462\,ad^2e}{}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/315*(315*c*d^8 + 315*b*d^6*e^2 + 315*a*d^4*e^4 - (840*c*d^7 + 630*b*d^5*
e^2 + 420*a*d^3*e^4 - (1932*c*d^6 + 1071*b*d^4*e^2 + 462*a*d^2*e^4 - (2952*
```

```
c*d^5 + 1116*b*d^3*e^2 + 252*a*d*e^4 - (3098*c*d^4 + 729*b*d^2*e^2 + 63*a*e
^4 - 5*(440*c*d^3 + 54*b*d*e^2 - (204*c*d^2 + 9*b*e^2 + 7*((e*x + d)*c - 8*
c*d)*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*(e*x
 + d))*sqrt(e*x + d)*sqrt(-e*x + d)/e^10
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.37

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\left(\frac{128\,c\,d^9+144\,b\,d^7\,e^2+168\,a\,d^5\,e^4}{315\,e^{10}}+\frac{x^7\,\left(40\,c\,d^2\,e^7+45\,b\,e^9\right)}{315\,e^{10}}+\frac{x^2\,\left(64\,c\,d^7\,e^2+72\,b\,d^5\,e^4+84\,a\,d^3\,e^6\right)}{315\,e^{10}}+\frac{x^3\,\left(64\,c\,d^6\,e^3+72\,b\,d^4\right.}{315\,e^{10}}\right.}{}$$

```
[In] int((x^5*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((128*c*d^9 + 168*a*d^5*e^4 + 144*b*d^7*e^2)/(315*e^10) +
 (x^7*(45*b*e^9 + 40*c*d^2*e^7))/(315*e^10) + (x^2*(84*a*d^3*e^6 + 72*b*d^5
*e^4 + 64*c*d^7*e^2))/(315*e^10) + (x^3*(84*a*d^2*e^7 + 72*b*d^4*e^5 + 64*c
*d^6*e^3))/(315*e^10) + (c*x^9)/(9*e) + (x^5*(63*a*e^9 + 54*b*d^2*e^7 + 48*
c*d^4*e^5))/(315*e^10) + (x*(168*a*d^4*e^5 + 144*b*d^6*e^3 + 128*c*d^8*e))/
(315*e^10) + (x^6*(40*c*d^3*e^6 + 45*b*d*e^8))/(315*e^10) + (x^4*(54*b*d^3*
e^6 + 48*c*d^5*e^4 + 63*a*d*e^8))/(315*e^10) + (c*d*x^8)/(9*e^2)))/(d + e*x
)^(1/2)
```

3.133 $\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 159

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{d^2(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^8} + \frac{(3cd^4+2bd^2e^2+ae^4)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^8} - \frac{(3cd^2+be^2)(d-ex)^{5/2}(d+ex)^{5/2}}{5e^8} + \frac{c(d-ex)^{7/2}(d+ex)^{7/2}}{7e^8}$$

```
[Out] 1/3*(a*e^4+2*b*d^2*e^2+3*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^8-1/5*(b*e^2
+3*c*d^2)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^8+1/7*c*(-e*x+d)^(7/2)*(e*x+d)^(7/
2)/e^8-d^2*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^8
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {534, 1265, 785}

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{(d^2-e^2x^2)^2(ae^4+2bd^2e^2+3cd^4)}{3e^8\sqrt{d-ex}\sqrt{d+ex}} - \frac{d^2(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^8\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)^3(be^2+3cd^2)}{5e^8\sqrt{d-ex}\sqrt{d+ex}} + \frac{c(d^2-e^2x^2)^4}{7e^8\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^8*Sqrt[d - e*x]*Sqrt[d + e*x])) + ((3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^2)/(3*e^8*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^3)/(5*e^8*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*(d^2 - e^2*x^2)^4)/(7*e^8*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int \frac{x(a+bx+cx^2)}{\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int \left(\frac{cd^6+bd^4e^2+ad^2e^4}{e^6\sqrt{d^2-e^2x}} + \frac{(-3cd^4-2bd^2e^2-ae^4)\sqrt{d^2-e^2x}}{e^6} + \frac{(3cd^2+be^2)(d^2-e^2x)^{3/2}}{e^6} - \frac{c(d^2-e^2x)^{5/\cdots}}{e^6} \cdots\right)\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{d^2(cd^4+bd^2e^2+ae^4)(d^2-e^2x^2)}{e^8\sqrt{d-ex}\sqrt{d+ex}} + \frac{(3cd^4+2bd^2e^2+ae^4)(d^2-e^2x^2)^2}{3e^8\sqrt{d-ex}\sqrt{d+ex}} - \frac{(3cd^2+be^2)(d^2-e^2x^2)^3}{5e^8\sqrt{d-ex}\sqrt{d+ex}} + \frac{c(d^2-e^2x^2)^4}{7e^8\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(35ae^4(2d^2+e^2x^2)+7b(8d^4e^2+4d^2e^4x^2+3e^6x^4)+3c(16d^6+8d^4e^2x^2+6d^2e^4x^4+5e^6x^6))}{105e^8}$$

[In] Integrate[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*a*e^4*(2*d^2 + e^2*x^2) + 7*b*(8*d^4*e^2 + 4*d^2*e^4*x^2 + 3*e^6*x^4) + 3*c*(16*d^6 + 8*d^4*e^2*x^2 + 6*d^2*e^4*x^4 + 5*e^6*x^6)))/e^8

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15c\,x^6e^6+21b\,e^6x^4+18c\,d^2e^4x^4+35a\,e^6x^2+28b\,d^2e^4x^2+24c\,d^4e^2x^2+70a\,d^2e^4+56b\,d^4e^2+48c\,d^6)}{105e^8}$	109
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15c\,x^6e^6+21b\,e^6x^4+18c\,d^2e^4x^4+35a\,e^6x^2+28b\,d^2e^4x^2+24c\,d^4e^2x^2+70a\,d^2e^4+56b\,d^4e^2+48c\,d^6)}{105e^8}$	109
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15c\,x^6e^6+21b\,e^6x^4+18c\,d^2e^4x^4+35a\,e^6x^2+28b\,d^2e^4x^2+24c\,d^4e^2x^2+70a\,d^2e^4+56b\,d^4e^2+48c\,d^6)}{105e^8}$	109

[In] int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*c*e^6*x^6+21*b*e^6*x^4+18*c*d^2*e^4*x^4+35*a*e^6*x^2+28*b*d^2*e^4*x^2+24*c*d^4*e^2*x^2+70*a*d^2*e^4+56*b*d^4*e^2+48*c*d^6)/e^8

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(15\,ce^6x^6+48\,cd^6+56\,bd^4e^2+70\,ad^2e^4+3\,(6\,cd^2e^4+7\,be^6)x^4+(24\,cd^4e^2+28\,bd^2e^4+35\,ae^6)x^2)\sqrt{ex+\cdots}}{105\,e^8}$$

[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

```
[Out] -1/105*(15*c*e^6*x^6 + 48*c*d^6 + 56*b*d^4*e^2 + 70*a*d^2*e^4 + 3*(6*c*d^2*
e^4 + 7*b*e^6)*x^4 + (24*c*d^4*e^2 + 28*b*d^2*e^4 + 35*a*e^6)*x^2)*sqrt(e*x
 + d)*sqrt(-e*x + d)/e^8
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.90 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.31

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iad^3 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{ad^3 G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{ibd^5 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{bd^5 G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{icd^7 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{13}{4},-\frac{11}{4} & -3,-3,-\frac{5}{2},1 \\ -\frac{7}{2},-\frac{13}{4},-3,-\frac{11}{4},-\frac{5}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{cd^7 G_{6,6}^{2,6}\left(\begin{matrix} -4,-\frac{15}{4},-\frac{7}{2},-\frac{13}{4},-3,1 & \\ -\frac{15}{4},-\frac{13}{4} & -4,-\frac{7}{2},-\frac{7}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8}$$

```
[In] integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4
, -1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - a*d**3*meijerg(((-2
, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*
exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4) - I*b*d**5*meijerg(((-9/
4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**6) - b*d**5*meijerg(((-3, -11/4, -5/2, -9/4, -2
, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e
```

```
**2*x**2))/(4*pi**(3/2)*e**6) - I*c*d**7*meijerg(((-13/4, -11/4), (-3, -3,
-5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0), ()), d**2/(e**2*x**2))/(4*pi
**(3/2)*e**8) - c*d**7*meijerg(((-4, -15/4, -7/2, -13/4, -3, 1), ()), ((-15
/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*p
i**(3/2)*e**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^6}{7\,e^2} - \frac{6\,\sqrt{-e^2x^2+d^2}cd^2x^4}{35\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^4}{5\,e^2} - \frac{8\,\sqrt{-e^2x^2+d^2}cd^4x^2}{35\,e^6} - \frac{4\,\sqrt{-e^2x^2+d^2}bd^2x^2}{15\,e^4} - \frac{\sqrt{-e^2x^2+d^2}ax^2}{3\,e^2} - \frac{16\,\sqrt{-e^2x^2+d^2}cd^6}{35\,e^8} - \frac{8\,\sqrt{-e^2x^2+d^2}bd^4}{15\,e^6} - \frac{2\,\sqrt{-e^2x^2+d^2}ad^2}{3\,e^4}$$

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/7*sqrt(-e^2*x^2 + d^2)*c*x^6/e^2 - 6/35*sqrt(-e^2*x^2 + d^2)*c*d^2*x^4/e
^4 - 1/5*sqrt(-e^2*x^2 + d^2)*b*x^4/e^2 - 8/35*sqrt(-e^2*x^2 + d^2)*c*d^4*x
^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*b*d^2*x^2/e^4 - 1/3*sqrt(-e^2*x^2 + d^2)
*a*x^2/e^2 - 16/35*sqrt(-e^2*x^2 + d^2)*c*d^6/e^8 - 8/15*sqrt(-e^2*x^2 + d^
2)*b*d^4/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*a*d^2/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(105\,cd^6+105\,bd^4e^2+105\,ad^2e^4-(210\,cd^5+140\,bd^3e^2+70\,ade^4-(357\,cd^4+154\,bd^2e^2+35\,ae^4-3\,(1}{}$$

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/105*(105*c*d^6 + 105*b*d^4*e^2 + 105*a*d^2*e^4 - (210*c*d^5 + 140*b*d^3*
e^2 + 70*a*d*e^4 - (357*c*d^4 + 154*b*d^2*e^2 + 35*a*e^4 - 3*(124*c*d^3 + 2
8*b*d*e^2 - (81*c*d^2 + 7*b*e^2 + 5*((e*x + d)*c - 6*c*d)*(e*x + d))*(e*x +
 d))*(e*x + d))*(e*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d)/e^8
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.35

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\,\left(\frac{48\,c\,d^7+56\,b\,d^5\,e^2+70\,a\,d^3\,e^4}{105\,e^8}+\frac{x^5\,\left(18\,c\,d^2\,e^5+21\,b\,e^7\right)}{105\,e^8}+\frac{c\,x^7}{7\,e}+\frac{x^3\,\left(24\,c\,d^4\,e^3+28\,b\,d^2\,e^5+35\,a\,e^7\right)}{105\,e^8}+\frac{x\,\left(48\,c\,d^6\,e+56\,b\,d^4 \cdots\right.}{105\,e \cdots}\right.}{\sqrt{d+e\,x}}$$

```
[In] int((x^3*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((48*c*d^7 + 70*a*d^3*e^4 + 56*b*d^5*e^2)/(105*e^8) + (x^
5*(21*b*e^7 + 18*c*d^2*e^5))/(105*e^8) + (c*x^7)/(7*e) + (x^3*(35*a*e^7 + 2
8*b*d^2*e^5 + 24*c*d^4*e^3))/(105*e^8) + (x*(70*a*d^2*e^5 + 56*b*d^4*e^3 +
48*c*d^6*e))/(105*e^8) + (x^4*(18*c*d^3*e^4 + 21*b*d*e^6))/(105*e^8) + (x^2
*(28*b*d^3*e^4 + 24*c*d^5*e^2 + 35*a*d*e^6))/(105*e^8) + (c*d*x^6)/(7*e^2))
)/(d + e*x)^(1/2)
```

3.134 $\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 109

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^6} + \frac{(2cd^2+be^2)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^6} - \frac{c(d-ex)^{5/2}(d+ex)^{5/2}}{5e^6}$$

```
[Out] 1/3*(b*e^2+2*c*d^2)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^6-1/5*c*(-e*x+d)^(5/2)*(
e*x+d)^(5/2)/e^6-(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {534, 1261, 712}

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^6\sqrt{d-ex}\sqrt{d+ex}} + \frac{(d^2-e^2x^2)^2(be^2+2cd^2)}{3e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{c(d^2-e^2x^2)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^6*Sqrt[d - e*x]*Sqrt[d +
 e*x])) + ((2*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^2)/(3*e^6*Sqrt[d - e*x]*Sqrt[d
 + e*x]) - (c*(d^2 - e^2*x^2)^3)/(5*e^6*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}\int \frac{x(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}} \\
&= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}} \\
&= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{cd^4+bd^2e^2+ae^4}{e^4\sqrt{d^2-e^2x}}+\frac{(-2cd^2-be^2)\sqrt{d^2-e^2x}}{e^4}+\frac{c(d^2-e^2x)^{3/2}}{e^4}\right)dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{(cd^4+bd^2e^2+ae^4)(d^2-e^2x^2)}{e^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{(2cd^2+be^2)(d^2-e^2x^2)^2}{3e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{c(d^2-e^2x^2)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(5(2bd^2e^2+3ae^4+be^4x^2)+c(8d^4+4d^2e^2x^2+3e^4x^4))}{15e^6}$$

```
[In] Integrate[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(5*(2*b*d^2*e^2 + 3*a*e^4 + b*e^4*x^2) +
 c*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)))/e^6
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10e^2d^2b+8d^4c)}{15e^6}$	73
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10e^2d^2b+8d^4c)}{15e^6}$	73
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10e^2d^2b+8d^4c)}{15e^6}$	73

```
[In] int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*c*e^4*x^4+5*b*e^4*x^2+4*c*d^2*e^2*x^2
+15*a*e^4+10*b*d^2*e^2+8*c*d^4)/e^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(3\,ce^4x^4+8\,cd^4+10\,bd^2e^2+15\,ae^4+(4\,cd^2e^2+5\,be^4)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{15\,e^6}$$

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/15*(3*c*e^4*x^4 + 8*c*d^4 + 10*b*d^2*e^2 + 15*a*e^4 + (4*c*d^2*e^2 + 5*b
*e^4)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/e^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.52 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.21

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iadG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{adG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{ibd^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{bd^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{icd^5G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{cd^5G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}$$

```
[In] integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0)
, ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - a*d*meijerg(((-1, -3/4, -1/2,
 -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I
*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*b*d**3*meijerg(((-5/4, -3/4), (-1,
 -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2))/(4
*pi**(3/2)*e**4) - b*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7
/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi
**(3/2)*e**4) - I*c*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2,
 -9/4, -2, -7/4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - c*d*
*5*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2,
 -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^4}{5\,e^2} - \frac{4\,\sqrt{-e^2x^2+d^2}cd^2x^2}{15\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^2}{3\,e^2} - \frac{8\,\sqrt{-e^2x^2+d^2}cd^4}{15\,e^6} - \frac{2\,\sqrt{-e^2x^2+d^2}bd^2}{3\,e^4} - \frac{\sqrt{-e^2x^2+d^2}a}{e^2}$$

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*c*x^4/e^2 - 4/15*sqrt(-e^2*x^2 + d^2)*c*d^2*x^2/e
^4 - 1/3*sqrt(-e^2*x^2 + d^2)*b*x^2/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*c*d^4/e
^6 - 2/3*sqrt(-e^2*x^2 + d^2)*b*d^2/e^4 - sqrt(-e^2*x^2 + d^2)*a/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(15\,cd^4+15\,bd^2e^2+15\,ae^4-(20\,cd^3+10\,bde^2-(22\,cd^2+5\,be^2+3\,((ex+d)c-4\,cd)(ex+d))(ex+d)}{15\,e^6}$$

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/15*(15*c*d^4 + 15*b*d^2*e^2 + 15*a*e^4 - (20*c*d^3 + 10*b*d*e^2 - (22*c*
d^2 + 5*b*e^2 + 3*((e*x + d)*c - 4*c*d)*(e*x + d))*(e*x + d))*(e*x + d))*sq
rt(e*x + d)*sqrt(-e*x + d)/e^6
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\,\left(\frac{8\,c\,d^5+10\,b\,d^3\,e^2+15\,a\,d\,e^4}{15\,e^6}+\frac{x^3\,\left(4\,c\,d^2\,e^3+5\,b\,e^5\right)}{15\,e^6}+\frac{c\,x^5}{5\,e}+\frac{x^2\,\left(4\,c\,d^3\,e^2+5\,b\,d\,e^4\right)}{15\,e^6}+\frac{x\,\left(8\,c\,d^4\,e+10\,b\,d^2\,e^3+15\,a\,e^5\right)}{15\,e^6}+\frac{c\,d}{5}\right.}{\sqrt{d+e\,x}}$$

```
[In] int((x*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((8*c*d^5 + 10*b*d^3*e^2 + 15*a*d*e^4)/(15*e^6) + (x^3*(5
*b*e^5 + 4*c*d^2*e^3))/(15*e^6) + (c*x^5)/(5*e) + (x^2*(4*c*d^3*e^2 + 5*b*d
*e^4))/(15*e^6) + (x*(15*a*e^5 + 10*b*d^2*e^3 + 8*c*d^4*e))/(15*e^6) + (c*d
*x^4)/(5*e^2)))/(d + e*x)^(1/2)
```

3.135 $\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 93

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx=-\frac{(cd^2+be^2)\sqrt{d-ex}\sqrt{d+ex}}{e^4}+\frac{c(d-ex)^{3/2}(d+ex)^{3/2}}{3e^4}-\frac{a\text{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{d}$$

```
[Out] 1/3*c*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^4-a*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/
2)/d)/d-(b*e^2+c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {534, 1265, 911, 1167, 214}

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx=-\frac{a\sqrt{d^2-e^2x^2}\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(be^2+cd^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^2 + b*e^2)*(d^2 - e^2*x^2))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*x])) + (c
*(d^2 - e^2*x^2)^2)/(3*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (a*Sqrt[d^2 - e^2
*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(d*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$
= -\frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}
$$

$$
= -\frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int \left(b+\frac{cd^2}{e^2}-\frac{cx^2}{e^2}+\frac{a}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right)dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}
$$

$$
= -\frac{(cd^2+be^2)(d^2-e^2x^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(a\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}
$$

$$
= -\frac{(cd^2+be^2)(d^2-e^2x^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$
\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(2cd^2+3be^2+ce^2x^2)}{3e^4} + \frac{a\log\left(-1+\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{d} - \frac{a\log\left(d+\frac{d\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{d}
$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(2*c*d^2 + 3*b*e^2 + c*e^2*x^2))/e^4 + (a
*Log[-1 + Sqrt[d + e*x]/Sqrt[d - e*x]])/d - (a*Log[d + (d*Sqrt[d + e*x])/Sq
rt[d - e*x]])/d
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(\operatorname{csgn}(d)cd\,e^2x^2\sqrt{-e^2x^2+d^2}+3\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(d)bd\,e^2+2\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(d)c\,d^3+3\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(\right.}{x}\right.\right.}{3d\sqrt{-e^2x^2+d^2}\,e^4}$

```
[In] int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d*(csgn(d)*c*d*e^2*x^2*(-e^2*x^2+d^2)^(1/
2)+3*(-e^2*x^2+d^2)^(1/2)*csgn(d)*b*d*e^2+2*(-e^2*x^2+d^2)^(1/2)*csgn(d)*c*
d^3+3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4)*csgn(d)/(-e^2*x^2+d
^2)^(1/2)/e^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{3\,ae^4\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-(cde^2x^2+2\,cd^3+3\,bde^2)\sqrt{ex+d}\sqrt{-ex+d}}{3\,de^4}$$

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/3*(3*a*e^4*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) - (c*d*e^2*x^2 + 2*c
*d^3 + 3*b*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d*e^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.14 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.27

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibdG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{bdG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{icd^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{cd^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}$$

```
[In] integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
d**2/(e**2*x**2))/(4*pi**(3/2)*d) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()
), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*
pi**(3/2)*d) - I*b*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0
, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - b*d*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*
exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*c*d**3*meijerg(((-5/
4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**4) - c*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1,
 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**
2*x**2))/(4*pi**(3/2)*e**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^2}{3\,e^2} - \frac{a\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{2\,\sqrt{-e^2x^2+d^2}cd^2}{3\,e^4} - \frac{\sqrt{-e^2x^2+d^2}b}{e^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*c*x^2/e^2 - a*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2
+ d^2)*d/abs(x))/d - 2/3*sqrt(-e^2*x^2 + d^2)*c*d^2/e^4 - sqrt(-e^2*x^2 + d
^2)*b/e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(79) = 158.

Time = 0.44 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.03

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{3\,ae^4\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d} - \frac{3\,ae^4\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}-2\right|\right)}{d} + (3\,cd^2+3\,be^2+((ex -}{3\,e^4}$$

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/3*(3*a*e^4*log(abs(-(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + s
qrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)) + 2))/d - 3*a*e^4*log(abs(-
(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*s
qrt(d) - sqrt(-e*x + d)) - 2))/d + (3*c*d^2 + 3*b*e^2 + ((e*x + d)*c - 2*c*
d)*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d))/e^4
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.73

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{a\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d} - \frac{\sqrt{d-e\,x}\left(\frac{2\,c\,d^3}{3\,e^4}+\frac{c\,x^3}{3\,e}+\frac{c\,d\,x^2}{3\,e^2}+\frac{2\,c\,d^2\,x}{3\,e^3}\right)}{\sqrt{d+e\,x}} - \frac{\left(\frac{b\,d}{e^2}+\frac{b\,x}{e}\right)\sqrt{d-e\,x}}{\sqrt{d+e\,x}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] (a*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - ((d - e*
x)^(1/2)*((2*c*d^3)/(3*e^4) + (c*x^3)/(3*e) + (c*d*x^2)/(3*e^2) + (2*c*d^2*
x)/(3*e^3)))/(d + e*x)^(1/2) - (((b*d)/e^2 + (b*x)/e)*(d - e*x)^(1/2))/(d +
 e*x)^(1/2)
```

3.136 $\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{c\sqrt{d-ex}\sqrt{d+ex}}{e^2} - \frac{a\sqrt{d-ex}\sqrt{d+ex}}{2d^2x^2} - \frac{(2bd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{2d^3}$$

```
[Out] -1/2*(a*e^2+2*b*d^2)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d^3-c*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/e^2-1/2*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {534, 1265, 911, 1171, 396, 214}

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(ae^2+2bd^2)\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2-e^2x^2)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{c(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((c*(d^2 - e^2*x^2))/(e^2*Sqrt[d - e*x]*Sqrt[d + e*x])) - (a*(d^2 - e^2*x^
2))/(2*d^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((2*b*d^2 + a*e^2)*Sqrt[d^2 -
 e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^3*Sqrt[d - e*x]*Sqrt[d + e*x
])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}\int\frac{a+bx^2+cx^4}{x^3\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int\frac{a+bx+cx^2}{x^2\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int\frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{\left(2cd^2+be^2\right)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\text{Subst}\left(\int\frac{-a-\frac{2\left(cd^4+bd^2e^2\right)}{e^4}+\frac{2cd^2x^2}{e^4}}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{c(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(e^2\left(\frac{2cd^4}{e^6}+\frac{-a-\frac{2\left(cd^4+bd^2e^2\right)}{e^4}}{e^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{c(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{(2bd^2+ae^2)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int\frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{\sqrt{d-ex}\sqrt{d+ex}(ade^2+2cd^3x^2)}{e^2x^2}+2(2bd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{2d^3}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/2*((Sqrt[d - e*x]*Sqrt[d + e*x]*(a*d*e^2 + 2*c*d^3*x^2))/(e^2*x^2) + 2*(
2*b*d^2 + a*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e*x]])/d^3
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{a\sqrt{-ex+d}\,\sqrt{ex+d}}{2d^2x^2}+\frac{\left(-\frac{2c\,d^2\sqrt{-(ex-d)(ex+d)}}{e^2}-\frac{\left(a\,e^2+2b\,d^2\right)\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\sqrt{(ex+d)(-ex+d)}}{2d^2\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(2\,\mathrm{csgn}(d)c\,d^3x^2\sqrt{-e^2x^2+d^2}+\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)a\,e^4x^2+2\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)b\,d^2e^2x\right.}{2d^3\sqrt{-e^2x^2+d^2}\,x^2e^2}$

```
[In] int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^2+1/2/d^2*(-2*c*d^2/e^2*(-(e*x-d)
*(e*x+d))^(1/2)-(a*e^2+2*b*d^2)/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x))*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\,cd^4x^2-(2\,bd^2e^2+ae^4)x^2\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)+(2\,cd^3x^2+ade^2)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^3e^2x^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*c*d^4*x^2 - (2*b*d^2*e^2 + a*e^4)*x^2*log((sqrt(e*x + d)*sqrt(-e*x
+ d) - d)/x) + (2*c*d^3*x^2 + a*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^3*e
^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.24

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{b\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d}-\frac{ae^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^3} - \frac{\sqrt{-e^2x^2+d^2}c}{e^2}-\frac{\sqrt{-e^2x^2+d^2}a}{2\,d^2x^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
[Out] -b*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*a*e^2*log(2*
d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - sqrt(-e^2*x^2 + d^2)*c/
e^2 - 1/2*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(83) = 166$.

Time = 0.46 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.78

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\,\sqrt{ex+d}\sqrt{-ex+d}c+\frac{(2\,bd^2e^2+ae^4)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d^3}-\frac{(2\,bd^2e^2+ae^4)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{}}{}\right.\right.}{d^3}}{2\,e^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
[Out] -1/2*(2*sqrt(e*x + d)*sqrt(-e*x + d)*c + (2*b*d^2*e^2 + a*e^4)*log(abs(-(sq
rt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt
```

```
(d) - sqrt(-e*x + d)) + 2))/d^3 - (2*b*d^2*e^2 + a*e^4)*log(abs(-(sqrt(2)*s
qrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - s
qrt(-e*x + d)) - 2))/d^3 - 4*(a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqr
t(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 + 4*a*e^4*
((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*
sqrt(d) - sqrt(-e*x + d))))/((((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x
+ d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2 - 4)^2*d^3))/e^2
```

Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 422, normalized size of antiderivative = 4.26

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{b\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d} - \frac{\left(\frac{c\,d}{e^2}+\frac{c\,x}{e}\right)\sqrt{d-e\,x}}{\sqrt{d+e\,x}} - \frac{\frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{a\,e^2}{2}+\frac{15\,a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}}{\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{32\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}} - \frac{a\,e^2\,\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,d^3} + \frac{a\,e^2\,\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)}{2\,d^3} + \frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{32\,d^3\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^3*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] (b*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - (((c*d)/
e^2 + (c*x)/e)*(d - e*x)^(1/2))/(d + e*x)^(1/2) - ((a*e^2*((d + e*x)^(1/2)
- d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (a*e^2)/2 + (15*a*e^2*((d + e
*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e
*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^
(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2
) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6) - (a*e^2*log(((d + e*x)^(1/2
) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (a*e^2*log(((d + e*x)^
(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(2*d^3) + (a*e^2*((d
 + e*x)^(1/2) - d^(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)
```

3.137 $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 126

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4} - \frac{(4bd^2+3ae^2)\sqrt{d-ex}\sqrt{d+ex}}{8d^4x^2} - \frac{(8cd^4+4bd^2e^2+3ae^4)\,\text{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{8d^5}$$

```
[Out] -1/8*(3*a*e^4+4*b*d^2*e^2+8*c*d^4)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/
d^5-1/4*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^4-1/8*(3*a*e^2+4*b*d^2)*(-e*x+
d)^(1/2)*(e*x+d)^(1/2)/d^4/x^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {534, 1265, 911, 1171, 393, 214}

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d^2-e^2x^2}\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)(3ae^4+4bd^2e^2+8cd^4)}{8d^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(3ae^2+4bd^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/4*(a*(d^2 - e^2*x^2))/(d^2*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((4*b*d^2
+ 3*a*e^2)*(d^2 - e^2*x^2))/(8*d^4*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*c
*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^
2]/d])/(8*d^5*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^3\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^3}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-3a-\frac{4(cd^4+bd^2e^2)}{e^4}+\frac{4cd^2x^2}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{(4bd^2+3ae^2)(d^2-e^2x^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad-\frac{\left(\left(4b+\frac{8cd^2}{e^2}+\frac{3ae^2}{d^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{8d^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{(4bd^2+3ae^2)(d^2-e^2x^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad-\frac{(8cd^4+4bd^2e^2+3ae^4)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^5\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{d\sqrt{d-ex}\sqrt{d+ex}(2ad^2+4bd^2x^2+3ae^2x^2)}{x^4}+2(8cd^4+4bd^2e^2+3ae^4)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{8d^5}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/8*((d*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*d^2 + 4*b*d^2*x^2 + 3*a*e^2*x^2))
/x^4 + 2*(8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e
*x]])/d^5
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3a\,e^2x^2+4b\,d^2x^2+2a\,d^2)}{8d^4x^4}-\frac{(3e^4a+4e^2d^2b+8d^4c)\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)\sqrt{(ex+d)(-ex+d)}}{8d^4\sqrt{d^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(3\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(d)+d\right)}{x}\right)a\,e^4x^4+4\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(d)+d\right)}{x}\right)b\,d^2e^2x^4+8\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(\right.}{x}\right.\right.}{8d^5\sqrt{-e^2x^2+d^2}\,x}$

```
[In] int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/8*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*a*e^2*x^2+4*b*d^2*x^2+2*a*d^2)/d^4/x^4
-1/8/d^4*(3*a*e^4+4*b*d^2*e^2+8*c*d^4)/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x)*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$=\frac{(8\,cd^4+4\,bd^2e^2+3\,ae^4)x^4\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-(2\,ad^3+(4\,bd^3+3\,ade^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{8\,d^5x^4}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/8*((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*log((sqrt(e*x + d)*sqrt(-e*x + d
) - d)/x) - (2*a*d^3 + (4*b*d^3 + 3*a*d*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x +
 d))/(d^5*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.53

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{c\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{be^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^3} - \frac{3\,ae^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d^5} - \frac{\sqrt{-e^2x^2+d^2}b}{2\,d^2x^2} - \frac{3\,\sqrt{-e^2x^2+d^2}ae^2}{8\,d^4x^2} - \frac{\sqrt{-e^2x^2+d^2}a}{4\,d^2x^4}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
[Out] -c*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*b*e^2*log(2*
d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*a*e^4*log(2*d^2/abs
(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 1/2*sqrt(-e^2*x^2 + d^2)*b/(d^
2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^2) - 1/4*sqrt(-e^2*x^2 + d^2
)*a/(d^2*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. 2(108) = 216.

Time = 0.61 (sec) , antiderivative size = 767, normalized size of antiderivative = 6.09

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{\left(8\,cd^4e+4\,bd^2e^3+3\,ae^5\right)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d^5} - \frac{\left(8\,cd^4e+4\,bd^2e^3+3\,ae^5\right)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}-\right.\right.}{d^5}}{}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/8*((8*c*d^4*e + 4*b*d^2*e^3 + 3*a*e^5)*log(abs(-(sqrt(2)*sqrt(d) - sqrt(
-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))
 + 2))/d^5 - (8*c*d^4*e + 4*b*d^2*e^3 + 3*a*e^5)*log(abs(-(sqrt(2)*sqrt(d)
- sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*
x + d)) - 2))/d^5 - 4*(4*b*d^2*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt
(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 5*a*e^5*(
(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*s
qrt(d) - sqrt(-e*x + d)))^7 - 16*b*d^2*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x +
d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 + 1
2*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(
sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 - 64*b*d^2*e^3*((sqrt(2)*sqrt(d) - sqr
t(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d
)))^3 + 48*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e
*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 + 256*b*d^2*e^3*((sqrt(2)*sqr
t(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqr
t(-e*x + d))) + 320*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d)
 - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))/((((sqrt(2)*sqrt(d) -
 sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x
 + d)))^2 - 4)^4*d^5))/e
```

Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 932, normalized size of antiderivative = 7.40

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{\frac{a\,e^4}{4}+\frac{6\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{53\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}-\frac{87\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}+\frac{657\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{4\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^8}-\frac{121\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{10}}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{10}}}{\frac{256\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}-\frac{1024\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}+\frac{1536\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^8}-\frac{1024\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{10}}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{10}}+\frac{256\,d^5\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{12}}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{12}}}$$
$$-\frac{\frac{b\,e^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{b\,e^2}{2}+\frac{15\,b\,e^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}}{\frac{16\,d^3\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{32\,d^3\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{16\,d^3\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}}$$
$$+\frac{c\,\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d}-\frac{3\,a\,e^4\,\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{8\,d^5}$$
$$-\frac{b\,e^2\,\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,d^3}+\frac{3\,a\,e^4\,\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)}{8\,d^5}+\frac{b\,e^2\,\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)}{2\,d^3}$$
$$+\frac{7\,a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{256\,d^5\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+\frac{a\,e^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{1024\,d^5\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{b\,e^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{32\,d^3\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^5*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((a*e^4)/4 + (6*a*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(
1/2))^2 - (53*a*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^
(1/2))^4) - (87*a*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(
1/2))^6 + (657*a*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2) - d
^(1/2))^8) - (121*a*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) -
d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^
(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1
/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2
))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2)
)^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))
^12) - ((b*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2
 - (b*e^2)/2 + (15*b*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2)
 - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1
/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))
```

```
^6) + (c*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 -
 1) - log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - (3
*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(8*d^5
) - (b*e^2*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2
*d^3) + (3*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/
2))^2 - 1))/(8*d^5) + (b*e^2*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(
1/2) - d^(1/2))^2 - 1))/(2*d^3) + (7*a*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/(
256*d^5*((d - e*x)^(1/2) - d^(1/2))^2) + (a*e^4*((d + e*x)^(1/2) - d^(1/2))
^4)/(1024*d^5*((d - e*x)^(1/2) - d^(1/2))^4) + (b*e^2*((d + e*x)^(1/2) - d^
(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)
```

3.138 $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 212

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{6d^2x^6}-\frac{(6bd^2+5ae^2)\sqrt{d-ex}\sqrt{d+ex}}{24d^4x^4}-\frac{(8cd^4+6bd^2e^2+5ae^4)\sqrt{d-ex}\sqrt{d+ex}}{16d^6x^2}-\frac{e^2(8cd^4+6bd^2e^2+5ae^4)\sqrt{d^2-e^2x^2}\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/6*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^6-1/24*(5*a*e^2+6*b*d^2)*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/d^4/x^4-1/16*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*(-e*x+d)^(1
/2)*(e*x+d)^(1/2)/d^6/x^2-1/16*e^2*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*arctanh((-
e^2*x^2+d^2)^(1/2)/d)*(-e^2*x^2+d^2)^(1/2)/d^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {534, 1265, 911, 1171, 393, 205, 214}

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{e^2\sqrt{d^2-e^2x^2}\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)(5ae^4+6bd^2e^2+8cd^4)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(5ae^4+6bd^2e^2+8cd^4)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(5ae^2+6bd^2)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/6*(a*(d^2 - e^2*x^2))/(d^2*x^6*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*b*d^2
+ 5*a*e^2)*(d^2 - e^2*x^2))/(24*d^4*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*
c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*(d^2 - e^2*x^2))/(16*d^6*x^2*Sqrt[d - e*x]*S
qrt[d + e*x]) - (e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Sqrt[d^2 - e^2*x^2]*
ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^7\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^4\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{\left(2cd^2+be^2\right)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^4}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-5a-\frac{6\left(cd^4+bd^2e^2\right)}{e^4}+\frac{6cd^2x^2}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^3}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{6d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{(6bd^2+5ae^2)\,(d^2-e^2x^2)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}}$$
$$-\frac{\left(\left(6b+\frac{8cd^2}{e^2}+\frac{5ae^2}{d^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{8d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{(6bd^2+5ae^2)\,(d^2-e^2x^2)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}}$$
$$-\frac{(8cd^4+6bd^2e^2+5ae^4)\,(d^2-e^2x^2)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(e^2\left(6b+\frac{8cd^2}{e^2}+\frac{5ae^2}{d^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x\right.}{16d^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{(6bd^2+5ae^2)(d^2-e^2x^2)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{(8cd^4+6bd^2e^2+5ae^4)(d^2-e^2x^2)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{e^2(8cd^4+6bd^2e^2+5ae^4)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{d\sqrt{d-ex}\sqrt{d+ex}(6(2bd^4x^2+4cd^4x^4+3bd^2e^2x^4)+a(8d^4+10d^2e^2x^2+15e^4x^4))}{x^6} + 6e^2(8cd^4+6bd^2e^2+5ae^4)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{48d^7}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/48*((d*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(2*b*d^4*x^2 + 4*c*d^4*x^4 + 3*b*d
^2*e^2*x^4) + a*(8*d^4 + 10*d^2*e^2*x^2 + 15*e^4*x^4)))/x^6 + 6*e^2*(8*c*d^
4 + 6*b*d^2*e^2 + 5*a*e^4)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e*x]])/d^7
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15a\,e^4x^4+18b\,d^2e^2x^4+24c\,d^4x^4+10a\,d^2e^2x^2+12b\,d^4x^2+8a\,d^4)}{48d^6x^6} - \frac{e^2(5e^4a+6e^2d^2b+8d^4c)\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2}}{x}\right.}{16d^6\sqrt{d^2}\,\sqrt{ex+d}\,\sqrt{-e}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(15\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\ \mathrm{csgn}(d)+d\right)}{x}\right)a\,e^6x^6+18\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\ \mathrm{csgn}(d)+d\right)}{x}\right)b\,d^2e^4x^6+24\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\ \mathrm{c}\right.}{x}\right.\right.}{}$

```
[In] int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/48*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*a*e^4*x^4+18*b*d^2*e^2*x^4+24*c*d^4*
x^4+10*a*d^2*e^2*x^2+12*b*d^4*x^2+8*a*d^4)/d^6/x^6-1/16*e^2*(5*a*e^4+6*b*d^
2*e^2+8*c*d^4)/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2)
)/x)*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{3\,(8\,cd^4e^2+6\,bd^2e^4+5\,ae^6)x^6\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-(8\,ad^5+3\,(8\,cd^5+6\,bd^3e^2+5\,ade^4)x^4+2\,(6\,bd^5+}{48\,d^7x^6}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(8*c*d^4*e^2 + 6*b*d^2*e^4 + 5*a*e^6)*x^6*log((sqrt(e*x + d)*sqrt(-
e*x + d) - d)/x) - (8*a*d^5 + 3*(8*c*d^5 + 6*b*d^3*e^2 + 5*a*d*e^4)*x^4 + 2
*(6*b*d^5 + 5*a*d^3*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^7*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.28

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{ce^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^3} - \frac{3\,be^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d^5} - \frac{5\,ae^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^7} - \frac{\sqrt{-e^2x^2+d^2}c}{2\,d^2x^2} - \frac{3\sqrt{-e^2x^2+d^2}be^2}{8\,d^4x^2} - \frac{5\sqrt{-e^2x^2+d^2}ae^4}{16\,d^6x^2} - \frac{\sqrt{-e^2x^2+d^2}b}{4\,d^2x^4} - \frac{5\sqrt{-e^2x^2+d^2}ae^2}{24\,d^4x^4} - \frac{\sqrt{-e^2x^2+d^2}a}{6\,d^2x^6}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*c*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*b*
e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 5/16*a*e^6*lo
g(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 - 1/2*sqrt(-e^2*x^2 +
 d^2)*c/(d^2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^2) - 5/16*sqrt(-e
^2*x^2 + d^2)*a*e^4/(d^6*x^2) - 1/4*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^4) - 5/24
*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^4) - 1/6*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^6
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1434 vs. $2(184) = 368$.

Time = 0.77 (sec) , antiderivative size = 1434, normalized size of antiderivative = 6.76

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/48*(3*(8*c*d^4*e^3 + 6*b*d^2*e^5 + 5*a*e^7)*log(abs(-(sqrt(2)*sqrt(d) -
sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x
+ d)) + 2))/d^7 - 3*(8*c*d^4*e^3 + 6*b*d^2*e^5 + 5*a*e^7)*log(abs(-(sqrt(2)
*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) -
 sqrt(-e*x + d)) - 2))/d^7 - 4*(24*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x
+ d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11
+ 30*b*d^2*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 33*a*e^7*((sqrt(2)*sqrt(d) -
 sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x
 + d)))^11 - 288*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d
) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 168*b*d^2*e^5*((s
qrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqr
t(d) - sqrt(-e*x + d)))^9 + 20*a*e^7*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 768*c*d
^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(s
qrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 192*b*d^2*e^5*((sqrt(2)*sqrt(d) - sqr
t(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d
)))^7 + 1440*a*e^7*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt
(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 3072*c*d^4*e^3*((sqrt(2)*
sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) -
sqrt(-e*x + d)))^5 + 768*b*d^2*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt
(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 + 5760*a*e^
7*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2
)*sqrt(d) - sqrt(-e*x + d)))^5 - 18432*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-
e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))
^3 - 10752*b*d^2*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sq
```

```
rt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 + 1280*a*e^7*((sqrt(2)*sq
rt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sq
rt(-e*x + d)))^3 + 24576*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt
(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))) + 30720*b*d^2
*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d))) + 33792*a*e^7*((sqrt(2)*sqrt(d) - sqrt(-e*x
 + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))/(
(((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)
*sqrt(d) - sqrt(-e*x + d)))^2 - 4)^6*d^7))/e
```

Mupad [B] (verification not implemented)

Time = 24.81 (sec) , antiderivative size = 1621, normalized size of antiderivative = 7.65

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^7*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((b*e^4)/4 + (6*b*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(
1/2))^2 - (53*b*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^
(1/2))^4) - (87*b*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(
1/2))^6 + (657*b*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2) - d
^(1/2))^8) - (121*b*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) -
d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^
(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1
/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2
))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2)
)^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))
^12) - ((c*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2
 - (c*e^2)/2 + (15*c*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2)
 - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1
/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))
^6) + ((a*e^6)/6 + (4*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2)
 - d^(1/2))^2 + (71*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) -
 d^(1/2))^4 - (1558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/(3*((d - e*x)^(1/2
) - d^(1/2))^6) - (540*a*e^6*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2
) - d^(1/2))^8 + (4248*a*e^6*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/
2) - d^(1/2))^10 - (7683*a*e^6*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(
1/2) - d^(1/2))^12 + (5558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^14)/((d - e*x)
^(1/2) - d^(1/2))^14 - (3643*a*e^6*((d + e*x)^(1/2) - d^(1/2))^16)/(2*((d -
 e*x)^(1/2) - d^(1/2))^16))/((4096*d^7*((d + e*x)^(1/2) - d^(1/2))^6)/((d -
 e*x)^(1/2) - d^(1/2))^6 - (24576*d^7*((d + e*x)^(1/2) - d^(1/2))^8)/((d -
e*x)^(1/2) - d^(1/2))^8 + (61440*d^7*((d + e*x)^(1/2) - d^(1/2))^10)/((d -
```

e*x)^(1/2) - d^(1/2))^10 - (81920*d^7*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))^12 + (61440*d^7*((d + e*x)^(1/2) - d^(1/2))^14)/((d - e*x)^(1/2) - d^(1/2))^14 - (24576*d^7*((d + e*x)^(1/2) - d^(1/2))^16)/((d - e*x)^(1/2) - d^(1/2))^16 + (4096*d^7*((d + e*x)^(1/2) - d^(1/2))^18)/((d - e*x)^(1/2) - d^(1/2))^18) - (5*a*e^6*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(16*d^7) - (3*b*e^4*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(8*d^5) - (c*e^2*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (5*a*e^6*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(16*d^7) + (3*b*e^4*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(8*d^5) + (c*e^2*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(2*d^3) + (197*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/(8192*d^7*((d - e*x)^(1/2) - d^(1/2))^2) + (5*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/(4096*d^7*((d - e*x)^(1/2) - d^(1/2))^4) + (a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/(24576*d^7*((d - e*x)^(1/2) - d^(1/2))^6) + (7*b*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/(256*d^5*((d - e*x)^(1/2) - d^(1/2))^2) + (b*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(1024*d^5*((d - e*x)^(1/2) - d^(1/2))^4) + (c*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)

3.139 $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 216

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(5cd^4+6bd^2e^2+8ae^4)\,x\sqrt{d-ex}\sqrt{d+ex}}{16e^6} - \frac{(5cd^2+6be^2)\,x^3\sqrt{d-ex}\sqrt{d+ex}}{24e^4} + \frac{cx^5(-d+ex)\sqrt{d+ex}}{6e^2\sqrt{d-ex}} + \frac{d^2(5cd^4+6bd^2e^2+8ae^4)\,\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] 1/6*c*x^5*(e*x-d)*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-1/16*(8*a*e^4+6*b*d^2*e^
2+5*c*d^4)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6-1/24*(6*b*e^2+5*c*d^2)*x^3*(-
e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4+1/16*d^2*(8*a*e^4+6*b*d^2*e^2+5*c*d^4)*arcta
n(e*x/(-e^2*x^2+d^2)^(1/2))*(-e^2*x^2+d^2)^(1/2)/e^7/(-e*x+d)^(1/2)/(e*x+d)
^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {534, 1281, 470, 327, 223, 209}

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{d^2\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(8ae^4+6bd^2e^2+5cd^4)}{16e^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2-e^2x^2)\,(8ae^4+6bd^2e^2+5cd^4)}{16e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{x^3(d^2-e^2x^2)\,(6be^2+5cd^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/16*((5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*x*(d^2 - e^2*x^2))/(e^6*Sqrt[d - e
*x]*Sqrt[d + e*x]) - ((5*c*d^2 + 6*b*e^2)*x^3*(d^2 - e^2*x^2))/(24*e^4*Sqrt
[d - e*x]*Sqrt[d + e*x]) - (c*x^5*(d^2 - e^2*x^2))/(6*e^2*Sqrt[d - e*x]*Sqr
t[d + e*x]) + (d^2*(5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
```

```
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{x^2(-6ae^2-(5cd^2+6be^2)x^2)}{\sqrt{d^2-e^2x^2}}\,dx}{6e^2\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{(5cd^2+6be^2)\,x^3(d^2-e^2x^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad + \frac{\left((5cd^4+6bd^2e^2+8ae^4)\sqrt{d^2-e^2x^2}\right)\int \frac{x^2}{\sqrt{d^2-e^2x^2}}\,dx}{8e^4\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{(5cd^4+6bd^2e^2+8ae^4)\,x(d^2-e^2x^2)}{16e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5cd^2+6be^2)\,x^3(d^2-e^2x^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad - \frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(d^2(5cd^4+6bd^2e^2+8ae^4)\sqrt{d^2-e^2x^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e^6\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{(5cd^4+6bd^2e^2+8ae^4)\,x(d^2-e^2x^2)}{16e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5cd^2+6be^2)\,x^3(d^2-e^2x^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad - \frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(d^2(5cd^4+6bd^2e^2+8ae^4)\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e^6\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{(5cd^4+6bd^2e^2+8ae^4)\,x(d^2-e^2x^2)}{16e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5cd^2+6be^2)\,x^3(d^2-e^2x^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad - \frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{d^2(5cd^4+6bd^2e^2+8ae^4)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{-ex\sqrt{d-ex}\sqrt{d+ex}(6(3bd^2e^2+4ae^4+2be^4x^2)+c(15d^4+10d^2e^2x^2+8e^4x^4))+6d^2(5cd^4+6bd^2e^2+8a}{48e^7}$$

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(3*b*d^2*e^2 + 4*a*e^4 + 2*b*e^4*x^2)
 + c*(15*d^4 + 10*d^2*e^2*x^2 + 8*e^4*x^4))) + 6*d^2*(5*c*d^4 + 6*b*d^2*e^2
 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(48*e^7)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{x(8c\,x^4e^4+12b\,e^4x^2+10c\,d^2e^2x^2+24e^4a+18e^2d^2b+15d^4c)\sqrt{-ex+d}\,\sqrt{ex+d}}{48e^6}+\frac{d^2(8e^4a+6e^2d^2b+5d^4c)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)\sqrt{(}}{16e^6\sqrt{e^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(8\,\mathrm{csgn}(e)c\,e^5x^5\sqrt{-e^2x^2+d^2}+12\,\mathrm{csgn}(e)b\,e^5x^3\sqrt{-e^2x^2+d^2}+10\,\mathrm{csgn}(e)c\,d^2e^3x^3\sqrt{-e^2x^2+d^2}+24\sqrt{-e^2x^2+d^2}\ \mathrm{cs}\right.}{}$

```
[In] int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/48*x*(8*c*e^4*x^4+12*b*e^4*x^2+10*c*d^2*e^2*x^2+24*a*e^4+18*b*d^2*e^2+15
*c*d^4)/e^6*(-e*x+d)^(1/2)*(e*x+d)^(1/2)+1/16*d^2*(8*a*e^4+6*b*d^2*e^2+5*c*
d^4)/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x+d)*(-
e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{(8\,ce^5x^5+2\,(5\,cd^2e^3+6\,be^5)x^3+3\,(5\,cd^4e+6\,bd^2e^3+8\,ae^5)x)\sqrt{ex+d}\sqrt{-ex+d}+6\,(5\,cd^6+6\,bd^4e^2+}{48\,e^7}$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/48*((8*c*e^5*x^5 + 2*(5*c*d^2*e^3 + 6*b*e^5)*x^3 + 3*(5*c*d^4*e + 6*b*d^
2*e^3 + 8*a*e^5)*x)*sqrt(e*x + d)*sqrt(-e*x + d) + 6*(5*c*d^6 + 6*b*d^4*e^2
 + 8*a*d^2*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/e^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{\sqrt{-e^2x^2+d^2}cx^5}{6\,e^2} - \frac{5\sqrt{-e^2x^2+d^2}cd^2x^3}{24\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^3}{4\,e^2} \\ &+ \frac{5\,cd^6 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e^6} + \frac{3\,bd^4 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^4} + \frac{ad^2 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} \\ &- \frac{5\sqrt{-e^2x^2+d^2}cd^4x}{16\,e^6} - \frac{3\sqrt{-e^2x^2+d^2}bd^2x}{8\,e^4} - \frac{\sqrt{-e^2x^2+d^2}ax}{2\,e^2}\end{aligned}$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*c*x^5/e^2 - 5/24*sqrt(-e^2*x^2 + d^2)*c*d^2*x^3/e
^4 - 1/4*sqrt(-e^2*x^2 + d^2)*b*x^3/e^2 + 5/16*c*d^6*arcsin(e^2*x/(d*sqrt(e
^2)))/(sqrt(e^2)*e^6) + 3/8*b*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^
4) + 1/2*a*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 5/16*sqrt(-e^2
*x^2 + d^2)*c*d^4*x/e^6 - 3/8*sqrt(-e^2*x^2 + d^2)*b*d^2*x/e^4 - 1/2*sqrt(-
e^2*x^2 + d^2)*a*x/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{(33\,cd^5+30\,bd^3e^2+24\,ade^4-(85\,cd^4+54\,bd^2e^2+24\,ae^4-2\,(55\,cd^3+18\,bde^2-(45\,cd^2+6\,be^2+4\,((ex-}{}$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*((33*c*d^5 + 30*b*d^3*e^2 + 24*a*d*e^4 - (85*c*d^4 + 54*b*d^2*e^2 + 24
*a*e^4 - 2*(55*c*d^3 + 18*b*d*e^2 - (45*c*d^2 + 6*b*e^2 + 4*((e*x + d)*c -
5*c*d)*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x
+ d) + 6*(5*c*d^6 + 6*b*d^4*e^2 + 8*a*d^2*e^4)*arcsin(1/2*sqrt(2)*sqrt(e*x
+ d)/sqrt(d)))/e^7
```

Mupad [B] (verification not implemented)

Time = 27.91 (sec) , antiderivative size = 1132, normalized size of antiderivative = 5.24

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
a*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*a*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - ((175*c*d^6*(
(d + e*x)^(1/2) - d^(1/2))^3)/(12*((d - e*x)^(1/2) - d^(1/2))^3) + (311*c*d
^6*((d + e*x)^(1/2) - d^(1/2))^5)/(4*((d - e*x)^(1/2) - d^(1/2))^5) - (8361
*c*d^6*((d + e*x)^(1/2) - d^(1/2))^7)/(4*((d - e*x)^(1/2) - d^(1/2))^7) + (
42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^9)/(6*((d - e*x)^(1/2) - d^(1/2))^9
) - (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d - e*x)^(1/2) - d^(1
/2))^11) + (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^13)/(2*((d - e*x)^(1/2)
 - d^(1/2))^13) - (42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^15)/(6*((d - e*x
)^(1/2) - d^(1/2))^15) + (8361*c*d^6*((d + e*x)^(1/2) - d^(1/2))^17)/(4*((d
 - e*x)^(1/2) - d^(1/2))^17) - (311*c*d^6*((d + e*x)^(1/2) - d^(1/2))^19)/(
4*((d - e*x)^(1/2) - d^(1/2))^19) - (175*c*d^6*((d + e*x)^(1/2) - d^(1/2))^
21)/(12*((d - e*x)^(1/2) - d^(1/2))^21) - (5*c*d^6*((d + e*x)^(1/2) - d^(1/
```

```
2))^23)/(4*((d - e*x)^(1/2) - d^(1/2))^23) + (5*c*d^6*((d + e*x)^(1/2) - d^
(1/2)))/(4*((d - e*x)^(1/2) - d^(1/2))))/(e^7*(((d + e*x)^(1/2) - d^(1/2))^
2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^12) - ((23*b*d^4*((d + e*x)^(1/2) - d^
(1/2))^3)/(2*((d - e*x)^(1/2) - d^(1/2))^3) - (333*b*d^4*((d + e*x)^(1/2) -
 d^(1/2))^5)/(2*((d - e*x)^(1/2) - d^(1/2))^5) + (671*b*d^4*((d + e*x)^(1/2
) - d^(1/2))^7)/(2*((d - e*x)^(1/2) - d^(1/2))^7) - (671*b*d^4*((d + e*x)^(
1/2) - d^(1/2))^9)/(2*((d - e*x)^(1/2) - d^(1/2))^9) + (333*b*d^4*((d + e*x
)^(1/2) - d^(1/2))^11)/(2*((d - e*x)^(1/2) - d^(1/2))^11) - (23*b*d^4*((d +
 e*x)^(1/2) - d^(1/2))^13)/(2*((d - e*x)^(1/2) - d^(1/2))^13) - (3*b*d^4*((
d + e*x)^(1/2) - d^(1/2))^15)/(2*((d - e*x)^(1/2) - d^(1/2))^15) + (3*b*d^4
*((d + e*x)^(1/2) - d^(1/2)))/(2*((d - e*x)^(1/2) - d^(1/2))))/(e^5*(((d +
e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^8) + (2*a*d^2*at
an(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/e^3 + (3*b*d^4
*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*e^5) + (
5*c*d^6*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(4*e
^7)
```

3.140 $\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 128

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(3cd^2+4be^2)\,x\sqrt{d-ex}\sqrt{d+ex}}{8e^4}+\frac{cx^3(-d+ex)\sqrt{d+ex}}{4e^2\sqrt{d-ex}} - \frac{(3cd^4+4bd^2e^2+8ae^4)\arctan\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{4e^5}$$

```
[Out] -1/4*(8*a*e^4+4*b*d^2*e^2+3*c*d^4)*arctan((-e*x+d)^(1/2)/(e*x+d)^(1/2))/e^5
+1/4*c*x^3*(e*x-d)*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-1/8*(4*b*e^2+3*c*d^2)*x
*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {534, 1173, 396, 223, 209}

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(8ae^4+4bd^2e^2+3cd^4)}{8e^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{x(d^2-e^2x^2)(4be^2+3cd^2)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/8*((3*c*d^2 + 4*b*e^2)*x*(d^2 - e^2*x^2))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*
x]) - (c*x^3*(d^2 - e^2*x^2))/(4*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((3*c*d
^4 + 4*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(8*e^5*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}} \\ &= -\frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{\sqrt{d^2-e^2x^2}\int \frac{-4ae^2-(3cd^2+4be^2)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2\sqrt{d-ex}\sqrt{d+ex}}\end{aligned}$$

$$= -\frac{(3cd^2+4be^2)\,x(d^2-e^2x^2)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left((-8ae^4+d^2(-3cd^2-4be^2))\sqrt{d^2-e^2x^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(3cd^2+4be^2)\,x(d^2-e^2x^2)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left((-8ae^4+d^2(-3cd^2-4be^2))\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(3cd^2+4be^2)\,x(d^2-e^2x^2)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{(3cd^4+4bd^2e^2+8ae^4)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{-ex\sqrt{d-ex}\sqrt{d+ex}(3cd^2+4be^2+2ce^2x^2)+2(3cd^4+4bd^2e^2+8ae^4)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{8e^5}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
[Out] (-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(3*c*d^2 + 4*b*e^2 + 2*c*e^2*x^2)) + 2*(
3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(8*e^
5)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{x(2cx^2e^2+4be^2+3cd^2)\sqrt{-ex+d}\,\sqrt{ex+d}}{8e^4}+\frac{(8e^4a+4e^2d^2b+3d^4c)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)\sqrt{(ex+d)(-ex+d)}}{8e^4\sqrt{e^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(2\,\mathrm{csgn}(e)ce^3x^3\sqrt{-e^2x^2+d^2}+4\,\mathrm{csgn}(e)e^3\sqrt{-e^2x^2+d^2}\,bx+3\,\mathrm{csgn}(e)e\sqrt{-e^2x^2+d^2}\,cd^2x-8\arctan\left(\frac{\mathrm{csgn}(e)ex}{\sqrt{-e^2x^2+d^2}}\right.\right.}{8e^5\sqrt{-e^2x^2+d^2}}$

```
[In] int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x*(2*c*e^2*x^2+4*b*e^2+3*c*d^2)/e^4*(-e*x+d)^(1/2)*(e*x+d)^(1/2)+1/8*(
8*a*e^4+4*b*d^2*e^2+3*c*d^4)/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(2\,ce^3x^3+(3\,cd^2e+4\,be^3)x)\sqrt{ex+d}\sqrt{-ex+d}+2\,(3\,cd^4+4\,bd^2e^2+8\,ae^4)\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)}{8\,e^5}$$

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/8*((2*c*e^3*x^3 + (3*c*d^2*e + 4*b*e^3)*x)*sqrt(e*x + d)*sqrt(-e*x + d)
+ 2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d)
- d)/(e*x)))/e^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^3}{4\,e^2}+\frac{a\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}+\frac{3\,cd^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}e^4}+\frac{bd^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2}-\frac{3\,\sqrt{-e^2x^2+d^2}cd^2x}{8\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx}{2\,e^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*c*x^3/e^2 + a*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^
2) + 3/8*c*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) + 1/2*b*d^2*arcs
in(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 3/8*sqrt(-e^2*x^2 + d^2)*c*d^2*x/
e^4 - 1/2*sqrt(-e^2*x^2 + d^2)*b*x/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{(5\,cd^3+4\,bde^2-(9\,cd^2+4\,be^2+2\,((ex+d)c-3\,cd)(ex+d))(ex+d))\sqrt{ex+d}\sqrt{-ex+d}+2\,(3\,cd^4+4\,b}{8\,e^5}$$

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((5*c*d^3 + 4*b*d*e^2 - (9*c*d^2 + 4*b*e^2 + 2*((e*x + d)*c - 3*c*d)*(e
*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d) + 2*(3*c*d^4 + 4*b*d^2*e^2
 + 8*a*e^4)*arcsin(1/2*sqrt(2)*sqrt(e*x + d)/sqrt(d)))/e^5
```

Mupad [B] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 651, normalized size of antiderivative = 5.09

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\frac{14\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{14\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{2\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{2\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\,\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^4}$$
$$-\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}$$
$$-\frac{\frac{23\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{333\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{671\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{671\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^9}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^9}+\frac{333\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{11}}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{11}}}{e^5\,\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^8}$$
$$+\frac{2\,b\,d^2\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{e^3}+\frac{3\,c\,d^4\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,e^5}$$

```
[In] int((a + b*x^2 + c*x^4)/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)

[Out] ((14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
b*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*b*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*a*atan((e*
((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(e
^2)^(1/2) - ((23*c*d^4*((d + e*x)^(1/2) - d^(1/2))^3)/(2*((d - e*x)^(1/2) -
 d^(1/2))^3) - (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^5)/(2*((d - e*x)^(1/2
) - d^(1/2))^5) + (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^7)/(2*((d - e*x)^(
1/2) - d^(1/2))^7) - (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^9)/(2*((d - e*x
)^(1/2) - d^(1/2))^9) + (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d -
 e*x)^(1/2) - d^(1/2))^11) - (23*c*d^4*((d + e*x)^(1/2) - d^(1/2))^13)/(2*(
(d - e*x)^(1/2) - d^(1/2))^13) - (3*c*d^4*((d + e*x)^(1/2) - d^(1/2))^15)/(
2*((d - e*x)^(1/2) - d^(1/2))^15) + (3*c*d^4*((d + e*x)^(1/2) - d^(1/2)))/(
2*((d - e*x)^(1/2) - d^(1/2))))/(e^5*(((d + e*x)^(1/2) - d^(1/2))^2/((d - e
*x)^(1/2) - d^(1/2))^2 + 1)^8) + (2*b*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/
((d - e*x)^(1/2) - d^(1/2))))/e^3 + (3*c*d^4*atan(((d + e*x)^(1/2) - d^(1/2
))/((d - e*x)^(1/2) - d^(1/2))))/(2*e^5)
```

3.141 $\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 102

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x}+\frac{cx(-d+ex)\sqrt{d+ex}}{2e^2\sqrt{d-ex}} - \frac{(cd^2+2be^2)\arctan\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^3}$$

```
[Out] -(2*b*e^2+c*d^2)*arctan((-e*x+d)^(1/2)/(e*x+d)^(1/2))/e^3+1/2*c*x*(e*x-d)*(
e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {534, 1279, 396, 223, 209}

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} + \frac{\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(2be^2+cd^2)}{2e^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx(d^2-e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((a*(d^2 - e^2*x^2))/(d^2*x*Sqrt[d - e*x]*Sqrt[d + e*x])) - (c*x*(d^2 - e^
2*x^2))/(2*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((c*d^2 + 2*b*e^2)*Sqrt[d^2 -
```

e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^2\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\ &= -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}}-\frac{\sqrt{d^2-e^2x^2}\int \frac{-bd^2-cd^2x^2}{\sqrt{d^2-e^2x^2}}\,dx}{d^2\sqrt{d-ex}\sqrt{d+ex}}\end{aligned}$$

$$= -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx(d^2-e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(2b+\frac{cd^2}{e^2}\right)\sqrt{d^2-e^2x^2}\right)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx(d^2-e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(2b+\frac{cd^2}{e^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx(d^2-e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{(cd^2+2be^2)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int\frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{-\frac{e\sqrt{d-ex}\sqrt{d+ex}(2ae^2+cd^2x^2)}{d^2x} + 2(cd^2+2be^2)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{2e^3}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-((e*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*e^2 + c*d^2*x^2))/(d^2*x)) + 2*(c*d^
2 + 2*b*e^2)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(2*e^3)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(c\,d^2x^2+2a\,e^2)}{2e^2d^2x} + \frac{(2b\,e^2+c\,d^2)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)\sqrt{(ex+d)(-ex+d)}}{2e^2\sqrt{e^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(\text{csgn}(e)c\,d^2e\,x^2\sqrt{-e^2x^2+d^2}-2\arctan\left(\frac{\text{csgn}(e)ex}{\sqrt{-e^2x^2+d^2}}\right)b\,d^2e^2x-\arctan\left(\frac{\text{csgn}(e)ex}{\sqrt{-e^2x^2+d^2}}\right)c\,d^4x+2\,\text{csgn}(e)e^3\sqrt{-e^2x^2}}{2d^2e^3\sqrt{-e^2x^2+d^2}\,x}$

```
[In] int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(c*d^2*x^2+2*a*e^2)/e^2/d^2/x+1/2*(2*b*e^
2+c*d^2)/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x+d
)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\,(cd^4+2\,bd^2e^2)x\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+(cd^2ex^2+2\,ae^3)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^2e^3x}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*(c*d^4 + 2*b*d^2*e^2)*x*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(
e*x)) + (c*d^2*e*x^2 + 2*a*e^3)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^2*e^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{b\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}+\frac{cd^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}cx}{2\,e^2} - \frac{\sqrt{-e^2x^2+d^2}a}{d^2x}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] b*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/2*c*d^2*arcsin(e^2*x/(d*sqrt(e^
2)))/(sqrt(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*c*x/e^2 - sqrt(-e^2*x^2 + d
^2)*a/(d^2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(88) = 176.

Time = 0.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.34

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{8\,ae^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)}{\left(\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^2-4\right)d^2}-\left(\pi+2\arctan\left(\frac{\sqrt{ex+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)^2}{ex+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)}\right)\right)(cd^2+2\,be^2)+((ex}{2\,e^3}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/2*(8*a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
+ d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))/((((sqrt(2)*sqrt(d) - sqrt(-e*x +
d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2 - 4
)*d^2) - (pi + 2*arctan(1/2*sqrt(e*x + d)*((sqrt(2)*sqrt(d) - sqrt(-e*x + d
))^2/(e*x + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))*(c*d^2 + 2*b*e^2)
+ ((e*x + d)*c - c*d)*sqrt(e*x + d)*sqrt(-e*x + d))/e^3
```

Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.00

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\frac{14\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{14\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{2\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{2\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^4} - \frac{4\,b\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}} + \frac{2\,c\,d^2\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{e^3} - \frac{\left(\frac{a}{d}+\frac{a\,e\,x}{d^2}\right)\sqrt{d-e\,x}}{x\,\sqrt{d+e\,x}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
c*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*c*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
```

```
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*b*atan((e*
((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(e
^2)^(1/2) + (2*c*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^
(1/2))))/e^3 - ((a/d + (a*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d + e*x)^(1/2))
```

3.142 $\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 157

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(3bd^2+2ae^2)\,(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{c\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/3*a*(-e^2*x^2+d^2)/d^2/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/3*(2*a*e^2+3*b
*d^2)*(-e^2*x^2+d^2)/d^4/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+c*arctan(e*x/(-e^2*
x^2+d^2)^(1/2))*(-e^2*x^2+d^2)^(1/2)/e/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {534, 1279, 462, 223, 209}

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(d^2-e^2x^2)\,(2ae^2+3bd^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{c\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/3*(a*(d^2 - e^2*x^2))/(d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*b*d^2
+ 2*a*e^2)*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*Sqrt
[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(e*Sqrt[d - e*x]*Sqrt[d
+ e*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^4\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-3bd^2-2ae^2-3cd^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$
= -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(3bd^2+2ae^2)(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(c\sqrt{d^2-e^2x^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}
$$

$$
= -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(3bd^2+2ae^2)(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(c\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d-ex}\sqrt{d+ex}}
$$

$$
= -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(3bd^2+2ae^2)(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.52

$$
\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(3bd^2x^2+a(d^2+2e^2x^2))}{3d^4x^3} + \frac{2c\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{e}
$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(3*b*d^2*x^2 + a*(d^2 + 2*e^2*x^2)))/(d^4
*x^3) + (2*c*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/e
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(2a\,e^2x^2+3b\,d^2x^2+a\,d^2)}{3d^4x^3}+\frac{c\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)\sqrt{(ex+d)(-ex+d)}}{\sqrt{e^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(-3\arctan\left(\frac{\text{csgn}(e)ex}{\sqrt{-e^2x^2+d^2}}\right)c\,d^4x^3+2\ \text{csgn}(e)e^3\sqrt{-e^2x^2+d^2}\,a\,x^2+3\ \text{csgn}(e)e\sqrt{-e^2x^2+d^2}\,b\,d^2x^2+a\sqrt{-e^2x^2+d^2}\,d^2}{3d^4\sqrt{-e^2x^2+d^2}\,x^3e}$

```
[In] int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/3*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(2*a*e^2*x^2+3*b*d^2*x^2+a*d^2)/d^4/x^3+c
/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x+d)*(-e*x+d))^
(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{6\,cd^4x^3\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+(ad^2e+(3\,bd^2e+2\,ae^3)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{3\,d^4ex^3}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
[Out] -1/3*(6*c*d^4*x^3*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) + (a*d^2
*e + (3*b*d^2*e + 2*a*e^3)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^4*e*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.64

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{iae^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3 \\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0 \end{matrix} \middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{ae^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0 \end{matrix} \middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{ibeG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix} \middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{beG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix} \middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e} + \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3),
(0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**4) + a*e**3*meijerg(((3/2, 7/4, 2,
 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), d**2*exp_polar(-2*I*pi)/(
e**2*x**2))/(4*pi**(3/2)*d**4) + I*b*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2
)), ((1, 5/4, 3/2, 7/4, 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + b
*e*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((1/4
, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d**2/(e**2*x**2)
)/(4*pi**(3/2)*e) + c*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1
/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{c\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}b}{d^2x} - \frac{2\sqrt{-e^2x^2+d^2}ae^2}{3\,d^4x} - \frac{\sqrt{-e^2x^2+d^2}a}{3\,d^2x^3}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] c*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - sqrt(-e^2*x^2 + d^2)*b/(d^2*x) -
2/3*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x) - 1/3*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^
3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(139) = 278.

Time = 0.41 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.38

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{3\left(\pi+2\arctan\left(\frac{\sqrt{ex+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)^2}{ex+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)}\right)\right)c - \frac{4\left(3\,bd^2e^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^5+3\,ae^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(pi + 2*arctan(1/2*sqrt(e*x + d)*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))
^2/(e*x + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))*c - 4*(3*b*d^2*e^2*(
(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*s
qrt(d) - sqrt(-e*x + d)))^5 + 3*a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/s
qrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 - 24*b*d
^2*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(s
qrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 8*a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x
 + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3
+ 48*b*d^2*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))) + 48*a*e^4*((sqrt(2)*sqrt(d) - sq
rt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
d))))/((((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(
sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2 - 4)^3*d^4))/e
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{4\,c\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}} - \frac{\left(\frac{b}{d}+\frac{b\,e\,x}{d^2}\right)\,\sqrt{d-e\,x}}{x\,\sqrt{d+e\,x}} - \frac{\sqrt{d-e\,x}\,\left(\frac{a}{3\,d}+\frac{2\,a\,e^2\,x^2}{3\,d^3}+\frac{2\,a\,e^3\,x^3}{3\,d^4}+\frac{a\,e\,x}{3\,d^2}\right)}{x^3\,\sqrt{d+e\,x}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] - (4*c*atan((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) -
 d^(1/2)))))/(e^2)^(1/2) - ((b/d + (b*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d + e*
x)^(1/2)) - ((d - e*x)^(1/2)*(a/(3*d) + (2*a*e^2*x^2)/(3*d^3) + (2*a*e^3*x^
3)/(3*d^4) + (a*e*x)/(3*d^2)))/(x^3*(d + e*x)^(1/2))
```

3.143 $\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 160

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5bd^2+4ae^2)\,(d^2-e^2x^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(15cd^4+10bd^2e^2+8ae^4)\,(d^2-e^2x^2)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/5*a*(-e^2*x^2+d^2)/d^2/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/15*(4*a*e^2+5*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/15*(8*a*e^4+10
*b*d^2*e^2+15*c*d^4)*(-e^2*x^2+d^2)/d^6/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {534, 1279, 464, 270}

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(d^2-e^2x^2)\,(8ae^4+10bd^2e^2+15cd^4)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)\,(4ae^2+5bd^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/5*(a*(d^2 - e^2*x^2))/(d^2*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((5*b*d^2
+ 4*a*e^2)*(d^2 - e^2*x^2))/(15*d^4*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((15
*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*(d^2 - e^2*x^2))/(15*d^6*x*Sqrt[d - e*x]*S
qrt[d + e*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}\int\frac{a+bx^2+cx^4}{x^6\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\sqrt{d^2-e^2x^2}\int\frac{-5bd^2-4ae^2-5cd^2x^2}{x^4\sqrt{d^2-e^2x^2}}\,dx}{5d^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{(5bd^2+4ae^2)\,(d^2-e^2x^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad+\frac{\left((15cd^4-2e^2(-5bd^2-4ae^2))\,\sqrt{d^2-e^2x^2}\right)\int\frac{1}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^4\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}
$$

$$= -\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5bd^2+4ae^2)(d^2-e^2x^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(15cd^4+10bd^2e^2+8ae^4)(d^2-e^2x^2)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(15cd^4x^4+5bd^2x^2(d^2+2e^2x^2)+a(3d^4+4d^2e^2x^2+8e^4x^4))}{15d^6x^5}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
[Out] -1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(15*c*d^4*x^4 + 5*b*d^2*x^2*(d^2 + 2*e^2
*x^2) + a*(3*d^4 + 4*d^2*e^2*x^2 + 8*e^4*x^4)))/(d^6*x^5)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3a\,d^4)}{15x^5d^6}$	82
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3a\,d^4)}{15x^5d^6}$	82
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\operatorname{csgn}(e)^2(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3a\,d^4)}{15d^6x^5}$	86

```
[In] int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(8*a*e^4*x^4+10*b*d^2*e^2*x^4+15*c*d^4*x
^4+4*a*d^2*e^2*x^2+5*b*d^4*x^2+3*a*d^4)/x^5/d^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(3\,ad^4+(15\,cd^4+10\,bd^2e^2+8\,ae^4)x^4+(5\,bd^4+4\,ad^2e^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{15\,d^6x^5}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/15*(3*a*d^4 + (15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*x^4 + (5*b*d^4 + 4*a*d
^2*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^6*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}c}{d^2x} - \frac{2\sqrt{-e^2x^2+d^2}be^2}{3\,d^4x} - \frac{8\sqrt{-e^2x^2+d^2}ae^4}{15\,d^6x} - \frac{\sqrt{-e^2x^2+d^2}b}{3\,d^2x^3} - \frac{4\sqrt{-e^2x^2+d^2}ae^2}{15\,d^4x^3} - \frac{\sqrt{-e^2x^2+d^2}a}{5\,d^2x^5}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*c/(d^2*x) - 2/3*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x) -
8/15*sqrt(-e^2*x^2 + d^2)*a*e^4/(d^6*x) - 1/3*sqrt(-e^2*x^2 + d^2)*b/(d^2*x
^3) - 4/15*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*
a/(d^2*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. 2(145) = 290.

Time = 0.53 (sec) , antiderivative size = 1055, normalized size of antiderivative = 6.59

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{4\left(15\,cd^4e^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^9+15\,bd^2e^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^9+15\,ae^6\left(\sqrt{2}\sqrt{\right.}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/15*(15*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqr
t(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 15*b*d^2*e^4*((sqrt(2)*s
qrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - s
qrt(-e*x + d)))^9 + 15*a*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x +
 d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 240*c*d^4*e^2*(
(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*s
qrt(d) - sqrt(-e*x + d)))^7 - 160*b*d^2*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x +
 d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 -
80*a*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/
(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 1440*c*d^4*e^2*((sqrt(2)*sqrt(d) -
sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x
+ d)))^5 + 800*b*d^2*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d)
- sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 + 928*a*e^6*((sqrt(2)
*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) -
 sqrt(-e*x + d)))^5 - 3840*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 2560*b*
d^2*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(
sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 1280*a*e^6*((sqrt(2)*sqrt(d) - sqrt(
-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))
)^3 + 3840*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sq
rt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))) + 3840*b*d^2*e^4*((sqrt(2)*
sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) -
sqrt(-e*x + d))) + 3840*a*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x
+ d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))/((((sqrt(2)*sqrt(
d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(
-e*x + d)))^2 - 4)^5*d^6*e)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\,\left(\frac{a}{5\,d}+\frac{x^4\,\left(15\,c\,d^5+10\,b\,d^3\,e^2+8\,a\,d\,e^4\right)}{15\,d^6}+\frac{x^5\,\left(15\,c\,d^4\,e+10\,b\,d^2\,e^3+8\,a\,e^5\right)}{15\,d^6}+\frac{x^2\,\left(5\,b\,d^5+4\,a\,d^3\,e^2\right)}{15\,d^6}+\frac{x^3\,\left(5\,b\,d^4\,e+4\,a\,d^2\,e^3\right)}{15\,d^6}\cdots\right.}{x^5\,\sqrt{d+e\,x}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(5*d) + (x^4*(15*c*d^5 + 10*b*d^3*e^2 + 8*a*d*e^4))/(1
5*d^6) + (x^5*(8*a*e^5 + 10*b*d^2*e^3 + 15*c*d^4*e))/(15*d^6) + (x^2*(5*b*d
^5 + 4*a*d^3*e^2))/(15*d^6) + (x^3*(4*a*d^2*e^3 + 5*b*d^4*e))/(15*d^6) + (a
*e*x)/(5*d^2)))/(x^5*(d + e*x)^(1/2))
```

3.144 $\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 226

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(7bd^2+6ae^2)(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(35cd^4+28bd^2e^2+24ae^4)(d^2-e^2x^2)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{2e^2(35cd^4+28bd^2e^2+24ae^4)(d^2-e^2x^2)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/7*a*(-e^2*x^2+d^2)/d^2/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/35*(6*a*e^2+7*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/105*(24*a*e^4+
28*b*d^2*e^2+35*c*d^4)*(-e^2*x^2+d^2)/d^6/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-
2/105*e^2*(24*a*e^4+28*b*d^2*e^2+35*c*d^4)*(-e^2*x^2+d^2)/d^8/x/(-e*x+d)^(1
/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {534, 1279, 464, 277, 270}

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2e^2(d^2-e^2x^2)(24ae^4+28bd^2e^2+35cd^4)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(24ae^4+28bd^2e^2+35cd^4)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)(6ae^2+7bd^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/7*(a*(d^2 - e^2*x^2))/(d^2*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((7*b*d^2
+ 6*a*e^2)*(d^2 - e^2*x^2))/(35*d^4*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((35
*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^3*Sqrt[d - e*
x]*Sqrt[d + e*x]) - (2*e^2*(35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*
x^2))/(105*d^8*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^8\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-7bd^2-6ae^2-7cd^2x^2}{x^6\sqrt{d^2-e^2x^2}}\,dx}{7d^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(7bd^2+6ae^2)(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad + \frac{\left((35cd^4-4e^2(-7bd^2-6ae^2))\sqrt{d^2-e^2x^2}\right)\int \frac{1}{x^4\sqrt{d^2-e^2x^2}}\,dx}{35d^4\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(7bd^2+6ae^2)(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad - \frac{(35cd^4+28bd^2e^2+24ae^4)(d^2-e^2x^2)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad + \frac{\left(2e^2(35cd^4-4e^2(-7bd^2-6ae^2))\sqrt{d^2-e^2x^2}\right)\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}\,dx}{105d^6\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(7bd^2+6ae^2)(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad - \frac{(35cd^4+28bd^2e^2+24ae^4)(d^2-e^2x^2)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}\\
&\quad - \frac{2e^2(35cd^4+28bd^2e^2+24ae^4)(d^2-e^2x^2)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.55

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(35cd^4x^4(d^2+2e^2x^2)+7b(3d^6x^2+4d^4e^2x^4+8d^2e^4x^6)+3a(5d^6+6d^4e^2x^2+8d^2e^4x^4}{105d^8x^7}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*c*d^4*x^4*(d^2 + 2*e^2*x^2) + 7*b*(
3*d^6*x^2 + 4*d^4*e^2*x^4 + 8*d^2*e^4*x^6) + 3*a*(5*d^6 + 6*d^4*e^2*x^2 + 8
*d^2*e^4*x^4 + 16*e^6*x^6)))/(d^8*x^7)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.52

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(48a\,e^{6}x^{6}+56b\,d^{2}e^{4}x^{6}+70c\,d^{4}e^{2}x^{6}+24a\,d^{2}e^{4}x^{4}+28b\,d^{4}e^{2}x^{4}+35c\,d^{6}x^{4}+18a\,d^{4}e^{2}x^{2}+21b\,d^{6}x^{2}+15a\,d^{6})}{105x^{7}d^{8}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(48a\,e^{6}x^{6}+56b\,d^{2}e^{4}x^{6}+70c\,d^{4}e^{2}x^{6}+24a\,d^{2}e^{4}x^{4}+28b\,d^{4}e^{2}x^{4}+35c\,d^{6}x^{4}+18a\,d^{4}e^{2}x^{2}+21b\,d^{6}x^{2}+15a\,d^{6})}{105x^{7}d^{8}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\mathrm{csgn}(e)^{2}(48a\,e^{6}x^{6}+56b\,d^{2}e^{4}x^{6}+70c\,d^{4}e^{2}x^{6}+24a\,d^{2}e^{4}x^{4}+28b\,d^{4}e^{2}x^{4}+35c\,d^{6}x^{4}+18a\,d^{4}e^{2}x^{2}+21b\,d^{6}x^{2}+15a\,d^{6})}{105d^{8}x^{7}}$

```
[In] int((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(48*a*e^6*x^6+56*b*d^2*e^4*x^6+70*c*d^4
*e^2*x^6+24*a*d^2*e^4*x^4+28*b*d^4*e^2*x^4+35*c*d^6*x^4+18*a*d^4*e^2*x^2+21
*b*d^6*x^2+15*a*d^6)/x^7/d^8
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.49

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(15\,ad^6+2\,(35\,cd^4e^2+28\,bd^2e^4+24\,ae^6)x^6+(35\,cd^6+28\,bd^4e^2+24\,ad^2e^4)x^4+3\,(7\,bd^6+6\,ad^4e^2)x^2)\surd}{105\,d^8x^7}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/105*(15*a*d^6 + 2*(35*c*d^4*e^2 + 28*b*d^2*e^4 + 24*a*e^6)*x^6 + (35*c*d
^6 + 28*b*d^4*e^2 + 24*a*d^2*e^4)*x^4 + 3*(7*b*d^6 + 6*a*d^4*e^2)*x^2)*sqrt
(e*x + d)*sqrt(-e*x + d)/(d^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\sqrt{-e^2x^2+d^2}ce^2}{3\,d^4x} - \frac{8\sqrt{-e^2x^2+d^2}be^4}{15\,d^6x} - \frac{16\sqrt{-e^2x^2+d^2}ae^6}{35\,d^8x} - \frac{\sqrt{-e^2x^2+d^2}c}{3\,d^2x^3} - \frac{4\sqrt{-e^2x^2+d^2}be^2}{15\,d^4x^3} - \frac{8\sqrt{-e^2x^2+d^2}ae^4}{35\,d^6x^3} - \frac{\sqrt{-e^2x^2+d^2}b}{5\,d^2x^5} - \frac{6\sqrt{-e^2x^2+d^2}ae^2}{35\,d^4x^5} - \frac{\sqrt{-e^2x^2+d^2}a}{7\,d^2x^7}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -2/3*sqrt(-e^2*x^2 + d^2)*c*e^2/(d^4*x) - 8/15*sqrt(-e^2*x^2 + d^2)*b*e^4/(
d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x) - 1/3*sqrt(-e^2*x^2 + d^2
)*c/(d^2*x^3) - 4/15*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^3) - 8/35*sqrt(-e^2*
x^2 + d^2)*a*e^4/(d^6*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^5) - 6/35*sq
rt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1451 vs. 2(206) = 412.

Time = 0.69 (sec) , antiderivative size = 1451, normalized size of antiderivative = 6.42

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/105*(105*c*d^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - s
qrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 + 105*b*d^2*e^6*((sqrt(
2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d)
 - sqrt(-e*x + d)))^13 + 105*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt
(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 - 1960*c*d
^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(s
qrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 - 1400*b*d^2*e^6*((sqrt(2)*sqrt(d) - s
qrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
 d)))^11 - 840*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sq
rt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 16240*c*d^4*e^4*((sqrt
(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d
) - sqrt(-e*x + d)))^9 + 12656*b*d^2*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d)
```

```
)/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 144
48*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/
(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 80640*c*d^4*e^4*((sqrt(2)*sqrt(d) -
 sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x
 + d)))^7 - 69888*b*d^2*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x +
d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 - 40704*a*e^8*((sq
rt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt
(d) - sqrt(-e*x + d)))^7 + 259840*c*d^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x +
 d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 +
202496*b*d^2*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e
*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 + 231168*a*e^8*((sqrt(2)*sqrt
(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt
(-e*x + d)))^5 - 501760*c*d^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(
e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 358400*b*d
^2*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(s
qrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 215040*a*e^8*((sqrt(2)*sqrt(d) - sqrt
(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)
))^3 + 430080*c*d^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
 sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))) + 430080*b*d^2*e^6*((sqr
t(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(
d) - sqrt(-e*x + d))) + 430080*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))/((((sqrt(2
)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d)
- sqrt(-e*x + d)))^2 - 4)^7*d^8*e)
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\left(\frac{a}{7\,d}+\frac{x^2\,\left(21\,b\,d^7+18\,a\,d^5\,e^2\right)}{105\,d^8}+\frac{x^4\,\left(35\,c\,d^7+28\,b\,d^5\,e^2+24\,a\,d^3\,e^4\right)}{105\,d^8}+\frac{x^7\,\left(70\,c\,d^4\,e^3+56\,b\,d^2\,e^5+48\,a\,e^7\right)}{105\,d^8}+\frac{x^3\,(21\,b\,d^6\,e+18}{105\,d^8}\right.}{x^7\,\sqrt{d+e\,x}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^8*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(7*d) + (x^2*(21*b*d^7 + 18*a*d^5*e^2))/(105*d^8) + (x
^4*(35*c*d^7 + 24*a*d^3*e^4 + 28*b*d^5*e^2))/(105*d^8) + (x^7*(48*a*e^7 + 5
6*b*d^2*e^5 + 70*c*d^4*e^3))/(105*d^8) + (x^3*(18*a*d^4*e^3 + 21*b*d^6*e))/
(105*d^8) + (x^5*(24*a*d^2*e^5 + 28*b*d^4*e^3 + 35*c*d^6*e))/(105*d^8) + (x
^6*(56*b*d^3*e^4 + 70*c*d^5*e^2 + 48*a*d*e^6))/(105*d^8) + (a*e*x)/(7*d^2))
)/(x^7*(d + e*x)^(1/2))
```

3.145 $\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 292

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)\,(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(21cd^4+18bd^2e^2+16ae^4)\,(d^2-e^2x^2)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{4e^2(21cd^4+18bd^2e^2+16ae^4)\,(d^2-e^2x^2)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{8e^4(21cd^4+18bd^2e^2+16ae^4)\,(d^2-e^2x^2)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/9*a*(-e^2*x^2+d^2)/d^2/x^9/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/63*(8*a*e^2+9*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/105*(16*a*e^4+
18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^2)/d^6/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-
4/315*e^2*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^2)/d^8/x^3/(-e*x+d)^
(1/2)/(e*x+d)^(1/2)-8/315*e^4*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^
2)/d^10/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

$= \{534, 1279, 464, 277, 270\}$

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{8e^4(d^2-e^2x^2)\,(16ae^4+18bd^2e^2+21cd^4)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}} - \frac{4e^2(d^2-e^2x^2)\,(16ae^4+18bd^2e^2+21cd^4)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)\,(16ae^4+18bd^2e^2+21cd^4)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(d^2-e^2x^2)\,(8ae^2+9bd^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/9*(a*(d^2 - e^2*x^2))/(d^2*x^9*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((9*b*d^2
+ 8*a*e^2)*(d^2 - e^2*x^2))/(63*d^4*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((21
*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^5*Sqrt[d - e*
x]*Sqrt[d + e*x]) - (4*e^2*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*
x^2))/(315*d^8*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (8*e^4*(21*c*d^4 + 18*b*d
^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^10*x*Sqrt[d - e*x]*Sqrt[d + e*x]
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
```

```
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-9bd^2-8ae^2-9cd^2x^2}{x^8\sqrt{d^2-e^2x^2}}\,dx}{9d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left((63cd^4-6e^2(-9bd^2-8ae^2))\sqrt{d^2-e^2x^2}\right)\int \frac{1}{x^6\sqrt{d^2-e^2x^2}}\,dx}{63d^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(21cd^4+18bd^2e^2+16ae^4)(d^2-e^2x^2)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(4e^2(63cd^4-6e^2(-9bd^2-8ae^2))\sqrt{d^2-e^2x^2}\right)\int \frac{1}{x^4\sqrt{d^2-e^2x^2}}\,dx}{315d^6\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(21cd^4+18bd^2e^2+16ae^4)(d^2-e^2x^2)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{4e^2(21cd^4+18bd^2e^2+16ae^4)(d^2-e^2x^2)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(8e^4(63cd^4-6e^2(-9bd^2-8ae^2))\sqrt{d^2-e^2x^2}\right)\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}\,dx}{945d^8\sqrt{d-ex}\sqrt{d+ex}}$$

$$
\begin{aligned}
&= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)\,(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad - \frac{(21cd^4+18bd^2e^2+16ae^4)\,(d^2-e^2x^2)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad - \frac{4e^2(21cd^4+18bd^2e^2+16ae^4)\,(d^2-e^2x^2)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}} \\
&\quad - \frac{8e^4(21cd^4+18bd^2e^2+16ae^4)\,(d^2-e^2x^2)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.54

$$
\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(21cd^4x^4(3d^4+4d^2e^2x^2+8e^4x^4)+9b(5d^8x^2+6d^6e^2x^4+8d^4e^4x^6+16d^2e^6x^8)+a(35d}{315d^{10}x^9}
$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*c*d^4*x^4*(3*d^4 + 4*d^2*e^2*x^2 +
8*e^4*x^4) + 9*b*(5*d^8*x^2 + 6*d^6*e^2*x^4 + 8*d^4*e^4*x^6 + 16*d^2*e^6*x^
8) + a*(35*d^8 + 40*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 64*d^2*e^6*x^6 + 128*e^8
*x^8)))/(d^10*x^9)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.53

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^8x^8+144b\,d^2e^6x^8+168c\,d^4e^4x^8+64a\,d^2e^6x^6+72b\,d^4e^4x^6+84c\,d^6e^2x^6+48a\,d^4e^4x^4+54b\,d^6e^2x^4+63c\,d^8x^4-}{315x^9d^{10}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^8x^8+144b\,d^2e^6x^8+168c\,d^4e^4x^8+64a\,d^2e^6x^6+72b\,d^4e^4x^6+84c\,d^6e^2x^6+48a\,d^4e^4x^4+54b\,d^6e^2x^4+63c\,d^8x^4-}{315x^9d^{10}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\mathrm{csgn}(e)^2(128a\,e^8x^8+144b\,d^2e^6x^8+168c\,d^4e^4x^8+64a\,d^2e^6x^6+72b\,d^4e^4x^6+84c\,d^6e^2x^6+48a\,d^4e^4x^4+54b\,d^6e^2x^4+6}{315d^{10}x^9}$

```
[In] int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(128*a*e^8*x^8+144*b*d^2*e^6*x^8+168*c*
d^4*e^4*x^8+64*a*d^2*e^6*x^6+72*b*d^4*e^4*x^6+84*c*d^6*e^2*x^6+48*a*d^4*e^4
*x^4+54*b*d^6*e^2*x^4+63*c*d^8*x^4+40*a*d^6*e^2*x^2+45*b*d^8*x^2+35*a*d^8)/
x^9/d^10
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.49

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(35\,ad^8+8\,(21\,cd^4e^4+18\,bd^2e^6+16\,ae^8)x^8+4\,(21\,cd^6e^2+18\,bd^4e^4+16\,ad^2e^6)x^6+3\,(21\,cd^8+18\,bd^6}{315\,d^{10}x^9}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/315*(35*a*d^8 + 8*(21*c*d^4*e^4 + 18*b*d^2*e^6 + 16*a*e^8)*x^8 + 4*(21*c
*d^6*e^2 + 18*b*d^4*e^4 + 16*a*d^2*e^6)*x^6 + 3*(21*c*d^8 + 18*b*d^6*e^2 +
16*a*d^4*e^4)*x^4 + 5*(9*b*d^8 + 8*a*d^6*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x
+ d)/(d^10*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{8\sqrt{-e^2x^2+d^2}ce^4}{15\,d^6x}-\frac{16\sqrt{-e^2x^2+d^2}be^6}{35\,d^8x}\\&-\frac{128\sqrt{-e^2x^2+d^2}ae^8}{315\,d^{10}x}-\frac{4\sqrt{-e^2x^2+d^2}ce^2}{15\,d^4x^3}\\&-\frac{8\sqrt{-e^2x^2+d^2}be^4}{35\,d^6x^3}-\frac{64\sqrt{-e^2x^2+d^2}ae^6}{315\,d^8x^3}\\&-\frac{\sqrt{-e^2x^2+d^2}c}{5\,d^2x^5}-\frac{6\sqrt{-e^2x^2+d^2}be^2}{35\,d^4x^5}-\frac{16\sqrt{-e^2x^2+d^2}ae^4}{105\,d^6x^5}\\&-\frac{\sqrt{-e^2x^2+d^2}b}{7\,d^2x^7}-\frac{8\sqrt{-e^2x^2+d^2}ae^2}{63\,d^4x^7}-\frac{\sqrt{-e^2x^2+d^2}a}{9\,d^2x^9}\end{aligned}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -8/15*sqrt(-e^2*x^2 + d^2)*c*e^4/(d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*b*e^6
/(d^8*x) - 128/315*sqrt(-e^2*x^2 + d^2)*a*e^8/(d^10*x) - 4/15*sqrt(-e^2*x^2
 + d^2)*c*e^2/(d^4*x^3) - 8/35*sqrt(-e^2*x^2 + d^2)*b*e^4/(d^6*x^3) - 64/31
5*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*c/(d^2*x^
5) - 6/35*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^5) - 16/105*sqrt(-e^2*x^2 + d^2
)*a*e^4/(d^6*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^7) - 8/63*sqrt(-e^2*x
^2 + d^2)*a*e^2/(d^4*x^7) - 1/9*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. 2(267) = 534.

Time = 0.94 (sec) , antiderivative size = 1847, normalized size of antiderivative = 6.33

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4/315*(315*c*d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - s
qrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^17 + 315*b*d^2*e^8*((sqrt(
2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d)
 - sqrt(-e*x + d)))^17 + 315*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqr
t(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^17 - 6720*c*
d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(
sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^15 - 5040*b*d^2*e^8*((sqrt(2)*sqrt(d) -
sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x
+ d)))^15 - 3360*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
 sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^15 + 76608*c*d^4*e^6*((s
qrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqr
t(d) - sqrt(-e*x + d)))^13 + 68544*b*d^2*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x
+ d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13
+ 76608*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 - 580608*c*d^4*e^6*((sqrt(2)*s
qrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - s
qrt(-e*x + d)))^11 - 509184*b*d^2*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/s
qrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 - 32716
8*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/
(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 2892288*c*d^4*e^6*((sqrt(2)*sqrt(d
) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-
e*x + d)))^9 + 2363904*b*d^2*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e
*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 2728448*a*e
```

```
^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt
(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 9289728*c*d^4*e^6*((sqrt(2)*sqrt(d) - sq
rt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
d)))^7 - 8146944*b*d^2*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d
) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 - 5234688*a*e^10*((
sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sq
rt(d) - sqrt(-e*x + d)))^7 + 19611648*c*d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e
*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^
5 + 17547264*b*d^2*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 + 19611648*a*e^10*((sqr
t(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(
d) - sqrt(-e*x + d)))^5 - 27525120*c*d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x
+ d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 -
 20643840*b*d^2*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqr
t(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 13762560*a*e^10*((sqrt(2
)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d)
- sqrt(-e*x + d)))^3 + 20643840*c*d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d
))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))) + 2064
3840*b*d^2*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))) + 20643840*a*e^10*((sqrt(2)*sqrt(
d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(
-e*x + d))))/((((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2 - 4)^9*d^10*e)
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\left(\frac{a}{9\,d}+\frac{x^2\,(45\,b\,d^9+40\,a\,d^7\,e^2)}{315\,d^{10}}+\frac{x^6\,(84\,c\,d^7\,e^2+72\,b\,d^5\,e^4+64\,a\,d^3\,e^6)}{315\,d^{10}}+\frac{x^7\,(84\,c\,d^6\,e^3+72\,b\,d^4\,e^5+64\,a\,d^2\,e^7)}{315\,d^{10}}+\frac{x^4\,(63\,c\,d \cdots}{\cdots}\cdots\right)}{\cdots}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^10*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(9*d) + (x^2*(45*b*d^9 + 40*a*d^7*e^2))/(315*d^10) + (
x^6*(64*a*d^3*e^6 + 72*b*d^5*e^4 + 84*c*d^7*e^2))/(315*d^10) + (x^7*(64*a*d
^2*e^7 + 72*b*d^4*e^5 + 84*c*d^6*e^3))/(315*d^10) + (x^4*(63*c*d^9 + 48*a*d
^5*e^4 + 54*b*d^7*e^2))/(315*d^10) + (x^9*(128*a*e^9 + 144*b*d^2*e^7 + 168*
c*d^4*e^5))/(315*d^10) + (x^3*(40*a*d^6*e^3 + 45*b*d^8*e))/(315*d^10) + (x^
5*(48*a*d^4*e^5 + 54*b*d^6*e^3 + 63*c*d^8*e))/(315*d^10) + (x^8*(144*b*d^3*
e^6 + 168*c*d^5*e^4 + 128*a*d*e^8))/(315*d^10) + (a*e*x)/(9*d^2)))/(x^9*(d
+ e*x)^(1/2))
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                      convert(ExpnType_result,string)," vs. order ",
                      convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
           'sin','cos','tan','cot','sec','csc',
           'arcsin','arccos','arctan','arccot','arcsec','arccsc',
           'sinh','cosh','tanh','coth','sech','csch',
           'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
       'arctan2','floor','abs'
       ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])  #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```